Water, Water, Everywhere

Summary

In order to determine the best water strategy for 2013 to meet the Chinese projected water needs in 2025, the following steps should be taken to build up this mathematical model.

Firstly, China is divided into 10 zones to focus on the water-scarce regions. According to the official data we gain from 2005 to 2011, we set up gray forecast model(GM(1,1)) by analyzing the total data of industry, agriculture, life and environment, and then we figure out the accurate relations between this group of data and the corresponding years so as to predict the water demand in 2025. After that, we test the trusting degree of this model to ensure the facticity by identifying the statistical significance of each data.

Secondly, according to the water resources available and the demand, we discuss the storage problems of rainwater and water on the ground surface. At present, the main strategy to deal with it is to transform water from one area to another. In this case, we select the transformation spot by analyzing the contrast among the degrees of water lack so as to simplify the calculation. Afterwards, we use single aim optimization with restrictions and figure out the minimum cost, $1.523 million.

At last, we hand in a non-technological promotion to the governmental leadership and prove why our model is feasible, cost-worthy and effective.

Key words: Gray Forecast Water Storage Single Objective Programming

Contents

1. Introduction

1.1 The introduction of Chinese water resource

Fresh water is the constraint of the development for every country, and China, of course, is not an exception. Actually, China is now a country of severe water shortage. Though we have a total sum of 2.8 trillion cubic meters, which accounts for 6% of global water resources, it is still a tragedy that we have only 2.2 thousand cubic meters per person on average, a quarter of the world average level. Hence it is no wonder that we are among the 10 poorest country in terms of water resources per person. Therefore, we endures a lot of regional drought disasters year after year, doing a great amount of harm to local citizens and the economic development. According to river distribution, China can be divided into 10 areas: The Songhua River area, The Liaohe area, The Haihe River area, The Yellow River area The Huaihe River area, The Yangtze River area, Southeastern River area, The Pearl River area, The Southwest River area and The Northwest River area. Recently, economic development and global warming has generated increasing need of water, which in turn brings drought disasters. They are taking on higher frequency, expanding area, and heavier loss. The stricken areas are expanding from north-west to south-east, mainly affecting industry and ecological environment. So it is necessary to build up a mathematical model to figure out an effective, feasible and cost-worthy water strategy for 2013 to meet the projected water needs of China in 2025, and the best water strategy should be explained where the advantages consist in.

1.2 Restatement of the Problem

The problem of global water resource shortage has become a constraint factor that restricts the development of many countries, and of course, China is not an exception. The water resource support people’s life, industry and agriculture. However, the water resource does not exist balancedly. The structure of water resource that is available is also undergoing huge changes. Thus, it is necessary to forecast the total storage and the demand of water resource of China in the near future. Then we will build up appropriate mathematical model to work out the method to restore, transform, produce and conserve according to the result that the model above predicts. After that, we will check whether the water strategy is feasible, effective and cost-worthy, and hand in a proposal to the governmental leadership.

2. Problem Analysis

The requirements of the subject are to determine a feasible, effective and cost-worthy water strategy for 2013 to meet the water demand in 2025. We break it into four main sub-problem:

1. To predict how the water demand is going in 2025;
2. To draft a feasible, effective and cost-worthy water strategy to deploy water resources so as to meet the need of people;
3. To analyze the significance of this model to economy, physics and environment;
4. To hand in a non-technological position paper to the governmental leadership and explain the advantages of your strategy.

Aimed at Problem I, we divided China into 10 areas according to Chinese ministry of water conservation. We adopt the gray model and sum up the original data of total water resources from 2005 to 2011, and then acquire a conclusion similar to exponential regular. Based on this, we manage to predict national water resources and water need between 2012 and 2025, and analyze how China is lacking water in detail.

In order to satisfy people's need of water, a feasible, cost-worthy and effective water strategy should be set up. In this case, we use multi-aim optimization model to conserve and transform water where this kind of resource is in lack.

After the strategy is determined, we also need to analyze whether the model is feasible and what effect it may have on economy, physics and environment.

3. Assumptions

In order to have a better study on this model, we simplify our model by giving the following assumptions:

1. All water demands are restricted to only industry, agriculture, life and ecological environment;

2. All data collected by the survey are definitely accurate;
3. There will not be huge changes of water usage in China in the future years;
4. There will be no abnormal phenomenon such as flood, drought and other meteorological disasters from 2012 to 2025.
5. Assume that transforming the same size of water for the same distance in different areas takes equal cost.

