For office use only	Team Control Number	For office use only
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T2 ________	Problem Chosen	F2 ________
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T4 ________		F4 ________

2019
MCM/ICM
Summary Sheet

The Best Strategy for Using A Non-SPA-style Bathtub

A non-SPA-style bathtub cannot be reheated by itself, so the water will get noticeably cooler and users should add hot water from time to time. Based on the existing Partial Differential Equation (PDE) technique, we construct time-space based temperature model which can simulate any common condition. And then propose an optimal strategy for users to keep the temperature even and close to initial temperature and decrease the water consumption.

Firstly, we construct the PDE-solving model to simulate the temperature distribution in the bathtub by analyzing the heat loss and confirming the corresponding parameters. In this part, we consider two kinds of heat transfer, (water-bathtub heat conduction, water-air heat convection), and the effect of inflow water on the faucet is shown by the heat source. So we can confirm the boundary conditions of PDE to carry out the next step.

Secondly, we do parameter testing by free cooling process to ensure the parameters are in line with the actual condition. Most of the parameters are applicable, and a few parameters are fine-tuned to make the model more accurate and practicable. Thirdly, we confirm the optimal strategy by analyzing and comparing the continuous and discontinuous flow methods, which vary from the temperature and flux. There are two feasible methods gained from the analyses, one is continuous 42 °C water inflow, the other is turning on and off the faucet with 42 °C inflow for 10 minutes.In addition, we give two more kinds of methods.

At last, we use our model to determine the extent to which our strategy depends on the factors of the bathtub, the factors of the user, and a bubble bath additive. And we test the methods by changing the initial temperature. From those results, we further suggest the users move less and use more bubble bath additive, and turning on the faucet at the beginning is also a good choice.

Contents

1 Introduction

1.1 Problem Background

Taking a bath is a daily activity to remove the stains and have a rest, the cooling of water during a bath has always been a big problem for our bathing experience. Of course it is perfectly possible to live a full and active life without knowing anything about bathing strategy at all. But how could we improve the quality of life through taking a better bath?

To solve this problem, we need to construct the time-space based bathtub temperature model to determine the optimal strategy which could achieve the following three objectives:

1. To keep the temperature **even** throughout the bathtub;
2. To keep the temperature as close as possible to the **initial** temperature;
3. **Not to waste** the water too much.

1.2 Our work

1. We construct a time-space based temperature model to research the change of the temperature in the bathtub.
2. We use free cooling process to test some unknown parameters' value.
3. Through controlling the change of the inflow temperature and flux, we make some effective strategies to meet the demand of optimum and uniform temperature in the bathtub.
4. In order to examine the extent of our model, we test whether good strategies in the initial model still have the same effect on the model which changes some parameters based on normal bathing.

2 Preparation of the Models

2.1 Assumptions

It is difficult to study the actual situation in the bathtub because of the complex principles about water-flowing and the heat conduction. In order to simplify the problem and to hold the correctness of results at the same time, we make the assumptions below to simplify the bath problem:

1. We ignore the loss of water through evaporation. According to [12], the evaporation capacity of water is only about 7.35×10^{-8}m/s, so it is a tiny quantity of the whole water.

2. We ignore the influence caused by different density of cold and hot water(such as the flow) in order to simplify the model, and with the consideration that people may move around in the bathtub.

3. Our model concentrates the bathtub with full water, which can eliminate the effect of the overflow drain.It is based on the consideration that the change of position is difficult for liquid, and the change of temperature in water is smooth (That is to say the excess water is almost the same with the upper water).

2.2 Notations

The primary notations used in this paper are listed in **Table 1**. There can be some other notations to be described in other parts of the paper.

Table 1: Notations

Symbol	Definition
I	The Intensity of the Heat Source
k	The Material's Conductivity
h	The Heat Transfer Coefficient
c	The Heat Capacity of Water (=4200 J/(kg·°C))
ρ	The Density of Water (=10^3 kg/m^3)

3 Time-space Based Temperature Model through PDE

Partial Differential Equation Method (**PDE-Method**) [10] is very useful in solving the problems about the variables changed both by position and by time. In the problem of the heat conduction in bathtubs, it is proper to use such a kind of method to find out the temperature distribution in a bathtub in different time points.

We use **PDE-Toolbox** in **MATLAB** software to solve the model of temperature distribution in the bathtub. **Figure 1** shows the structure of our model, which can be divided into two parts: **PDE Solving Model** and **Analysing Model**. The former helps us to get the temperature distribution in the bathtub in different situations, and the latter helps to decide which strategy is better for an joyful experience while bathing.

3.1 Construction of PDE-Solving Model

In order to construct a model more close to the normal condition, we need to establish a *particular* bathtub with some actual parameters and make some preparations

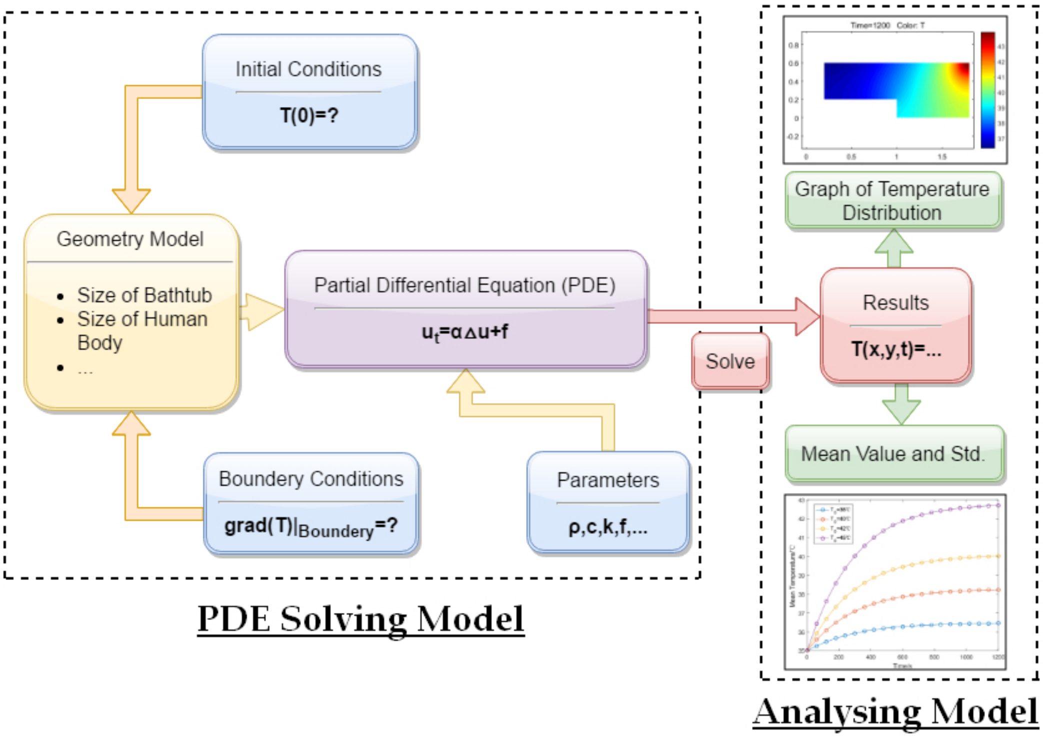

Figure 1: The Structure of Model

for the initial and optimum temperature. For an established condition of bathtub with specific faucet and inflow, we can use some laws about heat transmission to analyse the state of the water, such as the temperature and its stability.

