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习题选讲

作业:（第三版课本, 习题5.3） Page57

23,24(2)(4) , 26单号, 28,31,32,34 , 36

1. 已知 $f(x,y)\big|_{y=x^2}=1$，$f_1'(x,y)\big|_{y=x^2}=2x$，

求 $f_2'(x,y)\big|_{y=x^2}$．

解: 由 $f(x,x^2)=1$ 两边对 **x** 求导，得

$$f_1'(x,x^2)+f_2'(x,x^2)\cdot 2x=0$$

$$f_1'(x,x^2)=2x$$

$$f_2'(x,x^2)=-1$$

2. 设函数 $z=f(x,y)$ 在点 $(1,1)$ 处可微，且

$$f(1,1)=1,\quad \left.\frac{\partial f}{\partial x}\right|_{(1,1)}=2,\quad \left.\frac{\partial f}{\partial y}\right|_{(1,1)}=3,$$

$$\varphi(x)=f(x,\underline{f(x,x)}),\ \text{求} \left.\frac{\mathrm{d}}{\mathrm{d}x}\varphi^3(x)\right|_{x=1}.$$

解: 由题设 $\varphi(1)=f(1,f(1,1))=f(1,1)=1$

$$\left.\frac{\mathrm{d}}{\mathrm{d}x}\varphi^3(x)\right|_{x=1}=3\varphi^2(x)\left.\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right|_{x=1}$$

$$\begin{aligned}&=3\big[\underline{f_1(x,f(x,x))}\\&\quad\left.+\underline{f_2(x,f(x,x))}\big(\underline{f_1(x,x)+f_2(x,x)}\big)\big]\right|_{x=1}\end{aligned}$$

$$=3\cdot\big[2+3\cdot(2+3)\big]=51$$

3. 设 $u=f(x,y,z)$ 有连续的一阶偏导数，

又函数 $y=y(x)$ 及 $z=z(x)$ 分别由下列两式确定：

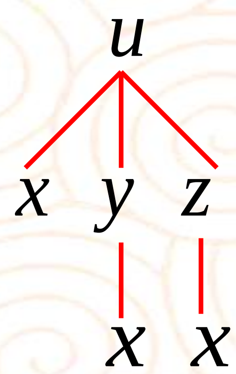
$$\underline{e^{xy}-xy=2},\ \underline{e^{x}=\int_0^{x-z}\frac{\sin t}{t}\,\mathrm{d}t},\ \text{求}\ \frac{\mathrm{d}u}{\mathrm{d}x}.$$

$$\boxed{\frac{\mathrm{d}u}{\mathrm{d}x}=f_1+y'f_2+z'f_3}$$

解：两个隐函数方程两边对 x 求导，得

$$e^{xy}(y+xy')-(y+xy')=0$$

$$e^{x}=\frac{\sin(x-z)}{x-z}(1-z')$$



解得 $y'=-\dfrac{y}{x}$， $z'=1-\dfrac{e^{x}(x-z)}{\sin(x-z)}$

因此 $$\frac{\mathrm{d}u}{\mathrm{d}x}=f_1-\frac{y}{x}f_2+\left[1-\frac{e^{x}(x-z)}{\sin(x-z)}\right]f_3$$

4、设函数 $u=f(x,y)$ 具有二阶连续偏导数, 且满足

$$4\frac{\partial^2 u}{\partial x^2}+12\frac{\partial^2 u}{\partial x\partial y}+5\frac{\partial^2 u}{\partial y^2}=0.$$ 确定 a,b 的值，使等式在变换

