How to Locate a Submarine by Detecting Changes in Ambient Noise

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Summary

We generate an artificial ambient noise field and give an algorithm for locating a submarine.

Our model for the ambient noise field allows any frequency range, but we use 30 kHz, where environmental sounds, such as surface activity and biologics, dominate. We assume a normal distribution of noise in the area of interest and that a submarine impedes measuring the noise beyond it.

Our recognition algorithm is simple: We look for contour lines on our ambient noise field, then for closed contours of the right size, and finally for the intensity patterns that match those of submarines. To help detect significant changes, we smooth the data. Since our algorithm is reasonably fast, it can also detect the changing locations of the center of the submarine as it moves and hence compute the speed and direction of the submarine. We calculate the size and the depth of the submarine by finding the maximum intensity of the dampening and the average dampening in the area around it.

Using the algorithm on our artificial data, we can spot a submarine within 25 m of the actual location, when we are working with an area of 7 km $\times$ 7 km, with the ratio of about 1.8 of maximum dampening effect to standard deviation of ambient noise. We were within a computer error scale for determining the speed of the submarine. Our model at the latest stage computed the depth of the submarine and the size of the submarine within a factor of 10^2.

Physics Facts

- Acoustic intensity is related to the distance r that sound travels and the attenuation coefficient α [Apel 1987, 368] via

$$I(r) = I_0 \left(\frac{r_0}{r}\right)^2 e^{-\alpha(r-r_0)},$$

where I_0 is the acoustic intensity at distance r_0.

- "[B]elow 20 Hz, a frequency-independent attenuation coefficient, α_1, occurs and is approximated by: $\alpha_1 = 6.9 \times 10^{-7} \mathrm{m}^{-1}$" [Apel 1987, 341, 369]. At higher frequencies, the attenuation constant is a function of frequency; at still higher frequencies, the dominant factor is water viscosity [Apel 1987, 368–371].
- "[T]he [acoustic] energy radiates in all directions as a spherical wave," which causes an attenuation proportional to 1 / distance^2. [Dera 1992, 434]

- "[T]he velocity of sound in the ocean varies from 1430–1540 m / s near the surface to 1580 m / s at great depths" [Dera 1992, 436].
- Different underwater sounds have their own specific frequency ranges, such as shipping and machinery (less than 2 kHz), biologics (0.1 to 100 kHz), and ice (several Hz to kHz) [Hassab 1989, 3].

Assumptions and Justifications

- We monitor a small area of ocean, say 10 km $\times$ 10 km, with a uniform depth of D. In a small area, ocean depth tends to vary less, and we do not have to consider the curvature of the earth. If we need to locate a submarine in a larger area, we divide up the area into 10 $\times$ 10 squares..
- Effects of submarines on ambient noise:
  - When there is a submarine between a sound and the observation point, the ambient noise field is dampened at nearby observation points, the same effect as the submarine absorbing almost all sounds (about 98%). Most of the sound reflected off an ellipsoid is scattered far away from our sensors.
  - The dampening effects are independent of the speed of the submarine.
- Assumptions about the ambient noise field:
  - We have instruments to measure acoustic intensity as a function of the point on the plane at a particular depth d.
  - Sensors do not malfunction.
  - Atmospheric sound does not propagate to the ocean [Pain 1983, 158].
  - The dominant effect of sound hitting the bottom of the ocean is scattering off bumps.
  - We set d = 100 m, for realism and for simplicity ("sources located in shallow surface ducts can give complex ray arrival patterns" [Munk et al. 1995, 382]).

- Just 14% of sound is transmitted through the steel-water boundary [Pain 1983]; so $(14\%)^2 = 2\%$ of the sound is transmitted completely through the submarine's two steel-water boundaries.
- We measure a small interval of frequencies. "At the high-frequency end, sound absorption by seawater is very high. ... At the low end, below 1 cps, one has great difficulty in generating sound (except with earthquakes and very large explosions)" [Tolstoy and Clay 1966, 3].
- We ignore the effects of ambient water velocity on sound propagation, the common practice [Keller 1977, 2].