3.1.1 General Preparations for PDE Solving Model

We decide a typical bathtub for our research. **Table 2** shows the size of the bathtub, together with the size of the human body settle in the bathtub (also very typical) and the temperature in different situations.

Table 2: The value of geometry size and temperature

Class	Quantity	Value
Size of the bathtub	Length	1.8m
	Width	0.8m
	Depth	0.6m
Size of the human body (consist of two cubiods)	Length	0.8m
	Width	0.2m
Temperature	Human skin	36°C
	External (the wall and the air)	20°C
	The Water in bathtub	35°C (Initial)

The 3-D Model of the bathtub is shown in **Figure 2**.

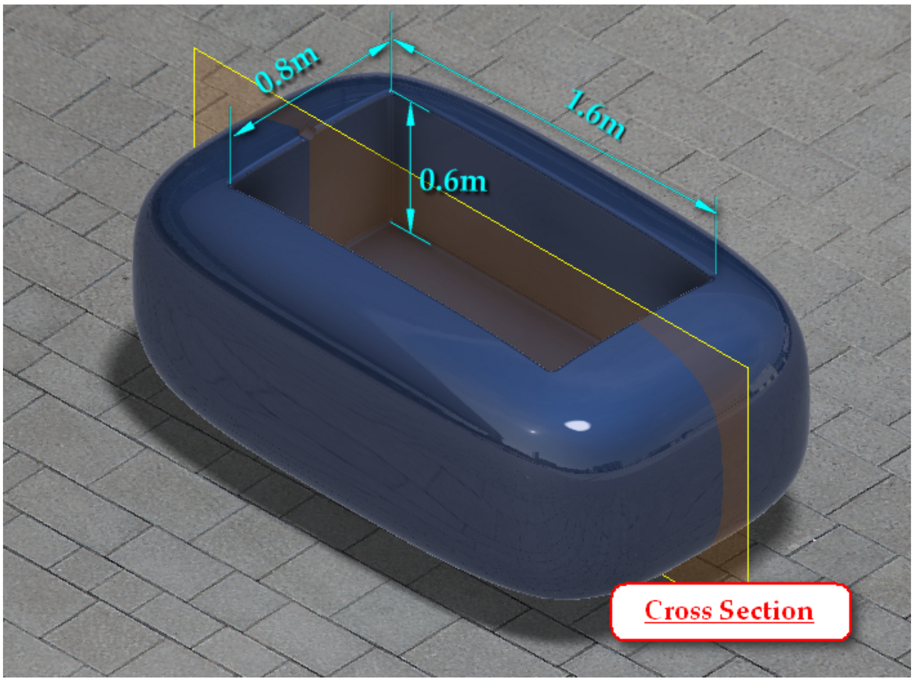

Figure 2: The 3-D model of the bathtub, designed with *Inventor 2017*

It is always difficult to study the 3-D space with PDE-Method. So in our study, we just research a particular **cross section** of the bathtub which can represent the distribution of temperature in the whole bathtub. The position of the cross section has been emphasized in **Figure 1**, and the component of the section is shown in **Figure 3**.

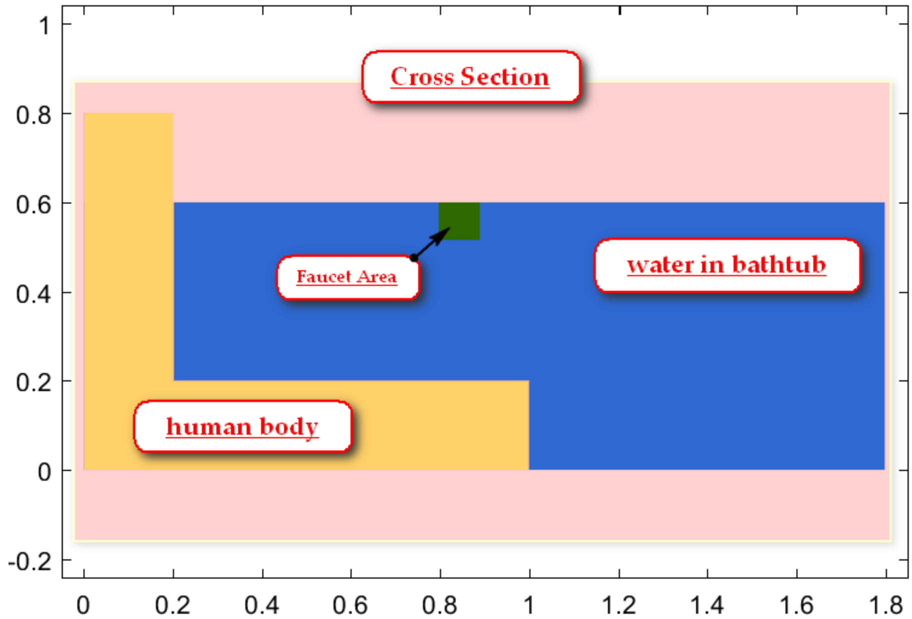

Figure 3: The component of the cross section

3.1.2 Partial Differential Equation of Heat Conduction

In order to calculate the change of temperature by time for each small water component, the **Heat Conduction Equation** is needed. It can be expressed as

$$u_t = \frac{k}{\rho c}\nabla^2 u + I, \tag{1}$$

where

- u_t is the temperature's derivative by time, which represent the change we want to know;
- k is the thermal conductivity of water (W/(m·K));
- $\nabla^2 u = \mathrm{div}(\nabla u)$ roughly means the change of temperature in different positions;
- ρ, c are the density and heat capacity of water, whose value has been given in **Table 1**.

To set the value of k, we consider the theoretical basis of the Equation (1), **the Fourier's Law**. The law states that the time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles to that gradient, through which the heat flows. It is shown as:

$$\vec{q} = -k\nabla T \tag{2}$$

where (including the SI units)

- $\vec{q}$ is the local heat flux density (W/m^2);
- k is the material's conductivity (W/(m·K));
- ∇T is the temperature gradient (K/m).

The thermal conductivity k is often treated as a constant, though this is not always true. When water is absolutely static, the value of k is very little. However if the value of k in static condition is used in the model, a bathtub of water can keep the temperature for even 10 minutes! The reason for this is that the water flow can carry much energy. Therefore we set:

$$k = 0.6 + M(\max u - \min u). \tag{3}$$

The first term is water's initial thermal conductivity [11]. The second term is to reinforce the influence of water flow, where the coefficient M will be given in the following part after simulating the model.

