Modeling Telephony Energy Consumption

Amrish Deshmukh
Rudolf Nikolaus Stahl
Matthew Guay

Cornell University
Ithaca, NY

Advisor: Alexander Vladimirsky

Summary

The energy consequences of rapidly changing telecommunications technology are a significant concern. While interpersonal communication is ever more important in the modern world, the need to conserve energy has also entered the social consciousness as prices and threats of global climate change continue to rise. Only 20 years after being introduced, cellphones have become a ubiquitous part of the modern world. Simultaneously, the infrastructure for traditional telephones is well in place and the energy costs of such phones may very well be less. As a superior technology, cellphones have gradually begun to replace the landline but consumer habits and perceptions have slowed this decline from being an outright abandonment.

general and sp backgroud

To evaluate the energy consequences of continued growth in cellphone use and a decline in landline use, we present a model that describes three processes—landline consumption, cellphone consumption, and landline abandonment—as economic diffusion processes. In addition, our model describes the changing energy demands of the two technologies and considers the use of companion electronics and consumer habits. Finally, we use these models to determine the energy consequences of the future uses of the two technologies, an optimal mode of delivering phone service, and the costs of wasteful consumer habits.

consideration of solving the problem and the general introduction to the step

---

The UMAP Journal 30 (3) (2009) 353–365.

Introduction

general and
sp backgroud

The telephone has become a fundamental part of our social fabric. In the past couple of decades, we have seen a shift from fixed landline telephones, generally one per household, to individual ownership of cellphones. We attempt to determine the impact of this change on American energy consumption.

The factors that go into accurately modeling telephony energy consumption are complex. We need to take into account also the energy consumption of peripheral devices, such as answering machines for landline phones and chargers for cellphones. Moreover, landline phones are not a uniform product. Cordless phones consume considerably more energy than their corded counterparts. Likewise, the total energy cost of cellphone usage is complicated by such factors as recharging, replacement, and battery recycling. Our model takes all of these factors into account, and additionally attempts to use the limited real-world data available to chart the changes in each of these factors over time.

Perhaps the most complex factor to model is adoption of technological innovations in a population. This is relevant not only to landline adoption and cellphone adoption, but additionally de-adoption of landline phones in the face of cellphone usage can be considered an independent innovation and modeled accordingly. Research into the phenomenon indicates that it can be modeled globally by the differential equation

$$\frac{dP}{dT} = rP\left(1 - \frac{P}{K}\right),$$

where P is the proportion of the population that has adopted the innovation at time t, r is the adoption rate, and K is the saturation point for the innovation.

Using the descriptions of such a model, we arrive at an accurate fit to available data and can predict future demand for cellphones and landlines. Determining the cost for these respective technologies we arrive at the total energy burden. Briefly, we explore how this question relates to the energy consumption of other household electronics, and how much waste is generated therein. Additionally, we explore the caveat that technological development has been and continues to be wildly unpredictable, and the consequences of this reality.

A separate question is how best to distribute landline and cellphones throughout a population committed to neither, so as to minimize energy consumption while not violating social preference. This problem is explored through an optimization with respect to energy usage, in which we discover that a country, here a "Pseudo-U.S.," which supports a cellphone-only communicative infrastructure minimizes its total energy consumption, and also does not violate social demand for novel technologies. Finally, we estimate the total energy consumption by such a nation over the next 50 years.

Model Overview

We examine two approaches to modeling technology diffusion through a population. The first attempts to gauge technology adoption at the household level and aggregate these results to model global trends. However, this approach is unsuccessful, and we explain why. The second approach models technology adoption at the global level; it

- accurately models past and present telephony energy consumption,
- makes future predictions of cellphone saturation and landline de-adoption consistent with previous technological replacement paradigms, and
- encompasses a broad range of pertinent factors in telephony energy consumption.

Model Derivation

Adoption of Innovations

Our model describes U.S. usage rates for landlines and cellphones as three diffusive innovation curves. Consider the adoption of an innovation Y. At small times after the development of this innovation, adoption of Y throughout a population is minimal. As the innovation spreads, demand increases until a saturation point is reached. Thus, the spread of Y throughout a population is proportional to its synchronous prevalence, but is checked from exponential growth by an upper bound to its saturation in a population. At its simplest, we can model this as

$$\frac{dY}{dt} = Y(1 - Y).$$

Of course, adoption is not uniform between different technologies, and saturation rates likewise vary. By introducing constants r for adoption rate and S for saturation rates, we can refine our model to

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{K}\right),$$

which has a solution in form of the logistic equation. Therefore, for each of the processes we assume a model of the form

$$Y(t) = \frac{A}{1 + Be^{-Ct}}.$$

The sigmoidal form of adoption processes is well-known and has been observed in the specific case of cellphone adoption and wireless-only lifestyle adoption.

