Detection of a Silent Submarine from Ambient Noise Field Fluctuations

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Summary

We developed a method for detecting intrinsically silent submarines in the ocean by measuring only the fluctuations in the ambient noise field. This method allows us to calculate the position, velocity, and approximate size of a submarine.

Our model relies on measuring the noise field at four different listening stations, with each station composed of four microphones a relatively small distance apart. We calculate an amplitude spectrum of the noise at each microphone using a Fourier analysis and compare this spectrum to the previously measured baseline spectrum for ambient noise. The difference between these spectra is the noise reflected from the submarine.

We use the four microphones at a particular station to measure the gradient of the peak amplitude from the submarine noise spectrum. Because amplitude varies inversely with distance from the submarine, we can compute the submarine's location from the amplitude and the gradient at each listening station. The approximate size, in terms of the radius of a similarly sized sphere, follows from the distance and peak amplitude.

Our comparison of the frequencies of the peak amplitudes of the submarine and ambient noise spectra provides a measure of the Doppler shift at each listening station caused by the submarine's motion. The Doppler shift gives us a component of the submarine's velocity in the direction of each station. We select a basis from among unit vectors in these four directions and convert the submarine's velocity into standard Cartesian components.

We wrote a Fortran program to implement our algorithm. Our simulations show that we can determine position with better than 8% accuracy in each dimension. Size calculations suggest a systematic error of roughly 20–30%. Error in the velocity computations varied for each component with changes in submarine position and in the dominant frequency of the ambient noise, but was within about 30% for a single frequency of 1,000 Hz.

The model could be modified to remove some of our assumptions, such as the absence of currents. Our model uses a minimum number of listening stations, but a larger number would significantly improve the results.

Assumptions

- The speed of sound in the ocean is constant. Though the speed of sound depends on temperature, the range of detection devices is small enough to render speed of sound fluctuations negligible.
- Ambient noise has the same frequency and amplitude everywhere, so reflections from the surface and bottom of the ocean need not be accounted for [Horton 1959].
- All submarines are approximately spherical, are made of steel, and reflect a fraction k of the sound energy incident upon them. Although the surface of the submarine, as a two-dimensional surface curved in three dimensions, is not a simple harmonic oscillator (SHO), an SHO is a reasonable analogy. In this case, the SHO is both forced (by ambient noise) and damped (by the water and by the flexibility of the metal). The steady-state solution for such a forced and damped SHO is vibration at the forcing frequency. The high damping coefficient likely with a submarine indicates that the response of the submarine should be independent of frequency (and Krasil'nikov [1963] lists a single reflectivity for all frequencies). Certainly, at distances large compared to the dimensions of the submarine, a sound wave reflected off it will approximate a spherical wave.
- Sound waves reflected from a submarine which then reflect off the surface or bottom of the ocean have negligible intensity. This is to say that non-ambient noises detected by our microphones can be considered to have reflected directly from a submarine, and not secondarily from the surface or bottom of the ocean.
- The ocean is of homogeneous consistency, so there are no large animals or objects, aside from the submarine, which significantly influence the transmission of sound waves. Furthermore, we assume that only one submarine is present at any given time.
- The submarine, in addition to creating no intrinsic noise, does not by its movement generate any turbulence that affects noise transmission.
- A typical submarine cannot move faster than 20 m/s.
- There is no appreciable current, and the detection stations are at rest with respect to the water.
- We begin by assuming that there is only one frequency (and corresponding amplitude) of ambient noise, and then we generalize to multiple frequencies.

Description of the Model

Our detection scheme consists of four clusters of microphones in a pyramidal orientation. We require four non-coplanar clusters in order to determine with certainty the submarine's direction of travel in case the submarine is located in the plane of three of the clusters. Four clusters is a minimum; see **Analysis of Error and Sensitivity** for advantages and disadvantages of having only four clusters. We define a Cartesian coordinate system with the xy-plane parallel to the surface of the ocean and the positive z-axis pointing up. We place one microphone cluster at the origin and at the points $(d, 0, 0)$, $(0, d, 0)$, and $(0, 0, d)$. (See **Figure 1**.)