• Other properties of submarines:

- The submarine shape is an ellipsoid. The thickness of the submarine is negligible in comparison to the depth of the ocean and the distance from which we are measuring the noise field.
- The speeds of submarines do not exceed 35 knots [Friedman 1984, 105].
- The lengths of submarines vary from 50 m to 150 m; the width is always about 10 m Friedman [1984].
- The submarine is parallel to the ocean surface; for operational reasons, submarines do not tilt by a very large angle.

Development of the Model

Choice of Frequency

We limit our noise detection to a convenient frequency range, near 30 kHz; our model can adapt to different choices.

Noise near 30 kHz is caused mostly by surface water movements, thermal activity, and biologics, which either affect relatively large areas uniformly, or are distributed randomly throughout the region. Thus, it is reasonable to assume that intensity of noise in the ambient noise field is distributed according to a smooth function with random fluctuations according to a normal or uniform distribution.

We could alternatively look at low frequencies, near a few Hz, for which we would need a different attenuation constant. The dominant noise in this range is seismic activity, and seismic information is readily available in real life.

Dampening Effects

We assume that reflections off the submarine and off the ocean bottom are dominated by scatterings, which means that the effects of reflection cannot be measured in the range in which we are working. This means that any sound that hits the submarine effectively disappears, i.e., it has the same effects as if the sound were absorbed by the submarine. Thus, we assume that the submarine blocks all sounds that come toward the observation point from the other side.

An ellipsoidal submarine presents an elliptical profile, which at depth d can be represented by the equation

$$\frac{(x-x')^2}{a^2}+\frac{(y-y')^2}{b^2}=1,$$

where (x', y') is the center of the ellipse. In addition, we assume that the ocean depth is uniformly D (see **Figure 1**).