Proceeding globally, we initially model the consumption of telephones from their inception by the equation:

$$p_l(t) = A\left(\frac{1}{1+Be^{-C(t-D)}} + \frac{1}{1+Ee^{-F(t-G)}} - 1\right), \tag{1}$$

where the D and G parameters are chosen so that time is shifted relative to the onset of cellphone adoption. This expression is essentially the addition of two sigmoid curves. The first models the adoption of the landline phone as a new innovation; and the second models the de-adoption of landlines as an independent innovation of a "wireless-only" lifestyle, which has a subtractive effect total landline usage.

Likewise, the consumption by cellphones is given by

$$p_c(t) = \frac{J}{1+e^{-K(t-L)}}, \tag{2}$$

where again L is a time shift chosen to make the model coincide with cellphone adoption.

We tried to model this at the microscopic level, but that proved to be an intractable approach. From census data, the number of households with m members over the course of history is readily available [U.S. Census Bureau 2007]. Equally accessible are the rates of penetration and average costs of cellular and landline communications penetration [U.S. Census Bureau 2001; Eisner 2008]. With this abundance of data, one may be tempted to propose an econometric forecast of telephony usage that is driven by the marginal cost-benefit analysis that a household performs. However, determining the functional form that defines the behaviors that are muddled by habits and irrationality are troubling. When reduced to a first-order approximation, such a model still requires the calibration of numerous parameters [Koyck 1954]. After attempting such an approach several times, we abandoned it. We believe the above model captures the data equally well without making undue assumptions.

Energy Cost of Landlines

Together these two functions model three processes: landline adoption, wireless adoption, and wireless only adoption. Additionally, they describe the long-term behavior of these processes as they reach a steady state. To approach the question of annual energy consumption by telephony products, we combine these functions with models for energy expenditure by landline phones and their peripherals, as well as cellphones and their peripherals. The formula for energy consumption by landline phones and peripherals is

$$E_l(t) = Pp_l h(\pi_a e_a + \pi_b e_b + \pi_c e_c + \pi_d e_d).$$

Table 1 delineates the variables and their explanations. The time variable t is normalized so that $t = 0$ denotes 1960.

Table 2.
Variables and their meanings.

Variable	Description
$P(t)$	Population of U.S. in year t
$p_l(t)$	Landlines per person in U.S.
$h(t)$	Handsets per landline
$\pi_a(t)$	Percentage of landline owners with corded phones
$e_a(t)$	Yearly Energy Consumption (YEC) (kWh) by corded phones
$\pi_b(t)$	Percentage of landline owners with cordless phones
$e_b(t)$	YEC by cordless phones
$\pi_c(t)$	Percentage of landline owners with combination cordless phone/answering machines
$e_c(t)$	YEC by combination cordless phone/answering machines
$\pi_d(t)$	Percentage of landline owners with separate answering machines
$e_d(t)$	YEC by separate answering machines

Due to a lack of relevant data, we make several assumptions:

- All yearly energy consumption functions are constant over time. Because corded phones draw their energy solely from phone lines, there is little room for variation in their power draws, so this at least seems reasonable. However, answering machines, cordless phones, and combinations of the two do not have this restriction, and it seems likely that they are becoming more energy efficient with time. However, no data were available to support this hypothesis, so we fixed YEC based on available sources.
- The adoption of cordless vs. corded phones and answering machines no doubt follows its own sigmoidal curve, but again no data are available. So the variables $h, \pi_a, \pi_b, \pi_c, \pi_d$ are all modeled as first-order linear approximations.

Regardless, results produced by the model agree well with available data for energy consumption.

Energy Cost of Cellphones

The energy cost for cellphones can be modeled as

$$E_C(t) = Pp_c(E_{c1} + E_{c2}),$$

where

$$E_{c1}(t) = f_C(C_{\text{charge}} t_{\text{charge}} + C_{\text{standby}} t_{\text{standby}})$$

and

$$E_{c2}(t) = R_{\text{cell}} R(t).$$

Table 2 describes each relevant variable.

Table 2.
Variables and their meanings.

Variable	Description
$P(t)$	Population of U.S. in year t
$p_c(t)$	Number of cellphones per person
E_{c1}	YEC by cellphones and chargers
E_{c2}	YEC by cellphone recyclers
f_C	Frequency of cellphone charging
C_{charge}	Charger wattage during charging
t_{charge}	Daily charger time spent charging
C_{standby}	Charger wattage during standby
t_{standby}	Daily charger time spent in standby
R_{cell}	Energy needed to recycle one cellphone battery
$R(t)$	Percentage of cellphones recycled in year t

The immediate contributions to cellphone energy consumption are charging the phone and leaving the charger plugged in with no phone attached. It is difficult to find data on cellphone charging frequency. Rosen et al. [2001] argue that people charge their phone 50 times each year at their residence (noting that many people charge the phone in their car); but this figure seems very low. Newer phones with a multitude of features require more-frequent charging. Since charging the cellphone has developed into a habit for most people, we assume that people charge the phone every night and keep the charger attached to an outlet all the time.