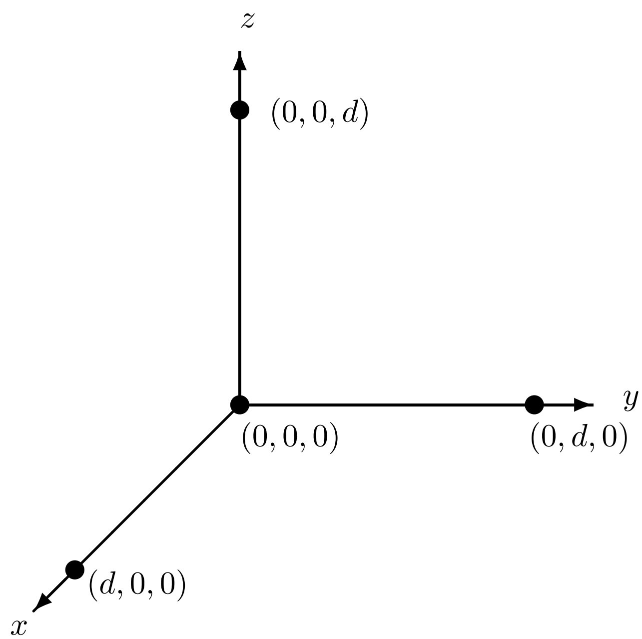

Figure 1. Array of microphone clusters.

Because submarines typically do not descend below 1,500 m, we place the origin of our system 1,000 m below the surface of the water, so that one microphone cluster is $(1{,}000 - d)$ m below the surface and the other three clusters are 1,000 m down. We chose to let d = 500 m so that the detection clusters are well-spaced throughout the potential depth-range of a submarine. We envision the detection clusters as either buoyant anchored rigs or as pods at the end of lines dropped from the surface (e.g., suspended from ships). However, the analysis is not affected by the location of the origin or the orientation of the coordinate axes, as long as the entire array is sufficiently submerged.

Each of the four clusters (listening stations) in turn consists of four microphones, one at the precise location we gave for the cluster and the other three a small distance δ away, one each in each of the coordinate directions. (See **Figure 2**.)

We first measure the ambient noise waveform when no submarines are present, so that we can determine the ambient noise's frequencies and associ-

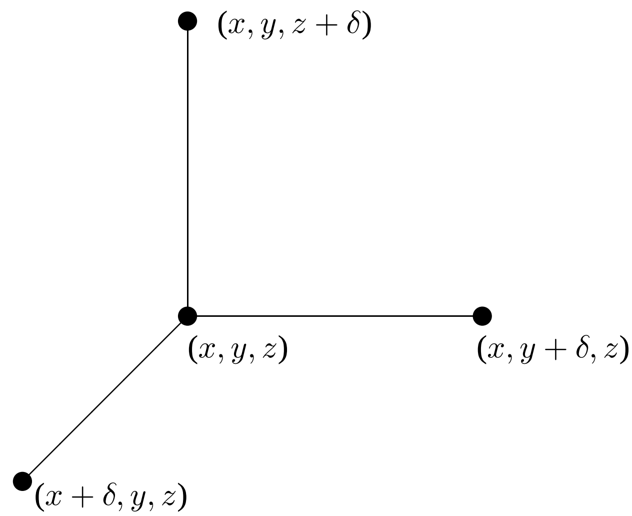

Figure 2. Microphone arrangement within each detection station.

ated amplitudes (at first, only one frequency was present). We then measure the sound at each microphone location for a short period of time, perform a Fourier analysis on the resulting wave pattern to determine the frequencies present and their respective amplitudes, and use these data to figure out the location, speed, direction of travel, and size of any submarines.

Data Required for Calculations

We seek the position of a submarine relative to our coordinate system, its velocity vector, and its size. Because we assume that a submarine can be treated as a sphere, its size is just its radius R, and its position can be described by its center. Hence, a complete solution to our detection problem is comprised of radius R, position coordinates (x, y, z), and velocity vector $\vec{v} = (v_x, v_y, v_z)$.

The data available to calculate these quantities consist of the frequencies and their respective amplitudes received by our array of microphones. We list the constants and variables that are required for our calculations:

f = the frequency of the ambient noise. We chose 1,000 Hz, a frequency well within the range of real oceanic ambient noise, for our single-frequency simulations.