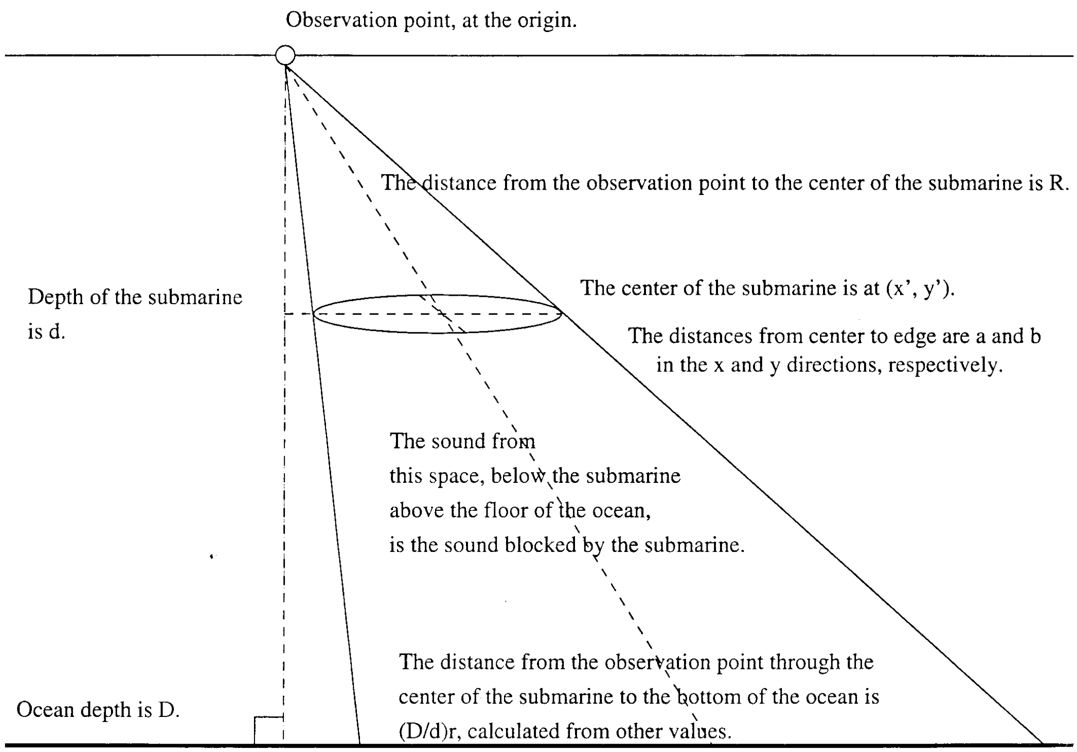

Figure 1. Setup for the integral to calculate the dampening caused by the submarine. The z-axis points downward, for convenience.

The sound intensity measured is inversely proportional to the square of the distance from the source. Ignoring seawater's absorption of sound, the total sound blocked is expressed by the integral

$$\iiint_V \frac{1}{(x^2+y^2+z^2)}\,dx\,dy\,dz \qquad \textbf{(1)}$$

where V is the volume of water blocked by the submarine. The evaluation of the integral proved exceedingly difficult, so we used the following approximation:

$$\int_R^{(D/d)R} \frac{ab\pi\left(\frac{r}{R}\right)^2\left(\frac{d}{R}\right)}{r^2}\,dr \qquad \textbf{(2)}$$

where $R = \sqrt{x'^2 + y'^2 + d^2}$ is the distance from the observation point to the center of the submarine. This integral reduces to

$$\frac{ab\pi}{R^2}\,(D-d). \tag{3}$$

Note that the dampening is *not* affected by the direction in which the submarine is pointing.

Water dampens the sound by a factor of $e^{-\alpha(r-r_0)}$, where r is the distance from the source of the sound, r_0 is a reference distance (usually taken to be 1), and α is a constant dependent on the frequency of the sound that we are measuring. For example, $\alpha = 3\times10^{-2}$ dB / km at 30 kHz [Apel 1987]. Incorporating this factor into **(1)** makes the integral even harder to evaluate. Thus, we use **(2)** to get

$$\int_R^{(D/d)R} \frac{ab\pi\left(\frac{r}{R}\right)^2\left(\frac{d}{R}\right)}{r^2}\, e^{-\alpha(r-r_0)}\,dr,$$

which reduces to

$$\begin{aligned}\int_R^{(D/d)R} \frac{ab\pi d}{R^3}\, e^{-\alpha(r-r_0)}\,dr &= \left.\frac{abe^{-\alpha(r-r_0)}\pi d}{-\alpha R^3}\right|_R^{(D/d)R}\\ &= \frac{abde^{\alpha r_0}\pi}{\alpha R^3}\left(e^{-\alpha R} - e^{-\alpha RD/d}\right).\end{aligned}$$

Setting $r_0 = 1$, we get

$$\frac{abde^{\alpha}\pi}{\alpha R^3}\left(e^{-\alpha R} - e^{-\alpha RD/d}\right). \tag{4}$$

We see that the amount of noise measured at each point is approximately proportional to $2\pi e^{\alpha}/\alpha$. For derivation of the integrals and these values, see the **Appendix**.

Our computer model reveals that **(4)** produces similar maximal dampening effects to **(3)**, but the effects of **(4)** are registered in an area roughly one-third to one-half the radius of **(3)**.

We use **(3)** with the additional constraint of a smaller dampening radius. Dampening is more than some criterion constant c when

$$h < \sqrt{\frac{ab\pi(D-d)}{c} - d^2} < \sqrt{\frac{ab\pi D}{c}},$$

where h is the horizontal distance from the observation point. Note that the middle expression depends on the unknown depth of the submarine; the expression on the right contains known quantities, except for the length of the submarine, which we assume varies by a factor of only 3. We can assume c to be a relatively small constant, such as the standard deviation of the ambient noise, properly scaled. Doing so produces an area of radius approximately 100 m where the effects of the submarine are detectable.