Rosen et al. [2001] observe that the average time to charge a cellphone is 2 hrs, which seems low in comparison to other data, which suggest 3–4 hrs to charge to 80% and an additional 8 hrs to charge to 100%. However, a phone charged every night is unlikely to have a nearly-empty battery. We assume that the overnight charging does not affect the 2-hr charging time. That 50% of cellphone batteries are lithium-ion batteries, which do not allow for overcharging, justifies this assumption [Fishbein 2002; Rosen et al. 2001]. Once a lithium-ion battery is charged, the power drawn differs negligibly from that when no phone is connected to the charger [Rosen et al. 2001]. Therefore, we feel justified in adopting Rosen et al.'s statistic.

To model the energy cost of recycling used cellphone batteries, we consider the batteries to be recycled by the Rechargeable Battery Recycling Corporation, justified by its significant market share and the fact that it recycles batteries in the U.S. [Office Depot 2004].

Energy Optimization

Given the above functions for energy costs for cellular and landline telephone usage, we can optimize energy consumption. A Pseudo U.S. with the approximate size of the U.S. would likely have a similar distribution of household size.

Let H_m be the number of households with m members and l_m the fraction of households with m members that have landline service. If we assume that the communication needs of every family are satisfied by either having a landline or by each member possessing a cellphone, the numbers of required cellphones T_c and landline phones T_l can be calculated as

$$T_l = \sum_{m=1}^{7} l_m H_m, \qquad T_c = \sum_{m=1}^{7} m(1 - l_m) H_m.$$

We believe that in the absence of a landline, members of a household will not share cellphones.

The total telephony energy demand of the proposed plan for Pseudo U.S. is

$$E(t) = E_l(t) + E_c(t).$$

Using only landlines would minimize the number of telephone units required; however, landline phones and their companion technologies are much less energy-efficient than cellphones. Using only cellphones would maximize the number of telephone units required; and though the energy cost per unit is reduced, the overall increase in units may have deleterious consequences. Therefore, we optimize the variables l_m to yield the best communications strategy from an energy perspective.

We could modify the above summations to consider roles played by cellphones that are not achievable by a landline. For example, suppose that a single landline cannot serve a large family. If n is the number of people a single landline can serve in a household, we may assume that a family of m with one landline will need to purchase $(m - n)$ cellphones. Then we have

$$T_c = \sum_{m=1}^{7} m(1 - l_m) H_m + \sum_{m=n+1}^{7} l_m H_m (m - n),$$

where the second term gives the fraction of families too large to be served by a single landline. Implicit in this formula is an assumption that no family obtains a second landline. This is reasonable, since the average number of landlines per household in the U.S. is only 1.118 [Eisner 2008].

Likewise, we could further complicate the cost function by asserting that not every family member requires a cellphone if a landline is absent.

However, we find that such a modification does not enrich the conclusions of our optimization.

Results

Energy Consumption

Using the above information, we create an energy consumption function:

$$E(t) = E_c(t) + E_l(t).$$

To make this specific, we must estimate parameter values for $A, \dots, G$ in **(1)** and **(2)**. Using an optimization algorithm described in the methods section below, we arrive at the conclusions in **Table 3**.

Table 3.
Values of parameters, as fitted from data in Eisner [2008].

Parameter	Value
A	1.1263
B	1.0924
C	0.0423
D	27
E	0.0109
F	0.1587
G	30

Moreover, functions can be described for parameters for E_l and E_c. **Tables 4** and **5** give values for the variables and parameters in **Tables 1** and **2**.

Table 4.
Values for variables in **Table 1**. Source: Rosen et al. [2001].

Variable	Value
$P(t)$	Population growth as predicted by the Census Bureau
$p_l(t)$	As defined in **(1)**
$h(t)$	$1.89\text{E}^{-3}t + 1.076$, $t \leq 40$; $-1.20\text{E}^{-3} + 1.152$, $t > 40$
$\pi_a(t)$	$1 - \pi_b(t) - \pi_c(t)$
$e_a(t)$	20 kWh
$\pi_b(t)$	max(0,1.45E$^{-2}t - 1.45\text{E}^{-1}$), $t \leq 40$; .44,$t > 40$
$e_b(t)$	28 kWh
$\pi_c(t)$	max(0,1.07E$^{-2}t - 1.07\text{E}^{-1}$), $t \leq 40$; .32,$t > 40$
$e_c(t)$	36 kWh
$\pi_d(t)$	max(0,2.31$^{-2}t - 2.31\text{E}^{-1}$), $t \leq 40$; .69,$t > 40$
$e_d(t)$	36 kWh

Table 5.
Values for variables and parameters in **Table 2**. Source: Rosen et al. [2001].

