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Exercises for Algorithmic Bioinformatics II

Assignment 10

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Task 1: Simple Orthodox and Bayesian Statistics (10P):

Coin tossing

You are modelling a set of fair and biased coins. Fair coins show head and tail with equal probability, whereas a biased coin will show head with a probability of 7/8.

(a) In order to find out, whether a given coin is fair, you throw it 10 times. How are the proba- bilities of each outcome for every coin (e.g. 5x head)?

outcome	**fair coin**	**biased coin**
0 head, 10 tail	$\binom{10}{0}(\frac{1}{2})^{10} = 0.0010$	$\binom{10}{0}(\frac{1}{8})^{10} = 0.0000$
1 head, 9 tail	$\binom{10}{1}(\frac{1}{2})^{10} = 0.0098$	$\binom{10}{1}(\frac{1}{8})^{9} \times \frac{7}{8} = 0.0000$
2 head, 8 tail	$\binom{10}{2}(\frac{1}{2})^{10} = 0.0439$	$\binom{10}{2}(\frac{1}{8})^{8} \times (\frac{7}{8})^{2} = 0.0000$
3 head, 7 tail	$\binom{10}{3}(\frac{1}{2})^{10} = 0.1172$	$\binom{10}{3}(\frac{1}{8})^{7} \times (\frac{7}{8})^{3} = 0.0000$
4 head, 6 tail	$\binom{10}{4}(\frac{1}{2})^{10} = 0.1172$	$\binom{10}{4}\binom{10}{4}(\frac{1}{8})^{6} \times (\frac{7}{8})^{4} = 0.0005$
5 head, 5 tail	$\binom{10}{5}(\frac{1}{2})^{10} = 0.2461$	$\binom{10}{5}(\frac{1}{8})^{5} \times (\frac{7}{8})^{5} = 0.0039$
6 head, 4 tail	$\binom{10}{6}(\frac{1}{2})^{10} = 0.1172$	$\binom{10}{6}(\frac{1}{8})^{4} \times (\frac{7}{8})^{6} = 0.0230$
7 head, 3 tail	$\binom{10}{7}(\frac{1}{2})^{10} = 0.1172$	$\binom{10}{7}(\frac{1}{8})^{3} \times (\frac{7}{8})^{7} = 0.0920$
8 head, 2 tail	$\binom{10}{8}(\frac{1}{2})^{10} = 0.0439$	$\binom{10}{8}(\frac{1}{8})^{2} \times (\frac{7}{8})^{8} = 0.2416$
9 head, 1 tail	$\binom{10}{9}(\frac{1}{2})^{10} = 0.0098$	$\binom{10}{9}(\frac{1}{8})^{1} \times (\frac{7}{8})^{9} = 0.3758$
10 head, 0 tail	$\binom{10}{10}(\frac{1}{2})^{10} = 0.0010$	$\binom{10}{10}(\frac{7}{8})^{10} = 0.2631$

(b) After how many tries would you be asserted that your coin is fair/not fair?

$X \sim binomial(10, \frac{1}{2})$

$H_0 : Pr(head) = 0.5, H_1 : Pr(head) \neq 0.5$

$p = Pr(x \leq 1) + Pr(x \geq 9) = 0.0010 + 0.0098 + 0.0098 + 0.0010 = 0.0216 < 0.05$

$p = Pr(x \leq 2) + Pr(x \geq 8) = 0.1094 > 0.05$

so we can only reject H_0 and assert coin is biased because of p-value < 0.05 significance when we tossed a coin 10 times and observed 0, 1, 9, 10 heads.

(c) Assume that you have 10 coins and that one of the coins is not fair. Would you adapt your trials from (b)? we use likelihood ratio to test whether the coin is fair or not. $H_0 : Pr(head) = 0.5, H_1 : Pr(head) \neq 0.5$

$$
\begin{aligned}
\Lambda &= \frac{\mathcal{L}(\theta_0|x)}{\mathcal{L}(\theta_1|x)} \\
&= \frac{P(d_1, \ldots, d_N|H)}{P(d_1, \ldots, d_N| - H)} \\
&= \frac{9 \cdot \mathcal{L}(p = \frac{1}{2}|k, N)}{1 \cdot \mathcal{L}(p = \frac{7}{8}|k, N)} \\
&= 9 \cdot (\frac{4}{7})^{N-k}
\end{aligned}
$$

$df = 10 \Longrightarrow \lambda_0 = x^2_{0.05} = 18.31 > \Lambda \Longrightarrow$ So we Reject H_0.

Bayes

Given a diagnostic method to determine a disease which appears in 1 of 10,000 in a population. The method has a false positive rate of 0.001 and a false negative rate of 0.01.

(d) What is the probability of having the disease, if the test is positive?

A : Having disease B : Test positive

$$
\begin{aligned}
&P(B|\overline{A}) = 0.001, P(\overline{B}|A) = 0.01 \Longrightarrow P(\overline{B}|\overline{A}) = 0.999, P(B|A) = 0.99 \\
&P(A) = \frac{1}{10000} = 0.0001 \Longrightarrow P(\overline{A}) = 0.9999 \\
&P(B) = \sum_n P(B|A_n) = P(B|A)P(A) + P(B|\overline{A})P(\overline{A}) = 1.0989 \times 10^{-3} \Longrightarrow \\
&P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{0.99 \cdot 0.0001}{1.0989 \times 10^{-3}} = 0.09009
\end{aligned}
$$

The probability of having the disease when test is positive is 0.09009.

(e) What is the probability of not having the disease, if the test is negative?

$$P(\overline{A}|\overline{B}) = \frac{P(\overline{B}|\overline{A})P(\overline{A})}{P(\overline{B})} = \frac{0.999 \cdot 0.9999}{1 - 1.0989 \times 10{-}3} = 0.9999$$

The probability of not having the disease when test is negative is 0.9999.