3.1.3 Inflow Process

As the bath problem shows,the person would add a constant trickle of hot water from the facet to reheat the bathing water. We consider the inflow of hot water as heat source in the top center under *the full water assumption*. The temperature of the bathtub would not change greatly as the total quantity of water is constant, so the effects of trickle of hot water and heat source are the same approximately.

In this part, we set the temperature of inflow water as T_{in}, and the maximum quantity of flow as V_{in}. We consider the **faucet area** as an area of 0.1 m $\times$ 0.1 m $\times$ 0.1 m, which means the heat of the inflow water initially distributes in such an area. The range of the faucet area in the researching cross section has been shown in **Figure 3**.

The intensity of the heat source I represents the extra heat transmission in per volume. We measure the effect of inflow water through turning it into quantity of heat added into the bathtub, which means the inflow water bring the heat with $T_{\text{in}} - T_{\text{avg}}$ higher temperature (here T_{avg} is the average temperature throughout the bathtub). So we can get the heat transfer formula:

$$I = \frac{Q_{\text{in}}}{V_{\text{per}}} = \frac{\rho c V_{\text{in}}}{V_{\text{per}}}(T_{\text{in}} - T_{\text{avg}}). \tag{4}$$

Here, $V_{\text{per}} = 1\text{dm}^3$ is the volume of faucet area. And V_{in} is the volume of inflow water per second.

3.1.4 Boundary Conditions of PDE

Water in the bathtub has three kinds of boundaries: the wall of bathtub, the air above the water surface and the person's skin. In order to restrain the boundary condition, we consider the **Newton's Cooling Law**.

The heat-transfer version of **Newton's Cooling Law**, states that the rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings. It can be stated as:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} \quad = hA\Delta T(t), \tag{5}$$

where

- Q is the thermal energy in Joules;
- h is the heat transfer coefficient (assumed independent of T here) (W/(m^2·K));
- A is the heat transfer surface area (m^2);
- T is the temperature of the object's surface and interior (since these are the same in this approximation). If we set T_{env} as the temperature of the environment (i.e. the temperature suitably far from the surface), then $\Delta T(t) = T(t) - T_{env}$ is the time-dependent thermal gradient between environment and object.

Table 3: The value of h in various interfaces

Interface	Value of h
Water - Wall of bathtub	20 W/(m·K)
Water - Human body	50 W/(m·K)
Water - Air	600 W/(m·K)

Base on the results from [11], the rate of heat transfer in such circumstances is derived. The values of these coefficients are shown below in **Table 3**.

The heat transfer coefficient h depends upon physical properties of the fluid and the physical situation in which convection occurs. Therefore, a single usable heat transfer coefficient (one that does not vary significantly across the temperature-difference ranges covered during cooling and heating) must be derived or found experimentally for every system that can be analysed using the presumption that Newton's law will hold.

3.2 Parameter Testing by Free Cooling Process

We use **PDE-Toolbox** in MATLAB software to stimulate the **free cooling process** in the bathtub with different value of M, which is defined in Equation (3).

We test the model with the value of parameters given below, except for M which is set to different value. After trying with several possible values, when $M = 600$ the speed of free cooling is close to the speed in real situation. **Figure 4** shows the temperature distribution of the water in the bathtub when $T = 1200$s (20 minutes later) and the mean temperature in different time points. It is appropriate to set the coefficient according to the results of simulation without support from the literature, so we will consider $M = 600$ in the following part.

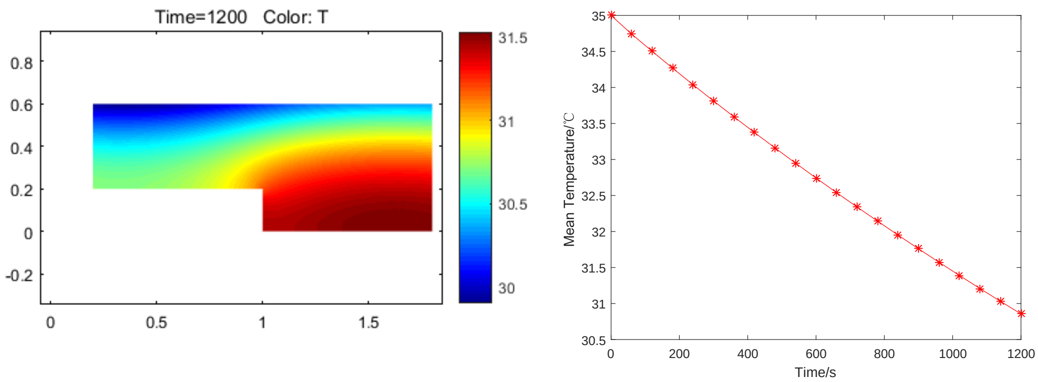

Figure 4: The free cooling process, with $M = 600$

4 Analysis of the Best Strategy

Different people have different demands on taking bath in the bathtub. For example, some people prefer to enjoy warm water early no matter how much hot water is used. Other people do not like the difference of the temperature, maybe they also want to save water. Therefore, it is impossible to point out which is more important: *quantity*, *stability*, or *optimality*. In order to put forward effective suggestions to consumers with different demands, we control two parameters, temperature and flux from the faucet, to analyse the effect of different inflow plans. By analyzing the graphs and linking them with normal lives, we can find some new methods to give consumers a better experience.

In this section, we will set the initial temperature of water in the bathtub $T(0) = 35°$C to simulate the situation by the problem sheet: the user begins to heat up the water when the water **has been** cooled down.

4.1 Different Temperature, Same Flux

According to normal lives, we set the temperature in four levels, which are shown in **Table 4**. It is usually impossible for the users to feel out the actual temperature of the water, we also give a series of description to express the feeling of different levels of temperature. In addition, we control the flux as V_{per} =0.25L/s.

Table 4: Levels of water temperature

Temperature	Description
38°C	"Optimum temperature"
40°C	"A little higher temperature"
42°C	"Scalding temperature"
45°C	"Highest temperature"

Optimum water By using PDE-Toolbox, we obtained the change in the bathtub by time for four kinds of water. We calculated the mean temperature of the water in the bathtub in 20-minute-time, and the results are shown in **Figure 5**. From the graph we can see and infer that by the increase of the temperature, the water attaches the optimum temperature more and more early.

If we turn on the faucet from the beginning to the end, 38°C and 40°C water cannot make the whole water attach the optimum temperature. So in the following concern about the temperature, 38°C and 40°C inflow cannot heat up the water. However, if we choose 45°C water, after a period of time, the water will be too hot to stand. For 42°C water, actually in the last 10 min, the temperature do not make any big influence.