Variable	Value
$p_c(t)$	as defined in **(2)**
E_{c1}	$0.365(4 \cdot 2 + 0.6 \cdot 24)$ kWh
E_{c2}	$-0.0283e^{\frac{-(t-1993)}{17.1573}} + 0.00037$ kWh
f_C	365
C_{charge}	4 W
t_{charge}	2 hr
C_{standby}	0.6 W
t_{standby}	24 hr
R_{cell}	0.0037 kWh
$R(t)$	$-7.639e^{-(t-1993)/17.1573} + 0.0999$

From our model, we expected that by 2050 cellphones will have completely replaced landlines in the U.S. Thus, we estimate steady-state energy consumption as $E(90) = 2.99$ TWh/yr, equivalent to 1.7 million bbl/yr of oil.

Energy Optimization Results

From our optimization results for the distribution of telephone types in the Pseudo U.S., we find that it is almost always preferable to have a cellphone-only state, in terms of energy efficiency. Even assuming a landline can service an unlimited number of people in a household, our optimization finds that only for families of size 7 or larger it is energy-efficient to own a single landline and peripherals in place of a cellphone for each family member.

The cost of leaving cellphone chargers on standby when not active would amount to approximately 62% of the total YEC, or 862,000 bbl/yr of oil.

Energy Waste by Other Household Electronics

We also discuss the impact of leaving devices plugged in when the device is not in use. From Rosen et al. [2001], we adopt the following approach. First, we investigate the average wattage used in standby mode by the devices under consideration and the time spent in standby mode, respectively. Then we find saturation and penetration values to find the total energy expenditure in the U.S. We consider computers, TVs, set-top boxes (digital and analog), wireless set-top boxes, and video-game consoles.

We take the data for the three types of set-top boxes and the video-game console from Rosen et al. [2001]. Furthermore, the average American spends an average of 4.66 hours watching television and 4.4 hours using a computer every day [Bureau of Labor Statistics 2009]. Average power drawn by computers and television sets turns out to be 4 and 5.1 W [Rosen et al. 2001]. The first two columns of **Table 6** give our data set. We use that information, along with saturation rates and household penetration rates [Eisner 2008], to arrive at the figures in the third column.

Table 6.

Data used for power consumption of household electronics. Source: Thorne and Suozzo [1998].

Device	Standby time (proportion)	Power drawn in use (W)	Standby power consumption (TWh/yr)
Set-top box, analog	.78	10.5	3.2
Set-top box, digital	.78	22.3	0.6
Wireless receiver	.78	10.2	1.4
Video-game console	.98	1.0	0.5
TV	.80	5.1	10.3
Computer	.81	4	3.3

We conclude that wasteful energy expenditure due to appliance standby in the U.S. consumes approximately 11.4 million bbl/yr of oil.

Future Predictions

Assuming moderate economic and population growth, **Table 7** shows results for the Pseudo U.S., using population projections from the U.S. Census Bureau [1996a; 1996b].

Table 7.

Projected energy use in Pseudo U.S.

Year	Energy ($\times 10^6$ bbl oil)
2010	1.14
2020	1.24
2030	1.66
2040	1.77
2050	1.89

However, we believe that such an analysis is of limited use. Predicting the future of so many variables for a 50-year period is extremely difficult, especially in the realm of technology, where it is commonplace for innovations to change social paradigms. For example, consider an attempt in the 1950s to model the growth of computer usage. Any such attempt would have been unlikely to foresee personal computers, the Internet, or cellphones (which today are rapidly replacing many of the functions of personal computers). Likewise, the energy cost of cellphones may vary greatly due to changes in technology: Social awareness about energy efficiency may drive them to ever-lesser energy consumption, but also they may gain additional features or be replaced by miniaturized computers that result in more energy consumption.

Conclusions

Recommendations

From and energy perspective, we find that it is more efficient to abandon landlines in favor of cellphones. This suggestion is reinforced by the model prediction, which suggests an elimination of landlines in the near future by consumer adoption of a wireless-only lifestyle.

Finally we find that the waste generated by chargers on standby (i.e., not charging a device) are a significant source of energy waste. We therefore advocate that efforts be made to forgo convenience and unplug devices when in standby.

Model Strengths and Weaknesses

Strengths

- The model reproduces sigmoidal innovation-adoption behavior without making undue assumptions about the underlying processes.
- The model incorporates a broad span of indirect sources of energy consumption: battery recycling, commuters with cellphones, landline companion technologies.