4. Symbols and Definitions

In this section, we will give some basic symbols and definitions used in the following pages for convenience.

Table4-1.Variable Definition

Variable Symbols	Definition
$\lambda(k)$	Grade ratio deviation
W	Total cost to transform
P	Cost to transform a certain water
x_{ij}	Water from i-spot to j-spot

5. The Model

5.1 Gray Forecast Model

5.1.1 The Regional Distribution of Water Resources in China

The rules of the regional division are as follows:

This distribution accords to the principle that rivers and regional areas are parallel considered so that the unity of provinces and be preserved.

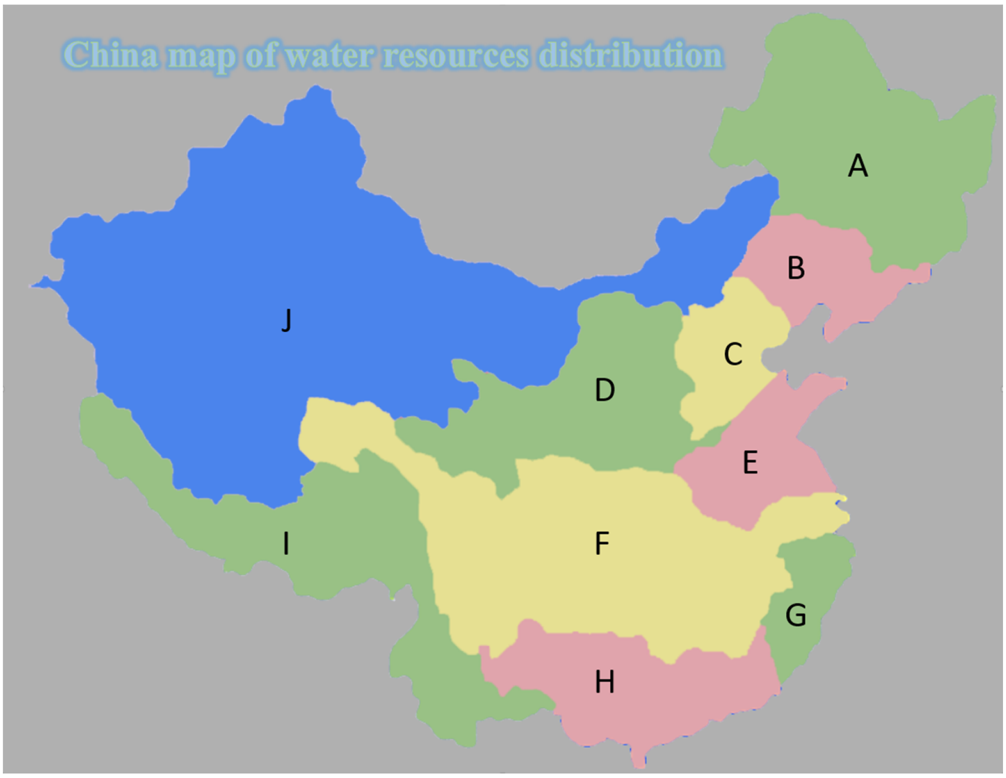
The distribution have considered mainly the series of rivers, and additionally landscape, economic development and the area.

China has 32 provinces. According to river distribution, China can be divided into

10 areas: A zone、B zone、C zone、D zone、E zone、F zone、G zone、H zone、I zone、J zone,

Specific dividing circumstances are as follows：

Picture 5-1 distribution of water resources in China



The letters above stand for the following areas:

A. The Songhua River area;

B. The Liaohe River area;

C. The Haihe River area;

D. The Yellow River area;

E. The Huaihe River area;

F. The Yangtze River area;

G. The Southeastern River area;

H. The Pearl River area;

I. The Southwest River area;

J. The Northwest River area.

5.1.2 Prediction of National Water Resources and water need

Let’s take the national water storage and water needs as an example:

Table5-1

NO.	Year	Total Water Resources	NO.	Year	Total Water Resources
1	2005	28053.1	5	2009	24180.3
2	2006	25330.1	6	2010	30906.3

