

Due: Tuesday 11/19 at 11:59pm.

Policy: Can be solved in groups (acknowledge collaborators) but must be submitted individually.

Make sure to show all your work and justify your answers.

Note: This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.

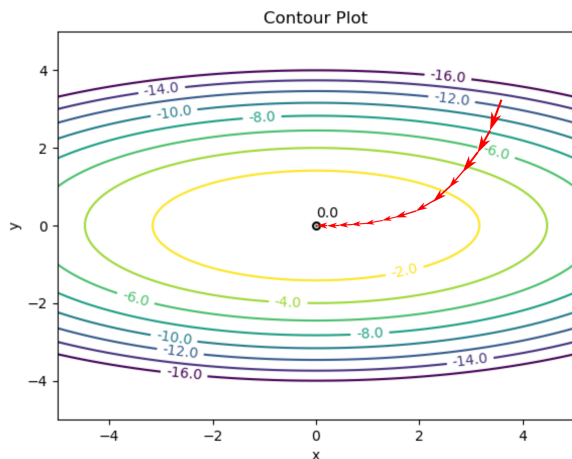
Note: Leave the self-assessment sections blank for the original submission of your homework. After the homework deadline passes, we will release the solutions. At that time, you will review the solutions, self-assess your initial response, and complete the self-assessment sections below. The deadline for the self-assessment is 1 week after the original submission deadline.

Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

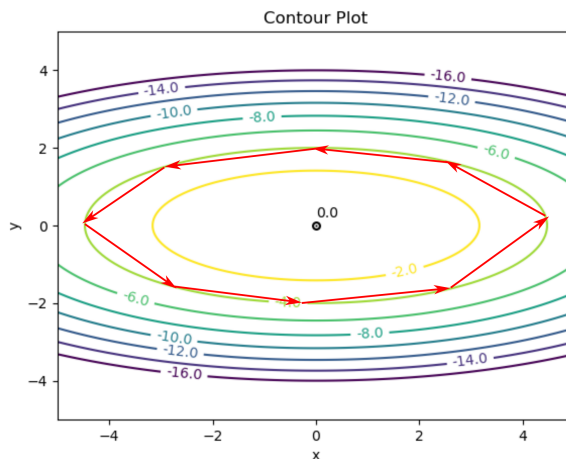
| | |
|---------------|--|
| First name | |
| Last name | |
| SID | |
| Collaborators | |

Q1. [10 pts] Machine Learning

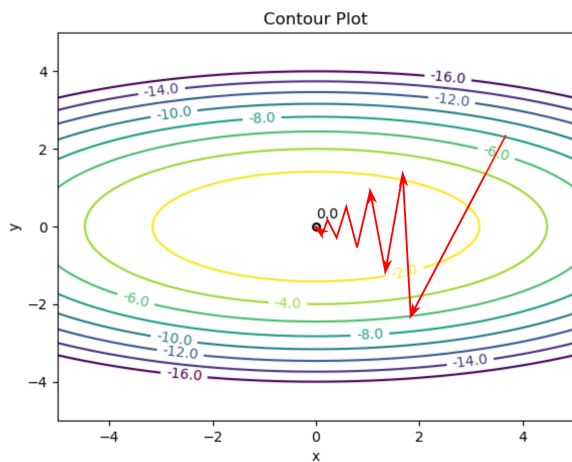
- (a) (i) [2 pts] The following four figures contain the steps of possible gradient ascent implementations each for some learning rate. Each arrow designates a step of gradient ascent. The numbers in the contour lines denote the value of the function we are maximizing on that contour. Which of the following figures, if any, contain paths that could be generated by gradient ascent?



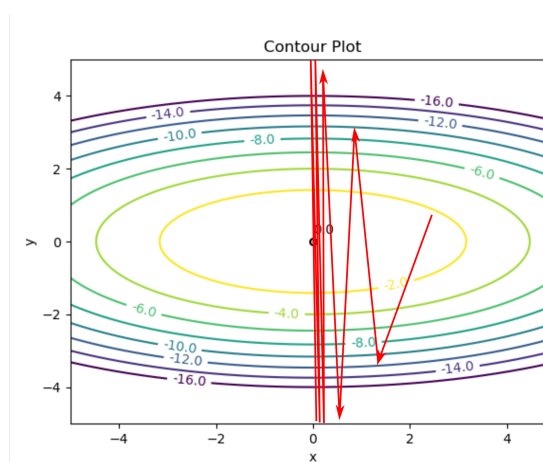
A.



B.



C.



D.

☐ A.☐ B.☐ C.☐ D.

- (ii) [2 pts] Assume that we are trying to maximize a function that has many local maxima using gradient ascent. Which of the following variations of gradient ascent are likely to help obtain a better solution than what would be achieved with basic gradient ascent that uses a fixed step size α ?