Weaknesses

- Our model captures only global adoption behavior. This exclusion of underlying behavior is a detriment in capturing deviations from the standard behavior, as was exemplified by the underestimation in the 1990s, when economic expansion may have driven telephone adoption.
- Due to lack of data, the model relies on interpolation of data related to cellphone and landline energy costs.
- For simplicity, the model excluded other possible communications technologies. As noted earlier, paradigm shifts in technology are commonplace yet hard to predict.
- The perspective excludes other communications technologies.
- The model fails to capture any benefit of landlines not provided by cellphones. It may be that landlines are associated with a certain degree of security, which mediates the current prediction that landlines will be completely abandoned.

Future Work

- We believe that a model at the microscopic level that takes into consideration consumer perceptions and habits, in addition to economic data, would perform the best.
- We also believe with Bagchi [2008] that modeling cellphones and landlines as more directly competing products with reference to economic data would provide better data fits and predictions.
- The analysis is limited to the household level. Landline phones will persist in many businesses, and we believe that this persistence will be a significant factor in energy consumption.

References

Bagchi, Kallol. 2008. The impact of price decreases on telephone and cell phone diffusion. *Information and Management* 45 (2008): 183–193.

Eisner, James. 2008. Table 16.2: Household Telephone Subscribership in the United States. In Trends in Telephone Service (March 2008), Federal Communications CommissionIndustry Analysis and Technology Division, Wireline Competiton Bureau. `http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-284932A1.pdf` .

Fishbein, Bette. 2002. *Waste in the Wireless World: The Challenge of Cell Phones*. New York: INFORM, Inc. `http://www.p2pays.org/ref/19/18713.htm` .

Koyck, L.M. 1954. *Distributed Lags and Investment Analysis*. Amsterdam: North-Holland.

Office Depot. 2004. Office Depot launches free cell phone and rechargeable battery recycling program. `http://mediarelations.officedepot.com/phoenix.zhtml?c=140162&p=irol-newsArticle&ID=674008` .

Rosen, Karen, Alan K. Meier, and Stephan Zandelin. 2001. Energy use of set-top boxes and telephony products in the U.S. `http://repositories.cdlib.org/cgi/viewcontent.cgi?article=1950&context=lbnl` .

Thorne, Jennnifer, and Margaret Suozzo. 1998. Leaking electricity: Standby and off-mode power consumption in consumer electronics and household appliances. `http://www.aceee.org/pubs/a981.htm` .

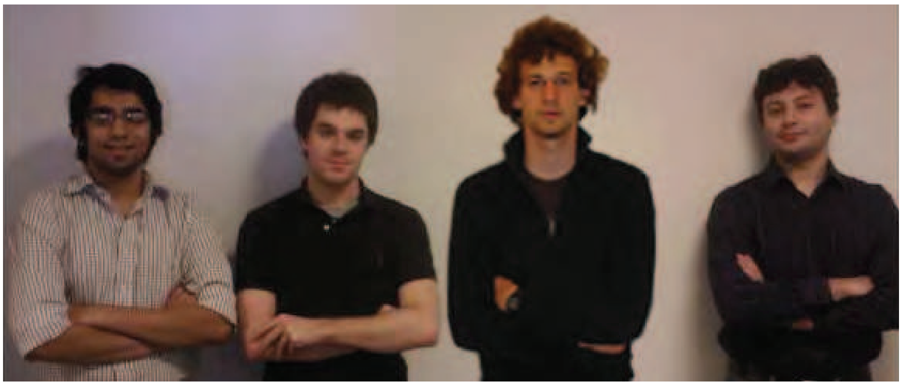
U.S. Bureau of Labor Statistics. 2009. American time use survey summary—2008 results. `http://www.bls.gov/news.release/atus.nr0.htm` .

U.S. Census Bureau.. 1996a. Projections of the number of households and families in the United States: 1995 to 2010. Current Population Reports P25–1129. `http://www.census.gov/prod/1/pop/p25-1129.pdf` .

________. 1996b. Population projections of the United States by age, sex, race, and Hispanic origin: 1995 to 2050. Current Population Reports P25–1130. http://www.census.gov/prod/1/pop/p25-1130.pdf .

________. 2001. Construction and housing. Chapter 25 in Statistical Abstract of the United States: 2000. http://www.census.gov/prod/2001pubs/statab/sec25.pdf .

________. 2007. U.S. households by size, 1790–2006. Reproduced at http://www.infoplease.com/ipa/A0884238.html .



Amrish Deshmukh, Matt Guay, Niko Stahl, and advisor Alexander Vladimirsky.