3	2007	25255.1	7	2011	23256.7
4	2008	27434.3			

Table5-2

NO	Year	The Use of Water	NO.	Year	The Use of Water
1	2005	5632.9	5	2009	5965.1
2	2006	5795.9	6	2010	6022
3	2007	5818.7	7	2011	6125
4	2008	5910			

1)Grade ratio deviation Test

The time sequence of total water resources：

$x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(7))=(28053.1,25330.1,25255.1,27434.3,24180.3,30906.3,23256.7)$

The time sequence of water consumption:

$x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(7))=\left(5632.9,5795.9,5818.7,5910,5965.1,6022,6125\right)$

（1） Grade ratio deviation $\lambda(k)$:

$$\lambda(k)=\frac{x^{(0)}(k-1)}{x^{0}(k)}$$

The Grade ratio deviation of national:

$$\lambda=(\lambda(1),\lambda(2),\cdots,\lambda(7))=\left(1.0259,1.0007,0.9797,1.0312,0.9394,1.0741\right)。$$

The Grade ratio deviation of water consumption：

$$\lambda=(\lambda(1),\lambda(2),\cdots,\lambda(7))=(0.9719,0.9961,0.9846,0.9908,0.9906,0.9832)$$

（2）（2）The Grade ratio deviation estimate. Due to $\lambda(k)\in\left[0.9394,\quad 1.0312\right]$, we can define $x^{(0)}$ as a satisfied GM(1,1) model.

2)GM（1,1）Modeling

（1） To sum $x^{(0)}$ up for the first time. We get:

1. $x^{(1)} = (28050, 133380, 238640, 450250, 561160, 664220)$;

2. $x^{(1)} = (5633, 11429, 17248, 23158, 29123, 35145, 41270)$.

(2) Construct the matrix B and vector Y:

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1)) + x^{(1)}(2) & 1 \\ -\frac{1}{2}(x^{(1)}(2)) + x^{(1)}(3) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(3)) + x^{(1)}(7) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(7) \end{bmatrix}$$

(3) calculate: 1. $\hat{u} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (\mathrm{B}^T\mathrm{B})^{-1} B^T Y = \begin{bmatrix} -0.00089 \\ 105680 \end{bmatrix}$

$$2. \hat{u} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (\mathrm{B}^T\mathrm{B})^{-1} B^T Y = \begin{bmatrix} -0.0111 \\ 5680.5 \end{bmatrix}$$

(4) Set up diff equation model: $\dfrac{dx^{(1)}}{dt} + \hat{a}x^{(1)} = b$

The result is:

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{\hat{b}}{\hat{a}})e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} = 1.188 \times 10^8 e^{0.00089k} - 1.187 \times 10^8 \backslash$$

(5) Solve the predicted value of sequence figured out $\hat{x}^{(1)}(k+1)$ and the restored value $\hat{x}^{(0)}(k+1)$, we define $k = 1, 2, 3, 4, 5, 6$, 7, from the equation of response time function we can work out $\hat{x}^{(1)}$, among which we pick up $\hat{x}^{(1)}(1) = \hat{x}^{(0)}(1) = x^{(0)}(1) = 5632.9$, $\hat{x}^{(0)}(\mathrm{k}+1) = \hat{x}^{(1)}(\mathrm{k}+1) - \hat{x}^{(1)}(\mathrm{k})$, when k=1,2,3,4,5,6,we get:

$$\hat{x}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(7)) = (28053.1, 25330.1, 25255.1, 27434.3, 24180.3, 30906.3, 23256.7).$$

3) Model Checking

The results of checking the model are as follows:

