KENTECH
Korea Institute of Energy Technology

Economic Dispatch, Unit Commitment, and Reserve

Power System Economics

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Outline of Question 1 & 2

All day-ahead optimization problems were performed over 24-hour time horizon of July 21, 2022. Reserve requirement, load shedding, and renewable curtailment were not considered. The system is reduced to a single bus.

Historical system demand and renewable data from the KPX were used. Only solar energy generation was considered for renewable data.

- This decision was made because the original plan for question 3 was performing DA-SUC with two distinct stochastic factors, one for the system demand the other for solar generation.
- Having wind or other types of renewable would have made the project too complex. The plan for question 3 has changed after all.

All optimization problems have the same objective: minimize the total system cost. However, the total system cost in each refers to different quantities. Different sets of constraints are applied.

- Full UC considers all possible constraints we learned in class, except for network constraints.
- Snapshot UC excludes all intertemporal constraints from full UC.
- Full ED refers to ED with all intertemporal constraints. It is basically full UC with all binary decision variables fixed, at minimum.
- Snapshot ED excludes all intertemporal constraints from full ED.

Two cases was experimented with snapshot ED, and they corresponds to my answer to question 1.

- Case 1 deals with all units being turned on. This was possible because thermal demands were between two sums, power minimums & maximums.
- Case 2 deals with a custom and specific case where units with cheapest linear generation cost coefficients were turned on one by one, until the thermal demand could be covered. This synthetic commitment decisions were produced such that it respects each unit's power minimum and maximum.

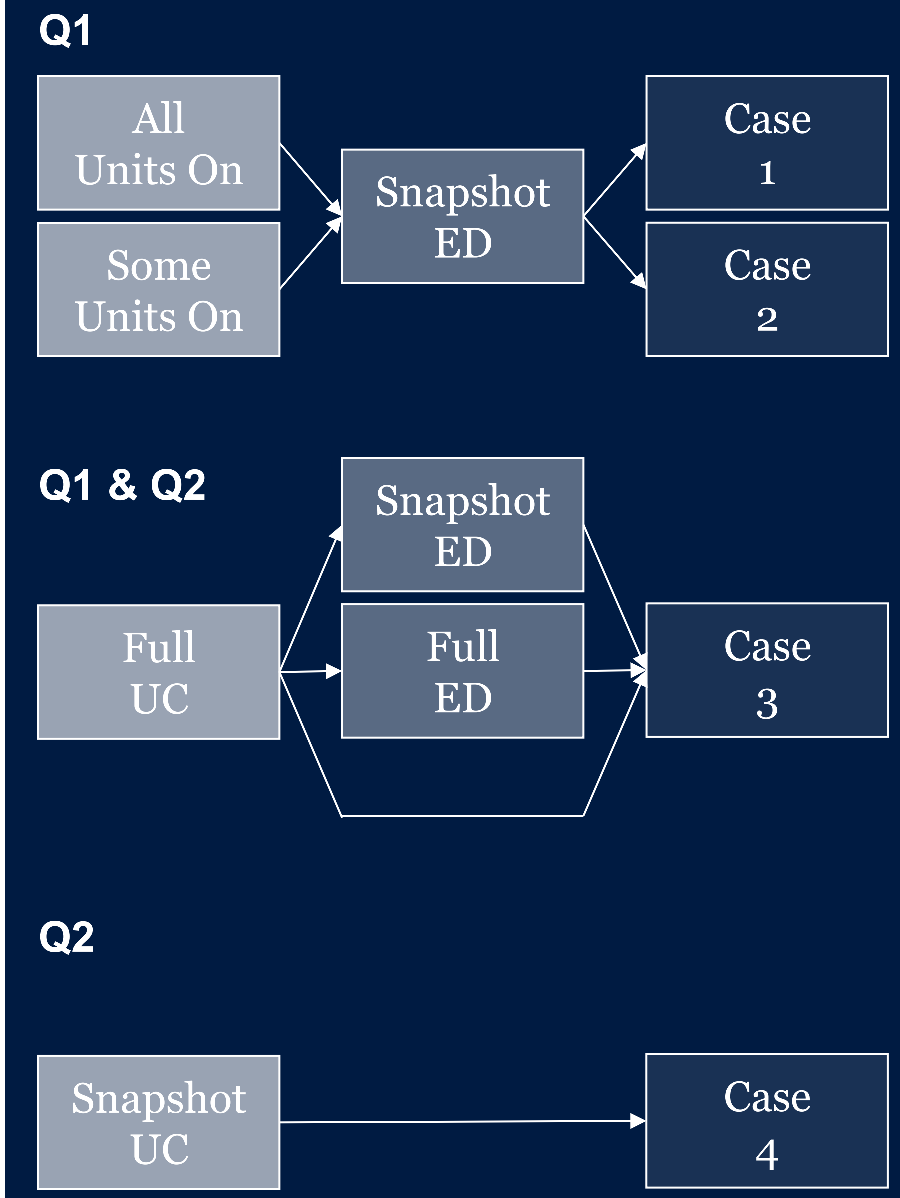
Case 3 corresponds to my answer to both question 1 and 2. The experiment shows the following:

- The result of 1. full UC & snapshot ED, 2. full UC & full ED, 3. full UC are identical in terms of schedules and dispatch outputs.
- The equality of the second and third make sense mathematically, since power dispatch outputs are the only decision variable in full ED while the binary variables were fixed. In code, this was achieved through a single-time recursive call of full UC function with a Boolean variable as an UC/ED indicator (while the duals, SMP, were obtained).
- The equality of the first and second, or third, is not as straight as the previous point. One necessary condition is that all ramp-related constraints are non-binding. This was true for the selected time horizon and timeseries data, and further experiments with different timeseries data confirmed the necessary condition.

Case 4 is the answer to question 2.

- It looks identical to snapshot ED, but commitment decisions are output, not input, of snapshot UC.

Note: For analysis, question 1 and 2 results were required to be precise. Relative and absolute MIP optimality gap were set to 0, while tolerance for integer, primal, and dual feasibility were set to minimum (1e-9). For question 3, default values were used.



Case 3 result from three methods involving full UC is explained with a brief guide on how to read a dispatch heatmap. We also experimented with different timeseries data to obtain interesting patterns.

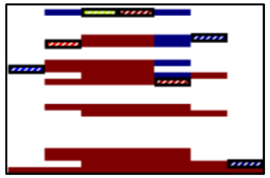
A dispatch heatmap shows both commitment decisions and power outputs based on the result of optimization for all units during the entire time horizon.

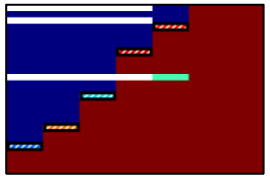
- On-units are colored. Dark blue indicates that the corresponding unit is set to produce the minimum power. Dark red indicates the maximum power. Off-units are white.
- Throughout the project, we deal with 122 generation units which are sorted by their unique linear generation cost coefficients in ascending order. The first 25 units are nuclear, the next 41 are coal, and the last 56 are LNG.
- The marginal unit for the load-balance constraint is marked by a black boundary and white hatch lines. Its linear cost parameter also equals the system marginal price for the hour as quadratic cost coefficients were not considered in generation cost functions.

Some key notes for case 3 are as follows:

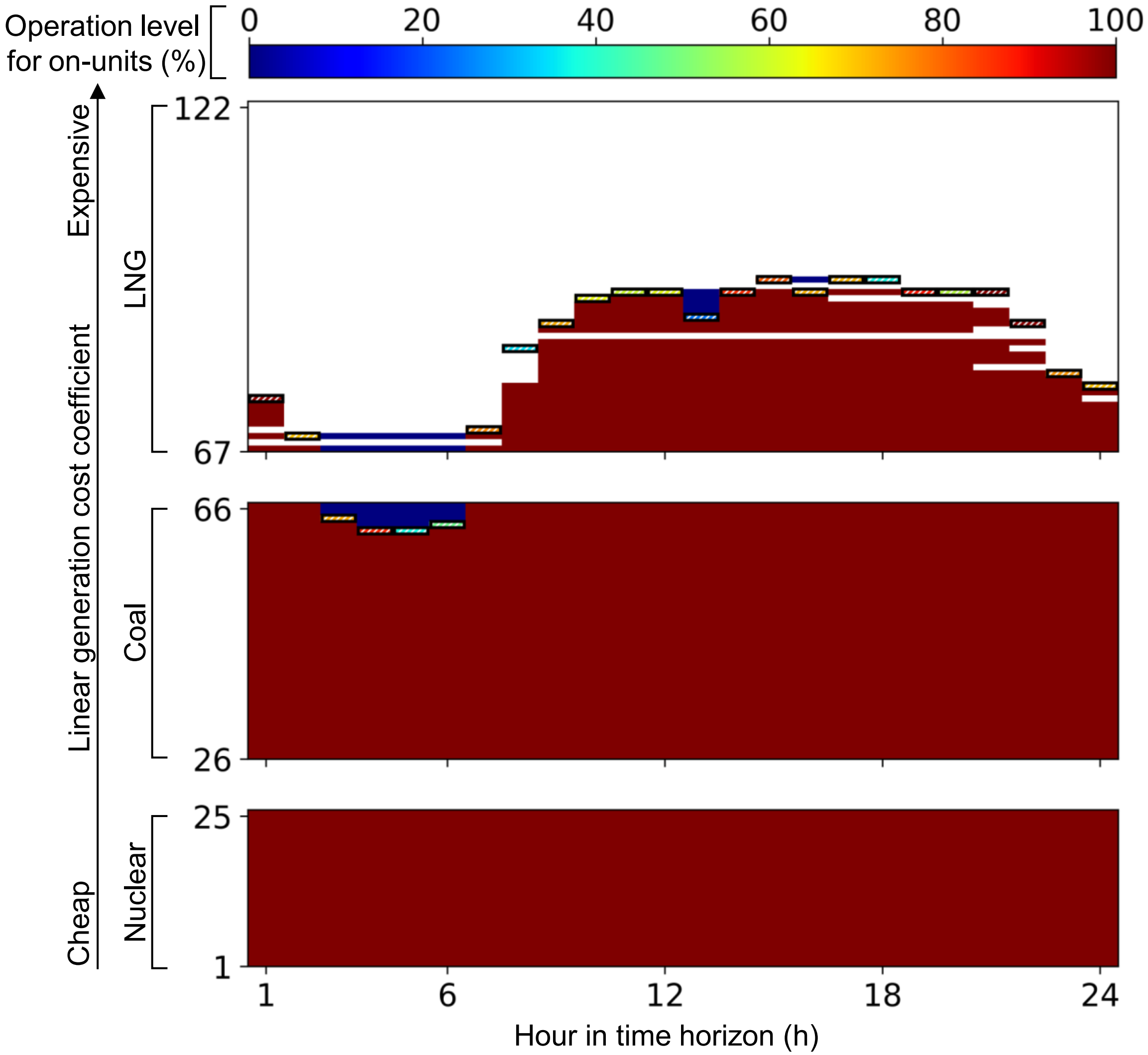
- Cheapest units are turned on, but not always due to three main reasons. At a specific hour, cheap units are not turned on while expensive ones are, because their no-load or startup cost parameters are relatively higher, which would result in suboptimal objective value otherwise. The other reasons are that either an intertemporal constraint is binding for the unit in the hour or the "residual" thermal demand after "filling up" with cheaper units were less than the minimum power, both of which are not shown in case 3.
- Some units are turned on at the minimum operation level, indicated by dark blue. These units are above the marginal unit for each hour, as their minimum power constraints are binding. These units often have high (hot) startup costs. The similar patterns emerge for units in other cases with nonzero reserve requirements.
- For the first several hours, LNG units with index 68 and 70 are off. They have been off since the initial condition. We can safely assume that their (cold) startup costs are high.

