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Exercises for Algorithmic Bioinformatics II

Assignment 13

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Exercise 2 (Markov-Chains I, 10P):

In an exam with only binary questions, the answers are distributed as follows: If the answer to the current question is *yes*, then the following answer will be *yes* with probability 3/4. If the answer to the current question is *no*, then the following answer will be *no* with probability 2/3.

(a) determine the fraction of questions with answer *yes* (you may assume a very long exam).

The state transition matrix A is defined as:

$$A = \begin{pmatrix} & yes & no \\ yes & \frac{3}{4} & \frac{1}{4} \\ no & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

As markov chain implies a invariant distribution π, thus

$$\pi \cdot A = \pi = \pi \cdot \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} = \pi$$

Denote x for initial state for *yes* and y for initial state for *no*.

$$(x, y) \cdot A = \pi \cdot \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} = (x, y)$$

From

$$\frac{3}{4}x + \frac{1}{3}y = x$$
$$\frac{1}{4}x + \frac{2}{3}y = y$$

we get

$$\frac{x}{y} = \frac{4}{3}$$

Thus, the fraction of questions with answer *yes* is $\frac{4}{3+4} = \frac{4}{7}$.

(b) is the Markov chain ergodic?

- The Markov chain follows a stationary distribution because the transition probabilities do not depend on t.
- The Markov chain is finite recurrent, it has finite number of states and each state is recurrent. s.t. for recurrent time $T_i = \min\{n : x_n = i | x_0 = i\} : P(T_i < \infty)$
- The Markov chain is irreducible because $\forall i, j, p_{ij} > 0$ each state is reachable from each other.
- The Markov chain is aperiodic because for each state it requires $1, 2, 3, 4, \ldots, n$ steps to get back to itself which means The GCD of the possible steps to return back is 1.

Thus, the Markov chain is ergodic.