I_0 = the intensity of the ambient noise. A reasonable intensity at the frequency 1,000 Hz is 5.4457×10^{-10} Pa^2 [Munk et al. 1995, 179].

A_0 = the amplitude of the ambient noise. Amplitude squared is proportional to intensity. Because the proportionality constant has already been taken into account in calculating I_0, we get $A_0 = \sqrt{I_0} = 2.3336 \times 10^{-5}$ Pa.

k = the percentage of the sound energy (or intensity) reflected by the surface of the submarine. We let $k = 0.86$ [Krasil'nikov 1963, 172]. Since amplitude is proportional to the square root of intensity, the amplitude of sound waves immediately after reflecting from the submarine surface will be $\sqrt{k}A_0 = 0.9274A_0$.

$A(s)$ = the amplitude of sound waves reflected from the submarine's surface at distance s from the center of the (spherical) submarine. Note that

$$A(s) = \frac{\sqrt{k}A_0R}{s}.$$

This formula is a well-known consequence of conservation of energy.

Step One: Detecting the Submarine

The raw data that we receive through each of the microphones consist of a waveform recorded over a short time interval (see **Figure 3** for a sample plot of multiple frequency noise).

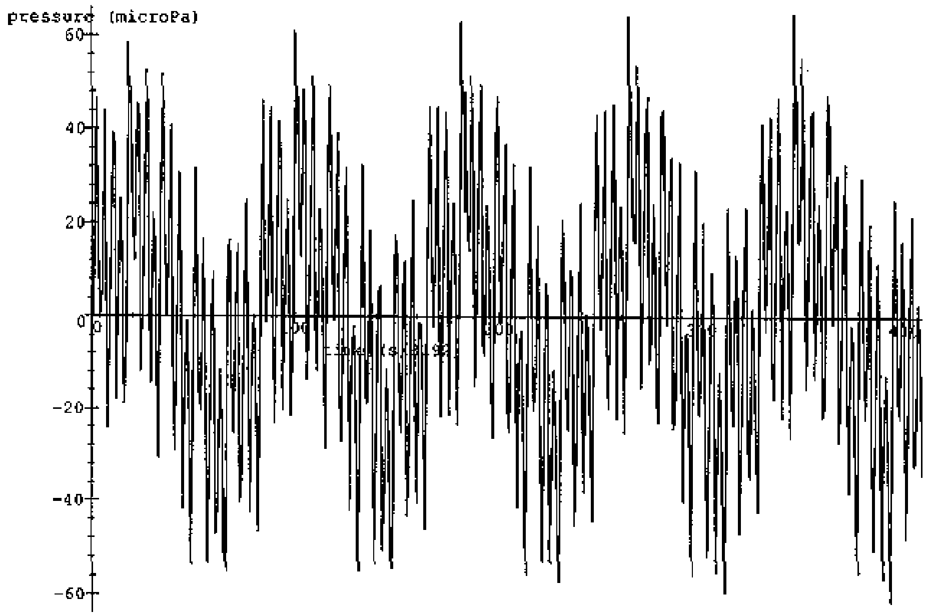

Figure 3. Ambient noise with five frequencies, showing pressure as a function of time.

To convert these data into useful frequency and amplitude figures, we use a fast Fourier transform, which isolates the particular frequencies in a given signal. The Fourier transforms that we used were the sine and cosine transforms provided with Press et al. [1986]. Once this computation is completed, we have amplitude values $A_{i,j}$ associated with the frequency f (with each frequency if there is more than one) of recorded noise for each microphone, where i is the station number and j is the microphone number within that station.

Our algorithm subtracts the ambient noise amplitude spectrum (determined as discussed previously) from the new amplitude spectrum recorded at each microphone. If all the differences are equal to zero, the current noise in the ocean is only the homogeneous ambient noise, so there must not be a submarine within a detectable distance. The microphones then collect a new set of data, and we begin the process again.