With other timeseries data for full UC, we observe that diverse patterns emerge.





- Left shows LNG units with their minimum up-time constraints binding (4h). The peak thermal demand was sharp for this data.
- Right shows a coal unit with its startup-ramp constraint binding (660 MW).

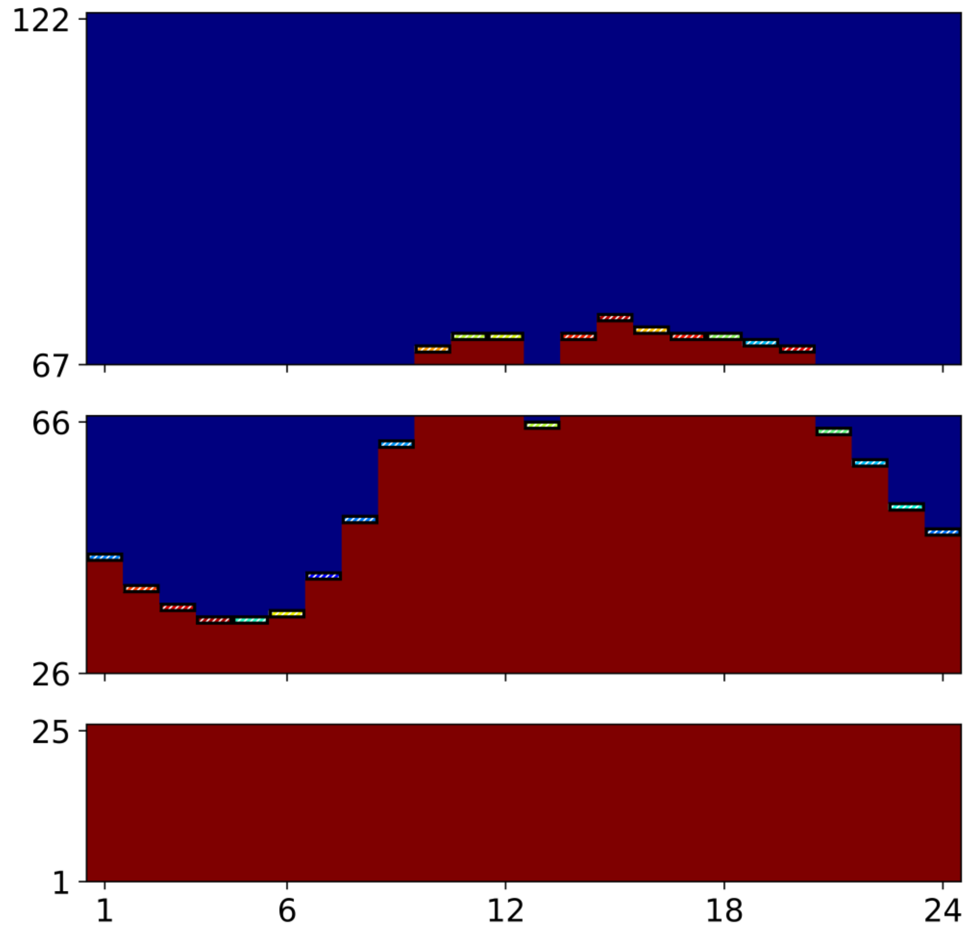


Note: No ramp-constraint-bound units in our main result reconfirms the equivalence of results from 3 different methods for this specific input.

In case 1 and 2, snapshot ED optimization for each hour is truly independent as all intertemporal constraints and startup cost are excluded. However, we must decide a meaningful combination of commitment decisions for later comparison.

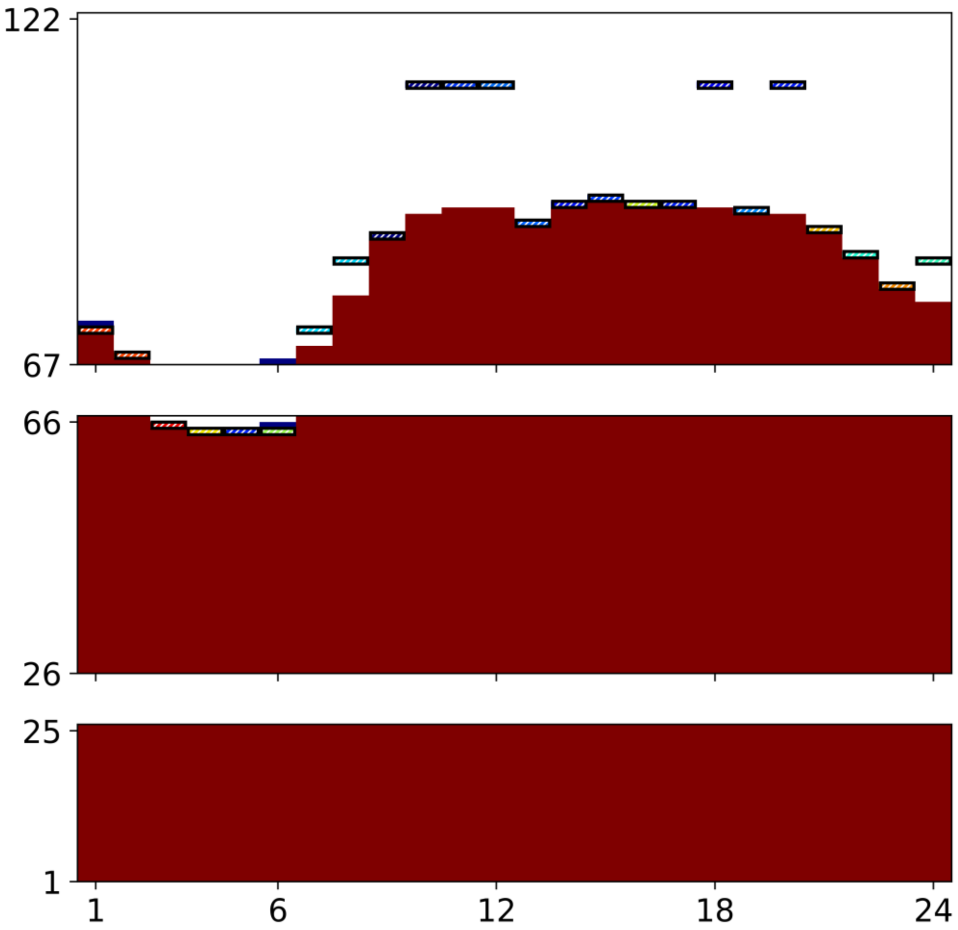
Dispatch Heatmap, Case 1

All 122 units are turned on. This resembles hand-solved EDs in the course, where all units in the problem were given as on-line.



Dispatch Heatmap, Case 2

Commitment decisions for each hour was produced based on this logic: turn on cheapest units one by one, but if a unit's power minimum is greater than the residual thermal demand to be filled, skip the unit.



Case 1 is far from reality but offers unique insights.

- Optimization fully disregards no-load costs for all 122 units (and so as snapshot ED itself); we completely exclude the possibility of units being "skipped" due to relatively high no-load costs.
- Hence, all units "cheaper" than the marginal one are at their highest operational levels, and vice versa. Due to expensive on units, the SMP is lower than case 2.

Case 2 is closer to reality.

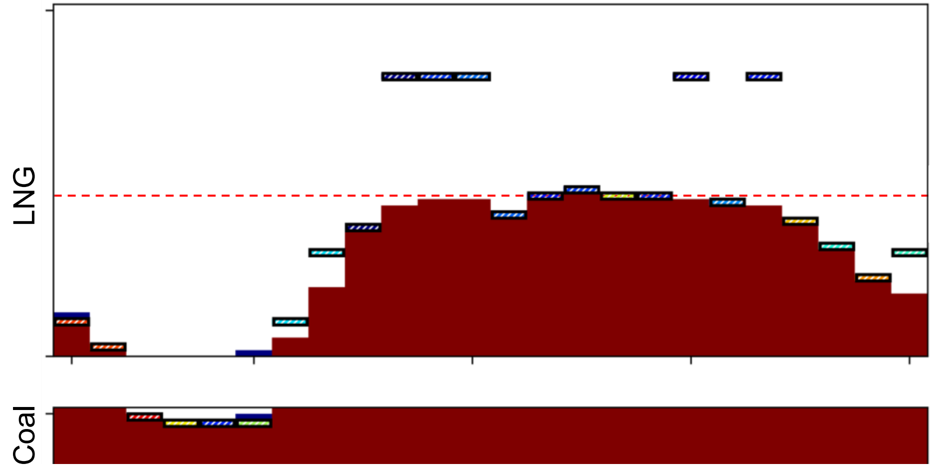
- However, the effects of commitment decision logic overwhelms the result, as any other combinations do.
- We note that the same LNG unit (index 110) have 209 MW power minimum, while all units turned off in-between have power minimum exceeding 300 MW. The residual thermal unit was between 209 and 300 MW.

Note: For case 2, no-load costs for the selected on-units are disregarded by snapshot ED, too. It is the units with power minimums higher than residual thermal demand that causes "skips".

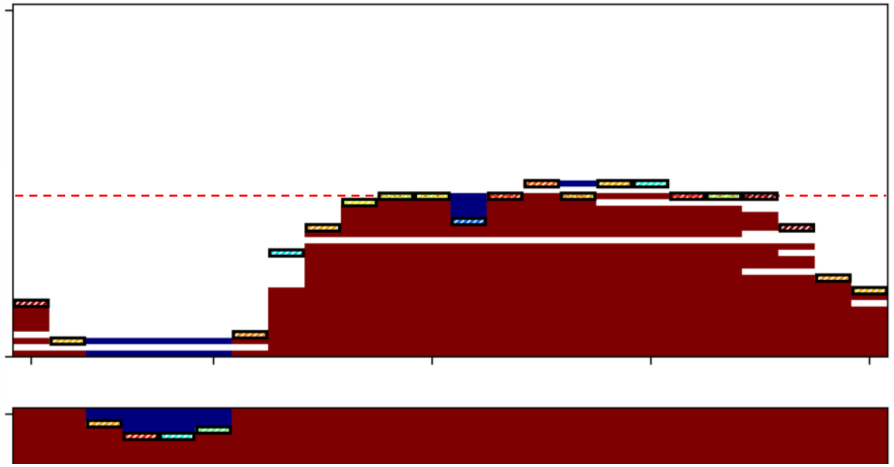
Note: At hour (and similar to 6) in case 2, it is actually optimal to have unit 72 to produce minimum (167.2 MW) and (marginal) unit 71 to produce 397.6 MW below the two's maximum (440 MW) than current marginal unit producing the maximum and skipping several already-expensive units.