$\xi=x+ay,\eta=x+by$ 下简化为 $\dfrac{\partial^2 u}{\partial\xi\,\partial\eta}=0.$

【解1】 $$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial\xi}+\frac{\partial u}{\partial\eta}\qquad \frac{\partial u}{\partial y}=a\frac{\partial u}{\partial\xi}+b\frac{\partial u}{\partial\eta}$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial\xi^2}+2\frac{\partial^2 u}{\partial\xi\partial\eta}+\frac{\partial^2 u}{\partial\eta^2}\qquad \frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial\xi^2}+2ab\frac{\partial^2 u}{\partial\xi\partial\eta}+b^2\frac{\partial^2 u}{\partial\eta^2}$$

$$\frac{\partial^2 u}{\partial x\partial y}=a\frac{\partial^2 u}{\partial\xi^2}+(a+b)\frac{\partial^2 u}{\partial\xi\partial\eta}+b\frac{\partial^2 u}{\partial\eta^2}$$

$$(5a^2+12a+4)\frac{\partial^2 u}{\partial\xi^2}+[10ab+12(a+b)+8]\frac{\partial^2 u}{\partial\xi\partial\eta}+(5b^2+12b+4)\frac{\partial^2 u}{\partial\eta^2}=0$$

$$\begin{cases}5a^2+12a+4=0\\5b^2+12b+4=0\end{cases}\qquad 10ab+12(a+b)+8\neq 0$$

$$\begin{cases} a=-2, \\ b=-\dfrac{2}{5}, \end{cases} \quad 或 \quad \begin{cases} a=-\dfrac{2}{5}, \\ b=-2, \end{cases}$$

【解2】 由 $\xi = x+ay, \eta = x+by$ **解得** $\begin{cases} x=\dfrac{a\eta - b\xi}{a-b}, \\ y=\dfrac{\xi-\eta}{a-b}, \end{cases}$

$$\frac{\partial u}{\partial \xi} = \frac{-b}{a-b}\frac{\partial u}{\partial x} + \frac{1}{a-b}\frac{\partial u}{\partial y}$$

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = \frac{-ab}{(a-b)^2}\frac{\partial^2 u}{\partial x^2} + \frac{a+b}{(a-b)^2}\frac{\partial^2 u}{\partial x \partial y} + \frac{-1}{(a-b)^2}\frac{\partial^2 u}{\partial y^2}$$

欲使 $\dfrac{\partial^2 u}{\partial \xi \partial \eta} = 0,$ **即** $-ab\dfrac{\partial^2 u}{\partial x^2} + (a+b)\dfrac{\partial^2 u}{\partial x \partial y} - \dfrac{\partial^2 u}{\partial y^2} = 0$

$$\frac{-ab}{4} = \frac{a+b}{12} = \frac{-1}{5}$$

由此解得 $\begin{cases} a=-2, \\ b=-\dfrac{2}{5}, \end{cases}$ 或 $\begin{cases} a=-\dfrac{2}{5}, \\ b=-2, \end{cases}$ $4\dfrac{\partial^2 u}{\partial x^2} + 12\dfrac{\partial^2 u}{\partial x \partial y} + 5\dfrac{\partial^2 u}{\partial y^2} = 0.$

5. 设$u=u(x)$由方程组$u=f(x,y,z),\varphi(x^2,e^y,z)=0,y=\sin(x)$确定,

其中f,φ都具有一阶连续偏导数, 且$\dfrac{\partial\varphi}{\partial z}\neq 0$,求$\dfrac{du}{dx}$.

解法1 $$\frac{du}{dx}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}\frac{dy}{dx}+\frac{\partial f}{\partial z}\frac{dz}{dx}=f_1+f_2\frac{dy}{dx}+f_3\frac{dz}{dx}$$

$\dfrac{dy}{dx}=\cos x,\quad \varphi(x^2,e^y,z)=0$两边对x求导得:

$$\varphi_1\cdot 2x+\varphi_2\cdot e^y\cdot\frac{dy}{dx}+\varphi_3\cdot\frac{dz}{dx}=0,\ \frac{dz}{dx}=-\frac{2x\cdot\varphi_1+e^y\cos x\cdot\varphi_2}{\varphi_3}$$

$$\boxed{\therefore\frac{du}{dx}=f_1+f_2\cos x-f_3\cdot\frac{2x\cdot\varphi_1+e^{\sin x}\cos x\varphi_2}{\varphi_3}}$$

5. 设$u=u(x)$由方程组$u=f(x,y,z),\varphi(x^2,e^y,z)=0,y=\sin(x)$确定,

其中f,φ都具有一阶连续偏导数, 且$\dfrac{\partial\varphi}{\partial z}\neq 0$,求$\dfrac{du}{dx}$.