If any differences are nonzero, we must determine whether they are caused by a change in the ambient noise or by the presence of a submarine. If the differences in the spectra are due to a change in the ambient noise, all of the microphones should have recorded the same data (by the definition of ambient noise). However, any changes caused by a submarine should vary from station to station and from microphone to microphone because of varying positions of the microphones relative to the submarine. Therefore, we compare the difference spectrum (the amplitude spectrum from the microphone minus the ambient noise spectrum) of the first microphone of each station to the difference spectra of the first microphones of the other stations. We could compare all the difference spectra for all the microphones, but since the greatest variation is from station to station, we need to compare only among stations.

If there is no variation among the difference spectra among the stations, the algorithm must take the change in the ambient noise into account. It replaces the ambient noise spectrum by the new ambient noise spectrum.

If there is a difference from station to station, our algorithm has detected a submarine! In this case, the difference spectra give us the amplitude values $A_{i,j}$ of the sound reflected by the submarine for each microphone. Now the algorithm finds the frequency with the greatest amplitude for each microphone. Because all frequencies of noise reflect off the submarine with the same proportionality constant $\sqrt{k}$, this frequency and the corresponding peak amplitude must be the reflection of the frequency with the peak amplitude in the ambient noise. We consider only these peak amplitudes and their corresponding frequencies throughout the remainder of the algorithm, whether the ambient noise is composed of one frequency or many (see **Figure 4**).

Step Two: The Submarine's Position

Our strategy now is to compute $\vec{\nabla} A_i$ for each of the four stations by approximating the derivatives of $A_{i,1} = A_i(x, y, z)$, where (x, y, z) are the coordinates of microphone $(i, 1)$, in each of the x, y, and z directions. This explains our rationale for having four microphones at each of the four stations, since we can compute

$$\frac{A_i(x+\delta, y, z) - A_i(x, y, z)}{\delta} \approx \frac{\partial A_i(x, y, z)}{\partial x},$$

as well as the derivatives in the y and z directions, to find $\vec{\nabla} A_i$.

We note that $|\vec{\nabla} A_i|$ is the absolute value of the derivative of A_i with respect to s, the distance from the center of the submarine. Since we now have values

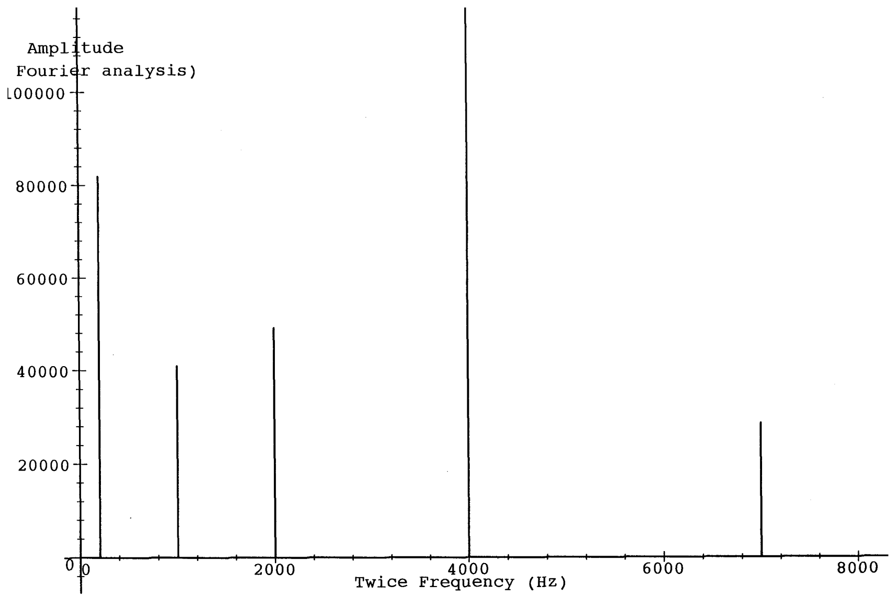

Figure 4. Amplitude spectrum of ambient noise, showing amplitude as a function of twice the frequency (in Hz).