Therefore, we thought out a new method. When the water attaches the optimum temperature, the user cannot turn off the faucet. This can also help save the water. Based on the results before, we give out three methods:

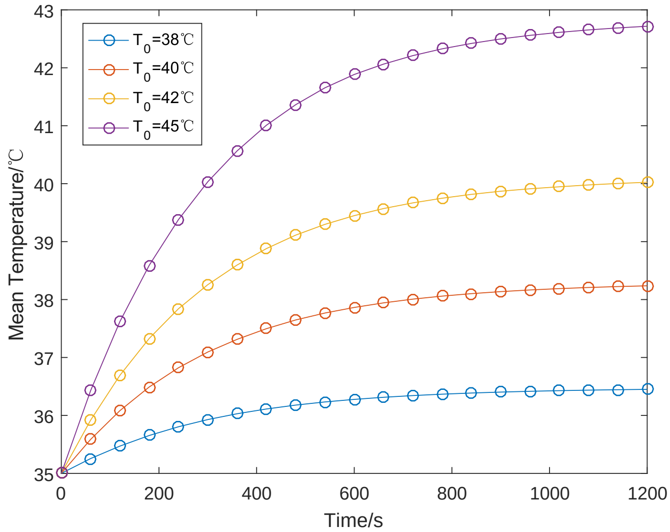

Figure 5: The mean temperature of bathtub water, with continuous inflow

- **Method 45 $\times$ 5**: Turn on the faucet with 45°C inflow for 5min, and then turn off the faucet.
- **Method 42 $\times$ 5**: Turn on the faucet with 42°C inflow for 5min, and then turn off the faucet.
- **Method 42 $\times$ 10**: Turn on the faucet with 42°C inflow for 10min, and then turn off the faucet.

The mean value of the water with these methods is given in **Figure 6**. From the graph we can see that in the former 10 minutes, water in the bathtub keep the temperature in a comfortable degree. However, along with the shut of the faucet, the temperature fell down at a quick speed. At the time point of T=1200s, the temperature of Method 45×5 and Method 42×5 both fall under 36°C, which is not accepted. However, Method 42×10 gives a better performance in maintaining the temperature in an acceptable level. So Method 42×10 is considered to be one of the recommended strategies.

It is seen that the water will still cool down too early with only a period of inflow. So why not turn on the faucet when the water fell down under the optimum temperature? Based on the results before, we choose the two possible test two methods:

- **Method 45-twice**: Turn on the faucet with 45°C inflow for 5min, then turn off for 5min, and repeat such process once.
- **Method 42-twice**: The same as below, just with 42°C inflow.

The mean temperature of water with these two methods is shown in **Figure 7**. It shows that Method 45-twice has a significant effect on the rising of the temperature. Compared with the former method, Method 42-twice makes the water keep the temperature lower than the optimum temperature for a period of time.

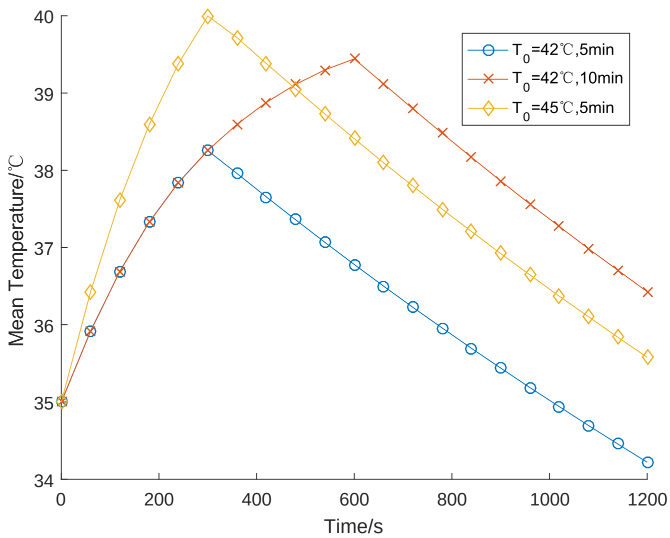

Figure 6: The mean temperature of bathtub water, with a period of inflow

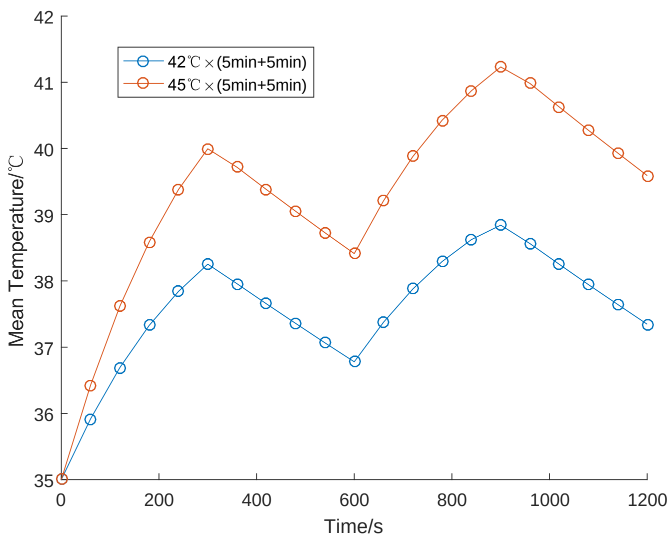

Figure 7: The mean temperature of bathtub water, with "twice"-methods

Uneven temperature We use standard deviation of the temperature data to evaluate the instability of the temperature. Actually, the standard deviation of water may not represent the instability completely, for there is always somewhere (near the faucet) with high temperature and somewhere (far from the faucet) with low temperature. For example, if our feet and our arms are around 39°C water and 37°C water all the time, respectively, maybe we would still not feel uncomfortable. Therefore the feeling of uneven temperature while bathing may also be linked with the stability of the standard deviation itself.

Firstly, we tested continuing inflow methods with various temperatures shown in **Table 4**. The standard deviations of the temperature under these methods are shown in **Figure 8**. From the graph we can see and infer that by the increase of the temperature, the standard deviation becomes larger and larger. That means hotter water will cause more uneven temperature. In normal lives, temperature within 1.5 °C difference is hard to be felt. Therefore, 45°C water may cause some trouble to people with high demand on uniform temperature.

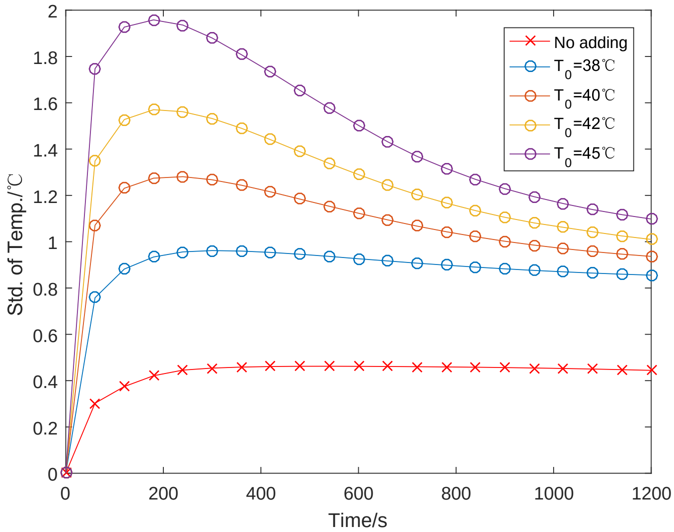

Figure 8: The standard deviation of the temperature, with continuous inflow

In addition, we also tested the "twice"-methods which we put forward before. The standard deviations under the two methods, Method 45-twice and Method 42-twice, are shown in **Figure 9**. We can see that not only the standard deviation of water is a little lager, but the stability of the standard deviation is much lower than the methods above. That is to say, discontinuous inflow may cause "*sometimes hot and sometimes cold*" water.