解法2 由一阶全微分形式不变性得:

$$du = f_1\,\mathrm{d}x + f_2\,\mathrm{d}y + f_3\,\mathrm{d}z \qquad dy = d\sin x$$

$$\varphi_1 dx^2 + \varphi_2 de^y + \varphi_3 dz = 2x\varphi_1 dx + e^y\varphi_2 dy + \varphi_3 dz = 0$$

将dy, dz用dx表示,并代入第一个式子,得:

$$du = \left[f_1 + f_2\cos x - \frac{f_3(2x\cdot\varphi_1 + e^{\sin x}\cos x\varphi_2)}{\varphi_3} \right]\mathrm{d}x$$

$$\therefore \frac{du}{dx} = f_1 + f_2\cos x - \frac{f_3(2x\cdot\varphi_1 + e^{\sin x}\cos x\varphi_2)}{\varphi_3}$$

6. (P21.第11题)

$f(x)$是区域$D\subseteq R^n$上的n元向量值函数，证明：

$$\lim_{x\to x_0} f(x)=f(x_0)\Leftrightarrow \forall\{x_k\}\subseteq D, x_k\to x_0, \lim_{k\to\infty} f(x_k)=f(x_0)$$

证明 $\Rightarrow$ $\lim\limits_{x\to x_0} f(x)=f(x_0)$ 即

$\forall\varepsilon>0,\exists\delta,$使当$\|x-x_0\|<\delta\cap D$时，恒有$\|f(x)-f(x_0)\|<\varepsilon$

再由 $\lim\limits_{k\to\infty} x_k=x_0$ 则对上述$\delta>0,\exists N,$

使当$k>N$时 有$\|x_k-x_0\|<\delta\cap D$

故 $\|f(x_k)-f(x_0)\|<\varepsilon$

$\therefore \lim\limits_{k\to\infty} f(x_k)=f(x_0)$

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$$\lim_{x\to x_0} f(x)=f(x_0)\Leftrightarrow\forall\{x_k\}\subseteq D, x_k\to x_0, \lim_{k\to\infty} f(x_k)=f(x_0)$$

设对 $\forall x_k \in D, x_k \to x_0$，都有 $\lim\limits_{n\to\infty} f(x_n)=f(x_0)$

要证 $\lim\limits_{x\to x_0} f(x)=f(x_0)$

用反证法

假设 $\lim\limits_{x\to x_0} f(x)\neq f(x_0)$

即 $\exists\varepsilon_0$ 使对 $\forall\delta>0$，$\exists x_\delta$ 满足 $\|x-x_0\|<\delta$

但 $|f(x_\delta)-f(x_0)|\geq\varepsilon_0$

现取 $\delta=\dfrac{1}{n}$ $\exists x_n$ 满足 $\|x_n-x_0\|<\dfrac{1}{n}$

即 $x_n\to x_0$，但 $\|f(x_n)-f(x_0)\|\geq\varepsilon_0$

与 $\lim\limits_{n\to\infty} f(x_n)=f(x_0)$ 矛盾 $\therefore \lim\limits_{x\to x_0} f(x)=f(x_0)$

7. 设函数$f(x,y)$有二阶连续偏导数，$\dfrac{\partial f}{\partial y}\neq 0$，

证明：$\forall C$，$f(x,y)=C$为一条直线的充要条件是：

$$(f_2)^2 f_{11}-2f_1 f_2 f_{12}+(f_1)^2 f_{22}=0.$$

$$\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x^2}-2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}=0.$$

$$y''=0\ \ ?$$

$$y'=-\frac{f_1}{f_2}$$

8. 设 $z = f(e^{x+y}, \dfrac{x}{y})$, f 具有二阶连续偏导数, 求 $\dfrac{\partial z}{\partial x}, \dfrac{\partial^2 z}{\partial x \partial y}$