for A_i and for $|\vec{\nabla} A_i|$, and since

$$A_i(s) = \frac{\sqrt{k} A_0 R}{s}, \qquad |\vec{\nabla} A_i| = \frac{\sqrt{k} A_0 R}{s^2}, \tag{1}$$

we see that $A_i / |\vec{\nabla} A_i| = s$. At this point, because we know the vector $\vec{\nabla} A_i$ that points to the submarine and the distance s to its center, we know the position of the submarine relative to detection station i. In fact, the coordinates of the submarine's center are given by

$$(a, b, c) = s \, \frac{\vec{\nabla} A_i}{|\vec{\nabla} A_i|} + (x, y, z).$$

While this calculation for one of the i stations is sufficient to get coordinates for the submarine's position, our four stations allow us to compute these coordinates four different ways. Since there will be some random error in each computation, averaging the four different points provides a better approximation of the submarine's position.

Step Three: The Submarine's Size

With values for the amplitude A_i at station i and the distance s of station i from the submarine, computing the radius of the submarine is easy. From **(1)**, we have

$$R = \frac{A_i s}{\sqrt{k} A_0}. \tag{2}$$

We obtain a better estimate by averaging the four values of R.

Step Four: The Submarine's Velocity

To calculate the velocity, we use the frequency f of sound reflected from the submarine's surface. Since we know the frequency of the ambient noise and thus the frequency shift between ambient and reflected sound, we can solve the equation describing the Doppler effect for the speed of the submarine in the direction of a particular detection station. The general Doppler effect for sound [Krane et al. 1992] is

$$f_o = f_s \left(\frac{c - v_o}{c + v_s} \right),$$

where f_o is the frequency received by the observer, f_s is the frequency of the source (the ambient frequency), v_o and v_s are the components of the velocities of the observer and the source along the line between them, and c is the speed of sound, in this case in water. Because our detection stations are stationary, we have $v_o = 0$. Also, we let v_s be positive if the submarine is moving away from station. Solving for v_s, we get

$$v_s = \left(\frac{f_s}{f_o} - 1 \right) c.$$

Once this v_s has been calculated for a particular listening station (let's rename it v_i), we can express this component of the submarine's velocity in terms of a vector. Since $\hat{u}_i = \vec{\nabla} A_i / |\vec{\nabla} A_i|$ is just the unit vector pointing from detection station i to the center of the submarine, the vector $\vec{v}_i = v_s \hat{u}_i$ is the component of the submarine's velocity in the direction of station i.

Note that we need velocity components in only three linearly independent directions to compute the velocity vector. However, we have four potential basis vectors, the four $\hat{u}_i$. To determine which set of three vectors is the most useful basis for our analysis, we consider the four matrices formed by taking combinations of the $\hat{u}_i$ as column vectors. First, we know that any coplanar set of three vectors will not form a basis at all, so the matrix composed of them will be singular. By perturbation, any set of three vectors that are almost coplanar will form an almost singular matrix; and, in the real world of measurement and computation errors, such a basis would not be useful. Therefore, we choose the set of vectors $\{\hat{u}_i, \hat{u}_j, \hat{u}_k\}$ whose matrix has the largest determinant as the basis likely to be most useful to our algorithm.

Now, the vector (v_i, v_j, v_k) is a coordinate vector in terms of our chosen basis. We want to change our basis to the standard $\{\vec{x}, \vec{y}, \vec{z}\}$ basis to determine the velocity vector of the submarine with respect to our coordinate system. This change of basis can be performed by a simple matrix multiplication,

$$\left[\begin{array}{c|c|c} \hat{u}_i & \hat{u}_j & \hat{u}_k \end{array} \right] \begin{bmatrix} v_i \\ v_j \\ v_k \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \vec{v}, \tag{3}$$

resulting in the velocity vector $\vec{v}$.

Extensions of the Model

Although most of our simulations were carried out with only one frequency in the ambient noise, we find a broad range of frequencies in the ambient noise of the ocean. Fortunately, the algorithm can process multiple frequencies because it uses only the peak amplitudes and corresponding frequencies. This technique reduces the scenario to a single-frequency problem.

The algorithm, as described above and encoded in the computer simulation, can adjust to fluctuations over time in the ambient noise field. However, if both the appearance of a submarine and a significant change in the ambient noise field coincide, the algorithm assumes that all of the difference spectra represent noise due to the presence of the submarine. This effect could cause some error.