4.2 Different Flux, Same Temperature

Another parameter needs to control is the flux of inflow, which means turning on the faucet "*bigger*" or "*thinner*". Here we give out the different flux which will be discussed below in **Table 5**. We also give a series of description about these choices, in order to describe the actual motion on the faucet.

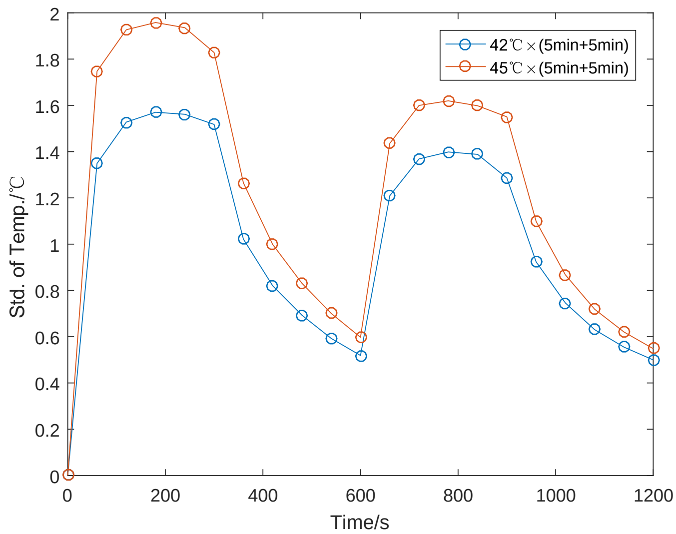

Figure 9: The standard deviation of the temperature, under "twice"-methods

Table 5: Levels of inflow flux

Flux	Description
No adding	"Faucet Off"
0.025L/s (10%)	"Trickle"
0.15L/s (60%)	"Turn on in half"
0.25L/s (100%)	"Maximum"

Optimum water By using PDE-Toolbox, we obtained the change in the bathtub by time for three kinds of flux of inflow. The mean value of these methods are shown in **Figure 10**. From the graph we can see that by the increase of the flux, the water attaches the optimum temperature more and more early. 10%-flux method cannot keep the temperature increase, so it is eliminated. 60%-flux and 100%-flux method can attach the optimum temperature, but with 100%-flux water all the time will make water scalding.

We also consider the effects of the temperature under the "twice"-methods. However, unlike the pair before, we give such a pair of methods:

- **Method 100%-twice**: Turn on the faucet with 45°C, 100%-flux inflow for 5min, then turn off for 5min, and repeat such process once.
- **Method 60%-twice**: Turn on the faucet with 45°C, 60%-flux inflow for 5min, then turn off for 5min, and repeat such process once.

The mean temperature of the water is shown in **Figure 11**. The graph shows that inflow with 100% (maximum) flux can make the water keep the temperature around the optimum while 60% flux may make people feel a little cold at the beginning of the bath.

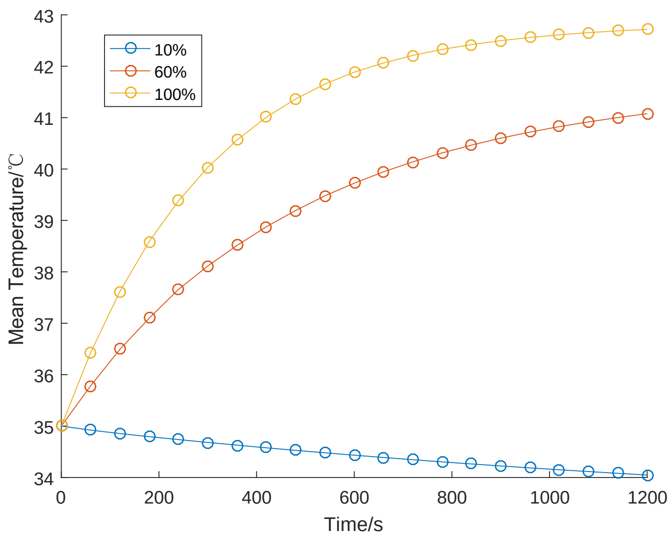

Figure 10: The mean temperature with continuous 45°C inflow (different flux)

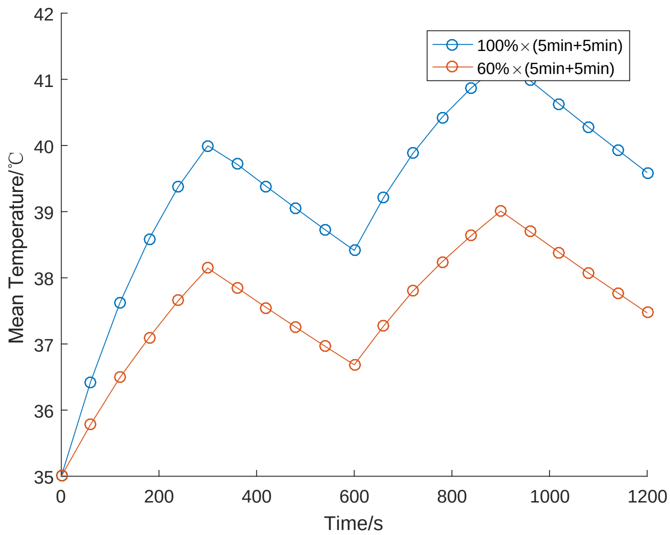

Figure 11: The mean temperature with 45°C inflow, under "twice"-methods

Uneven temperature We tested continuing inflow methods, and the results are shown in **Figure 12**. From the graph, we can see that by the increase of the flux, the standard deviation became more and more larger, which is so say, more water per second from the faucet will cause more uneven temperature.

4.3 Confirm the inflow methods

From the analysis above, we have gained two feasible methods: Continuous water inflow with lower temperature, and Method 42×10. There are also two kinds of "twice"-methods which are feasible, but here we will give two more kinds of methods. Actually, both the increase of inflow temperature and flux have the same effect on the whole water (based on our research). In normal lives, we usually open the faucet for water with high temperature and big flux to heat the water at the beginning. When the temperature falls down after we relax for a period of time, we often turn on the faucet