1. $\dfrac{\partial z}{\partial x} = f_1 \cdot e^{x+y} + f_2 \cdot \dfrac{1}{y}$

2. $\dfrac{\partial^2 z}{\partial x \partial y} = e^{x+y} f_1 + e^{x+y}\left(f_{11} \cdot e^{x+y} + f_{12} \dfrac{-x}{y^2}\right) - \dfrac{1}{y^2} f_2 + \dfrac{1}{y}\left(f_{21} e^{x+y} - \dfrac{x}{y^2} f_{22}\right)$

9. 若$\forall t>0$,有$f(tx,ty)=t^n f(x,y)$,则函数$f(x,y)$为**n 次齐次函数.**

证明：若$f(x,y)$可微,

则$f(x,y)$是n次齐次函数的充要条件是:

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf(x,y).$$

必要性 $f(tx,ty)=t^n f(x,y).\quad (t>0)$,

两边对t求导：$xf_1(tx,ty)+yf_2(tx,ty)=nt^{n-1}f(x,y).$

令 $t=1$ 得

$$xf_1(x,y)+yf_2(x,y)=nf(x,y).$$

即$x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}=nf(x,y).$

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9. 若$\forall t>0$,有$f(tx,ty)=t^n f(x,y)$,则函数$f(x,y)$为**n 次齐次函数.**

证明:若$f(x,y)$可微,则$f(x,y)$是n次齐次函数的充要条件是

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf(x,y).$$

充分性 令 $F(t)=f(tx,ty)\ (t>0)$,

$$\frac{\mathrm{d}F}{\mathrm{d}t}=xf_1(tx,ty)+yf_2(tx,ty).$$

上式两端乘以 t 得$t\frac{\mathrm{d}F}{\mathrm{d}t}=txf_1(tx,ty)+tyf_2(tx,ty)$

$$\because xf_1(x,y)+yf_2(x,y)=nf(x,y)$$

$$\therefore txf_1(tx,ty)+tyf_2(tx,ty)=nf(tx,ty)=nF(t)$$

于是$t\frac{\mathrm{d}F}{\mathrm{d}t}=nF(t)$, $\quad \frac{\mathrm{d}F}{F}=\frac{n}{t}\,\mathrm{d}t$

充分性 令 $F(t)=f(tx,ty)\ (t>0)$,

$$\frac{\mathrm{d}F}{\mathrm{d}t}=xf_1(tx,ty)+yf_2(tx,ty).$$

上式两端乘以 t 得 $t\frac{\mathrm{d}F}{\mathrm{d}t}=txf_1(tx,ty)+tyf_2(tx,ty)$

$$\because xf_1(x,y)+yf_2(x,y)=nf(x,y)$$

$$\therefore txf_1(tx,ty)+tyf_2(tx,ty)=nf(tx,ty)=nF(t)$$

于是 $t\frac{\mathrm{d}F}{\mathrm{d}t}=nF(t),\frac{\mathrm{d}F}{F}=\frac{n}{t}\,\mathrm{d}t$

由此解得 $F(t)=Ct^n$, 令 $t=1$, 得 $F(1)=C$.

又 $F(t)=f(tx,ty)$, 令 $t=1$, 得 $F(1)=f(x,y)$,

则 $C=f(x,y)$,

$\because F(t)=\begin{cases}Ct^n\\ f(tx,ty)\end{cases}$ 故 $f(tx,ty)=t^nf(x,y)$.

则函数 $f(x,y)$ 为 n 次齐次函数.

10. 证明极限 $\lim\limits_{\substack{x\to0\\y\to0}}\frac{\sqrt{xy+1}-1}{x+y}$ 不存在

$\sqrt[n]{1+x}-1\sim\frac{1}{n}x$

$$a.\ \lim_{\substack{x\to0\\y=x}}\frac{\sqrt{xy+1}-1}{x+y}=\lim_{\substack{x\to0\\y=x}}\frac{\frac{(xy)}{2}}{x+y}=0$$

$$\lim_{\substack{x\to0\\y=x^2-x}}\frac{\sqrt{xy+1}-1}{x+y}=\lim_{\substack{x\to0\\y=x^2-x}}\frac{xy}{x+y}\cdot\frac{1}{\sqrt{xy+1}+1}=-\frac{1}{2}$$

