第 8 讲 VINS 回顾与展望

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Section 1

课程简单回顾

从概率到最小二乘

已知一堆传感器数据，如何概率建模？

传感器测量：在某个状态发生的条件下，得到一些观测，对应似然概率 $p\left(\mathbf{r}_i|\boldsymbol{\xi}\right)$.

利用最大后验概率估计状态量：

$$\boldsymbol{\xi}_{\text{MAP}} = \arg\max_{\boldsymbol{\xi}} \prod_i p\left(\mathbf{r}_i|\boldsymbol{\xi}\right) p(\boldsymbol{\xi}) \tag{1}$$

假设观测值服从多元高斯分布：

$$p\left(\mathbf{r}_i|\boldsymbol{\xi}\right) = \mathcal{N}\left(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i\right), p\left(\boldsymbol{\xi}\right) = \mathcal{N}\left(\boldsymbol{\mu}_{\boldsymbol{\xi}}, \boldsymbol{\Sigma}_{\boldsymbol{\xi}}\right) \tag{2}$$

则转化为最小二乘问题：

$$\boldsymbol{\xi}_{\text{MAP}} = \underset{\boldsymbol{\xi}}{\operatorname{argmin}} \sum_i \left\|\mathbf{r}_i - \boldsymbol{\mu}_i\right\|^2_{\boldsymbol{\Sigma}_i} + \left\|\boldsymbol{\xi} - \boldsymbol{\mu}_{\boldsymbol{\xi}}\right\|^2_{\boldsymbol{\Sigma}_{\boldsymbol{\xi}}} \tag{3}$$

最小二乘的建模关键

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最小二乘问题建模的两个要素：残差函数的构建，协方差矩阵计算.

$$\boldsymbol{\xi}_{\mathrm{MAP}} = \arg\min_{\boldsymbol{\xi}} \sum_i \|\mathbf{r}_i - \boldsymbol{\mu}_i\|^2_{\boldsymbol{\Sigma}_i} + \left\|\boldsymbol{\xi} - \boldsymbol{\mu}_{\boldsymbol{\xi}}\right\|^2_{\boldsymbol{\Sigma}_{\boldsymbol{\xi}}} \tag{4}$$

残差函数的构建

- 视觉重投影误差：视觉特征提取，匹配，几何约束.
- IMU 预积分误差：IMU 动力模型，误差模型.
- 扩展：gps 误差，轮速计误差，uwb 等等.

协方差的计算

- 噪声方差的标定，如艾伦方差标定 IMU 随机噪声的方差.
- 协方差的传递.

最小二乘问题的求解关键

最小二乘问题求解的两个要素：求解器，雅克比.

$$\mathbf{J}^\top \boldsymbol{\Sigma}^{-1} \mathbf{J} \delta \boldsymbol{\xi} = -\mathbf{J}^\top \boldsymbol{\Sigma}^{-1} \mathbf{r} \tag{5}$$

求解器

- 基础：最速下降，高斯牛顿.
- 进阶：LM，DogLeg 等等.
- 高级：Schur Complement，鲁棒核函数的实现.

雅克比

- 链式法则，雅克比矩阵的组成.
- 对 $so3, se3$ ，四元数微小增量的导数.

Section 2

VIO 拓展

综述论文推荐

一些综述性质的工作:

- 2019, arxiv, Visual-inertial navigation: A concise review[1]. 全面系统的介绍了 VIO 相关研究, 值得一读.
- 2018,ICRA,A benchmark comparison of monocular visual-inertial odometry algorithms for flying robots[2]. 全面评测了各个 VIO 系统的精度, 有待商榷.
- 2018,IROS,Tutorial on Quantitative Trajectory Evaluation for Visual(-inertial) Odometry[3]. VIO 精度评估工具.

[1] Guoquan Huang. "Visual-inertial navigation: A concise review". In: *arXiv preprint arXiv:1906.02650* (2019).

[2] Jeffrey Delmerico and Davide Scaramuzza. "A benchmark comparison of monocular visual-inertial odometry algorithms for flying robots". In: *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE. 2018, pp. 2502–2509.

[3] Zichao Zhang and Davide Scaramuzza. "A tutorial on quantitative trajectory evaluation for visual (-inertial) odometry". In: *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE. 2018, pp. 7244–7251.

一些经典的 VIO 系统

基于滤波或者优化的经典 VIO 系统: 基于滤波的 VIO:

- 2017, IJRR, Iterated extended Kalman filter based visual-inertial odometry using direct photometric feedback[4]. ROVIO, 基于 EKF 和光度误差的 VIO 系统。代码开源。
- 2007, ICRA, A multi-state constraint Kalman filter for vision-aided inertial navigation[5]. MSCKF, 经典 VIO 之作.
- 2018, RAL, Robust Stereo Visual Inertial Odometry for Fast Autonomous Flight[6]. Stereo-MSCKF, 代码开源。
- 2014, THESIS, Visual-inertial odometry on resource-constrained systems[7]. MSCKF 的各种改进和完善， li mingyang 的博士论文。

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[4] Michael Bloesch et al. "Iterated extended Kalman filter based visual-inertial odometry using direct photometric feedback". In: *The International Journal of Robotics Research* 36.10 (2017), pp. 1053–1072.

[5] Anastasios I Mourikis and Stergios I Roumeliotis. "A multi-state constraint Kalman filter for vision-aided inertial navigation". In: *Proceedings 2007 IEEE International Conference on Robotics and Automation*. IEEE. 2007, pp. 3565–3572.