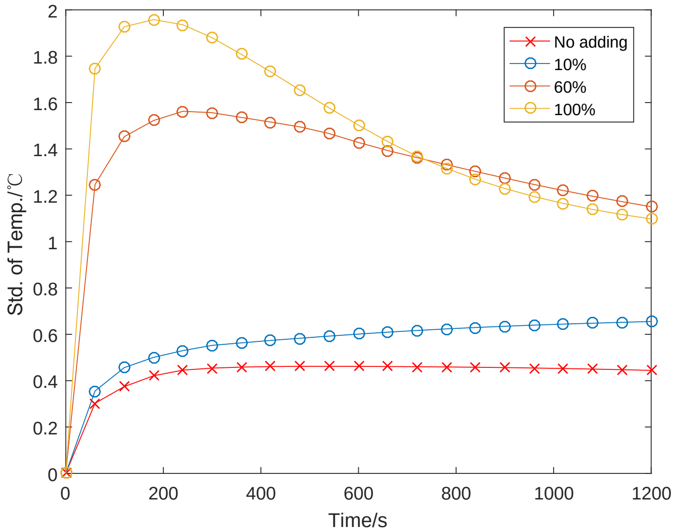

Figure 12: The mean temperature with 45°C inflow, under "twice"-methods

and let water with warm temperature and mediate flux enter the bathtub to keep the temperature. Therefore, we also make two more effective methods to add in the final list. So we have four feasible methods, which are:

- **Method 42**: Continuous 42°C water inflow;
- **Method 42×10**: Turn on the faucet with 42°C inflow for 10min,and then turn off the faucet.
- **Method 45+42**: Turn on the faucet with 45°C inflow for 5min, and then turn off. Turn on with 42°C inflow 5min later, still for 5min.
- **Method 45+42-little**: As same as the method before, but the 42°C inflow is with 60% flux.

We tested the four methods about the "optimum temperature" and "uneven temperature", and the results are shown in **Figure 13** (mean and std. of temperature). From the graph we can see that **Method 42** can lead to more optimum and uniform temperature, but this method waste a lot of water. **Method 45+42** will save a lot of water , meanwhile the temperature is more uneven. **Method 42×10** cannot give very comfortable water in the end of the bath. Compared with the three methods above, **Method 45+42little** is a mediate strategy.

Actually, the users can make their choices neatly according to their demands. According to the results, we give our suggestion in **Figure 14**.

5 Analysis of Various Conditions

We simplify our time-space based temperature model by setting down the shape volume and material of the bathtub,the position of the faucet,the shape, volume and temperature of the person and assuming that the person is static. But there are multiple

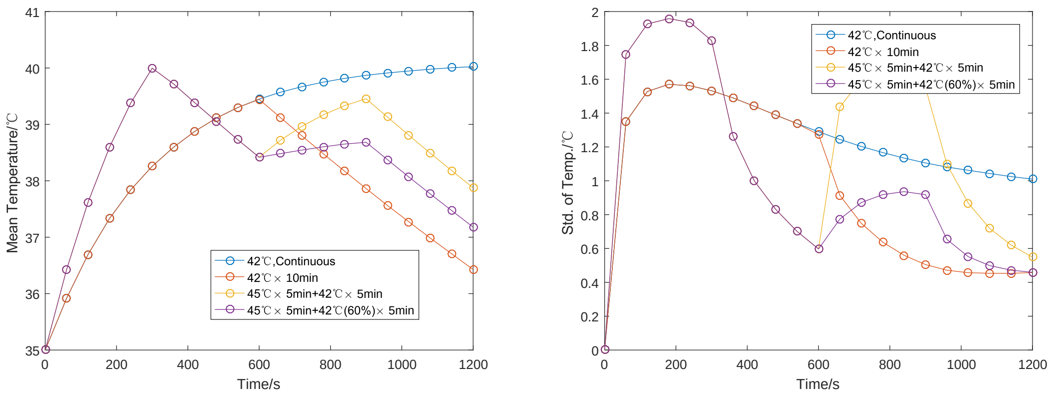

Figure 13: The mean and std. of temperature, under the four recommended methods

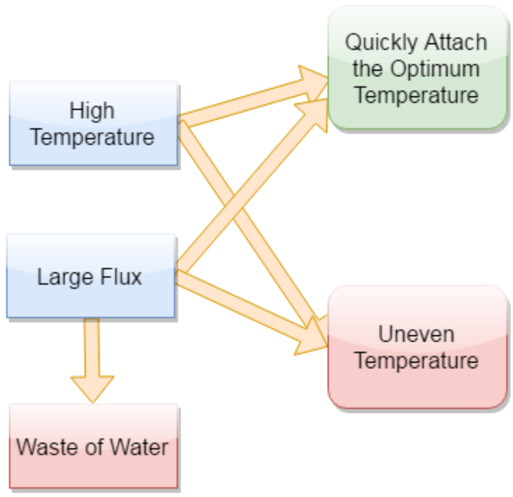

Figure 14: Diagram of actions & results

kinds of bathing in real life, and different kinds of people may have different strategies to have a better experience. So in this part, we use our model to determine the extent to which our strategy depends on the complex and multiple factors in order to give corresponding and optimum suggestions on different conditions.

Another way to say, we change several parameters' value in the model to make a **sensibility analysis**.

5.1 The Factors of the Bathtub

There are four factors of the bathtub which can affect the effects of different strategies and determine the best strategy: shape,volume,material and the position of the faucet.

Shape For the shape of the bathtub, we divide the bathtubs into two kinds based on the shape of their bottom: plane-style (which has been studied before) and concave-style. Under the prerequisite of having the same volume, we reshape the regular plane-style bathtub to build a concave-style bathtub. The image of the cross section is shown in shown in **Figure 15**. Based on the parameters before, we test the free cooling process of concave-style bathtub, which can illustrate the actual influence made by the difference of shape. The results (mean temperature and std. of temperature) are shown in **Figure 16**, which shows that two curves are almost coincided. That means the extent of the model depends little on the shape of the bathtub.

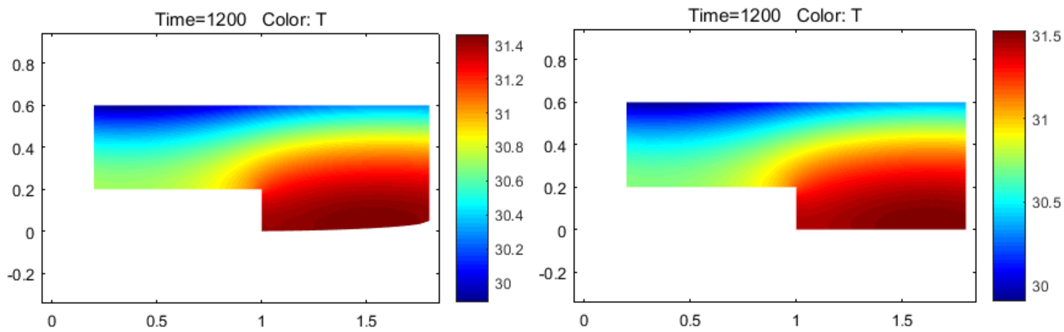

Figure 15: The concave-style bathtub together with the plane-style one

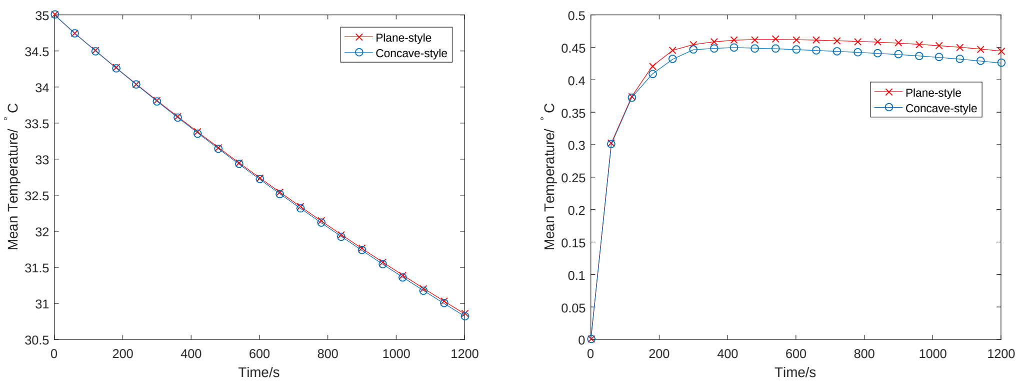

Figure 16: The mean and std. of the temperature in concave-style and plane-style bathtub