GM(1,1) test table of total water resource

		Original	Predict	Residual	Relative	Grade

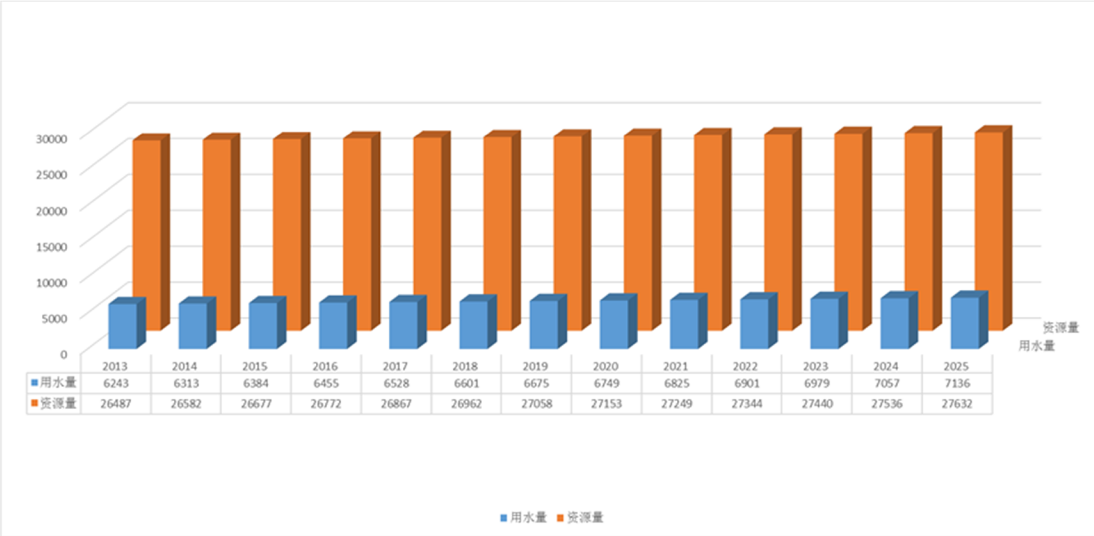
No.	Year	Value	Value	Error	Error	ratio deviation
1	2005	28053.1	28052.3	0	0	
2	2006	25330.1	25331.2	-0.0047	0.000004%	1.0259
3	2007	25255.1	25253.4	-0.0063	0.000006%	1.0007
4	2008	27434.3	27431.0	0.0132	0.000012%	0.9797
5	2009	24180.3	24178.2	-0.0185	0.000017%	1.0312
6	2010	30906.3	30907.3	-0.0424	0.000038%	0.9394
7	2011	23256.7	23259.1	0.0294	0.000029%	1.0741

GM(1,1) test table of total water demand

No.	Year	Original Value	Predict Value	Residual Error	Relative Error	Grade ratio deviation
1	2005	5632.9	5632.9	0	0	
2	2006	5795.9	5791.8	0.0036	0.000061%	0.0172
3	2007	5818.7	5814.1	-0.0037	0.000062%	-0.0072
4	2008	5910	5911.6	0.0008	0.000013%	0.0044
5	2009	5965.1	5964.7	-0.0011	0.000018%	-0.0019
6	2010	6022	6022.9	-0.0027	0.000045%	-0.0016
7	2011	6125	6127.1	0.0031	0.000051%	0.0058

Researches have proved that the accuracy of this model is adequate to satisfy the forecast.

According to this model, we predict the water storage and the demand in the near future and now it is vividly shown here in the form of chart:



From the chart we can conclude that people's need of water will undergo a constant increase, while the storage of water will not appear some apparent changes. Thus, it is necessary for us to determine an appropriate water strategy, so that all the people can be blessed with enough water. Besides, economic, physical and ecological factors must also be taken into consideration.

According to the method above, we manage to predict the total storage and demand of water for the 10 areas above in 2025. The result presents here in the table below:

Table：2025 water_demand prediction

Area	A	B	C	D	E
Demand	378.3	191.1	379.1	381.5	543.8
Area	F	G	H	I	G
Demand	1842.2	325.1	873.6	101.6	616.6

5.2 Storage Model

In the flood season of rainy year, a large quantity of water supply exceeds demand. Excess rainwater would be stored in the intake area, in order to be taken advantage of in the dry season. The excess water is usually stored in two ways: groundwater storage that makes the most of the hydrogeological conditions and the pre-existing groundwater extraction wells of the water-starved city to replenish groundwater; surface water storage that makes full use of the pre-existing reservoirs of water-starved city, the best of which supplements the surface water from the source region of the city's water supply (which can take advantage of the pre-existing water supply system).

The underground reservoir is a special reservoir, built with the natural underground water storage space, playing the roles of impoundment、 regulation and utilizing groundwater flow. As shown in the following figure:

According to the thickness of the aquifer, the distribution area and rock storage coefficient, it can be calculated that the maximum regulation and storage capacity of underground reservoir as:

$$Vmax = SA\Delta H \qquad (6)$$

Where: $Vmax$ refers to the regulation and storage capacity of underground reservoirs(m3); is storage coefficient of the underground water-bearing medium in reservoir storage layer, for the no pressure water-bearing body $S = \mu + \mu *$, for the pressure water-bearing body$S = \mu *$,where is gravity specific yield(dimensionless),$\mu *$ is elastic storage coefficient(dimensionless);is reservoir area (km2);ΔH is the maximum thickness the regulation and storage layer, that is, the water level difference value between the extreme high water level and the planning regulation and storage layer bottom interface.