We still need to consider the motion of the submarine. Since the typical speed of about 30 knots (15 m / s) is greatly less than the speed of the sound (1430–1540 m / s), the time delay due to movement is small enough to ignore.

Analysis of the Problem and Model Design

We developed a graphical simulation in MATLAB, with data on a grid (each square representing a sensor) and with color to indicate acoustic intensity.[1]

Generation of Simple Random Noise

We had MATLAB generate a data set containing only random noise. To start, we used 1 as the mean of the noise intensity. The adjustment values at each point came from a uniform distribution on the interval $(-0.00005, 0.00005)$. Later, we used a normal distribution mean 0 and standard deviation 0.00005 to create random noise.

Dampening Effect of the Submarine

We first guessed that the submarine would create an ellipse-shaped area of dampening, with greater dampening in the center than at the edges.

Later, we derived two dampening functions using **(3)** and **(4)**. The "attenuated" model included the effect of sound absorption by seawater, while the "unattenuated" model did not.

Smoothing Functions

Having simulated the data, we sought a way to locate the submarine.

We began by removing the random noise via a smoothing function. The first smoothing method, "consecutive," used least-squares to fit a line to the first 5 points in each row, and then to the next 5 points, etc. We specified an acceptable range of "noise" and then checked each point to see whether it was within that (vertical) distance from the least-squares line. If so, the value of the point was reassigned to the value on the regression line. We also wrote a routine that performed the same type of smoothing on each column. This method proved ineffective, but it inspired a second method.

The second method, "overlapping," was very similar to the consecutive method, except instead of doing the process on points 1 to 5, then 6 to 10, then 11 to 15, we did the process on points 1 to 5, then 2 to 6, then 3 to 7, etc. This was much more effective. We found that running the overlapping row and column

[1] AUTHORS' NOTE: The source code, together with a fuller version of this paper and additional figures, is available at `http://www.math.unc.edu/Undergrads/cleitner/ambient` .

smoothers several times each did a very good job of enhancing the contrast between average noise and dampening effect.

If we take the average of 5 points at each step, we theoretically expect the standard deviation of the mean to be reduced by a factor of $1/\sqrt{5}$ at each step. After 6 smoothings, the standard deviation was reduced by about $1/12$, which is greater than $(1/\sqrt{5})^6$. However, since there are many points that we are not affecting when they are outside of our noise tolerance (twice the standard deviation), this is consistent with the theoretical result.

Location of the Submarine

We first had MATLAB produce a contour map of the data. Then we located contours that might represent a submarine's detectable dampening radius. Finally, we checked these "suspicious" contours for whether they behaved like submarine-dampening contours.

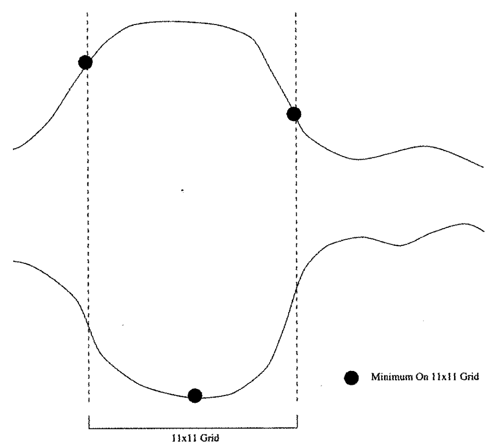

Figure 2. Cross-section view of the ambient field on an 11×11 grid. The marked points are minima.

We identified the contours that stayed within the grid. Then we eliminated all contours with diameters significantly greater than the detectable dampening radius. We found the "center of mass" of each contour by averaging all of the contour's vectors. Then we found clusters of contours by grouping those whose "centers of mass" were within one-fourth of the detectable dampening radius. Within each cluster, we averaged the contour centers to get a cluster center.