Case 3 marginally increased total generation cost from case 2, although it includes many constraints in UC. On the other hand, case 3's system marginal price was lower, attributed to the model design of both case 2 and 3.

Partial Dispatch Heatmaps, Case 2 (snapshot ED)



Partial Dispatch Heatmaps, Case 3 (full UC & snapshot ED)

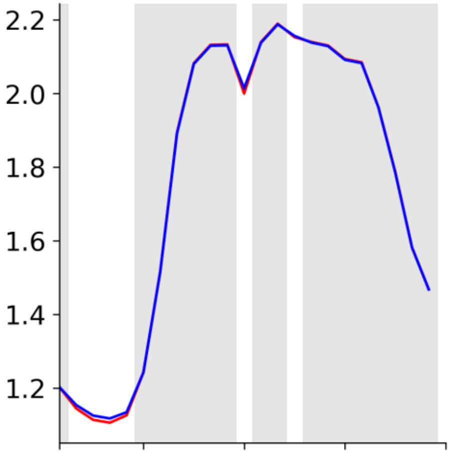


Total Generation Cost, B KRW

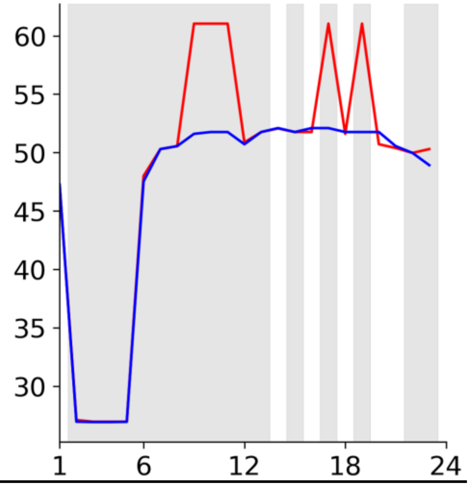
Case 2 (red · snapshot ED): 41.77

Case 3 (blue · full UC): 41.81

Generation Cost, B KRW



System Marginal Price, KRW/kWh

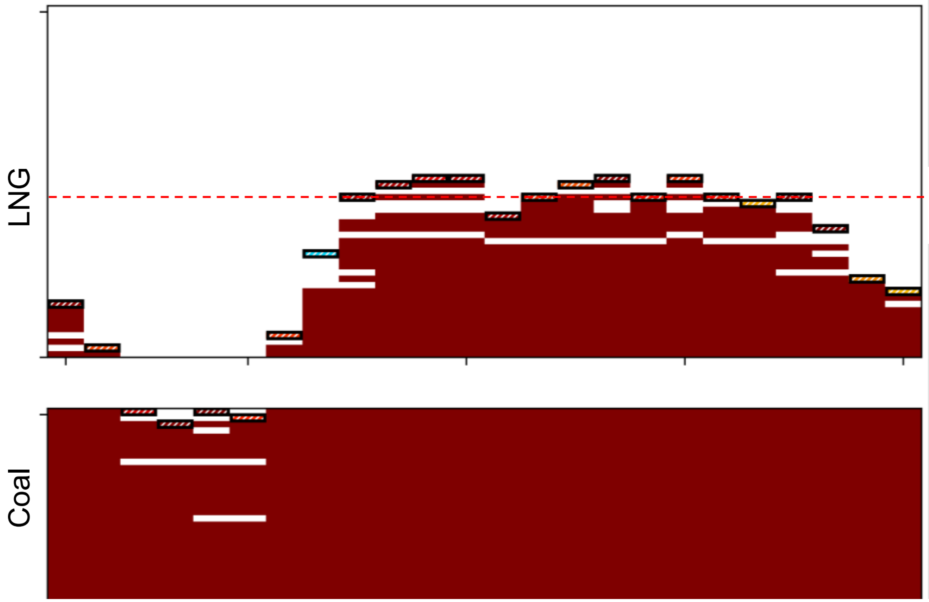


- Reality check: case 2 model (snapshot ED) resembles pre-revision DA-ED in Korea, which is followed up by separate considerations of intertemporal and network constraints. Case 3 model (full UC) resembles post-revision DA-UC in Korea, without key factors such as reserve and network constraints.
- The differences in dispatch schedules between the two become clearer with the side-by-side comparison. It reveals the effect of case 3's seemingly out-of-nowhere turn-off for one specific LNG unit (single white horizontal line). In case 2, the marginal unit was "pushed higher" during 9~12 and 18, 20 hour due to those in-between off-units with high power minimums being unable to serve residual thermal demand. In case 3, full UC's decision to off a single cheaper unit prevented SMP spikes. We note that this is not "intended" by the solver as it only focuses to minimize the total generation cost.
- The valley-like pattern during the first 6 hours have lower trough in case 3. Due to startup cost consideration in full UC, coal and 2 LNG units were kept on, snapshot ED, based on full UC's binary decision variables fixed, decided to dispatch minimum power for these. Thermal demands to be served by cheaper coal units are reduced as a result, and the marginal units are cheaper than case 2. The details are invisible in the SMP graph as the difference is small. Differences are larger in simulations with greater renewable generation.
- Generation costs for the first 6 hours are greater in case 3 (blue), due to no-load costs of the coal units. However, generation costs in case 3 were mostly equal or less than case 2 for the rest of the time horizon, highlighted by grey backgrounds. We believe that highly suboptimal logic behind commitment decision generation is responsible for relatively higher generation costs.
- We finally note that the total generation cost for case 3 was greater by approximately 33M KRW, which is surprisingly small considering that the addition of all the intertemporal constraints did not cause a significant increase.

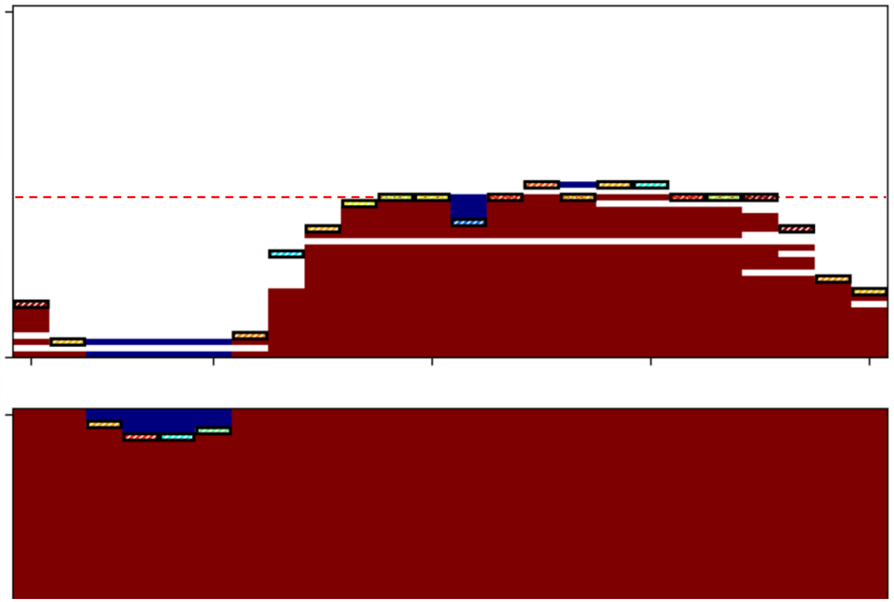
Source: Ok K. et al., *Details and implications of reformed Korean day-ahead electricity market*, 2021
Note: valley-like patterns in dispatch heatmaps are due to decrease in thermal demands for short time intervals.

Case 4 is by far the most optimal result. Case 4 basically removes costly intertemporal constraints from case 3. The addition of commitment decisions as binary decision variables empowers case 4 compared to case 2.

Partial Dispatch Heatmaps, Case 4 (snapshot UC)

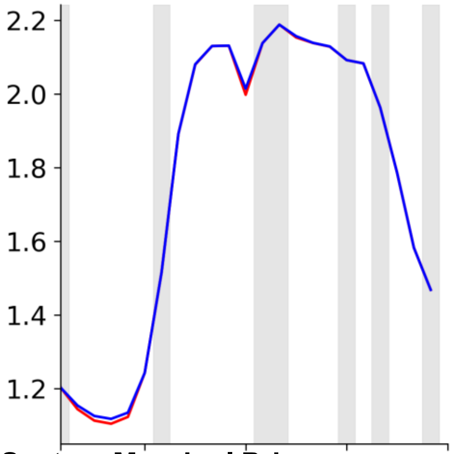


Partial Dispatch Heatmaps, Case 3 (full UC)

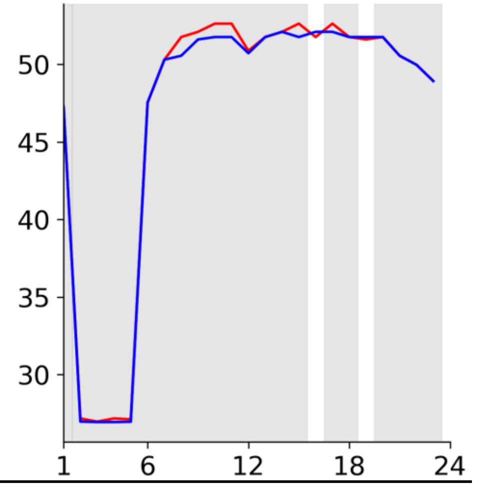


Total Generation Cost, B KRW
Case 4 (red · snapshot UC): 41.74
Case 3 (blue · full UC): 41.81

Generation Cost, B KRW



System Marginal Price, KRW/kWh



- We get deeper insights from inspecting case 4 off-units under marginal units. Since there is no intertemporal constraints, case 4 freely attempts to exclude units with high no-load costs for each hour. These off-units may also include those with high power minimums; it ensures the number of skipped units are minimal, unlike case 2, such that residual thermal demands can be served by next units with lower power minimums “at bulk”.
- Case 4 is impossible in the real world, but the result is the most optimal among 4 cases, if we were to compare. There is not a single unit bound by its power minimum constraint (as previously explained). Hence, the marginal unit is always the most expensive on-unit.
- The side effect of many discontinuous white lines previously explained is that it most certainly elevates system marginal price.
- However, this is a different story for generation costs. Since both snapshot and full UC can decide commitment decisions, the effect of including no-load costs in the objective function is direct and not negligible, compared to question 1 comparison. For only 7 hours, down from 18, full UC’s generation costs were equal or less than snapshot UC.
- The total generation cost for case 3 was greater by approximately 70M KRW, greater than the previous comparison. Snapshot UC (case 4) is basically an extension of snapshot ED (case 2) with the addition of commitments decision as binary variables. Hence, case 4 can find the best possible combination of commitment decisions by minimizing the total no-load cost for each hour, too.