故极限不存在

$$b.\ \frac{\sqrt{xy+1}-1}{x+y}=\frac{xy}{x+y}\cdot\frac{1}{\sqrt{xy+1}+1}$$

$\lim\limits_{\substack{x\to0\\y\to0}}\frac{1}{\sqrt{xy+1}+1}=\frac{1}{2}$, $\lim\limits_{\substack{x\to0\\y\to0}}\frac{xy}{x+y}$ 不存在,

$\lim\limits_{\substack{x\to0\\y=x}}\frac{xy}{x+y}=0,$

$\lim\limits_{\substack{x\to0\\y=x^2-x}}\frac{xy}{x+y}=-1$

由反证法可知 $\lim\limits_{\substack{x\to0\\y\to0}}\frac{\sqrt{xy+1}-1}{x+y}$ 不存在

11.讨论极限 $\lim\limits_{\substack{x\to 1\\ y\to 0}}(x+y)^{\frac{1}{\sin(x-1)}}$

12.讨论极限 $\lim\limits_{(x,y)\to(\infty,\infty)}\dfrac{x+y}{x^2-xy+y^2}$

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P59.B 7. 设 $\mathbf{u}=\mathbf{u}\left(\sqrt{\mathbf{x}^2+\mathbf{y}^2}\right)$ 具有连续二阶偏导数, 且满足

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{1}{x}\left(\frac{\partial u}{\partial x}\right)+u=x^2+y^2$$ 试求函数 $\mathbf{u}$ 的表达式.

解析:

令 $\sqrt{\mathbf{x}^2+\mathbf{y}^2}=\mathbf{t}$, 则 $\mathbf{u}=\mathbf{u}(\mathbf{t})$, 根据复合函数求导的链式法则可将题中的等式化简为:

$$\frac{\partial}{\partial x}\left[u'(t)\cdot\frac{x}{\sqrt{x^2+y^2}}\right]+\frac{\partial}{\partial y}\left[u'(t)\cdot\frac{y}{\sqrt{x^2+y^2}}\right]-\frac{1}{x}\cdot u'(t)\cdot\frac{x}{\sqrt{x^2+y^2}}+u(t)=x^2+y^2$$

进一步化简可得到:

$$\mathbf{u}''(\mathbf{t})\cdot\frac{\mathbf{x}^2}{\mathbf{x}^2+\mathbf{y}^2}+\left[\mathbf{u}'(\mathbf{t})\cdot\frac{\sqrt{\mathbf{x}^2+\mathbf{y}^2}-\frac{\mathbf{x}^2}{\sqrt{\mathbf{x}^2+\mathbf{y}^2}}}{\mathbf{x}^2+\mathbf{y}^2}\right]+\mathbf{u}''(\mathbf{t})\cdot\frac{\mathbf{y}^2}{\mathbf{x}^2+\mathbf{y}^2}$$

$$+\left[\mathbf{u}'(\mathbf{t})\cdot\frac{\sqrt{\mathbf{x}^2+\mathbf{y}^2}-\frac{\mathbf{y}^2}{\sqrt{\mathbf{x}^2+\mathbf{y}^2}}}{\mathbf{x}^2+\mathbf{y}^2}\right]-\frac{\sqrt{\mathbf{x}^2+\mathbf{y}^2}}{\mathbf{x}^2+\mathbf{y}^2}\cdot\mathbf{u}'(\mathbf{t})+\mathbf{u}(\mathbf{t})=\mathbf{x}^2+\mathbf{y}^2$$

整理得: $u''(t)+u'(t)=t^2$

P59.B

7. 设 $\mathbf{u}=\mathbf{u}\left(\sqrt{\mathbf{x}^2+\mathbf{y}^2}\right)$ 具有连续二阶偏导数, 且满足

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{1}{x}\left(\frac{\partial u}{\partial x}\right)+u=x^2+y^2$$ 试求函数 $\mathbf{u}$ 的表达式.