We eliminated contour clusters with fewer than 4 contours because submarines tend to have 4 or more contours associated with them.

For each cluster, we then searched the 11×11 square around the center and found the absolute minimum intensity in that square. We looked at the 4 grid squares adjacent to the minimum to see whether the absolute minimum was a local minimum (the 4 adjacent squares might not all be part of the 11×11 square). If the center was not a local minimum, then the cluster marked an area of high intensity rather than an area of dampening, so we eliminated such clusters from our set.

The remaining clusters should indicate the presence of submarines at their centers.

Speed and Direction

To approximate the speed of the submarine, we locate it at two different times and use the time-distance formula from algebra to find the speed. We would approximate the direction as a vector from the first location of the submarine to the second.

Size and Depth

We use the unattenuated model, **(3)**, to approximate depth and size. Recall that the dampening is

$$I = \frac{ab\pi}{R^2}(D-d).$$

The maximum of this function occurs when R is smallest, which is when $R = d$. Also, by rewriting the equation, we see that

$$\begin{aligned} ab\pi(D-d) &= I(h^2+d^2) \\ &= ab\pi(D-d) = m\big(I(h^2+d^2)\big) \\ &= ab\pi(D-d) = d^2 m(I) + m(Ih^2), \end{aligned}$$

where h is the horizontal distance and m is the averaging function over some region. By substitution, we see that

$$d^2 \max(I) = ab\pi(D-d) = d^2 m(I) + m(Ih^2),$$

or

$$d = \sqrt{\frac{m(Ih^2)}{\max(I) - m(I)}}.$$

Since we assume the width to be constant, we can assume a to be constant. So the length b of the submarine is

$$\frac{d^2 \max(I)}{a(D-d)\pi}.$$

Figures

Figures 3–6 are a clear visual representation of our algorithm. The mean of the measurements in these figures is scaled by $2\pi e^{\alpha}/\alpha \approx 2.095 \times 10^4$.

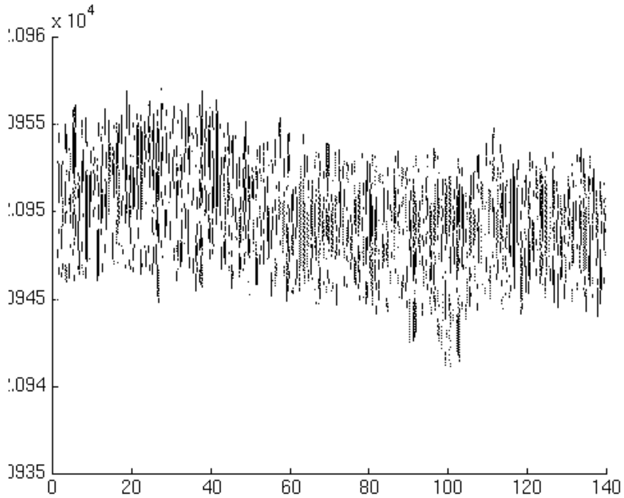

Figure 3. Plot of the sample data.

- **Figure 3** is a plot of our sample data. Where the graph dips down is where the submarine is. (The figure is a horizontal view of the 3-D graph.)
- **Figure 4** is a contour map of the 3-D data from which **Figure 3** was made.

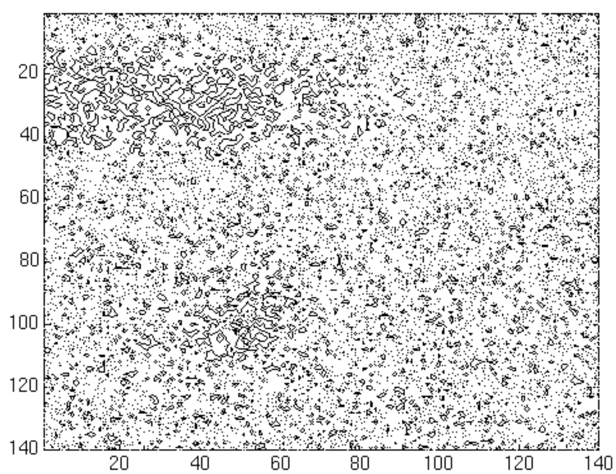

Figure 4. Contour map of the sample data.