Note: Marginal units’ positions do not directly translate into system marginal price. Although all 122 units have unique linear cost parameters, the values for two units may be close.
Note: We also acknowledge that addition of binding constraints increase the objective value most times, but not necessarily at some conditions (although it is non-slack); the “cost” of adding this specific constraint would be zero just like any other slack constraints.

Outline of Question 3

Our upward spinning reserve requirement consists of three deterministic and a single stochastic components. The objective of DA-SUC is to minimize the total system cost based on six scenarios and a single forecast thermal demand for tomorrow. The value of lost load is set to 3.5M KRW/MWh. The SUC formulation is in Appendix I.

Deterministic reserve components are as follows:

- **N-1 contingency reserve** guarantees reliability for instant loss of the single largest on-unit. Despite the certainty in the value, I made sure it is determined during run-time. This reserve is superposed with below frequency-regulation reserve due to the possibility of independent events.

$$R_t^{\text{N-1}} = \max_{i \in \mathcal{I}} P_i^{\max} u_{i,t} \quad \forall t$$

- **Frequency-regulation reserve** aims to increase sub-hour-level reliability. We do not consider any sub-hour-level operations, but I included this as 2% of the forecasted thermal demand of each hour.

$$R_t^{\text{FR}} = 0.02 \cdot {}^{\text{the}}D_t^{\text{for}} \quad \forall t$$

- **Minimum operation-level reserve** comes from all online units keeping at least 5% of their maximum power outputs. This is inspired from Korea's practice. This can not only protect units but also guarantee reliability for up to 5.263% surge in thermal demand alone. This is not superposed with the other reserve components; hence it may be covered by forecast-error reserve.

$$r_{i,t} \geq 0.05 \cdot P_i^{\max} u_{i,t} \quad \forall i, \forall t$$

Forecast-error reserve is stochastic in that it is formulated as a recourse decision: it attempts to trade off the cost of procuring additional reserve against the penalty for load shedding.

- **Forecast error** is the positive deviation between each scenario and forecast thermal demand. If the forecast underestimates, the error will be larger.

$$\epsilon_{t,k} = [{}^{\text{the}}D_{t,k}^{\text{sce}} - {}^{\text{the}}D_t^{\text{for}}]_+ \quad \forall t, \forall k$$

- **Non-served energy** refers to the amount of forecast error which cannot be covered by reserve.

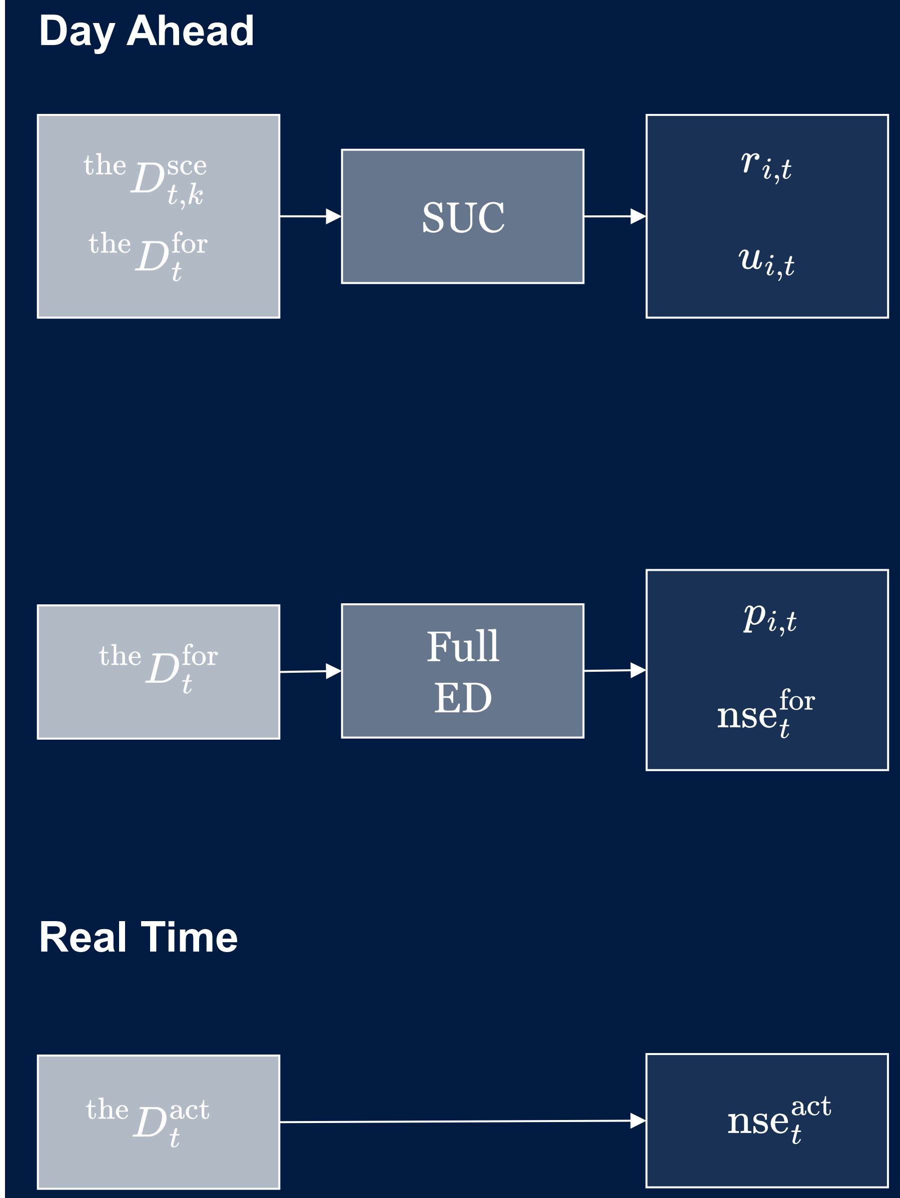
$$\text{nse}_{t,k} \geq [\epsilon_{t,k} - \textstyle\sum_i r_{i,t}]_+ \quad \forall t, \forall k$$

- **Expected cost of load shedding** is the weighted sum of penalty for non-served energy across all scenarios. Its inclusion in the objective forces the model to procure additional reserve, if the marginal increase in the total system cost is less than the marginal reduction in expected load-shedding penalty, unless operational constraints are binding.

$$\textstyle\sum_t \sum_k (\kappa_k \cdot \text{nse}_{t,k} \cdot \text{VoLL})$$

The full simulation steps are as follows:

- First, we solve DA-SUC based on thermal demand scenarios and forecast to decide commitment decision and reserve for each unit to be used next.
- We solve DA-ED with all intertemporal constraints based on thermal demand forecast, deciding power dispatch and non-served energy forecast.
- Lastly, we evaluate DA optimization results based on actual thermal demand, assuming that the plan is not changed until the end of the time horizon.



Source: K. Ok et al., *Details and implications of reformed Korean day-ahead electricity market*, 2021
Note: Assuming that all units are at 95% operation level, 5% over 95% equals 5.263%; N-1 contingency reserve equals the largest on-unit's power maximum, not its 95%. Note: I did not explicitly adjust the objective of DA-SUC to observe nonzero non-served energy.

Six demand and renewable generation scenarios for July 21, 2022 were generated via quantile-binning of historical weekday profiles followed by linear trend projection and averaging. Each scenario carries the same probability weight.

We used 42 weekdays' demand and renewable ratio profile from a ±30-day window around July 21 in 2019 and 2021.

$$|\mathcal{N}^{\text{yr}}| = 42, \quad |\mathcal{K}| = 6, \quad |\mathcal{B}_k| = 7, \quad \bigcup_{k\in\mathcal{K}} \mathcal{B}_k = \mathcal{N}^{\text{yr}}$$

$$\mathcal{N}^{\text{yr}} = \left\{ n \mid |\text{date}_n - \text{July } 21| \le 30 \ \wedge \ \text{Mon} \sim \text{Fri} \right\} \qquad \text{yr} \in \{2019, 2021\}$$

$${}^{\text{sys}}D_{t,n}^{\text{raw,yr}}, \ {}^{\text{ren}}\text{rat}_{t,n}^{\text{raw,yr}} \qquad \forall t \in \mathcal{T}, \, n \in \mathcal{N}^{\text{yr}}$$

We quantile-binned them into 6 bins of 7 data points each. Although the scenarios are uniformly distributed for an hour, their six centroids are NOT equally spaced.

$${}^{\text{sys}}D_{t,k}^{\text{sce,yr}} = {}^{1}\!/_{|\mathcal{B}_k|} \sum_{n\in\mathcal{B}_k} {}^{\text{sys}}D_{t,n}^{\text{raw,yr}} \qquad \forall t \in \mathcal{T}, \, k \in \mathcal{K}, \, \text{yr} \in \{2019, 2021\}$$

$${}^{\text{ren}}\text{rat}_{t,k}^{\text{sce,yr}} = {}^{1}\!/_{|\mathcal{B}_k|} \sum_{n\in\mathcal{B}_k} {}^{\text{ren}}\text{rat}_{t,n}^{\text{raw,yr}} \qquad \forall t \in \mathcal{T}, \, k \in \mathcal{K}, \, \text{yr} \in \{2019, 2021\}$$

We performed linear trend projection for system demand while ignoring 2022. For renewable ratio, we took the average then computed renewable generation.

$${}^{\text{sys}}D_{t,k}^{\text{sce,2022}} = 1.5 \cdot \left({}^{\text{sys}}D_{t,k}^{\text{sce,2021}} - {}^{\text{sys}}D_{t,k}^{\text{sce,2019}} \right) + {}^{\text{sys}}D_{t,k}^{\text{sce,2019}} \qquad \forall t, k$$

$${}^{\text{ren}}\text{rat}_{t,k}^{\text{sce,2022}} = 0.5 \cdot \left({}^{\text{ren}}\text{rat}_{t,k}^{\text{sce,2019}} + {}^{\text{ren}}\text{rat}_{t,k}^{\text{sce,2021}} \right) \qquad \forall t, k$$

