Xiheng He

Lisanne Friedrich

Exercises for Algorithmic Bioinformatics II

Assignment 11

Xiheng He

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Exercise 2 (Entropy, 10P):

Given the following joint distribution of two random variables X and Y :

	Y = 0	Y = 1
X = 0	$\frac{1}{3}$	$\frac{1}{3}$
X = 1	0	$\frac{1}{3}$

and entropy defined as $H(Y|X) := \sum_x P(x)H(Y|X = x)$ and $H(X,Y) := -\sum_{x,y} P(x,y)\log P(x,y)$.

Calculate the entropies, relative entropies and mutual information:

(a) $H(X)$, $H(Y)$

$$H(X) = -\sum_i P(x_i) \log P(x_i)$$

$$= -(1/3 \cdot \log(1/3) + (1 - 1/3) \cdot \log(1 - 1/3))$$

$$= \log(3) - \frac{2}{3}$$

$$H(Y) = -\sum_i P(y_i) \log P(y_i)$$

$$= -(1/3 \cdot \log(1/3) + (1 - 1/3) \cdot \log(1 - 1/3))$$

$$= \log(3) - \frac{2}{3}$$

(b) $H(X|Y)$, $H(Y|X)$

$$H(X|Y) = H(Y|X) = \sum_{x_i} P(x_i) H(Y|X = x_i)$$

$$= 0 \times 1/3 + 1 \times 2/3 = \frac{2}{3}$$

(c) $H(X, Y)$

$$H(X, Y) = -\sum_{x,y} P(x, y) \log P(x, y)$$

$$= -(1/3 \cdot \log(1/3) + 1/3 \cdot \log(1/3) + 1/3 \cdot \log(1/3)) = \log 3$$

(d) $H(X||Y)$

$$H(X||Y) = \sum_i P(x_i) \log(\frac{P_X(x_i)}{P_Y(x_i)})$$

$$= 2/3 \times \log(2/3 \div 1/3) + 1/3 \times \log(1/3 \div 2/3) = 1/3$$

(e) $M(X; Y)$

$$M(X; Y) = \sum_{i,j} P(x_i, y_j) \log(\frac{P(x_i, y_j)}{P(x_i)P(y_j)})$$

$$= log(3) - 3/4$$