Judges' Commentary: The Outstanding Cellphone Energy Papers

Marie Vanisko
Dept. of Mathematics, Engineering, and Computer Science
Carroll College
Helena, MT 59625
mvanisko@carroll.edu

General Remarks

As in past years, the diverse backgrounds of the undergraduate participants yielded many interesting modeling approaches to the stated problem. The judges assessed the papers on the breadth and depth of analysis for all major issues raised, on the validity of proposed models, and on the overall clarity and presentation of solutions.

The Executive Summary is often still below par; it should motivate the reader to read the paper. It must not merely restate the problem, but indicate how it was modeled and what conclusions were reached, without being unduly technical.

Assumptions must be clearly stated and justified where appropriate. Some teams overlook important factors and/or make unrealistic assumptions with no rationale. It should be made clear in the model precisely where those assumptions are used.

Graphs need labels and/or legends and should provide information about what is referred to in the paper. Tables and graphs that are taken from other sources need to have specific references. Failure to use reliable resources and to properly document those resources kept some papers from rising to the top. The best papers not only list trustworthy resources but also document their use throughout the paper.

The UMAP Journal 30 (3) (2009) 403–407.

Requirements and Selected Modeling Approaches

The Cellphone Problem involved the "energy" consequences of the cellphone revolution, and five Requirements were delineated. To receive an Outstanding or Meritorious designation, teams had to address issues raised in each of these Requirements. Additionally, Outstanding papers considered both wireless and wired landlines and the infrastructure to support cellphones and landlines.

Requirement 1

Teams were first asked to estimate the number of U.S. households in the past that were served by landlines and also to estimate the average size of those households. They were then to consider the energy consequences, in terms of electricity use, of a complete transition from landline phones to cellphones, with the understanding that each member of each household would get a cellphone.

To address this problem, the energy used by current landlines had to be considered. Whereas corded landline phones use relatively little electricity, the same cannot be assumed about cordless landline phones. The top papers researched this issue and arrived at documented estimates of the number of corded vs. cordless landline phones and the average number of each per household. This led to a more realistic appraisal of the energy used by the landline phone system.

With regard to cellphones, teams that rose to the top considered the infrastructure necessary—for example, the building of numerous additional communication towers if cellphones are to replace landline phones completely. This was of special importance in the analysis of the transitional phase. Also, the varying amount of electricity required by different types of cellphones was a consideration in the transitional and steady-state phases.

Interesting models were constructed for the transitional phase of the cellphone "takeover." Some teams considered the spread of cellphones as the spread of a disease and used the Verhulst model for logistic growth, using the population of the U.S. as the carrying capacity and estimating the rate of growth of cellphones from published reports on the growth of cellphone use. Other teams generalized this to an SIR model or used the Lotka-Volterra predator-prey model, with cellphones as the predators and landline phones as the prey. A few used the competing-species model. The judges looked very favorably upon models for which sufficient rationale was given as to why that model might be appropriate in this circumstance. Interpretation of the parameters and solutions as they applied to the problem at hand was essential.

Many papers ignored the transition phase and only considered the energy comparison for the steady state in order to determine the energy consequence. Some teams merely talked their way through the issues and did not construct a mathematical model. After estimating energy costs associated with landline phones and cellphones, many teams used linear equations to model the total costs associated with the numbers of phones.

Requirement 2

Teams were asked to consider a "Pseudo U.S."—a country similar to the current U.S. in population and economic status, but with neither landlines or cellphones. They were to determine the optimal way, from an energy perspective, to provide phone service to this country. The teams were also to take into account the social advantages that cellphones offer and the broad consequences of having only landlines or only cellphones.

Once again, consideration of the infrastructure for phones was important. In addition to landline phones and cellphones, many teams considered the VoIP (Voice over Internet Protocol) communication option. Not every team that considered VoIP took into account the costs for laying the cables; some assumed that such cables were already in place (a questionable assumption). However, failure to consider the VoIP method of phone service may have kept a Meritorious paper from becoming an Outstanding paper. If one were to assume that households would already have one or more computers with Internet access, the energy costs associated with VoIP would be quite small.

In terms of finding the optimal way to provide phone service from an energy perspective, some teams used linear programming, using the costs determined in Requirement 1 and quantifying in various ways the social advantages of cellphones, as well as the preference for landlines in certain circumstances. Other teams used AHP (Analytic Hierarchy Process), which worked well to get parameters used in the optimization routine but did not work as an optimization tool in itself. Teams that considered the advantages and disadvantages of various phone types not just for individuals, but for businesses also, demonstrated a thoroughness that was commendable. Another factor that some teams considered was the number of children under 5 who would have no need for cellphones.

Requirement 3

This was an extension of Requirement 2, asking teams to consider the electricity wasted when cellphones are plugged in that do not need charging and when chargers are left plugged in after the phone is removed. Teams were to continue to assume that they were in the Pseudo U.S. and were to interpret energy wasted in terms of barrels of oil used.