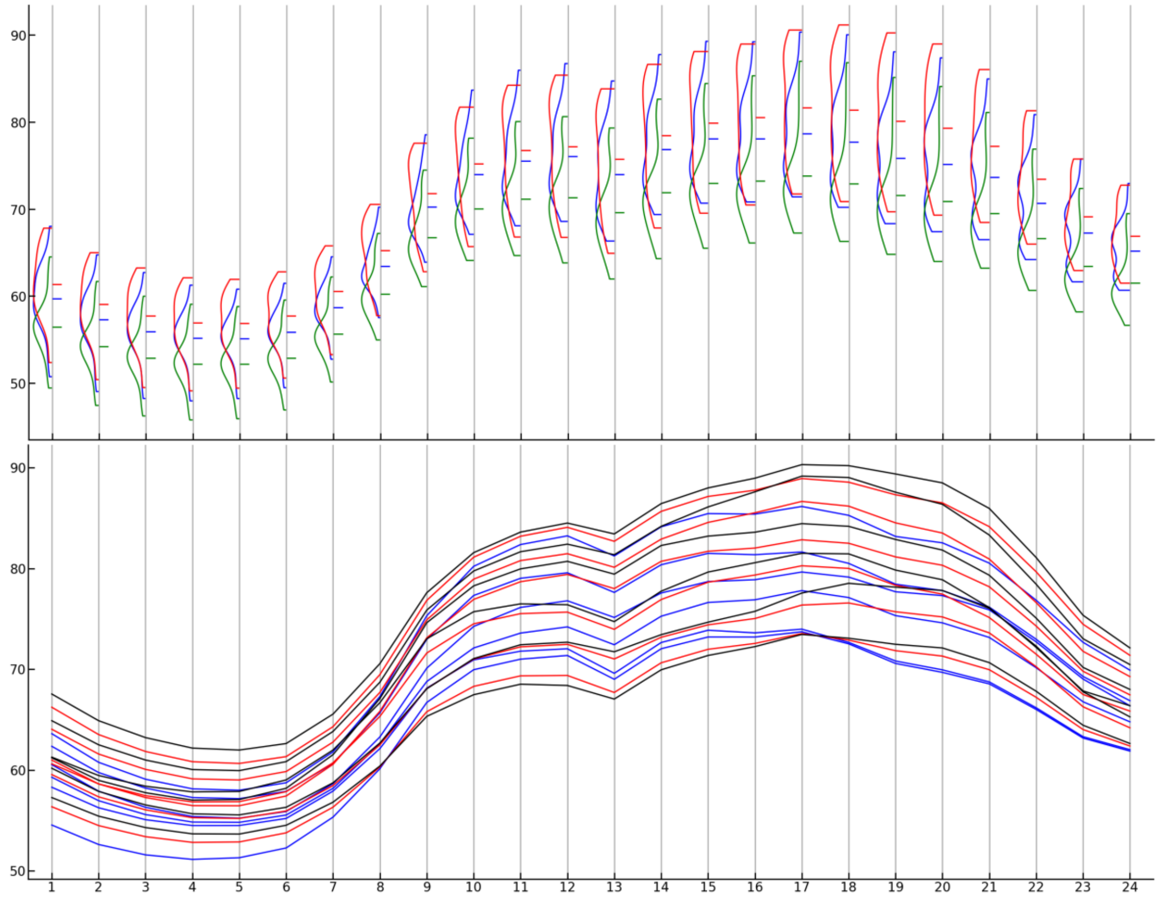
$${}^{\text{ren}}D_{t,k}^{\text{sce,2022}} = {}^{\text{ren}}\text{cap}_{t}^{2022} \times {}^{\text{ren}}\text{rat}_{t,k}^{\text{sce,2022}} \qquad \forall t, k$$

$${}^{\text{the}}D_{t,k}^{\text{sce,2022}} = {}^{\text{sys}}D_{t,k}^{\text{sce,2022}} - {}^{\text{ren}}D_{t,k}^{\text{sce,2022}} \qquad \forall t, k$$

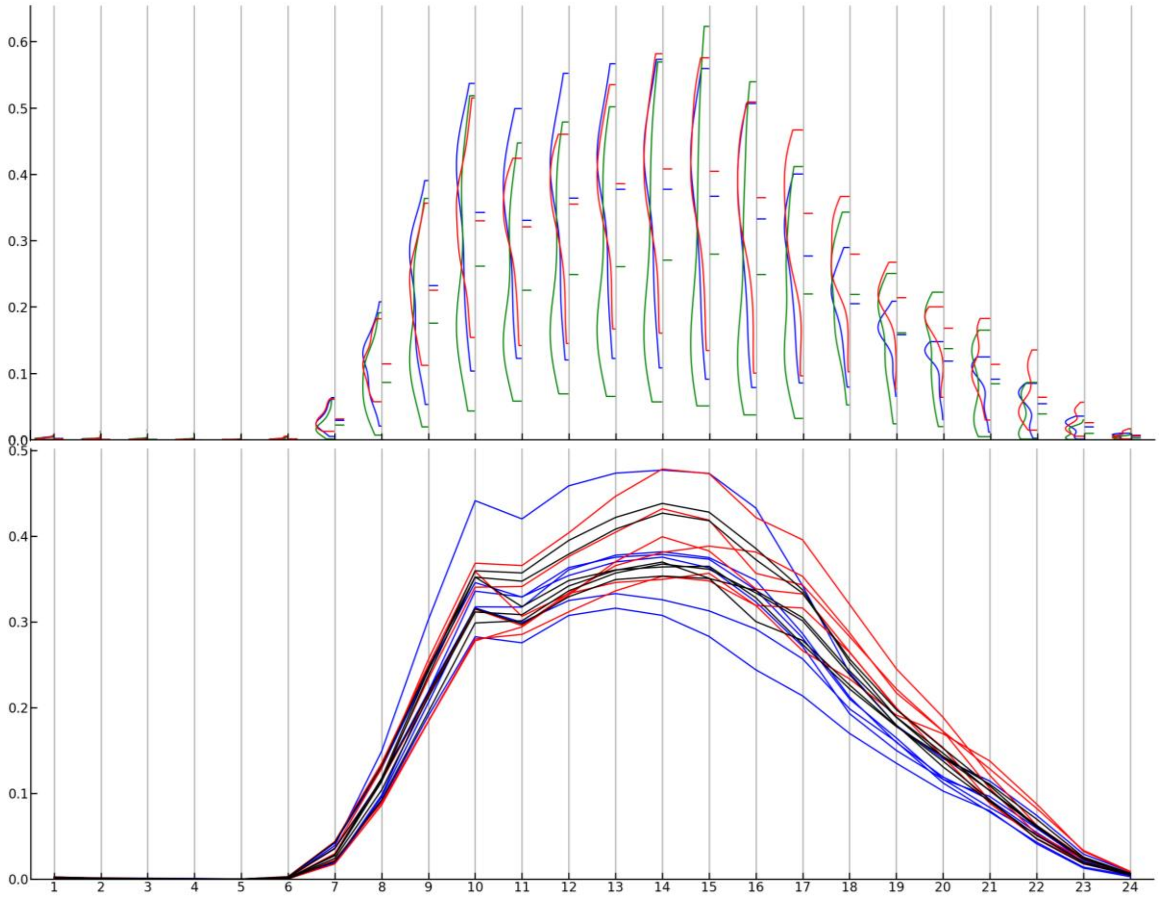
Note: Among 43 and 45 weekdays in the two windows, we removed 1 and 3 data points for each year, respectively. The removed data points corresponds to weekdays in June.
Note: Basically, linear trend projection (linear extrapolation) is used for estimating 2022 demand scenarios, which is sufficient for our project, because we are essentially estimating rough distributions, not a single data point. This part serves as “forecasting” the distribution of thermal demand on July 21, 2022, contrary to superscript “sce” rather than “for” in next parts.

COVID-19 is over, but historical data from 2020 still haunt us. We completely erased 2020 and was able to obtain nice scenarios for demand and renewable ratio.

Hourly Distribution (T) **& Six Scenarios** (B), system demand, **2019** · **2020** · **2021** · **2022**
Hour in time horizon (X, h) · System demand (Y, GW)
Demand distributions are based on raw data points · Horizontal line indicates mean demands for the hour · 2022 scenarios are linear trend projections of 2019 and 2021.



Hourly Distribution (T) & **Six Scenarios** (B), renewable ratio, **2019** · **2020** · **2021** · **2022**
Hour in time horizon (X, h) · Renewable ratio (Y, p.u.)
Renewable ratio distributions are based on raw data points · Horizontal line indicates mean renewable ratios for each hour · 2022 scenarios are averages of 2019 and 2021.

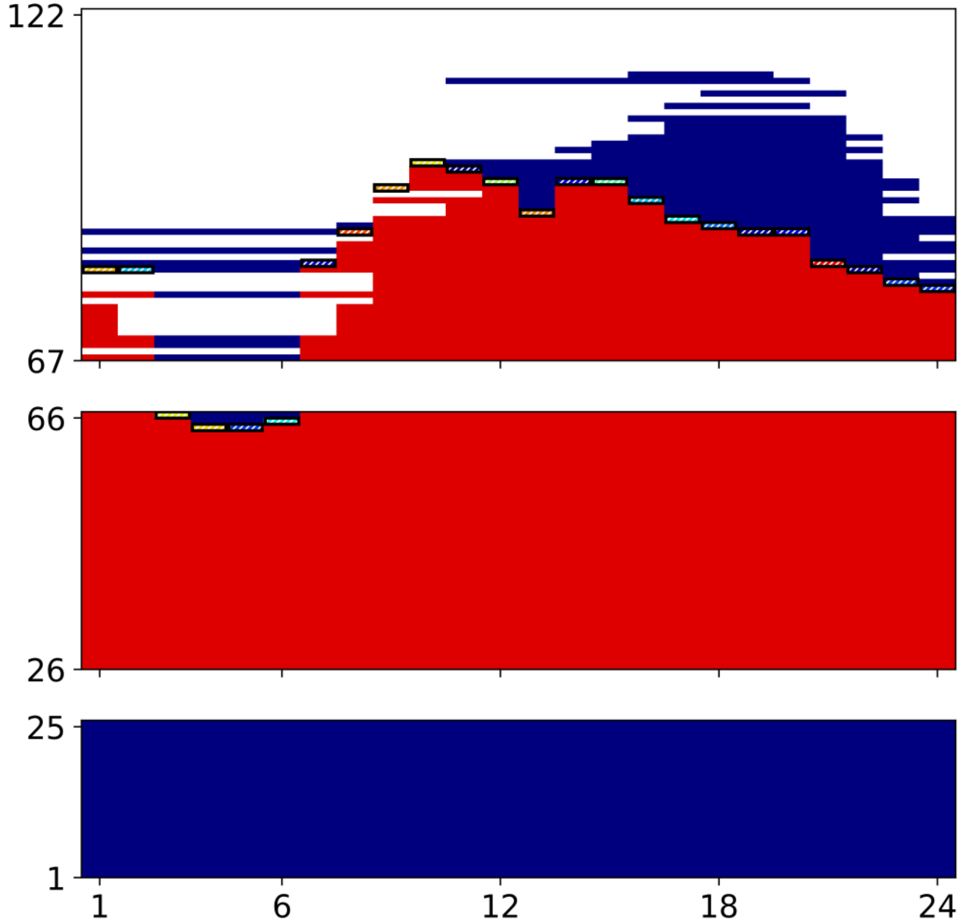


Note: “Renewable” throughout the project in fact refers to “solar”. This is discussed later.

We first used a perfect-forecast case as the baseline. Adding reserve and stochasticity to the model completely reshaped the dispatch heatmap, providing deeper into the system.

