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Comparison of transmittance and distance estimators in light transport

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Abstract

In this assignment we have implemented multiple transmittance estimators, some that estimate optical thickness and some that utilize null collisions. We have performed evaluation on a sample volumetric cube and visualized results.

1. Introduction

Transmittance evaluation is an important part of light transport. When light travels through volumes, the photons can absorb or out-scatter, so the ratio of photons that are left after some distance gives us transmittance. The loss of photons on every unit of distance traveled is determined by extinction coefficient $\mu(x)$ which is equal to a sum of absorption $\mu_a(x)$ and scattering $\mu_s(x)$ coefficients. Then transmittance is defined as $T(t) = \frac{L_o}{L_i} = e^{-\tau(t)}$, where $\tau(t) = \int_0^t \mu(x)dx$ is optical thickness, L_i is entering light, L_o is remaining light after distance t.

Another option is to utilize null collision methods where fictitious matter is added in optically thinner segments of original material and in each iteration of algorithm a random number is generated and based on it we determine if there was a real or null collision.

If we have homogeneous material, the $\mu(x) = \mu$ is a constant, so it can be calculated directly. If we have a heterogeneous volume, we have to use other methods for transmittance computation. When estimating transmittance we can compute it explicitly or include it in our distance sampling techniques.

There are basic ray marching and stratified Monte Carlo [PKK00] approaches where optical thickness is evaluated. In [NSJ14] they describe delta, ratio and residual tracking, which are all types of null collision methods. In [NGHJ18] they describe next flight versions of delta and ratio tracking, which try to address the weaknesses of base methods.

2. Methods

In the following chapters we will describe methods we used to estimate transmittance.

2.1. Analytically

In this approach we directly calculate the optical-thickness integral if we have defined function $\mu(x)$, which is usually not the case.

2.2. Ray Marching

Here we do not need to compute boundaries, we just move in ray direction by equal sized steps assuming that in one step the extinction coefficient does not change. We calculate optical thickness as in Equation (1), where $t_i = i\Delta_t, \Delta_t = t/k$.

$$\langle \tau(t) \rangle = \sum_{i=1}^{k} \mu(t_i)\Delta_t \quad (1)$$

2.3. Stratified Monte Carlo

This approach is similar to previous, except when calculating t_i we add random jitter to remove possible artefacts. The step is calculated as $t_i = (i + \xi)\Delta_t$

2.4. Delta Tracking

This method is the first of null collision methods we used. These methods define a constant majorant $\bar{\mu}$ which bounds extinction coefficient, so $\bar{\mu} = \mu(x) + \mu_f(x)$, where $\mu_f(x)$ represents fictitious matter.

Here we perform random walk and in each iteration sample free path and add it to already traveled distance like in $t = -\frac{ln(1-\xi)}{\bar{\mu}}$. We repeat this until we travel over determined distance $t \leq d$ or we come to a real collision. Every iteration another random number δ is generated and if $\delta < \frac{\mu(x)}{\bar{\mu}}$ the collision is real and we stop. If the collision is with fictitious matter, we continue. number of null collisions is dependent on how close the majorant is to $\mu(x)$.

The output of the algorithm is 1 if the traveled distance t is larger than d, else it is 0.

2.5. Ratio Tracking

In this method we sample free path in the same way as above, but we do not stop when we encounter real collision. Instead we calculate the joint probability of null collisions along whole distance d as in Equation (2). So in each iteration the estimate is reduced proportionally to the fictitious matter ratio at that point.

$$\langle T(d) \rangle = \prod_{i=1}^{K} (1 - \frac{\mu(x_i)}{\bar{\mu}}) \tag{2}$$

The output is transmittance estimation that is not only binary compared to delta tracking.