To determine the amount of energy wasted, teams had to first estimate the number of hours that a "typical" cellphone charger is in the state of charging a phone, left plugged into a phone not in need of charging, and left plugged in when the phone is not connected to it. Some teams did their own informal surveys, but better estimates were arrived at from publications and surveys.

In some papers, probability distributions were used to describe this behavior, but use of such distributions was not always justified.

Teams that were more comprehensive took into account the fact that some cellphones and chargers use less power than to do others, based on brands, age, and capabilities of the phones and chargers. Assuming that all electrical energy is generated by oil, translating kilowatts of energy into barrels of oil used was a straightforward transformation.

Requirement 4

This requirement extended the concepts in Requirement 3 and asked teams to estimate the amount of energy wasted by all electronic chargers. Since this question was very open-ended, contest papers showed a wide variety of estimates for the energy wasted in terms of barrels of oil. The top teams estimated the average hours that appliances are left plugged in, charging and not charging, and also the number of hours that chargers are plugged in without the appliance.

More-comprehensive papers considered many other kinds of electronic devices and by comparison showed that the amount of energy wasted by cellphones is relatively small.

Requirement 5

For this part, students were to consider the population and economic growth of the Pseudo U.S. for the next 50 years and predict energy needs for providing phone service based on their analysis in the first three Requirements. Predictions were to be interpreted in terms of barrels of oil used.

Papers needed to consider both economic growth and population growth in order to estimate energy needs in the future. Various types of regression fits were applied to historical population data and economic data such as GDP. Using earlier estimates of energy requirements, coupled with the regression equations from historical data, predictions were made for the amount of energy used every decade for the next 50 years. Some teams predicted greater efficiency in future phones and the development of chargers that would use less electricity.

Papers showed estimates for the number of barrels of oil used on a per-day basis or perhaps on a per-year basis. There was no one right answer, and answers given depended on assumptions made at the start. Some papers contained graphs displaying future values but did not give tables. A table together with a graph is a better way to display information.

Concluding Remarks

Mathematical modeling is an art that requires considerable skill and practice in order to develop proficiency. The big problems that we face now and in the future will be solved in large part by those with the talent, the insight, and the will to model these real-world problems and continuously refine those models.

The judges are very proud of all participants in this Mathematical Contest in Modeling, and we commend you for your hard work and dedication.

About the Author

Marie Vanisko is a Mathematics Professor Emerita from Carroll College in Helena, Montana, where she has taught for more than 30 years. She was also a visiting professor at the U.S. Military Academy at West Point and taught for five years at California State University Stanislaus. While in California, she co-directed MAA Tensor Foundation grants on Preparing Women for Mathematical Modeling, a program encouraging more high school girls to select careers involving mathematics, and was also active in the MAA PMET (Preparing Mathematicians to Educate Teachers) project. Marie serves as a member of the Engineering Advisory Board at Carroll College, is on the advisory board for the Montana Learning Center for mathematics and science education, and is a judge for both the MCM and HiMCM COMAP contests.

Judges' Commentary: The Fusaro Award for the Cellphone Energy Problem

Marie Vanisko
Dept. of Mathematics, Engineering, and Computer Science
Carroll College
Helena, MT 59625
mvanisko@carroll.edu

Peter Anspach
National Security Agency
Ft. Meade, MD
anspach@aol.com

Introduction

MCM Founding Director Fusaro attributes the competition's popularity in part to the challenge of working on practical problems. "Students generally like a challenge and probably are attracted by the opportunity, for perhaps the first time in their mathematical lives, to work as a team on a realistic applied problem," he says. The most important aspect of the MCM is the impact that it has on its participants, and, as Fusaro puts it, "the confidence that this experience engenders."

The Ben Fusaro Award for the 2009 Cellphone Energy problem went to a team from the Lawrence Technological University in Southfield, MI. This solution paper was among the top Meritorious papers and exemplified some outstanding characteristics:

- It presented a high-quality application of the complete modeling process.
- It demonstrated noteworthy originality and creativity in the modeling effort to solve the problem as given.
- It was well written, in a clear expository style, making it a pleasure to read.

The UMAP Journal 30 (3) (2009) 409–412.

The Problem: Energy Consequences of the Cellphone Revolution

The Cellphone Energy Problem involved many facets of the "energy" consequences of replacing landlines with cellphones and five Requirements were delineated. Teams had to address issues raised in each of the five Requirements. Additionally, the best papers considered both wireless and wired landlines and the infrastructure to support cellphones and landlines.