Dispatch Heatmap, forecast equals actual thermal demand

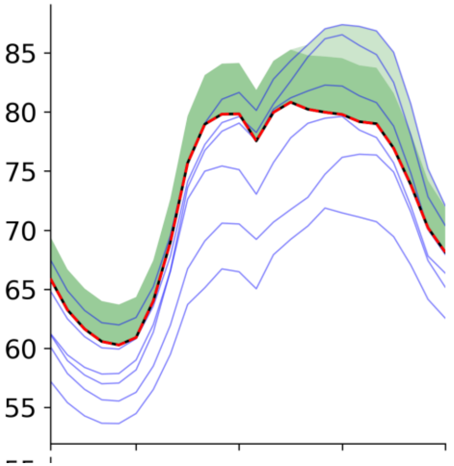
There is no unit producing its power maximum due to minimum operation-level reserve. With greater reserve at 18 hour, more expensive LNG units are turned on at minimum operation levels to keep large headroom. Minimum up-time constraints keep online a few of these units, too.



Thermal Demand Profiles, B KRW

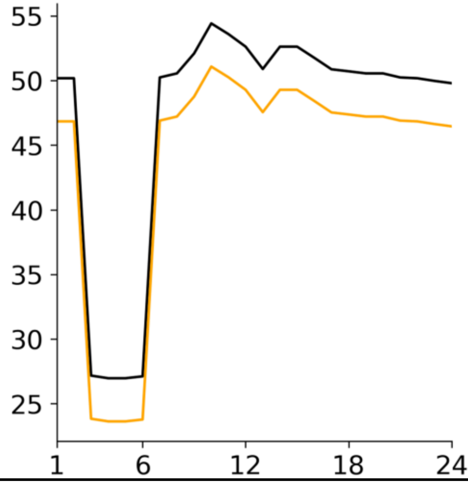
Scenarios · Forecast · Actual

Deterministic & Stochastic reserve



Marginal Prices, KRW/kWh

Generation · **Reserve** (Opportunity cost)



Reserve margin: 5.21 - 10.53%

Total generation, startup cost: 45.65B, 1.74B KRW

Total reserve cost: 5.03B KRW (none was useful)

Total cost for retailor (revenue for generators): 88.66B KRW

The perfect forecast case means absolutely no load shedding at real-time stage, and the dispatch is perfect in that load-generation balance is met. No reserve has to be deployed throughout the day. This is unrealistic but serves as the base case for next. The day-ahead plan is analyzed below.

- The top graph on left shows the position of red forecast thermal demand relative to six blue scenarios. During the first 15 hours, the forecast overestimates thermal demand relative to scenarios, which resulted in lower expected forecast error. Scenarios under the red line have zero non-served energy, and vice versa. Hence, stochastic reserve component, highlighted by the lighter green shade, is little to none when deterministic reserve can cover small or zero forecast errors.
- As expected forecasting errors become larger in later hours, stochastic reserve component increases. We can safely assume that expected load shedding penalty was higher than increased cost of LNG unit's startups and generation. Thermal demand decreased, which is the reason for the downhill in the heatmap.
- The reserve price is the opportunity cost for reserve-producing units, which exactly reflects the marginal reserve price formula used by Korea. It is simply a vertically shifted SMP because the marginal unit for reserve with the highest opportunity cost is the first cheapest nuclear unit (3.34 KRW/kWh). All reserve-procuring units are paid at this price.
- Detailed investigation of the result revealed that it was “stochastically” economical to execute load shedding of 1.24 MW for the sixth (highest) scenario at 19 hour.

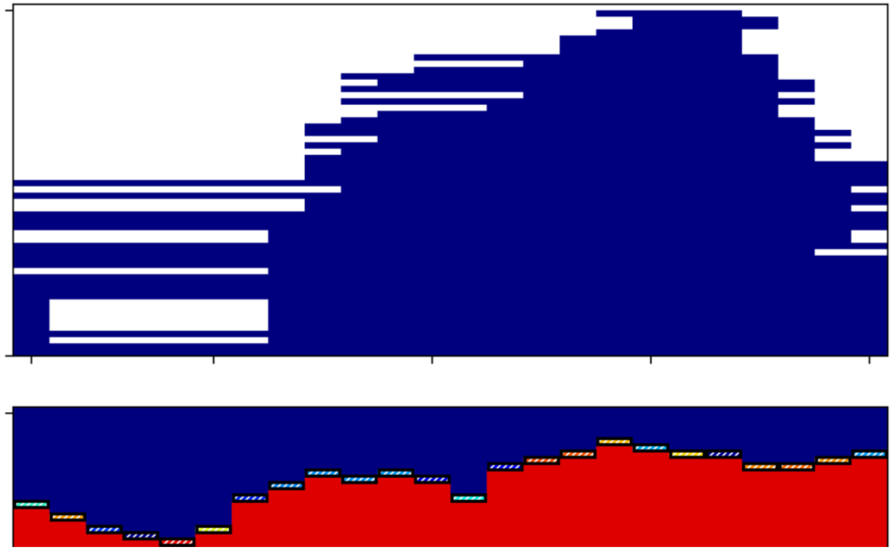
Note: Reserve margin is relative to system demand, not thermal demand.

Note: Nuclear units are at their minimum operating levels, and this turned out to be correct because 95% of their power maximums exactly equal their power minimums. We acknowledge that colors can leave room for misinterpretation. Note: Forecast errors only consider scenarios higher than the forecast as we are only focused on procuring upward spinning reserve.

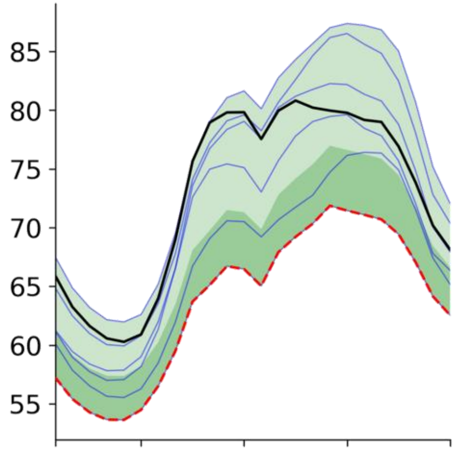
Simple stress test of our reserve policy reveals no load shedding at real-time stage. Essentially, the full cost of reliability for uncertainty was paid at day-ahead stage, because scenarios covered a road range.

Forecast underestimation means reserve is mostly from stochastic one. Paying 6.92B KRW for reserve procurement saves 763.44B KRW of load-shedding cost on that day.

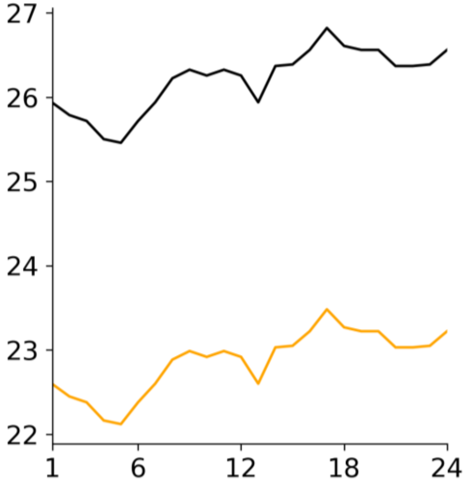
- Thanks to the sixth scenario, the reserve was fully prepared for such high forecast errors at real-time stage. Reserve was deployed, and such operation would have resulted in payments on top of the reserve procurement.
- Otherwise, we would have faced a disastrous amount of the cost of unexpected load shedding.



Thermal Demand Profiles, B KRW
Scenarios · Forecast · Actual
Deterministic & Stochastic reserve

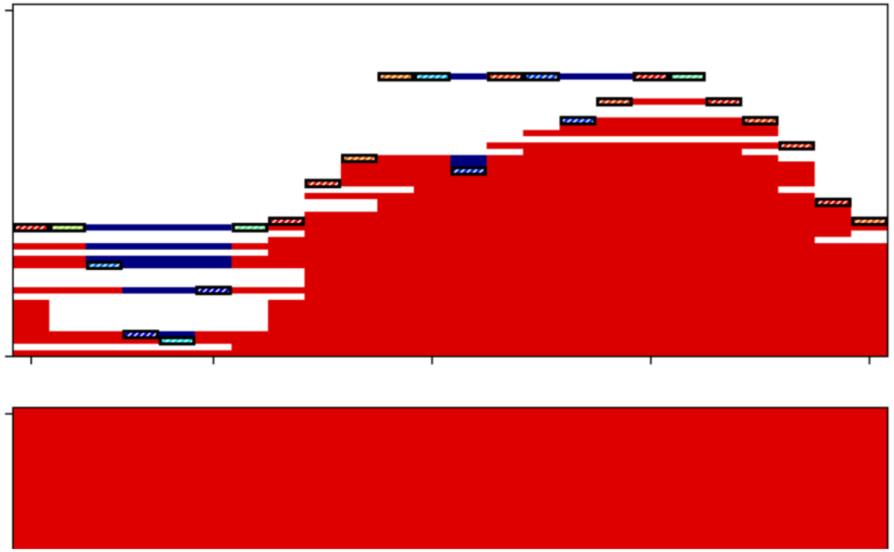


Marginal Prices, KRW/kWh
Generation · Reserve (Opportunity cost)

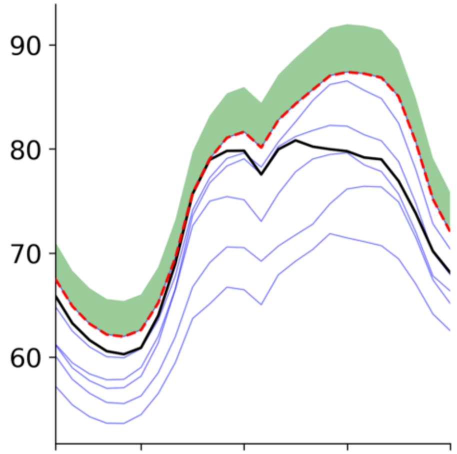


Forecast overestimation relative to scenarios means there is only deterministic reserve.

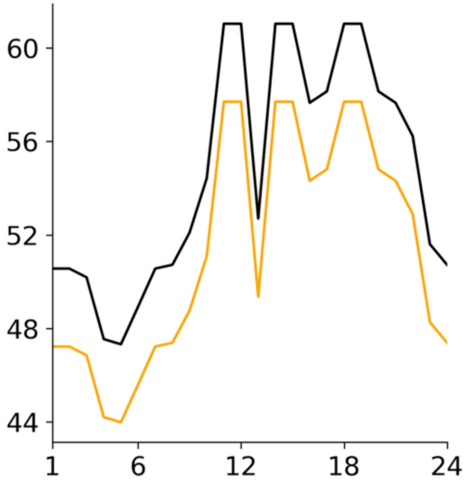
- In reality, downward spinning reserve must be deployed.
- It is insufficient to overestimate thermal demand but is cheaper than potential load shedding.
- Moreover, reserve based on forecast overestimation is more sensitive to a steeper incline of thermal demand. It can exceed small procured deterministic reserve at real-time stage, leading to unexpected load shedding, too.