Superiorities：

① Underground reservoir taking natural aquifer as water storage spaces, so it has relatively simple construction conditions and lower construction costs;

② Underground reservoirs has little effect on other ground buildings, relatively safe, low environmental impact;

③ Groundwater in reservoir regions has smaller evaporation loss, easy to conserve;

④ It provides the basis and conditions for the joint scheduling of surface water and groundwater, and it slows down the flood pressure.

Problems：It has relative larger errors to calculate the amount of resources of underground reservoirs; and it has a larger workload to survey, with higher costs and longer periods .

Considering that the underground reservoir just saves the local region's water supply, rather than additional water source, it can be used to solve water supply-demand imbalance issues in the short term, so that in the dry season of the year, the residents can use the water of the flood season. However, it is not suitable as a long-term water strategy to resolve the regional water supply-demand imbalance situation.

5.3.Mobilization model

By the first prediction model, it can be found that the distribution of water resources in China is extremely imbalanced, with more Water in Southern, less Water in Northern. And the water resources are concentrated in the Yangtze River basin.

In order to solve the water shortage problem, it is feasible to transfer the water from the wet area to the water-poor areas, by the redistribution of water resources through a canal or aqueduct.

Statistics show that the water transfer costs in the south-to-north water diversion project center line is 2.6 yuan/m^3. The water transfer costs in western route of water diversion from South to North is 1yuan/m^3， and it is 3.4 yuan/m^3 in the east route.

Here the diversion costs take the average value$(2.6 + 1 +3.4) \div 3=2.3$yuan/$m^3$

Abide by the principle of cost minimization, the determination of the water transfer schemes need linear programming to solve.

5.3.1 Linear programming

As the figure shows, in order to simplify the transfer process, take one big city in each area as a centralized point to study their water mobilization.

Those centralized point turn to be respectively:

area	Available water	Demand water
A	343.51	378.30
B	452.24	191.10

C	371.95	379.10
D	521.37	381.50
E	532.56	1543.80
F	1932.45	1842.20
G	273.32	325.10
H	1687.83	873.60
I	585.81	101.60
J	823.31	616.60

A: The Songhua river area, (-34.79)

B:Liaohe area (261.14);

C:Haihe River area(7.15);

D:The Yellow River area (139.87);

E:The Huaihe River area(-1011.24);

F: The Yangtze River area (90.25);

G:Southeastern Rivers area(-41.78);

H:The Pearl River area(814.23);

I:Southwest Rivers area(484.21);

