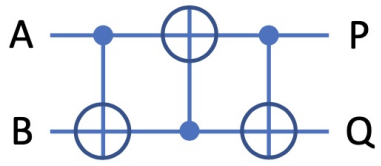
Exercise #2

Weeks 2

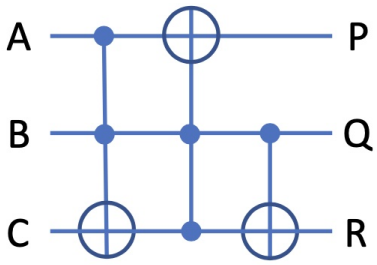
1. **Reversible circuits**

   Find the truth table of the following reversible circuits:

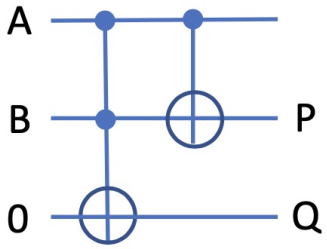
   (a)



   (b)



   (c)



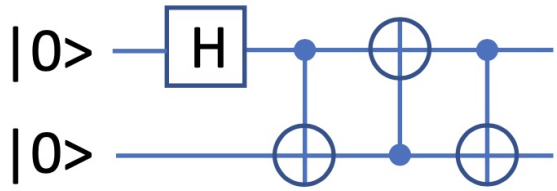
2. **Quantum circuits with Hadamard gate**

   The "Hadamard" gate, denoted by H, is given by

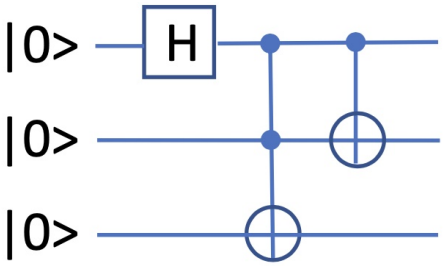
$$\begin{cases} H|0\rangle &= |0\rangle + |1\rangle\,, \\ H|1\rangle &= |0\rangle - |1\rangle\,. \end{cases}$$

   Find the output of the following quantum circuits:

(a)



(b)



3. **Quantum circuits with NOT and Phase gates**

The "NOT" gate, denoted by X, is given by

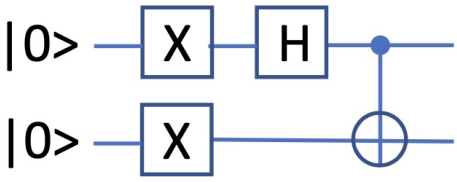
$$\begin{cases} X|0\rangle &= |1\rangle\,, \\ X|1\rangle &= |0\rangle\,. \end{cases}$$

The "Phase" gate, denoted by Z, is given by

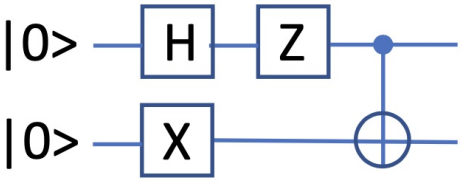
$$\begin{cases} Z|0\rangle &= |0\rangle\,, \\ Z|1\rangle &= -|1\rangle\,. \end{cases}$$

Find the output of the following quantum circuits:

(a)



(b)

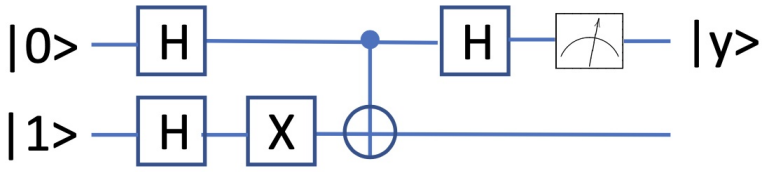


4. **Deutsch's algorithm**

Given a function $f(x)$, where x is a single bit string (i.e. $x = \{0, 1\}$) and $f(x)$ has only two possible values (i.e. $f(x) = \{0, 1\}$). Determine whether

$$f(0) = f(1) \text{ or } f(0) \neq f(1).$$

The Deutsch's algorithm solves this problem using only a single query of the function f. For some function f, the quantum circuits is given by:



Find the output y of the circuit. So is this function f balanced or constant?