Thermal Demand Profiles, B KRW
Scenarios · Forecast · Actual
Deterministic & Stochastic reserve



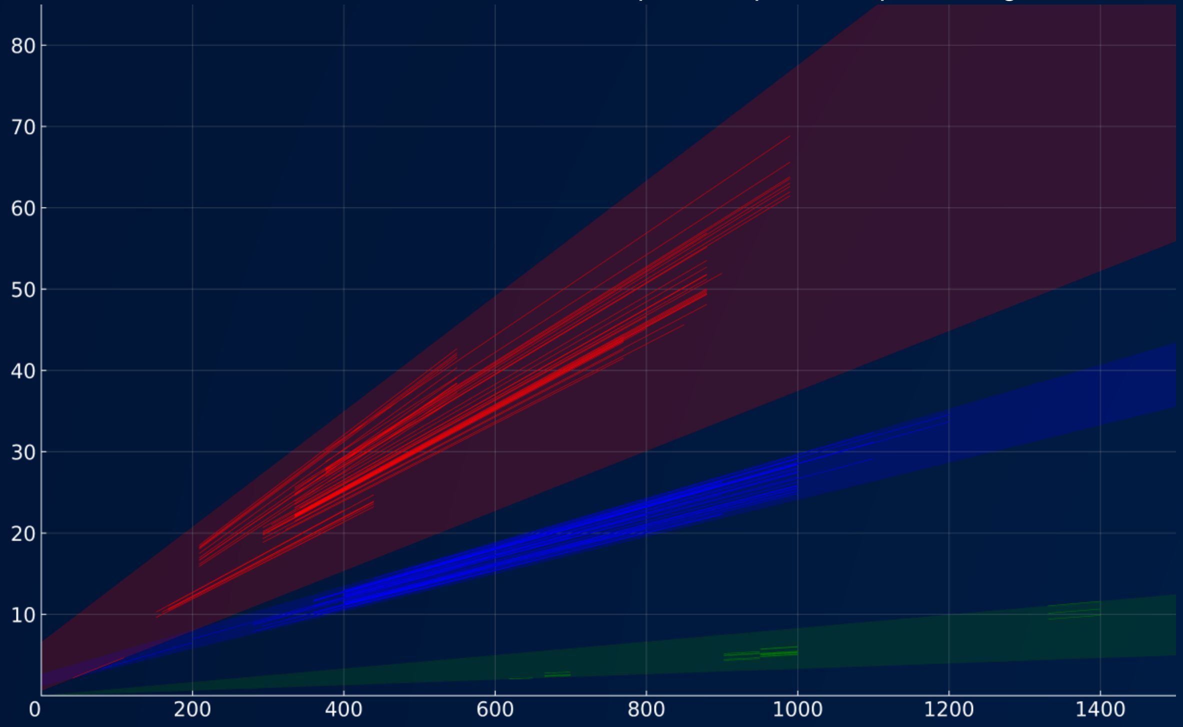
Marginal Prices, KRW/kWh
Generation · Reserve (Opportunity cost)



Note: The underestimation case is unrealistic because of our assumption: no (sub-)hour level (re-)scheduling and (re-)dispatch after day-ahead stage. This is further discussed later.
Note: For the overestimation case, the actual thermal demand did not reach the forecast at hour 9 and 10, meaning there was no stochastic reserve, and no reserve was actually deployed.
Note: We acknowledge that scenarios have a precise and significant impact on deterministic and stochastic reserve procurement.

We have linearized the cost function from the KPG dataset. This greatly reduced computation time and made analysis simpler, despite the departure from reality. Other generator-related data were taken without modification.

Post-modification Generation Cost Functions of 122 Units, LNG · Coal · Nuclear
Power dispatch Output (X, MW) · Generation cost (Y, M KRW) · Each colored area showing the minimum and maximum gradients (linear cost parameters) of each unit type · Generation cost for each unit is shown within its possible power dispatch range



Range of Linear Cost Parameters, KRW/kWh

	Nuclear	Coal	LNG
Minimum	3.339	22.912	36.872
Maximum	8.282	27.174	80.756

Generation unit-related inputs are from the KPG dataset. However, we completely excluded quadratic generation cost parameters in the project.

- Generation cost functions are linearized by simply removing quadratic coefficients from consideration. No further modification is made.
- UC computation is significantly faster as the optimization problem is now MILP.
- Sorting the units by their linear cost parameters correctly groups them into their type, as shown in the table. These values are also unique.
- The system marginal price for each hour exactly equals the linear cost parameter of the marginal unit, allowing us to map the SMP visually in the dispatch heatmap.

We are now farther away from reality.

- Every coal unit stays cheaper than LNG at all output levels. In practice, it is reasonable to model the system such that coal units are as expensive as LNG, due to coal's higher heat rate degradation and lower thermal efficiency.
- I experimented with steeper cost functions for coal units, but this impractically increased UC computation time. I believe this is due to increase in the number of reasonable combinations to consider when the solver attempts to choose unit(s) for adding 900 MW, for example.

Other Generation-unit-related inputs are from the KPG dataset.

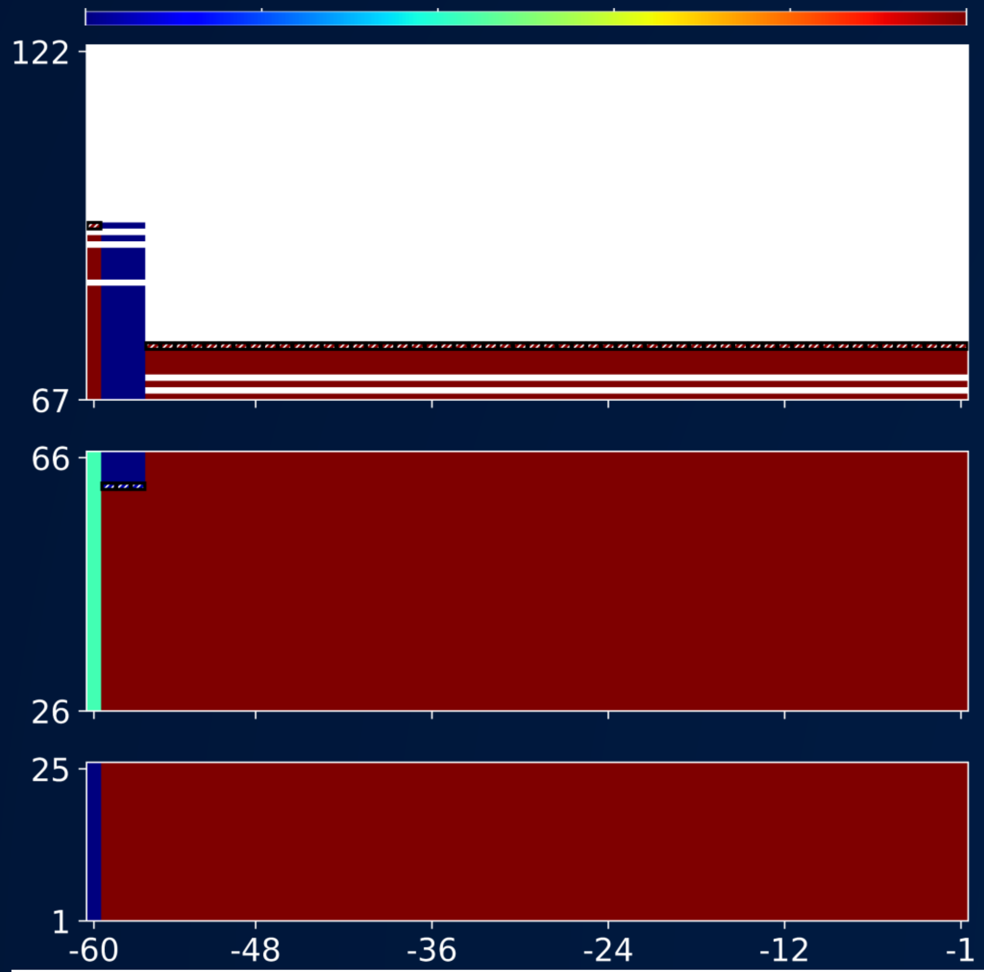
- There was no modification made.
- Must-off units are not considered for smoother and consistent analysis.
- Commitment decisions have been ignored, and initial conditions involving instant-black-start was computed instead.
- Linearization of the cost function does not mean that units with low linear cost parameters are turned on, one by one, to meet the required demand. This is because no-load costs and startup costs are not in order when sorted by the linear cost of the units. There may be other constraints that result in similar behavior.

Note: Nuclear units' ramp-up/down, startup-/shutdown-ramp are modified to their minimum power output. In the KPG dataset, the 4 quantities were less than their minimum power outputs, resulting in eternal turn-offs after serving their must-offs. Additionally, there is a mismatch between the KPG's commitment decisions and the 4 quantities under ramp-related constraints, as nuclear units were shown to successfully turn on after serving their must-offs.

For initial conditions, we performed instant-black-start 60 hours before the target horizon to produce a steady state at first-target-hour thermal demand. This completely removes undesirable effects of previous time horizon in question 1 and 2.

Dispatch Heatmap for Initial Condition of Question 1 & 2 & 3

Time period from start of the target horizon (X, hour) · Unit ranked by linear cost (Y)
LNG (T) · Coal (M) · Nuclear (B) · Operation level of on-units (color, %) · Off-units (white) · Marginal unit for generation (black boundary and white hatches)



Although we could fully rely on the KPG's datasets from generators to timeseries data, this poses several modeling logical flaws. We outlined just a few below:

- For Q3, we want to extract at least seasonal distributions for renewable generation or ratio. However, the KPG dataset offers synthetic ones limited to year 2022.
- Even if we relied on the KPG's renewable dataset, its commitment decisions data were certainly produced from different models which is expected to involve 1. multi-bus formulations with network constraints, 2. different reserve policy, and 3. renewable curtailment, at minimum.
- Essentially, we would be feeding initial conditions from the KPG dataset into a different system and models, which would have resulted in unacceptable, fundamental, and logical flaws in modeling.

A strategy for obtaining initial conditions, unique for our system and models, was discovered.

- We performed UC for the time horizon 60 hours before the target horizon. At -60 hour, the system, assumed to have no load previously, is "waken up" to meet the thermal demand, which equals the target horizon's first hour thermal demand. This demand is set constant for all 60 hours.
- At -60 hour, nuclear and coal units are at their minimum and mid operation levels, respectively, due to their ramp constraints. Extra LNG units are turned on to meet the residual thermal demand and kept on for a few hours due to their minimum up-time constraints.

This is not perfect for many reasons, but realism is not one of the objectives in the project.

- I simply set no minimum reserve requirement for initial condition UC. For Q1 and Q2, we study the system with actual thermal demand and no reserve.
- For Q3, the same initial conditions were used for simplicity. Until DA-SUC is solved, reserve is not known yet. I could have simply set minimum reserve requirement to DA-SUC's approximate deterministic reserve (roughly 5% of thermal demand from minimum operation-level reserve if thermal demand is above 1.4 GW / 3% = 46.666 GW). To include stochastic reserve, we may solve iteratively initial condition UC and DA-SUC until their system reserve values are close enough, only if we must use this initial condition design (it cannot be equal if thermal demand increases or decreases at 2 hour).
- In reality, such a steady state is never reached in this context. However, we can now obtain initial conditions such that effects of the previous horizon optimization on the target horizon optimization is removed and minimum for question 1, 2, and question 3, respectively, which cannot be obtained with varying thermal demand in previous time horizon. This is suitable for our project.