$\boxed{\text{整理得：} \mathbf{u}''(\mathbf{t})+\mathbf{u}(\mathbf{t})=\mathbf{t}^2,}$

求这个微分方程的解，

先求齐次方程的通解

再设非齐次方程的特解，进而得到非齐次方程的通解：

$$\mathbf{u}(\mathbf{t})=\mathbf{C}_1\cdot\sin\sqrt{\mathbf{x}^2+\mathbf{y}^2}+\mathbf{C}_2\cdot\cos\sqrt{\mathbf{x}^2+\mathbf{y}^2}+\mathbf{x}^2+\mathbf{y}^2-2$$

（其中 $\mathbf{C}_1$、$\mathbf{C}_2$ 为任意常数）.

$\left(x'' + a_1 x' + a_2 x = 0\right)$的特征方程是:$\lambda^2 + a_1\lambda + a_2 = 0$

情形3　特征方程有一对共轭复根 $(\Delta < 0)$

特征根为 $\lambda_1 = \alpha + i\beta,\ \lambda_2 = \alpha - i\beta,$

$$x_1 = e^{(\alpha+i\beta)t}, \qquad x_2 = e^{(\alpha-i\beta)t},$$

重新组合 $\bar{x}_1 = \frac{1}{2}(x_1 + x_2) = e^{\alpha t}\cos\beta t,$

$$\bar{x}_2 = \frac{1}{2i}(x_1 - x_2) = e^{\alpha t}\sin\beta t,$$

得齐次方程的通解为

$$x = e^{\alpha t}\left(C_1\cos\beta t + C_2\sin\beta t\right).$$

二、二阶常系数齐次线性方程解法(特征方程法)

设二阶常系数齐次线性方程为 $x'' + a_1 x' + a_2 x = 0$

设 $x = e^{\lambda t}$，　将其代入上方程, 得

$$\left(\lambda^2 + a_1\lambda + a_2\right)e^{\lambda t} = 0 \quad \because e^{\lambda t} \neq 0,$$

故有 $\lambda^2 + a_1\lambda + a_2 = 0$　特征方程

特征根 $\lambda_{1,2} = \dfrac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2}$,

情形1　特征方程有两个不相等的实根 $(\Delta > 0)$

特征根为: $\lambda_1 = \dfrac{-a_1 + \sqrt{a_1^2 - 4a_2}}{2}$,　$\lambda_2 = \dfrac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2}$,

两个线性无关的特解 $x_1 = e^{\lambda_1 t}$,　$x_2 = e^{\lambda_2 t}$,

得齐次方程的通解为 $x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$;

$$x''+a_1x'+a_2x=F(t)$$

情形1 $F(t)=\varphi(t)e^{\mu t}$，其中μ为常数,

$$\varphi(t)=b_mt^m+b_{m-1}t^{m-1}+\cdots+b_1t+b_0,\ m\geq 0$$

猜测非齐方程特解为$x^*(t)=Z(t)e^{\mu t}$.代入原方程

$$Z''(t)+\underline{\underline{(2\mu+a_1)}}Z'(t)+\underline{\underline{(\mu^2+a_1\mu+a_2)}}Z(t)=\varphi(t)$$

(1). 若μ不是特征方程的根 $\mu^2+a_1\mu+a_2\neq 0$,

设 Z(t)为与φ(t)次数相同的多项式, $Z(t)=B_mt^m+B_{m-1}t^{m-1}+\cdots+B_1t+B_0$,

则 $x^*(t)=Z(t)e^{\mu t}$代入原方程待定B_i, 从而得到特解.

(2). 若μ是特征方程的单根 则$\mu^2+a_1\mu+a_2=0$, $2\mu+a_1\neq 0$,

可设 $Z(t)=t\left(b_mt^m+b_{m-1}t^{m-1}+\cdots+b_1t+b_0\right)$,

则 $x^*(t)=t\left(b_mt^m+b_{m-1}t^{m-1}+\cdots+b_1t+b_0\right)e^{\mu t}$ 代入原方程待定B_i,可得特解.