- **Figure 5** shows the effect of "overlapping" smoothing on the data from **Figure 3**. Notice that the noise-to-dampening ratio has drastically improved. Unfortunately, the maximum dampening has also decreased, so it is necessary to return to the original data for information on submarine size.
- **Figure 6** is a contour map of the data in **Figure 5**. Notice how many fewer contours appear on **Figure 6** than on **Figure 4**.

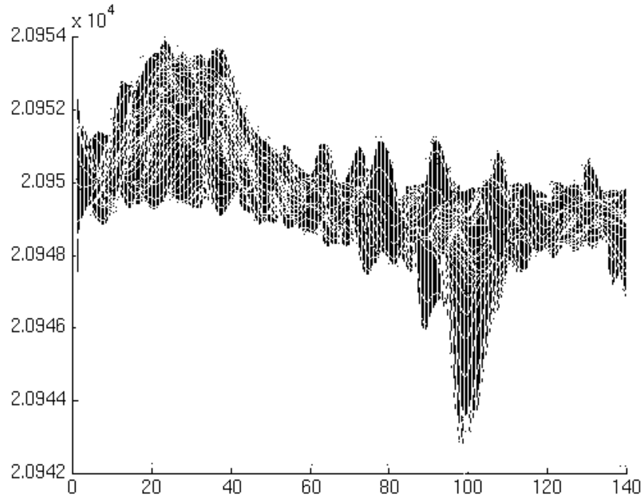

Figure 5. The effect of "overlapping" smoothing on the sample data from **Figure 3**.

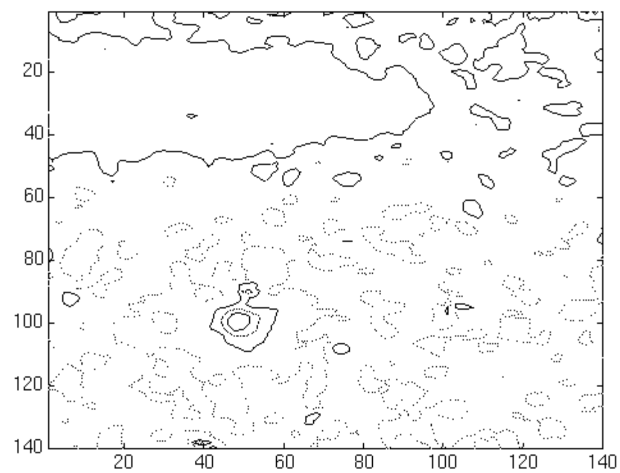

Figure 6. Contour map of the data in **Figure 5**.

Results and Model Testing

Using our algorithm on our modeled data, we located a submarine within 25 m of the actual location, working with an area of 7 km by 7 km, with a ratio of about 1.8 of maximum dampening effect to standard deviation of ambient noise. Considering that our scale is 25 m to a computer unit, this is the best that we can hope for. This remained true when we assumed that the mean noise level was not constant and replaced it with some smooth function with relatively small variation.

We were also within this same range for determining the speed of the submarine. Our model correctly located the center of the submarine at each subsequent step in the model, even when we assumed that the submarine is moving.

Since our model is functional with a ratio of about 1.8, we expect it to work just as well when the ratio is lower.

Unfortunately, our model at the latest stage computed the depth of the submarine and the size of the submarine only to within a factor of 10^2.

We need to test the model with various standard deviations in the random noise and with various means for the random noise as a function of grid location. We also need to do real-world experimentation.