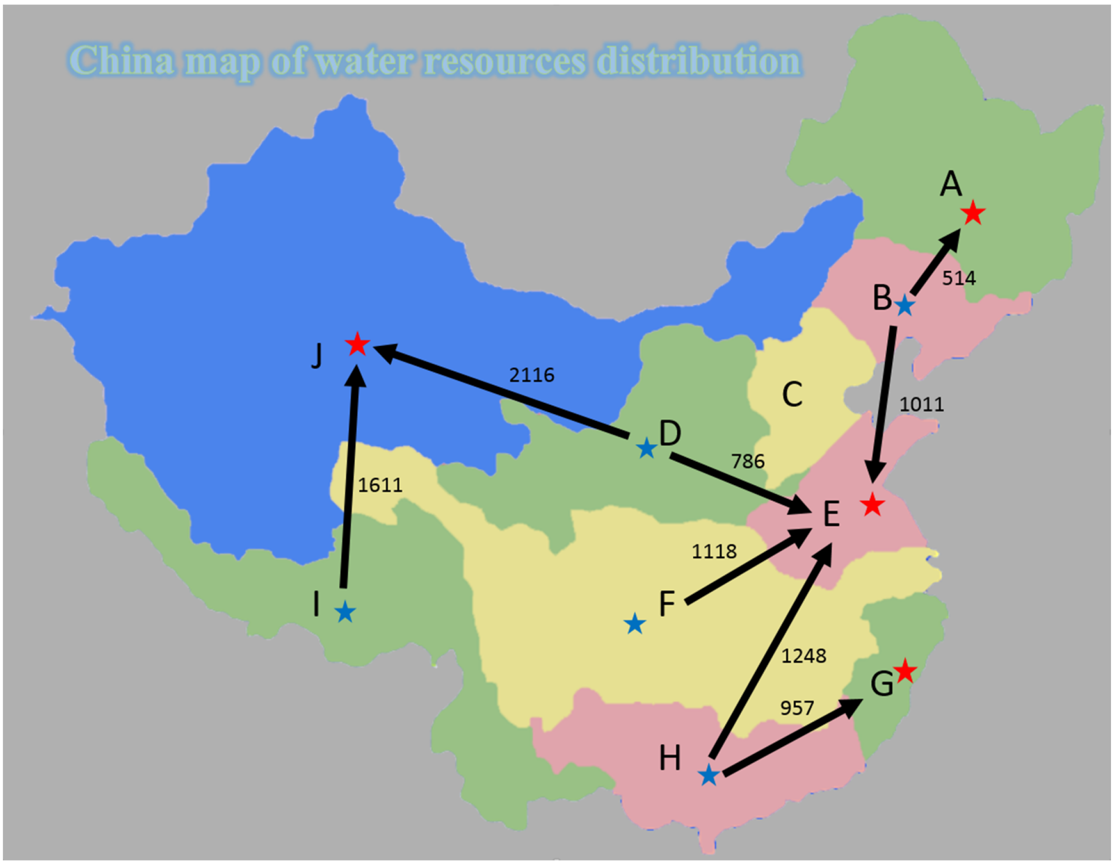
J:Northwest Rivers area(-206.71);

The data in blacks present the value of water resource available minus water needed. (A positive number indicates there to be water available, while a negative one implies lack of water)

We follow the principles below to determine how we deal with it:

1. Nearer areas have privilege over the others.
2. Water supply is always adequate.
3. Transformations are based on the leading city of the corresponding area.
4. C spot can be ignored for it can right make its living itself.

It can be sought by measuring tape through Baidu map that AB=514，BC=1011，DE=786，DJ=2116,IJ=1611，FE=1118，HE=1248，HG=957.



5.3.2Set single objective programming:

$$W = \mathrm{P}(514x_{21} + 1011x_{25} + 786x_{45} + 1118x_{65} + 1248x_{85} + 957x_{87} + 2116x_{40} + 1611x_{90})$$

$$s.t. = \begin{cases} x_{65} \le 90.25 \\ x_{21} \ge 34.779 \\ x_{87} \ge 41.78 \\ x_{40} + x_{90} \ge 206.71 \\ x_{25} + x_{45} + x_{65} + x_{85} \ge 1011.24 \\ x_{21} + x_{25} \le 261.14 \\ x_{40} + x_{45} \le 139.87 \\ x_{85} + x_{87} \le 814.23 \\ x_{90} \le 484.2 \end{cases}$$

through calculation, the following results are obtained:

$$\begin{cases} x_{21} = 34.73 \\ x_{25} = 226.35 \\ x_{45} = 139.87 \\ x_{65} = 90.25 \\ x_{85} = 554.77 \\ x_{87} = 41.87 \\ x_{40} = 0 \\ x_{90} = 206.71 \end{cases} \qquad W = 1523000$$

6. Evaluation and Improvement of the Model

6.1 Strengths and Weaknesses

6.1.1 Strengths

The study of water resources in China took the pattern of partition first and then studying within each region, which determined the significant water-scarce regions and seized the key points of the problem.

Replenishment measures are not only qualitative but also quantitative, along with the quantitative modeling and costs calculation.

Integrated to determine the water strategy in 2025, after the permutations and combinations of various replenishment measures, the priority of each measure is assessed to determine a complete and feasible water strategy.

Various replenishment measures have followed principles of the lowest cost and the best combined effect, to ensure the best strategy.

6.1.2 Weaknesses

When build the prediction models, only seven years of data are collected. Because the measurable data is respectively little, the fitting results existed certain errors.

Partition number is not enough, 3 more can be added to increase the accuracy of the model. Optimal allocation of regional freshwater lacked for experimental data, without quantitative calculations.

6.2 Promotion of the Model

Because the water data collected is less, in order to improve the accuracy of prediction model, related data of the 2014,2015,2016,2017 can be predicted firstly, then setting these data as the known data to re-establish the prediction model. Then the new function of water amount and time can be obtained. And what' more, water demand in 2015 can be calculated by the new function - iterative method.