The Lawrence Technological University Paper

Assumptions

The team began with a page of assumptions, most of which were well-founded and enabled them to determine parameters in their models. However, certain assumptions made were unrealistic and these led to results that did not reflect the real-world situation. In particular, in the eyes of the judges, assuming that all landline phones are cordless was a serious shortcoming that greatly impacted the issue of energy use. Furthermore, while the team did address the issue of infrastructure, the assumption that infrastructure for cellphones is equal to that for landline phones seemed to ignore the need for the large number of additional communication towers if cellphones were to replace landlines.

Requirement 1: Mathematical Formulation for the Transition

In Requirement 1, teams were to consider the energy consequences in terms of electricity utilization of a complete transition from landline phones to cellphones, with the understanding that each member of each household would get a cellphone. The Lawrence Tech team shone in mathematically modeling this transition! For their first model representing the transition from landline to cellphones, the team used the basic logistic differential equation to model the rate of change in the number of cellphones over time. They used the total population as the carrying capacity and determined the intrinsic rate of growth of cellphones from published results. This was very well done, though references for the tables and graphs should have been included. The second model introduced was a predator-prey system of differential equations, and the team is to be commended on their clear statement of rationale for using this model, with cellphones causing the demise of landlines. However, this model quickly became complicated, so they headed "down a different route." And, once again, their rationale for the equations used and parameters arrived at was commendable.

In modeling the energy used by cellphones, the team considered three distinct models of cellphones and did a good job of researching the habits of individuals of different ages regarding talking times. The assumption they made that the average number of calls is directly related to the talk time per call might be questionable, but this was not considered a serious deficiency and it enabled them to estimate needed parameters in their model.

Requirements 2, 3, 4, and 5

Documented sources were used to estimate energy used for charging batteries, and these were translated into barrels of oil used. Energy usage comparisons were demonstrated for landline cordless phones and cellphones. This was taken forward into Requirement 2 and they seemed to conclude that the optimal mix of landline and cellphones would be the state where the same amount of energy was used by cordless landline and cellphones.

For Requirement 3, after gathering data on energy consumption by phone chargers, the team demonstrated an interesting comparison of energy consumed by daily vs. weekly charging and charger left plugged in or not, and from this they estimated the long-term consequences of avoiding wasteful practices in the charging of cellphones. The team introduced a percentage comparison of energy wasted by various charging methods.

Requirement 4 extended the concepts in Requirement 3 and asked teams to estimate the amount of energy wasted by all idle electronic appliances. Since this question was very open-ended, contest papers showed a wide variety of estimates for the energy wasted. The Lawrence Tech team limited themselves to the average hours that computers, televisions, DVD players/VCRs, and game consoles are left plugged in and the resulting annual oil consumption from such wasteful practices. A linear pattern of growth was projected up to 2059. More-comprehensive papers considered many more electronics and, by comparison, showed that the amount of energy wasted by cellphones is relatively small compared to many other electronic devices. Thus, when the team referred to cellphones as the "most energy consuming devices" in the Executive Summary, judges questioned the credibility of the paper.

For Requirement 5, students were to consider the population and economic growth of a Pseudo U.S. for the next 50 years and predict energy needs for providing phone service based on their analysis in the first three Requirements. Predictions were to be interpreted in terms of barrels of oil used. To their credit, the Lawrence Tech team had numerous appendices with data tables (but again without reference).

Recognizing Limitations of the Model

Recognizing the limitations of a model is an important last step in the completion of the modeling process. The students recognized that their model failed to look at technological changes, including advances in battery and cellphone technology. They also acknowledged that assuming that every member of a population has a cellphone puts cellphones into the hands of infants and ignores the fact that some individuals have more than one cellphone.

Conclusion

Although there were some deficiencies in Requirements 2–5, the quality of the mathematical modeling done in Requirement 1, coupled with the excellent use of resources to answer the questions posed throughout, made the Lawrence Technological University paper one that the judges felt was worthy of the Meritorious designation. The team is to be congratulated on their analysis, their clarity, and their use of the mathematics that they knew to create and justify their own model for the cellphone revolution problem.

About the Authors

Marie Vanisko is a Mathematics Professor Emerita from Carroll College in Helena, Montana, where she has taught for more than 30 years. She was also a visiting professor at the U.S. Military Academy at West Point and taught for five years at California State University Stanislaus. While in California, she co-directed MAA Tensor Foundation grants on Preparing Women for Mathematical Modeling, a program encouraging more high school girls to select careers involving mathematics, and was also active in the MAA PMET (Preparing Mathematicians to Educate Teachers) project. Marie serves as a member of the Engineering Advisory Board at Carroll College, is on the advisory board for the Montana Learning Center for mathematics and science education, and is a judge for both the MCM and HiMCM COMAP contests.

Peter Anspach was born and raised in the Chicago area. He graduated from Amherst College, then went on to get a Ph.D. in Mathematics from the University of Chicago. After a post-doc at the University of Oklahoma, he joined the National Security Agency to work as a mathematician.