Strengths and Weaknesses of the Model

Strengths

- Our computer program contains many constants that are easy to adjust. These include:
  - Scale: We considered one unit equal to 25 m of real scale.
  - Size of the field we observe: We chose a grid of 140 $\times$ 140, but MATLAB is capable of handling a much larger matrix.
  - Depth of ocean.
  - Attenuation constant: We used the 3×10^{-2} dB / m^2 as the default, since that is Apel's value for 30 kHz sound [1987].
  - Conversion factor from computer scale noise to real noise: We chose a value that was theoretically easy to work with (2.095×10^4), but it could be replaced with a real average amount of noise.
  - Absorption / reflection factor: In our current model, the amount of noise that is actually transmitted rather than blocked is 98%.
  - Radius of the area affected: This is a constant relative to several factors, including what noise level is expected and what level of statistical accuracy we want.
- The graphical interface is a very intuitive way to organize data and to see the effects of each stage of the modeling.
- Our computer model of ambient noise considers dampening caused by a submarine; the model is faithful to the sound wave propagation and absorption patterns in the ocean.
- The computer algorithm is fast; it ran in a few seconds on a SPARC 20 station. Hence, our model can be applied in real time or near real time.
- We considered many possible factors that could affect the noise field, including marine biology, surface activities, human-generated noise, geothermal activity, and seismic activity.

- Our calculations are independent of scale, as long as the submarine is more than 1 grid square in length.
- We could incorporate other smoothing functions, such as polynomial smoothing or spline approximation on 20 points or so.

Weaknesses

- Our model assumes a very large number of sensors (the square of the number of grid units on a side), which would mean a high cost of implementation. To get adequate data from fewer sensors would require more-sophisticated sensors and might pose more difficulty if a sensor failed. Having fewer sensors would also mean a more"bumpy" field of data points.
- We could not test our model on real data.
- Our assumptions may make our approximations inaccurate. Such assumptions include the regular shape of the submarine, uniform depth of the ocean within our area of interest, and sound reflection / refraction patterns.
- Our integral approximations may be inaccurate, which would cause our computer-generated noise field to be inaccurate.
- Our model could not calculate the size of the submarine or its depth very accurately. We believe, however, that we have the relative scale on these numbers right, and it is a matter of finding the right corrective factor.
- Our ambient noise field model is just for a single submarine, though incorporating more than one submarine should be a relatively easy project.

Appendix: Derivation of Integrals

To approximate the sound blocked by the submarine, we use two techniques:

- We draw a line l_1 from the observation point, through the center of the submarine, to the bottom of the ocean (see **Figure 7**). We attempt to integrate along that line, on a portion of a spherical surface. That is, let the distance from the point of observation to the point on l_1 be r, consider the sphere of radius r, and consider the portion of the sphere that would project onto the submarine. We take the limits of integration on r to be the center of the submarine and the point where l_1 hits the ocean floor.
- It is very hard to approximate the portion of the sphere that projects onto the submarine. When r is at the center of the submarine, we can approximate the area by projecting the ellipse of the submarine onto the plane perpendicular

to l_1. This reduces the length in some direction by a factor of d/R, where d is the depth of the submarine and $R = \sqrt{x^2 + y^2 + d^2}$ is the distance from the observation point to the center of the submarine. On the other hand, in the direction perpendicular to that direction, the length is not reduced and the area is reduced by d/r.