We also can track a single submarine over a period of time, because the algorithm, in the form of a computer program, runs quickly enough to provide frequent data on the position and velocity of the submarine.

The presence of two or more submarines in the observation region presents more of a problem for the algorithm because it would not recognize the presence of the submarine with the smaller effect on the ambient noise field. However, the algorithm could be modified to detect the presence of a single submarine, calculate its effects on the ambient noise field, and compare the recorded data with those effects.

Our model could be extended to eliminate the assumptions of no current and stationary listening stations. The general Doppler equation already provides for a moving observer, so moving listening stations would be relatively easy to handle. Also, a constant surrounding current could be integrated into the Doppler equation, though this computation would be a bit more complicated.

Simulation Results

We wrote a Fortran program to simulate the implementation of our algorithm. The program uses only the parameter k, the positions of all the microphones, a waveform representing the ambient noise, and a waveform for each of the microphones representing the noise field with the presence of a submarine.

To run the simulation program, we needed to create sound data for the microphones to receive. We used another Fortran program to produce a discretized version of the soundwaves, with more than 8,000 samples per second. We first created a data set with only the ambient noise present, as a benchmark, and then added in the additional sound caused by a submarine of a particular size with specific position and velocity vectors. This data-generation scheme provided us with an easy check on the accuracy of our simulation program.

At first, we created an ambient noise data set with one fixed frequency and amplitude. We then created data files for three different selections of submarine radius, position, and velocity. We ran our simulation program for each of these three data sets in the presence of a submarine. The results of these simulations are provided in **Table 1**. These results show that our simulation was relatively successful in picking out the position and the velocity of the submarine, though appreciable error was present. While our simulation apparently did not do a particularly good job of calculating the radius of the submarine, the percentage error for the three simulations was fairly consistent, suggesting that this error may be systematic and thus correctable.

Since Fourier analysis can result in an apparent smearing of a particular frequency over a frequency range, the amplitude we calculate for this frequency may be consistently smaller than it should be. This seems like the sort of problem that could be corrected through more sophisticated, more powerful Fourier analysis, or at least through the inclusion of an amplitude correction factor.

Table 1.

Computer program output of radius, position coordinates, and velocity coordinates for three different submarine data sets, with one ambient frequency.

Simulation		R (m)	x (m)	y (m)	z (m)	v_x (m/s)	v_y (m/s)	v_z (m/s)
1	Input	13.8	−3,000	−2,000	300	15	10	5
	Output	9.73	−2,765	−1,843	281	14.9	10.5	4.7
2	Input	5	200	1,000	−500	2	10	2
	Output	4.02	190	953	−464	1.3	9.5	1.6
3	Input	10	2,000	2,000	−500	5	5	5
	Output	7.95	1,871	1,871	−468	4.8	4.8	3.9

While the results of our simulations are encouraging, we point out that our computer program is merely one realization of our general mathematical algorithm for finding a submarine's radius, position, and velocity. The results would be improved by providing more complete and accurate sound data (corresponding to better microphones) or using a more accurate and perhaps more appropriate fast Fourier transform algorithm.

We also ran the same simulations using ambient noise with multiple frequencies. The results are in **Table 2**.

Analysis of Error and Sensitivity

Table 2 shows the differences between actual and calculated position, radius, and velocity of a submarine. The error in the position coordinates most likely arises due to the dependence of our algorithm on measured amplitude figures and the amplitude derivative that we compute using them, which appear not to be accurate. At least part of this error may be an artifact of our need to use discrete data. Furthermore, our numerical calculation of $\vec{\nabla} A_i$ for station i introduces more error, since it is based on a finite, and in fact quite large,

Table 2.
Radius, position coordinates, and velocity coordinates for three different simulated submarine data sets with ambient noise of five different frequencies.

