西安交通大学本科生课程考试试题标准答案与评分标准

课程名称:数学物理方程(A) 课时：32 考试时间：2019 年 5 月 11 日

一、(5 分/题×11 题=55 分)

1	2	3	4	5	6	7	8	9	10	11
A	**C**	**C**	**A**	**D**	**D**	**B**	**D**	**C**	**D**	**A**

二、(10 分) 解：$u(x,t)=X(x)T(t)$，特征值问题

$\begin{cases} X''(x)+\lambda X(x)=0, 0<x<l \\ X(0)=0, X(l)=0 \end{cases}$，$\lambda_n=\left(\dfrac{n\pi}{l}\right)^2$，$X_n(\mathrm{x})=\sin\left(\dfrac{n\pi}{4}x\right)$，$n>0$，(5 分)

$\begin{cases} T_n''(t)+a^2\lambda_n T_n(t)=0 \\ T_n(0)=\varphi_n, T_n'(0)=0 \end{cases}$，$T_n(t)=\varphi_n\cos a\sqrt{\lambda_n}t$ ，$\varphi_n=\dfrac{2}{l}\displaystyle\int_0^l \varphi X_n dx$ (9 分)

$$u=\sum_{n=1}^{\infty}\varphi_n\cos a\sqrt{\lambda_n}t\sin\left(\frac{n\pi}{l}x\right),$$ (10 分)

三、(10 分) 设 $f(x)=\displaystyle\sum_{m=1}^{\infty}A_m J_2(\mu_m^{(2)}x)$ ，$A_m=\dfrac{2\int_0^1 x(x^2+1)J_2\left(\mu_m^{(2)}x\right)dx}{[J_2'\left(\mu_m^{(2)}\right)]^2}$ (4 分)

$$A_m=\frac{2}{[J_2'\left(\mu_m^{(2)}\right)]^2}[\frac{1}{\left(\mu_m^{(2)}\right)^4}\int_0^{\mu_m^{(2)}}t^3J_2(t)\mathrm{d}t+\frac{1}{\left(\mu_m^{(2)}\right)^2}\int_0^{\mu_m^{(2)}}t\ \ J_2(t)\mathrm{d}t]$$

$$=\frac{2}{[J_2'\left(\mu_m^{(2)}\right)]^2}\frac{1}{\left(\mu_m^{(2)}\right)^2}[\mu_m^{(2)}J_3(\mu_m^{(2)})-\int_0^{\mu_m^{(2)}}t^2(t^{-1}J_1(t))'\mathrm{d}t]$$

$$=\frac{2}{[J_2'\left(\mu_m^{(2)}\right)]^2}\frac{1}{\left(\mu_m^{(2)}\right)^2}[\mu_m^{(2)}J_3(\mu_m^{(2)})-\mu_m^{(2)}J_1(\mu_m^{(2)})+2\int_0^{\mu_m^{(2)}}J_1(t)\mathrm{d}t]$$

$$=\frac{2}{[J_2'\left(\mu_m^{(2)}\right)]^2}\frac{1}{\left(\mu_m^{(2)}\right)^2}[\mu_m^{(2)}J_3(\mu_m^{(2)})-\mu_m^{(2)}J_1(\mu_m^{(2)})-2J_0(\mu_m^{(2)})+2]$$

$$J_3=-J_1+\frac{4}{x}J_2, J_3(\mu_m^{(2)})=-J_1(\mu_m^{(2)}), J_0=-J_2+\frac{2}{x}J_1, 2J_1(\mu_m^{(2)})=\mu_m^{(2)}J_0(\mu_m^{(2)})$$

$$A_m=\frac{2}{[J_2'\left(\mu_m^{(2)}\right)]^2}\frac{1}{\left(\mu_m^{(2)}\right)^2}[-\mu_m^{(2)}J_1(\mu_m^{(2)})-\mu_m^{(2)}J_1(\mu_m^{(2)})-2J_0(\mu_m^{(2)})+2]$$

$$=\frac{-4}{[J_2'\left(\mu_m^{(2)}\right)]^2}\frac{1}{\left(\mu_m^{(2)}\right)^2}[\mu_m^{(2)}J_1(\mu_m^{(2)})+J_0(\mu_m^{(2)})-1]$$

$$=\frac{-4}{[J_2'\left(\mu_m^{(2)}\right)]^2}\frac{1}{\left(\mu_m^{(2)}\right)^2}[(\mu_m^{(2)})^2 J_0(\mu_m^{(2)})/2+J_0(\mu_m^{(2)})-1]$$ (8 分)

$$f(x)=-4\sum_{m=1}^{\infty}\frac{1}{[J_2'\left(\mu_m^{(2)}\right)]^2}\frac{1}{\left(\mu_m^{(2)}\right)^2}[((\mu_m^{(2)})^2/2+1)J_0(\mu_m^{(2)})-1]J_2(\mu_m^{(2)}x)$$ (10 分)

四、(10 分) 取一点 $M_0(x_0,y_0),y_0>0,$ 其对称点为 $M_1(x_0,-y_0)$ (2 分)

$$G(M,M_0)=\frac{1}{2\pi}\ln\frac{1}{r_{MM_0}}-\frac{1}{2\pi}\ln\frac{1}{r_{MM_1}}=\frac{1}{4\pi}\ln\frac{\left(x-x_0\right)^2+\left(y+y_0\right)^2}{\left(x-x_0\right)^2+\left(y-y_0\right)^2}$$

(5 分)

$$\frac{\partial G(M,M_0)}{\partial n}\Big|_{y=0}=-\frac{\partial G(M,M_0)}{\partial y}=-\frac{1}{4\pi}\frac{4y_0}{(x-x_0)^2+y_0^2}$$

$$u(M_0)=-\int_{y=0}\frac{\partial G(M,M_0)}{\partial n}\varphi(M)ds+\int_{y>0}G(M,M_0)f(M)dxdy$$

(10 分)

$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{y_0}{(x-x_0)^2+y_0^2}\varphi(x)dx$$

五、(15 分) 特征值和特征函数分别为 $\lambda_n=\left(\frac{n\pi}{l}\right)^2$，$X_n(\mathrm{x})=\cos\left(\sqrt{\lambda_n}x\right)$， $n\ge 0$, (5 分)

$$u=\sum_{n=0}^{\infty}T_n(t)X_n(x),,\quad \varphi=\sum_{n=0}^{\infty}\varphi_n(t)X_n(x),$$

$$\varphi_n=\frac{2}{l}\int_0^l\varphi X_n dx,\ n>0,\ \varphi_0=\frac{1}{l}\int_0^l\varphi dx$$

$$1=\sum_{n=0}^{\infty}f_n(t)X_n(x),\ f_n=\begin{cases}1,n=0\\0,n>0\end{cases}$$ (10 分)

$$\begin{cases}T_n'(t)+a^2\lambda_nT_n(t)=f_n\\T_n(0)=\varphi_n\end{cases},\quad T_0(t)=t+\varphi_0,\quad T_n(t)=\varphi_ne^{-a^2\sqrt{\lambda_n}t},\quad n>0$$

$$u=t+\varphi_0+\sum_{n=1}^{\infty}\varphi_ne^{-a^2\sqrt{\lambda_n}t}X_n(x),$$ (15 分)