The cost of water diversion has taken the average value of the south-to-north water diversion project center line、 western line and eastern line, with big errors. In fact, the average value should be calculated by some more collected water diversion project data. Or it is also feasible to collect the cost sources, establish a function of its distance and water transfer volume respectively, and finally get a function about the total costs .

7. The non-technical position paper

Among the ten zones, some of the zones have enough water supply in 2025 while the others not.

Faced with this problem, we work out a plan to transform water from where this resource is relatively rich to the areas in severe lack.

Replenishment measures taken by the article is the mainstream replenishment strategy nowadays, which has passed through the experts' research. In the water mobilization model, the water diversion route was determined using Matlab program, for solving the optimal solution to minimize the cost. The other replenishment strategies were obtained by reference to the actual production data, which is true and reliable. 5. The establishment of the regional freshwater allocation model provided possibility for the optimization of water resources allocation within the zone, which ensured that the region could maximize the economic benefits.

8. References

[1]Defeng Zheng & Bende Wang. 2004. Survey on the comprehensive evalution of the regulation and storage capacity of the ground reservoirs, SHUILIXUEBAO.0559-9350(2004)10-0056-07.

[2]Tianshi Zhao. 2002. Discussion on some problems of ground reservoirs, Hydrogeology and Engineering Geology. 1000-3665(2002)05-0065-03.
[3]Jianhua Chen. 2009. Alternative to coal-fired electricity stakeholder win-win path based on the external straw, Xian: xian jiaotong university press.
[4]Zhongnan Qiu. 2000. Price analysis on the water supply of south-to-north water diversion project center line south-to-north, Yangtze River, 31 -07.
[5]Xiaofeng Tu & Haizheng Wang & Dafa Ding& Meiying Tang,2001. Preliminary estimates of the economic benefit analysis and water cost of the first phase of the Western Route, YELOWRIVER.23-10.

9. Appendix

9.1 GM(1,1) codes

```
clear all;
clc;
format compact;
Data_Size=7;
A=xlsread('water.xlsx');
Start=[12 1];
Const_Var=80000;
m=menu('Select the data you use:','Total Resource','Water demand');
while m==0
    m=menu('Select the data you use:','Total Resource','Water demand');
end
if m==2
    Const_Var=0;
    Start=[23 1];
end
x0=A(Start(1),Start(2):Start(2)+Data_Size-1)';
x0=x0+Const_Var;
n=length(x0);
lamda=x0(1:n-1)./x0(2:n);
range=minmax(lamda');
x1=cumsum(x0);
B=[-(x1(1:n-1)+x1(2:n))./2,ones(n-1,1)];
Y=x0(2:n);
u=B\Y;
x=dsolve('Dx+a*x=b','x(0)=x0');
```

```
x=subs(x,{'a','b','x0'},{u(1),u(2),x0(1)});
yuce1=double(subs(x,'t',0:n-1));
yuce=[x0(1),diff(yuce1)];
eplison=(x0'-yuce)./(x0');
delta=abs(eplison./x0');
rho=1-(2-u(1))/(2+u(1))*lamda';
disp('Lamda=');disp(lamda');
disp('Eplison=');disp(eplison);
disp('Delta=');disp(delta);
disp('Rho=');disp(rho);
yuce2=double(subs(x,'t',0:20));
yuce_2=[x0(1),diff(yuce2)]-Const_Var;
disp('x(1)=');disp((x1-Const_Var)');
disp('Result=');disp(yuce_2);
```

9.2 Single Objective Programming

```
c=[514;1011;786;1118;1248;957;2116;1611];
a=[
     -1 0 0 0 0 0 0 0
     0 0 0 0 0 -1 0 0
     0 0 0 0 0 0 -1 -1
     0 -1 -1 -1 -1 0 0 0
     1 1 0 0 0 0 0 0
     0 0 0 0 1 1 0 0
     0 0 1 0 0 0 1 0
     0 0 0 0 0 0 0 1
     0 0 0 1 0 0 0 0
     ];
b=[-34.79;-41.87;-206.71;-1011.24;...
     261.14;814.23;139.87;484.21;90.25];
[x,y]=linprog(c,a,b,[],[],zeros(8,1))
```