2.6. Next Flight Delta Tracking

To improve regular delta tracking we use next flight method where we start with evaluation of transmittance over entire distance $T_{\bar{\mu}}(0,d)$ and add weighted free flight estimate at each collision $T_{\bar{\mu}}(x_i,d)$ as in Equation (3).

$$\langle T(d) \rangle = T_{\bar{\mu}}(0,d) + \sum_{i=1}^{n} (1 - \frac{\mu(x_i)}{\bar{\mu}}) T_{\bar{\mu}}(x_i,d) \tag{3}$$

We stop the iteration process when first real collision is encountered (n) or we reach whole distance d.

2.7. Next Flight Ratio Tracking

This is similar to previous, but it relies on ratio tracking, so it does not stop at first real collision. Here it weighs the free flight estimate using the product of all null collisions encountered as in Equation (4).

$$\langle T(d) \rangle = T_{\bar{\mu}}(0,d) + \sum_{i=1}^{K} \prod_{j=1}^{i} (1 - \frac{\mu(x_i)}{\bar{\mu}}) T_{\bar{\mu}}(x_i,d) \tag{4}$$

3. Implementation

The methods describes previously were implemented in Unity using shaders written in HLSL that execute on GPU. We implemented each methods in its own shader, which outputs the transmittance values as RGB pixel values. We created a simple scene with orthographic camera directed at a cube and saved the resulting render recorded by camera as an image. We also passed cost, defined as number of collisions with fictitious matter, as pixel values in null collision methods.

From these saved images with transmission and cost values we got our final renders and analysis results.

4. Results

First we computed renders of a same scene using different methods shown in Figure 1. The scene features a cube with exponentially decreasing extinction coefficient $\mu(x)$ from left to right, so that the transmission decreases linearly. The null collision methods also have decreasing collision sampling efficiency from bottom to top.

Collision sampling efficiency is defined as $\eta(x) = \frac{\mu(x)}{\bar{\mu}}$ to determine how closely the majorant bounds extinction coefficient, but in reality we directly set $\eta(x) = \frac{bar\mu}{10}$, which results in lower quality evaluations especially of next flight methods. This also means the results are not directly comparable to results of [?].

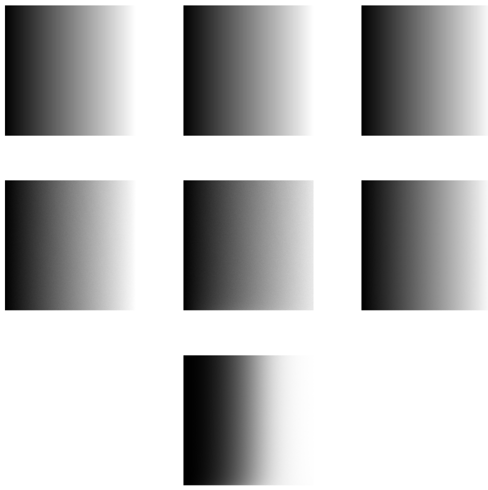

Figure 1: *Render of a cube with decreasing extinction coefficient from left to right and decreasing collision sampling efficiency from bottom to top (where used). In first row: analytical, ray marching, stratified Monte Carlo. Second row: delta, next flight delta, ratio. Third row: next flight ratio. In second and third row 300 samples were used.*

On the same scene we evaluated variance in Figure 2 and cost in Figure 3 of null collision methods at all parameters.

Next we evaluated variance in Figure 5 and error in Figure 4 over time on a simpler scene with constant parameters, $\bar{\mu} = 5$ and extinction coefficient $\mu = 1$.