☐ Once gradient ascent has converged, store the solution, randomly perturb the solution, and run gradient ascent again until convergence. Repeat this process K times and return the best solution.

- ☐ Run N independent gradient ascent methods starting from N different initial solutions. Out of the ones that converge choose the best solution.
- ☐ For the chosen magnitude of the learning rate $|\alpha|$, at each step choose its sign with equal probability.
- ☐ After each iteration, increase the learning rate linearly.

- (b) (i) [1 pt] Assume that we observe some data features $x_1^{(i)} \in \mathbb{R}$, $i = 1, \dots, N$ and outputs $y^{(i)} \in \mathbb{R}$, $i = 1, \dots, N$. We will use a linear model of the form $\hat{y}^{(i)} = c + bx_1^{(i)}$ to predict the output y . If we use the squared euclidean norm as an objective, i.e. $loss = \frac{1}{2} \sum_{i=1}^N (\hat{y}^{(i)} - y^{(i)})^2$ and a learning rate α , which of the following is the correct gradient descent rule for finding the optimal value of b ?
- ☐ $b \leftarrow b + \alpha \sum_{i=1}^N (c + bx_1^{(i)} - y^{(i)})x_1^{(i)}$
☐ $b \leftarrow b - \alpha \sum_{i=1}^N (c + bx_1^{(i)} - y^{(i)})x_1^{(i)}$
☐ $b \leftarrow b + \alpha \sum_{i=1}^N (c + bx_1^{(i)} - y^{(i)})$
☐ $b \leftarrow b - \alpha \sum_{i=1}^N y^{(i)}x_1^{(i)}$

Now we observe the following dataset in which the outputs y are binary. Furthermore, we are given three features x_1 , x_2 , and x_3 . The training dataset is as follows:

| | | | | |
|-------|---|---|---|---|
| y | 1 | 1 | 0 | 0 |
| x_1 | 1 | 0 | 1 | 0 |
| x_2 | 0 | 1 | 1 | 0 |
| x_3 | 0 | 0 | 0 | 0 |

- (ii) [1 pt] We decide to use logistic regression for this classification task. Using only these features, what is the shape of the decision boundary we obtain from logistic regression?

- ☐ Linear
 ☐ Nonlinear in general
☐ Sigmoid
 ☐ None of the above

- (iii) [2 pts] We decide to train a neural network to perform classification on the whole dataset shown above. We propose using three hidden layers each of dimension 10 with a ReLu activation function except for the output layer which uses a sigmoid activation function. We will test the classifier on new unseen test data. Which of the following scenarios are likely to occur?

- ☐ The network will achieve zero classification error on the training set.
 ☐ The network will overfit the data.
☐ The network will underfit the data.
 ☐ The network will achieve zero classification error on the test set.

- (iv) [2 pts] This is a more general question independent of the aforementioned datasets. Assume that we have trained a logistic regression classifier on the last dataset obtaining the weight vector $\mathbf{w} = [w_1, w_2, w_3]$ and we observe a new feature vector $\mathbf{x}' = [x'_1, x'_2, x'_3]$. Let $P(y = 1|\mathbf{x}; \mathbf{w}) = \frac{1}{1+e^{-\mathbf{w} \cdot \mathbf{x}}}$, with \cdot denoting the inner product. Which of the following formulas describe the correct classification rule for the label y' of this data point?

- ☐ $y' = \underset{y' \in \{-1, 1\}}{\operatorname{argmax}} P(y'|\mathbf{x}'; \mathbf{w})$
☐ $y' = \begin{cases} 1, & \text{if } P(y' = 1|\mathbf{x}'; \mathbf{w}) \geq 0 \\ -1, & \text{otherwise} \end{cases}$
☐ $y' = \begin{cases} 1, & \text{if } P(y' = 1|\mathbf{x}'; \mathbf{w}) \geq 0.5 \\ -1, & \text{otherwise} \end{cases}$
☐ $y' = \begin{cases} 1, & \text{if } P(y' = 1|\mathbf{x}'; \mathbf{w})(1 - P(y' = 1|\mathbf{x}'; \mathbf{w})) \geq 0 \\ -1, & \text{otherwise} \end{cases}$

Q1 Self-Assessment - leave this section blank for your original submission. We will release the solutions to this problem after the deadline for this assignment has passed. After reviewing the solutions for this problem, assess your initial response by checking one of the following options:

- ☐ I fully solved the problem correctly, including fully correct logic and sufficient work (if applicable).
- ☐ I got part or all of the question incorrect.

If you selected the second option, explain the mistake(s) you made and why your initial reasoning was incorrect (do not re-iterate the solution. Instead, reflect on the errors in your original submission). Approximately 2-3 sentences for *each* incorrect sub-question.