Note: "Instant-black-start", as we do not steadily increase thermal demand. It is deliberate because we want no initial minimum up/down-time requirement for the target horizon, for example.
Note: The code is written such that minimum reserve requirement can be set; Attempts to implement SUC which uses different initial conditions for each scenario were made, but I failed.
Note: Scenarios with higher thermal demands at the first hour incurred greater startup costs, and the dispatch would be suboptimal with the previous time horizon is taken into account.

Questions & Answers

WHERE is your code?

- github.com/realgyeongminnoh/power_system_economics

WHY no real-time redispatch after DA-ED?

- Our evaluation metric is the intrinsic quality of the day-ahead schedule and reserve policy. By freezing the DA commitments and dispatch, any load-following error shows up directly as non-served energy. This reveals whether SUC and reserve strategy were sufficient on their own to deal with uncertainty (given from scenarios).
- For other studies, we may be able to add a layer of desired optimization at real-time stage, which receive the power system state as inputs to adjust the day-ahead plan sequentially as time progresses.

WHY keep the six scenarios uniformly-weighted, making centroids uniformly distributed for each hour? Why not normal or any other distribution?

- I intended this equal-probability, unequal-spacing design. Quantile binning forces every bin to hold the same share of historical points. The bin means (centroids) therefore sit at the fixed 8, 25, 42, 58, 75, 92% percentiles. Because the underlying hourly distribution is somewhat wide-bell-shaped, these means are closer together near the median and father apart in the tails.
- When all weights are equal, it is also easier to experiment with the data. We can modify specific scenarios and observe the difference in results. We can also make sure that the modified scenario is definitely included in reserve procurement decision just like any other scenarios.
- Modelling system demand and renewable ratio as gaussian distributions is not automatically superior. In fact, I believe it is inferior, because it is not strictly normal when we observed the distribution graphs in page 9. It is sometimes skewed to one end. Other times, the two ends have higher probabilities than the means, which could be further distinguished into two types for example: hot and cold hours.
- Feeding gaussian or other distributions with long tails into SUC will further complicate our analysis, because scenarios with less probabilities are treated “less of a scenario” by the solver (as intended), which isn’t straightforward to humans.

Why only 42 historical profiles per hour?

- This is unfortunately the maximum I could do, given 2 years. A ±30-day window centered on July 21 and weekday translates to approximately 42.8 (60 · 5 / 7) profiles per hour.
- A larger window (e.g., ±40-day window, a full season, etc.) would certainly add more data points, but we risk blending multiple detailed seasonal characteristics in a single window and treating multiple distributions as one. This is similar for years.

WHY pick 21 July as the study date?

- I tested several dates in July 2022 including the original time horizon (July 2, 2022; my birthday). It turned out my birthday was Saturday in 2022. I selected this specific date while making sure there was sunny and renewable generation was high, in an effort to obtain interesting optimization result.

WHY omit wind in the stochastic scenario?

- Wind generation was nearly invisible when I inspected the generation profile. For other countries with high wind generation, it would be nice to add wind on top of solar because wind ratio distribution is often said to be Weibull (KPG dataset).
- We have lost approximately half of actual renewable generations by disregarding everything other than solar generation. I tested deterministic UC with full renewable generation just in case, but the result did not vary much, so I moved on with this decision at the end.
- I actually planned to have two distinct stochastic factors in the beginning, separate into system demand and solar generation. The addition of wind would have added to already-heavy model complexity. Modeling other renewable types’ ratio as probability distributions did not look valid. This original plan affected my data processing stage.

WHY do a single-bus formation and ignore network constraints?

- The model was heavy even after many micro- and macro-optimizations in terms of both math and code (which have minimal effects on the objective value). I will do it.

Note: “Renewable ratio” refers to capacity-utilization ratio.

Thank You for Reading!

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Appendix I: Nomenclature

$\mathcal{I}$	Set of thermal generation units	P_i^{min}	Minimum generation capacity
$\mathcal{T}$	Set of hourly time periods	P_i^{SU}	Startup ramp limit
$\mathcal{K}$	Set of thermal demand scenarios	P_i^{SD}	Shutdown ramp limit
$\mathcal{S}$	Set of startup cost segments	P_i^{RU}	Ramp-up rate
$\mathcal{S}_{i,s}^{\text{lag}}$	Time lags for startup segment s	P_i^{RD}	Ramp-down rate
$u_{i,t}$	Commitment status	TU_i	Minimum up-time requirement
$v_{i,t}$	Startup indicator	TD_i	Minimum down-time requirement
$w_{i,t}$	Shutdown indicator	TU_i^{R}	Initial up-time requirement
$\delta_{i,t,s}$	Startup segment selection	TD_i^{R}	Initial down-time requirement
$\tilde{p}_{i,t,k}$	Dispatch amount above P_i^{min}	${}^{1}C_i^{\text{G}}$	Linear generation cost
$r_{i,t}$	Reserve provision amount	${}^{0}C_i^{\text{G}}$	No-load cost
$\text{nse}_{t,k}$	Non-served thermal demand	$C_{i,s}^{\text{SU}}$	Startup cost for segment s
$\epsilon_{t,k}$	Forecast error in thermal demand	κ_k	Scenario weight
$R_t^{\text{N-1}}$	N-1 reserve	VoLL	Value of lost load
R_t^{FR}	Forecast-response reserve	${}^{\text{the}}D_{t,k}^{\text{sce}}$	Scenario thermal demand
P_i^{max}	Maximum generation capacity	${}^{\text{the}}D_t^{\text{for}}$	Forecasted thermal demand

Note: Other than unit status(u), three binary decision variables were used for startup status (v), shutdown status (w), and startup-type (δ). Continuous decision variables are dispatch output above minimum generation, reserve, non-served energy, and N-1 contingency reserve.

Appendix II: Formulation of Stochastic Unit Commitment (1)

$$\min \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \kappa_k \Big[\sum_{i\in\mathcal{I}} \big({}^1C_i^{\mathrm{G}} \left(\tilde{p}_{i,t,k} + P_i^{\min} u_{i,t}\right) + {}^0C_i^{\mathrm{G}} u_{i,t} \big) + \mathrm{nse}_{t,k} \cdot \mathrm{VoLL} \Big] + \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} C_{i,s}^{\mathrm{SU}} \delta_{i,t,s}$$

Subject to:

$\tilde{p}_{i,t,k} \leq u_{i,t} \cdot (P_i^{\max} - P_i^{\min})$	Generation	$\forall i, t, k$	(1)
$\tilde{p}_{i,t,k} + r_{i,t} \leq u_{i,t} \cdot (P_i^{\max} - P_i^{\min})$		$\forall i, t, k$	(2)
$\sum_{i\in\mathcal{I}} \left(\tilde{p}_{i,t,k} + u_{i,t} P_i^{\min}\right) + \mathrm{nse}_{t,k} = {}^{\mathrm{the}}D_{t,k}^{\mathrm{sce}}$		$\forall t, k$	(3)
$\epsilon_{t,k} \geq {}^{\mathrm{the}}D_{t,k}^{\mathrm{sce}} - {}^{\mathrm{the}}D_t^{\mathrm{for}}$	Reserve	$\forall t, k$	(4)
$\mathrm{nse}_{t,k} \geq \epsilon_{t,k} - \sum_i r_{i,t}$		$\forall t, k$	(5)
$R_t^{\text{N-1}} = \max_{i\in\mathcal{I}} P_i^{\max} u_{i,t}$		$\forall t$	(6)
$R_t^{\mathrm{FR}} = 0.02 \cdot {}^{\mathrm{the}}D_t^{\mathrm{for}}$		$\forall t$	(7)
$\sum_i r_{i,t} \geq R_t^{\text{N-1}} + R_t^{\mathrm{FR}}$		$\forall t$	(8)
$r_{i,t} \geq 0.05 \cdot P_i^{\max} u_{i,t}$		$\forall i, t$	(9)
$u_{i,t} - u_{i,t-1} = v_{i,t} - w_{i,t}$	Startup	$\forall i, t$	(10)

Note: Other optimization formulations are subsets of this full SUC formulation.
Source: G. Morales-España et al, *Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem*, 2013

Appendix II: Formulation of Stochastic Unit Commitment (2)

$$\min \sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}} \kappa_k \Big[\sum_{i\in\mathcal{I}} \big({}^1C_i^{\mathrm{G}}\,(\tilde{p}_{i,t,k} + P_i^{\min}\,u_{i,t}) + {}^0C_i^{\mathrm{G}}\,u_{i,t}\big) + \mathrm{nse}_{t,k}\cdot \mathrm{VoLL}\Big] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} C_{i,s}^{\mathrm{SU}}\,\delta_{i,t,s}$$

Subject to:

$\sum_{s\in\mathcal{S}}\delta_{i,t,s} = v_{i,t}$	Startup	$\forall i,t$	(11)
$\delta_{i,t,s} \le \sum_{\tau\in\mathcal{S}_{i,s}^{\mathrm{lag}}} w_{i,t-\tau}$		$\forall i,t,s$	(12)
$\tilde{p}_{i,t,k} + r_{i,t} \le u_{i,t}(P_i^{\max} - P_i^{\min}) - v_{i,t}(P_i^{\max} - P_i^{\mathrm{SU}}) - w_{i,t+1}(P_i^{\max} - P_i^{\mathrm{SD}})$	Ramp	$\forall i, t < T, k$	(13)
$\tilde{p}_{i,t,k} + r_{i,t} \le u_{i,t}(P_i^{\max} - P_i^{\min}) - v_{i,T}(P_i^{\max} - P_i^{\mathrm{SU}})$		$\forall i,k$	(14)
$\tilde{p}_{i,t,k} + r_{i,t} - \tilde{p}_{i,t-1,k} \le P_i^{\mathrm{RU}}$		$\forall i,t,k$	(15)
$\tilde{p}_{i,t-1,k} - \tilde{p}_{i,t,k} \le P_i^{\mathrm{RD}}$		$\forall i,t,k$	(16)
$u_{i,t} \ge 1$	Time	$\forall i, t \le \mathrm{TU}_i^{\mathrm{R}}$	(17)
$u_{i,t} \le 0$		$\forall i, t \le \mathrm{TD}_i^{\mathrm{R}}$	(18)
$\sum_{\tau=t-\mathrm{TU}_i+1}^{t} v_{i,\tau} \le u_{i,t}$		$\forall i, t \ge \mathrm{TU}_i$	(19)
$\sum_{\tau=t-\mathrm{TD}_i+1}^{t} w_{i,\tau} \le 1 - u_{i,t}$		$\forall i, t \ge \mathrm{TD}_i$	(20)

Note: For 0-time index, initial conditions are accessed.
Note: From instant-black-start initial condition design, initial minimum up-/down-time requirements are zero.
Source: G. Morales-España et al, *Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem*, 2013