We compare the analytically computed image with results

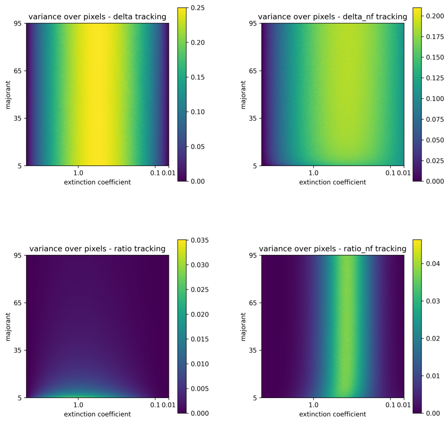

Figure 2: *Variance of null collision tracking methods.*

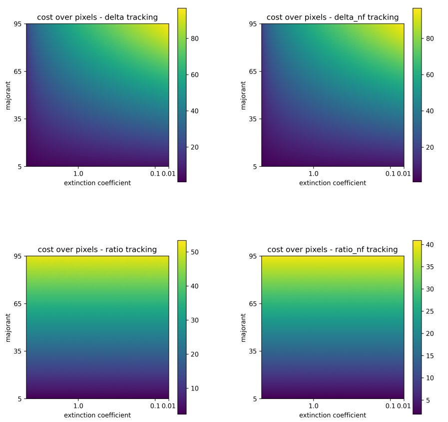

Figure 3: *Cost of null collision tracking methods calculated as number of null collisions encountered.*

of implemented methods and compute root mean square error for each pair in Table 1. The scene with a cube that has exponentially decreasing extinction coefficient, the images of the renders are shown in Figure 1.

5. Discussion

Overall the methods that compute optical thickness with stepping give much better results compared to null collision methods. The ray marching and stratified Monte Carlo give very similar results. Among the null collision methods, the lowest RMSE is achieved by ratio tracking, followed by delta tracking. Worst preform both next flight methods, which have visible differences even in the renders in Figure 1 (middle bottom two).

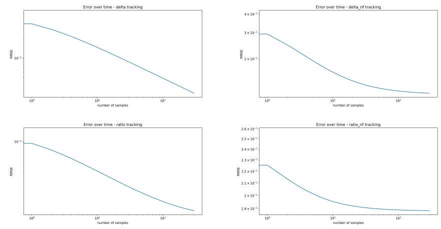

Figure 4: *Error of null collision tracking methods over time.*

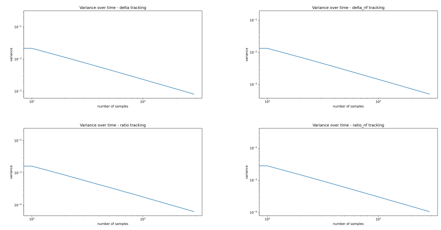

Figure 5: *Variance of null collision tracking methods over time.*

The variance of delta tracking is not dependent on tightness of majorant as seen in Figure 2, similarly but not as much goes for next flight delta, which also shows higher variance at low μ, where the probability of terminating algorithm is much smaller. Ratio tracking shows visible variance only at very low majorant values, compared to next flight ratio which has visible variance at all majorant values.

The costs of delta and ratio tracking are very similar among both of them as seen in Figure 3. The cost of delta methods is highest at high majorant with sparse real matter, so the probability of stopping is very low, so we encounter a lot of null collisions. In case of delta tracking we can see that the cost is highest at highest majorants irregardless of extinction coefficients, due to scaling distances with majorant and moving more slowly.

The variance plots in Figure 5 are pretty consistent, the variance expectantly decreases linearly in a log-log plot. Variance of ratio methods achieves lower values of below 10^{-4}, compared to ratio methods which achieve lows around

Method	RMSE
Ray marching	0.00081
Stratified Monte Carlo	0.00079
Delta	0.02404
Next flight delta	0.15833
Ratio	0.00981
Next flight ratio	0.10546

Table 1: *Errors of methods when compared to analytical result.*

10^{-3}. The RMSE in Figure 4 also shows that the error decreases with time, in next flight methods it also starts to converge slowly.

6. Conclusion

We have implemented multiple transmission estimators based on different principles, computation of optical thickness and null collision detection. We have analysed variance and errors of null collision estimators over time. Ratio tracking achieves very good accuracy but at high cost, because we must travel over the whole distance, compared to delta tracking which is less accurate, but also less costly, since we can finish early. Next flight methods give higher errors, which can be due to majorant estimation of remaining transmission.

References

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