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Judge's Commentary: The Outstanding Steiner Tree Papers

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The rectilinear minimum spanning tree problem in this year's competition is both intrinsically interesting and of practical importance. As such, it evinced a spectrum of responses which revealed the creative potential of inquisitive undergraduate students. Judging so many fine papers, therefore, was a challenging task. Broadly speaking, however, four factors distinguished the exceptional papers from those that were merely very good: maturity, respect for the problem, generality, and flexibility.

Maturity manifested itself in straightforward ways, such as a review of the relevant literature and a presentation of professional quality in both form and writing as well as mathematical substance. More importantly, the maturity of the superior teams was apparent in their approach to the problem. They fully understood the problem (evidenced, for example, by recognizing the differences between the Euclidean and rectilinear minimum spanning tree problems); they explicitly listed the assumptions underlying their analysis; they formally proved intermediate results; they communicated their intuition for the problem and their proposed solution(s); and they forthrightly described the limitations of their approach.

The best papers were also characterized by a healthy respect for the problem. Other papers spoke of "efficient algorithms for finding the optimal solutions." The better ones recognized that the rectilinear minimum spanning tree problem is NP-complete and were correspondingly more circumspect in their claims. When optimality is elusive, bounding arguments are a natural recourse. The relatively few teams that used bounding arguments distinguished themselves by doing so.

The utility of solutions often is decided by their generality. Some teams recognized and addressed the difficulty of generalizing their procedures to larger problems. Others overlooked the issue; still others offered only manual procedures that were not formally specified or relied extensively on human judgment or image-processing skills and hence could not easily be implemented on a computer for larger applications. Similarly, some teams embarked upon brute-force exhaustive searches. The clever ones pruned their search beforehand.

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Flexibility often characterizes genius, and such was the case with this contest. Many teams found one procedure for obtaining a rectilinear spanning tree, documented it, and submitted their solution. More-creative teams developed two or more approaches and compared their merits. The best teams' comparisons included a sensitivity analysis with respect to both problem parameters and problem size.

It is worth noting that the four factors that differentiated various teams' performances do not represent hoops through which they must leap to win a prize; there is no such "formula for success" or "checklist of desiderata." Rather, they are signatures of quality that become most visible after the judging when one looks back over the field of competitors. Such characteristics help teams win contests, and this contest wins inasmuch as it helps stimulate such characteristics.

About the Author

Jonathan P. Caulkins did his undergraduate work at Washington University's School of Engineering and Applied Science, where he participated in the MCM. His teams' papers were rated Outstanding in the first two competitions, in 1985 and 1986.

Jon went on to earn a masters in electrical engineering and computer science and a doctorate in operations research from MIT. Now he is an assistant professor of operations research and public policy at Carnegie Mellon University's Heinz School of Public Policy and Management, where his research focuses on developing mathematical models of illicit drug markets.

Jon was an associate judge for the Steiner Tree Problem.