Volume To judge how much the effect of strategy is influenced by the volume of the bathtub, we choose two particular size of the bathtub. The **regular** size is as same as before (1.8m$\times$0.8m$\times$0.6m), while the **smaller** size is defined as 1.5m$\times$0.6m$\times$0.5m. As two typical instances, we test these two kinds of volume under two methods: nature cooling process, and continuous 42°C inflow (Method "42", which is a feasible method in the normal bathtub).

The mean temperature in different situations are shown in **Figure 17**. We can see that the temperature rise up falls down more quickly under the same method when using a smaller bathtub. We can see that the method is still effective to the small bathtub, which proves that our model depends not much on the flux of the bathtub. However, the *details* of the recommended methods such as temperature or inflow flux, are supposed to have some slight changes in a smaller bathtub.

Material Different material can cause differences in the coefficient k of thermal conduction, which is defined in Section 3.1.2. However, the thermal conductivity coeffi-

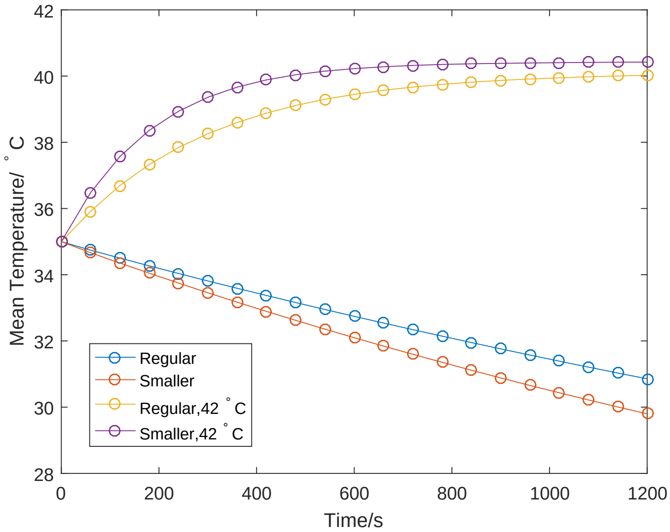

Figure 17: The mean temperature in regular and smaller bathtub

cient values of different materials are small, considering the fast loss through the water surface. So materials make little difference on the thermal conduction and we can infer that the strategy has small dependence on material based on the analysis above.

The Position of the Faucet We consider two kinds of the faucet's position: one is in the *upper corner*, while the other is in the *upper center* which is used in the bathtub studied before. We assume that the faucet would not in the same side of the person.(Who will let the burning water pouring on his or her back?) And we simulate the two conditions under **Method 42** in PDE-Toolbox.

The distribution of temperature with two different positions are shown in **Figure 18**, and the mean and std. of temperature under different conditions are shown in **Figure 19**. The two graphs show that if the faucet is in the upper corner, the water is at a lower temperature (under 38 °C), and it can cause more uneven temperature. If the faucet settles at the central side (upper center) of the bathtub, it is better for comfortable bath experience, since the whole bathtub is symmetrical and the heat is concentrated in the center of the bathtub.

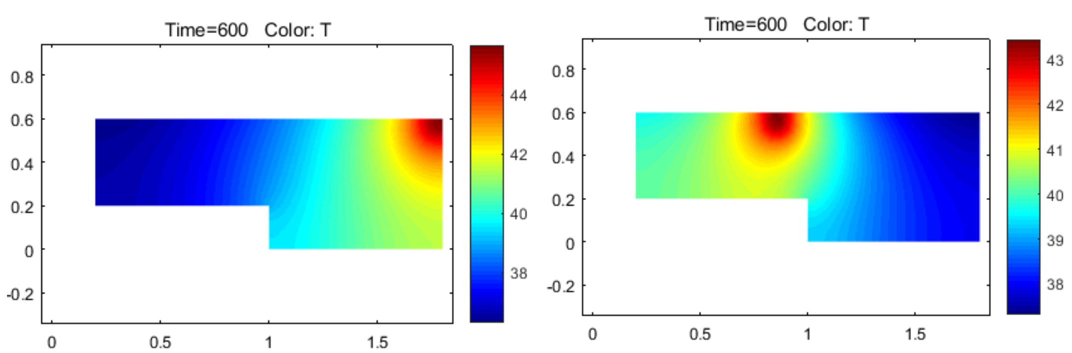

Figure 18: The temperature distribution with different position of faucet ($T = 600$s)

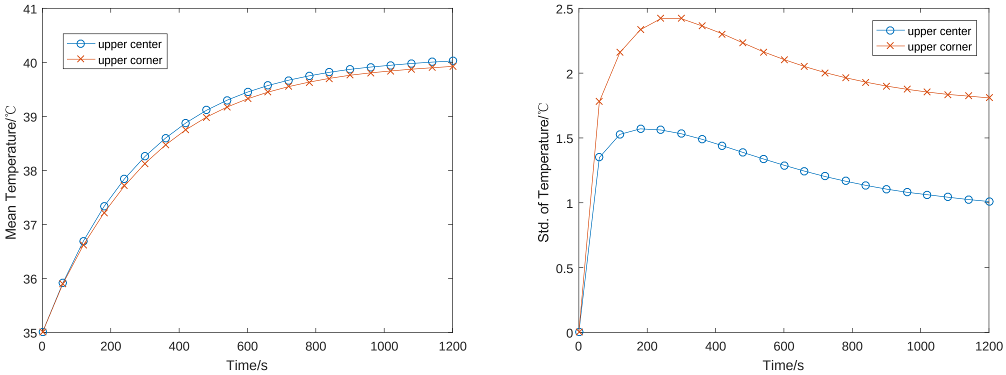

Figure 19: The mean and std. of temperature with different position of faucet

Generally speaking, the position of faucet makes a big difference on the strategy, and the upper center position of the faucet is more likely to provide a better bath experience with optimum and uniform temperature. So in fact, most of the bathtubs have their faucets on the central side, which is shown in **Figure 20**.