Simulation		R (m)	x (m)	y (m)	z (m)	v_x (m/s)	v_y (m/s)	v_z (m/s)
1	Input	13.8	−3,000	−2,000	300	15	10	5
	Output	8.48	−2,826	−1,886	287	14.6	11.0	52.8
2	Input	5	200	1,000	−500	2	10	2
	Output	4.19	192	939	−454	−0.3	6.1	−4.2
3	Input	10	2,000	2,000	−500	5	5	5
	Output	7.72	1,839	1,839	−448	5.0	5.0	−1.5
Frequencies (Hz):				100	500	1,000	2,000	3,500
Amplitudes (μPa):				30	10	20	12	7

distance δ between microphones. This error is difficult to eliminate, since increasing δ creates a worse approximation of a derivative, while decreasing δ to very small distances requires unreasonably sensitive microphones to perceive tiny differences in amplitude. Because of this practical consideration, we are forced to accept some error in our calculation of a submarine's position.

We have a nice measure of position error, since our algorithm computes the position vector four times, each time using only data from the four microphones at one listening station. For the same reason, we also have a measure of the error in the radius measurements (neglecting the apparent systematic error). The relative errors (standard deviation divided by mean) listed in **Table 3** are all quite small.

Table 3.
Relative error for radius and position calculations.

Simulation	R	x-coordinate	y-coordinate	z-coordinate
1	.043	.034	.014	.023
2	.078	.009	.009	.023

Error in our radius calculation is largely inherited from the problem with amplitudes discussed above. From **(2)**, we see that the radius is determined by the measured values of A_i and s_i. Thus, if the A_i calculated by Fourier analysis is too small, the calculated radius value will similarly be too small. We suspect that this fact is the cause of the fairly systematic error in our values of R shown in **Table 1**.

Velocity error arises primarily because of two factors:

- error in our position calculation, since the velocity computation relies upon the submarine's location, and

Table 4.

Relative error of calculations for two different simulations for different values of δ, the distance separating microphones in each station lattice.

δ (m)	Simulation	R (m)	x (m)	y (m)	z (m)	$\|\vec{v}\|$ (m / s)
5	1	−.195	−.041	−.046	−.068	−.058
	2	−.295	−.078	−.080	−.054	.007
10	1	−.192	−.049	−.046	−.070	−.058
	2	−.294	−.078	−.079	−.061	.007
15	1	−.197	−.057	−.047	−.072	−.057
	2	−.278	−.078	−.077	−.068	.007
20	1	−.198	−.067	−.047	−.075	−.056
	2	−.272	−.077	−.076	−.075	.007

- error in the observed frequency f of noise reflected from the submarine as determined through the Fourier analysis.

The error in the observed frequency becomes particularly important when low frequencies predominate the ambient noise. The error is amplified because a small absolute error in the frequency measurement becomes a large relative error in the low frequency range, and the measurement error is a constant absolute error for all frequencies. Note the errors in the velocity components in **Table 4**, the results of a simulation in which a frequency of 100 Hz dominates the ambient noise.

We performed some additional simulations in which we permuted the parameter δ, the distance between microphones within stations, in order to determine the sensitivity of our calculations to this parameter. The relative errors between our calculated values and the actual values are shown in **Table 2**. From this table, it appears that our model is not particularly sensitive to fluctuations in δ.

Using the fast sine and cosine transforms limits our model; their use assumes that the data have an initial phase of zero, so we do not take into account phase shift in the reflected noise due to travel time from the submarine to the stations.

Finally, we note that some of the error in our results arose because of the small number of microphones we arranged in our stations. We designed the model with some frugality, using only four stations because four is the minimum number needed to guarantee that we can pinpoint a submarine's velocity (three fail if the submarine is in the same plane as all three stations). Our calculations would have benefitted from some redundancy in our measurements, for instance, using eight stations arranged as vertices of a cube. However, our minimal scheme is cheaper, requires less superstructure, and provides for simpler computer calculations than would a more redundant arrangement.

Conclusions

This model successfully detects a silent submarine using only distortions of the ambient noise field as data. It accomplishes this task with a small number of microphones (16) arranged in a lattice structure beneath the surface of the ocean, and it provides relatively accurate data for a range of submarine sizes, positions, and velocities. The model may lack realism in that it requires such assumptions as a homogeneous ocean, nearly spherical submarines, and isolated ambient noise frequencies. However, the first two of these assumptions have fairly solid physical bases in most normal circumstances, and our algorithm does provide a solid foundation that can be extended to take into account such complicating factors as a continuous frequency distribution.

Acknowledgments

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