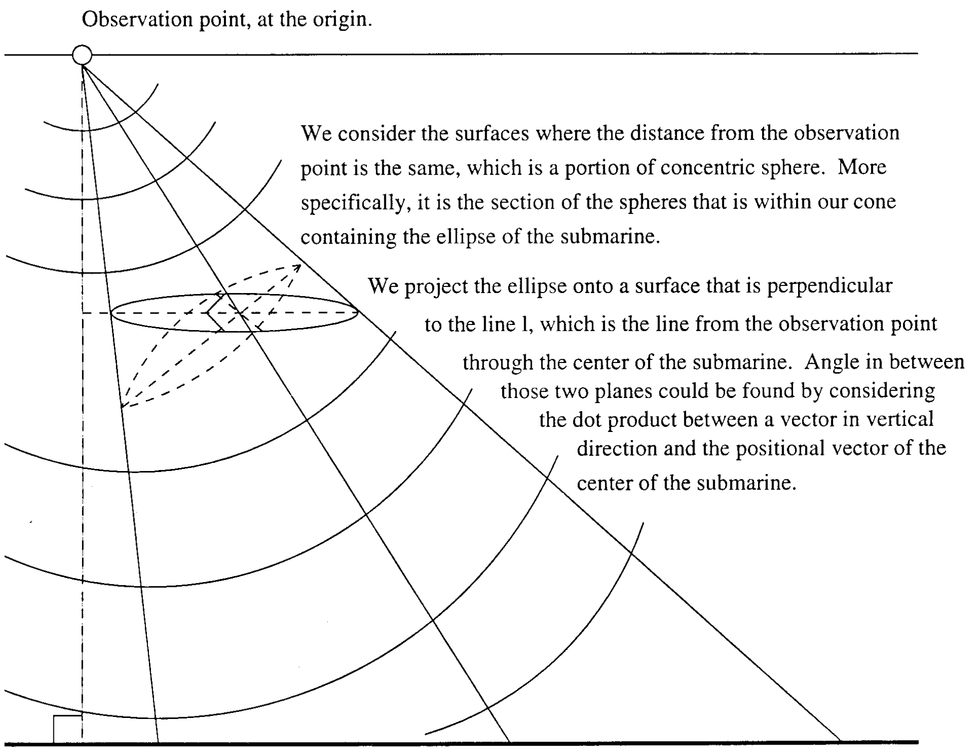

Figure 7. Approximation of the noise dampened by the submarine.

The area of the ellipse is $ab\pi$. We approximate the section of the sphere by a projection of this ellipse, which gives the area of $ab\pi(r/R)^2$, since the dimension in each direction changes by the factor of r/R as we move along l_1. Thus, we have the integral

$$\int_R^{R(D/d)} \frac{ab\pi\left(\frac{r}{R}\right)^2\left(\frac{d}{R}\right)e^{-\alpha(r-r_0)}\,dr}{r^2} = \int_R^{R(D/d)} \frac{abd\pi}{R^3}\,e^{-\alpha(r-r_0)}\,dr.$$

To calculate the total ambient noise at a point in this scale, we set up a similar calculation. We ignore the noise that might be coming from above the sensor. By doing so, we have

$$\int_0^D 2\pi r^2(1/r^2)e^{-\alpha(r-r_0)}\,dr + \int_D^\infty (A(r)/r^2)e^{-\alpha(r-r_0)}\,dr,$$

where $A(r)$ is the appropriate area function. We approximate $A(r) = 2\pi r D$, which results in

$$\int_0^D 2\pi e^{-\alpha(r-r_0)}\,dr + \int_D^\infty 2\pi r D(1/r^2)e^{-\alpha(r-r_0)}\,dr.$$

We again cannot integrate this immediately, so now we approximate $1/r$ by $1/D$, since this is the definite upper bound (also, we lost some noise in the approximation, so this will make up for the loss). This results in

$$\begin{aligned}\int_0^D 2\pi e^{-\alpha(r-r_0)}\,dr + \int_D^\infty 2\pi e^{-\alpha(r-r_0)}\,dr &= \int_0^\infty 2\pi e^{-\alpha(r-r_0)}\,dr \\ &= \frac{2\pi}{-\alpha}\Big[e^{-\alpha(r-r_0)}\Big]_0^\infty \\ &= 2\pi e^\alpha/\alpha.\end{aligned}$$

This integral treats the ocean as infinitely deep, which may make sense due to the constant reflections of sound off the bottom and surface.

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