Figure 20: A typical bathtub with faucet on central side

`http://useful999.com/product/image/pics/LS-170B.jpg`

5.2 The Factors of the person

Different people may have different suitable strategy to take a comfortable bath, the main factors are shape which means the posture of the person, volume, temperature of the human body and motions. And we would analyse the extent which our strategy depends on them one by one.

Shape The effect caused by human body is not so much in the temperature changing process, so we think that the different of the human shape will not make a difference in the recommended strategies.

Volume Since the volume of the person affect the volume of water and further reflect the volume of the bathtub, we think that the effects of the volume of the person is similar to those of the volume of the bathtub. So following the previous results about the volume of the bathtub, we can infer that the strategy has small dependence on the volume of the person based on the analysis of the volume of the bathtub.

Temperature The temperature of the person would not have a great impact on the strategy for the following reasons:

1. The heat transfer coefficient between the human body and water is too tiny so that the heat transfer effects are inconspicuous;

2. The person is almost exothermic and at a constant temperature.

We can infer that the strategy may rely less on the temperature of the person.

Motions Since the motions of the person are too complex and various to analyse, we can quantify the effects of the motions through the changes of parameter. Based on the heat transfer and heat convection theories, the motions of the person would increase the heat transfer coefficient and the heat convection coefficient. It means that the motions accelerate heat transfer,and the temperature would decline rapidly and acutely. We can infer that the best strategy rely on the motions slightly, and more specifically, higher temperature of inflow will be necessary.

5.3 A Bubble Bath Additive

A bubble bath additive is a kind of surfactant which is compound that lower the surface tension (or interfacial tension) between two liquids, between a gas and a liquid.Surfactants can form a foam, facilitate the removal of dirt, and enable the mixture of water and oil. From the view about the reduction of the surface tension, adding some bubbles can increase the evaporation of water.

However, shampoo bubbles have very large surface areas with very little mass. The bubbles prevent the heat transmission a lot between the water and the outside environment. And compared to the increasing evaporation, the reduction of heat transmission occupies the dominant place. Therefore, a bubble bath can keep the temperature more stable than a normal bath. The bathing strategies change towards less flux, less time and lower temperature in a corresponding way.

Figure 21 shows the photograph of a bathtub with bubbles and the diagram of how bubbles slow down the cooling process.

5.4 New Consideration: Different Initial Temperature

In the original problem, the user heat up the water after the water in the bathtub has cooled down, so we set the initial temperature as 35°C. If we do not follow the

Figure 21: A bathtub with bubbles and the diagram about the effect of bubbles
source of the photograph:
`http://you.ctrip.com/travels/mauritius444/2355353.html`

original problem and suppose that the user turn on the faucet when the water is still at the optimum temperature, is there anything to be different?

To find out the answer, we assume that the initial temperature is 38 °C (The optimum temperature). We test two methods with continuous inflow:

1. **Method 40**: Continuous 40 °C inflow, with 100%-flux (0.25L/s).
2. **Method 42-little**: Continuous 42°C inflow, with 60%-flux (0.15L/s).

We test these two methods under such condition, and compare the results with the data under the former condition ($T_0 = 35$°C), Method 40. The mean temperature in these three situations are shown in **Figure 22**. We can see that Method 40 and Method 42-little can both keep the temperature, while these two methods are unrecommended when the initial temperature is 35°C. And from the change of the curves we can see that adding water earlier can keep the temperature more uniform. So we can see that when consumers enter the bathtub, maybe turning on the faucet directly is a better choice, comparing with turning it on when the water is not hot enough any longer.

6 Conclusion

In this study, we construct the Time-Space Based Temperature Model through PDE-Method to give consumers effective strategies of having a bath in the bathtub. We control two parameters, temperature and flux of inflow, to find out how these parameters influence the effect of the inflow. In the end, we give four recommended methods to get optimum and uniform temperature while not wasting too much water, which are **Method 42**, **Method 42×10**, **Method 45+42** and **Method 45+42-little**.

By examining the model with the change of some parameters, our model depends little on the shape/volume/material of the bathtub and the shape/volume/temperature of the person. If the position of the faucet is changed, it is proved that attaching the same demand on temperature by this bathtub will waste more water than our initial bathtub. In addition, a bubble bath will keep the temperature of the water in a certain extent because of the prevention of water's evaporation.

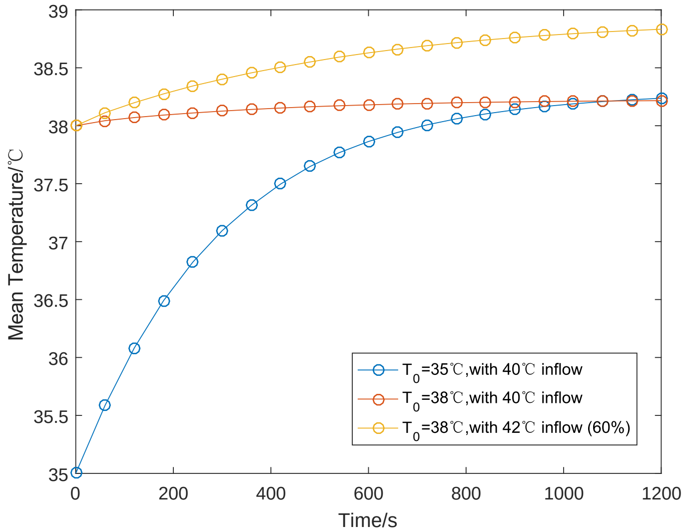

Figure 22: The mean temperature of the water, with different initial temperature and methods

Different methods may show different ranks of effect according to the users' demand, the conditions outside the bathtub and of the bathtub, and so on. However, according to our research in Section 5, these recommended methods are still useful under various conditions, if the details (e.g. the temperature and the time of period) are properly adjusted. So the users can choose and adjust the bath strategy based on the rules below:

Rule I *High temperature and large flux can make the water attach the optimum temperature earlier, meanwhile it may cause uneven temperature and waste of water.*

Rule II *Evenness comes from continuous inflow with appropriate temperature, a little higher than the optimum temperature in usual.*

Rule III *The best strategy to have an excellent bath is to heat up the water before it cool down.*

7 Strengths and Weaknesses

7.1 Strengths

- The model can reflect the change of temperature in every place of the bathtub, when the parameters of bathtub,people and water are input in the model.
- Through the analysis of the static temperature pattern with a continuous heat producer, our model successfully simulate the temperature's change with the water flow, with the help of our adjusting some parameters in order to correspond the real condition.

- Our model do not give only one strategy while bathing which may restrict the consumers' "freedom". According to different consumers's demand about the water, our model can give specific and effective strategies.

7.2 Weaknesses

- The model use the cross section of the bathtub to analyze the temperature of the water. However,the shape of the bathtub can't make sure that each section is the same,and with the consideration of different people's shape, there must be some deprivation of our results.

- Some parameters in our model (such as the thermal conductivity between two materials) are adjusted several times by using PDE-toolbox in the MATLAB for there is no research that give these parameters certain values. The value of these parameters in our model may have some deprivation from the real value.

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