[6] Ke Sun et al. "Robust stereo visual inertial odometry for fast autonomous flight". In: *IEEE Robotics and Automation Letters* 3.2 (2018), pp. 965–972.

[7] Mingyang Li. "Visual-inertial odometry on resource-constrained systems". PhD thesis. UC Riverside, 2014.

基于优化的 VIO:

- 2015, IJRR, Keyframe-based visual–inertial odometry using nonlinear optimization[8]. OKVIS, 最早的基于优化的 VIO 系统，支持双目、单目。代码开源。
- 2016, TRO, On-Manifold Preintegration for Real-Time Visual–Inertial Odometry[9]. SVO + 预积分, 非常系统的从 SO3 出发推导了预积分 VIO 系统。值得一读。
- 当然还有 VINS-Mono 的相关工作已在课程中进行推荐。

[8] Stefan Leutenegger et al. “Keyframe-based visual–inertial odometry using nonlinear optimization”. In: *The International Journal of Robotics Research* 34.3 (2015), pp. 314–334.

[9] Christian Forster et al. “On-Manifold Preintegration for Real-Time Visual–Inertial Odometry”. In: *IEEE Transactions on Robotics* 33.1 (2016), pp. 1–21.

VIO 一致性相关的论文

可观性:

- 2008, ICRA, Analysis and improvement of the consistency of extended Kalman filter based SLAM[10]. FEJ 论文.
- 2011, IROS, An observability-constrained sliding window filter for SLAM[11]. 可观性约束，开源的 MSCKF 代码中均有使用。

不变性:

- 2015, arxiv, An EKF-SLAM algorithm with consistency properties[12]. 对理解不变性非常好的一篇论文。
- 2017, IROS, An invariant-EKF VINS algorithm for improving consistency[13]. invariant-EKF VINS。

[10] Guoquan P Huang, Anastasios I Mourikis, and Stergios I Roumeliotis. "Analysis and improvement of the consistency of extended Kalman filter based SLAM". In: *2008 IEEE International Conference on Robotics and Automation*. IEEE. 2008, pp. 473–479.

[11] Guoquan P Huang, Anastasios I Mourikis, and Stergios I Roumeliotis. "An observability-constrained sliding window filter for SLAM". In: *2011 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE. 2011, pp. 65–72.

[12] Axel Barrau and Silvere Bonnabel. "An EKF-SLAM algorithm with consistency properties". In: *arXiv preprint arXiv:1510.06263* (2015).

[13] Kanzhi Wu et al. "An invariant-EKF VINS algorithm for improving consistency". In: *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE. 2017, pp. 1578–1585.

VIO 中利用图像数据构建不同残差模型

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光度误差或者点、线、面、曲线误差:

- 2016, ICRA, Direct visual-inertial odometry with stereo cameras[14]. 基于直接法的 VIO。
- 2018, TRO, Observability Analysis of Aided INS with Heterogeneous Features of Points, Lines and Planes[15]. 基于点，线，面的 VIO 系统的可观性。
- 2017, IROS, Edge-based visual-inertial odometry[16]. MSCKF + Edge。
- 2018, JIRS, Realtime edge based visual inertial odometry for MAV teleoperation in indoor environments[17]. REBVO, 代码开源。

[14] Vladyslav Usenko et al. "Direct visual-inertial odometry with stereo cameras". In: *2016 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE. 2016, pp. 1885–1892.

[15] Yulin Yang and Guoquan Huang. "Observability Analysis of Aided INS with Heterogeneous Features of Points, Lines and Planes". In: *IEEE Transactions on Robotics* (Aug. 2019).

[16] Hongsheng Yu and Anastasios I Mourikis. "Edge-based visual-inertial odometry". In: *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE. 2017, pp. 6670–6677.

[17] Juan José Tarrio and Sol Pedre. "Realtime edge based visual inertial odometry for MAV teleoperation in indoor environments". In: *Journal of Intelligent & Robotic Systems* 90.1-2 (2018), pp. 235–252.

VIO 和其他传感器融合

激光，码盘，gps 等多传感器系统：

- 2019, RAL, Visual-Inertial Localization with Prior LiDAR Map Constraints[18]. 视觉 VIO 和激光地图。
- 2018, arxiv, A General Optimization-based Framework for Global Pose Estimation with Multiple Sensors[19].VINS-Mono 的扩展版，能融合 GPS、单目、双目等信息。
- 2017, ICRA, Vins on wheels[20]. 系统分析了 VIO + 轮速计系统的可观性，做机器人的小伙伴值得一读。
- 2019, ICRA, Visual-Odometric Localization and Mapping for Ground Vehicles Using SE(2)-XYZ Constraints[21]. SE2-XYZ 的模型来对地面轮速机器人进行参数化，虽然不是 VIO 系统但是也值得一读。

[18] Xingxing Zuo et al. "Visual-Inertial Localization with Prior LiDAR Map Constraints". In: *IEEE Robotics and Automation Letters (RA-L)* (July 2019).

[19] Tong Qin et al. "A General Optimization-based Framework for Global Pose Estimation with Multiple Sensors". In: *arXiv preprint arXiv:1901.03642* (2019).

[20] Kejian J Wu et al. "Vins on wheels". In: *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE. 2017, pp. 5155–5162.

[21] Fan Zheng and Yun-Hui Liu. "Visual-Odometric Localization and Mapping for Ground Vehicles Using SE(2)-XYZ Constraints". In: *Proc. IEEE Int. Conf. Robot. Autom (ICRA)*. 2019.

谢谢，祝好。