Paying Professors What They're Worth

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Introduction

We develop a model with two variations: One encourages early retirement, the other does not.

Our model is generous to those who are promoted or retire later than is typical. It does not allow the college to change gracefully the real starting salaries, although nominal starting salaries are adjusted each year for inflation. The model also takes into account those that it considers to be overpaid. Instead of not giving them any raises, it gives them cost-of-living raises.

When the model is applied to the existing faculty at Aluacha Balaclava College with simulated hirings, promotions, and retirements, the faculty is separated into clearly different salary bands by the year 2010 (with the exception of two overpaid full professors), replacing the present muddle.

Constraints

For convenience in reference, we note here the constraints:

1. If there is enough money for raises, then everyone gets a raise.

2. Instructors who are promoted according to the usual schedule of seven years as an assistant professor and seven years as an associate professor and who work 25 years or more should receive at retirement twice as much as a new assistant professor's salary.

3. Although there should be a reward for years of experience, the salaries of two faculty members with equal rank should approach each other as they gain experience.

4. The salary for a newly promoted faculty member should be about what it would have been in seven years, without the promotion.

Assumptions

Although payments are actually made throughout the school year, and salary decisions are made in March, we assume that the decisions are made between discrete yearly salary payments.

We also assume that when the decisions are made, the Provost has the budget for next year and an estimate of the cost-of-living increase. However, no information is available for years beyond the one for which salaries are being decided.

Since we are prohibited from decreasing anyone's salary when moving the current faculty to the new scheme, we assume that there is always enough money to pay everyone's salaries from the previous year. That is, there might be no money for raises, but we can at least pay the faculty at last year's nominal level.

We must give everyone a raise if anyone gets a raise, but we assume that we can give unequal raises. Otherwise, we would just split the money evenly among the faculty, and the current salary system would not change very much at all.

Although Constraint 2 mentions 25 years and retirement, instructors are not forced to retire either at 25 years of experience or at 65 years of age. However, we assume an upper limit of 60 on the number of years of experience.

Constraints 2 and 4 refer to real dollar values. Otherwise, it would be extremely difficult to guarantee promising new Ph.D.s that they will retire at twice their current salary. Besides, doubling the nominal salary in 25 years won't even keep up with 3% inflation.

Constraints 1 and 3 refer to nominal dollar values. The college always deals in nominal amounts when calculating budgets. There might not be enough to give the full cost-of-living increase; this would amount to a decrease in real salary.

Faculty members who take longer to be promoted than usual may receive more than a seven-year raise if and when they finally do get promoted. That is, there is no penalty for being promoted late other than the salary lost while waiting.

New faculty are hired only if there is enough other money to pay their starting salaries, which take into account any previous experience and are not taken from the amount available for raises.

The salaries of retiring faculty do not get thrown into the pool for raises but can be applied to hiring new faculty.

Since no information was given about the transition from assistant professor to associate professor, we will assume that one must have seven years of experience total (not necessarily with this college, or as an assistant professor) in order to be promoted.

Analysis of the Problem

It is currently late in the winter of 1995, too late for next year's salaries to be decided by the model proposed in this paper. The first year that will be on the new salary model is 1996–97. Even then, salaries will gradually move toward the target curves, since some faculty members are overpaid and cannot have their salaries reduced.

One solution is to set target salaries so high that everyone needs a raise to attain the target. This is clearly not a solution that the college would favor, although it would be rather popular with the faculty.

It is difficult to conceive of a salary scheme that had no view of the future yet managed to reliably satisfy the constraints regarding promotions and retirement. Therefore, the model should contain an overall view of how much a faculty member at a certain rank with a certain number of years of experience should be paid.

This feature is certain to cause some friction between the college and the faculty; how much is a full professor with ten years of experience worth? Moreover, at what rate should salaries increase, within the framework of the constraints? Should the model be set up to encourage or discourage retirement? Can someone be hired as a full professor with no experience? These are political questions for the Provost and faculty to negotiate; the model must handle whatever answers that negotiation produces. Faculty members must become accustomed to the the Provost placing a certain value on them.

Design of the Model

Our model presents an overall goal for a salary system and then adapts it to the real world. We offer a pair of core systems (logarithmic and linear) for the administration and faculty to decide between. These cores are what the college will pay the faculty if it has enough money to do so. These cores are in real dollars, not adjusted to inflation. They are then adjusted for inflation (both historical and predicted) and adjusted to meet a finite budget. There are two options to take care of faculty members who, according to the new system, are being overpaid. Finally, we deal with the unlikely event of a budget excess.

Variables

Let t be the current year; $t+1$ is the year for which salaries are being computed. Let t_i be the year that faculty member i started at the college, adjusted by the number of years of experience credited upon entrance into the plan. Thus, if faculty member i joined the college in 1994 with four years of experience, then $t_i = 1990$.

Let $T(i,t)$ denote the amount in real dollars that faculty member i should get paid in year t, the Target for i for that year. This target will depend on the rank and number of years of experience of i.

In the cores that follow, a_0, b_0, c_0, d_0 denote the initial salaries of a full professor, associate professor, assistant professor, and instructor, respectively. This is the amount paid to a faculty member with no experience on entering the plan. According to the problem statement, no one can be hired at a rank higher than assistant professor; however, the model could easily handle such an event. Indeed, one needs to estimate the "initial" salaries of associate professors and full professors to start the salary system.

The initial salaries for no experience, $d_0 = \$27{,}000$ for an instructor and $c_0 = \$32{,}000$ for an assistant professor, are of some concern. We must have $a_0 < 2c_0$, or else full professors would have a decreasing salary in order to hit $2c_0$ at 25 years. Convention forces $d_0 < c_0 < b_0 < a_0$. It will turn out (see the **Appendix**) that $a_0 = \$40{,}000$ and $b_0 = \$36{,}000$ are good estimates.

The Two Cores

According to Constraint 3, faculty members of equal rank but different experience should have their salaries approach each other as time goes on. This leaves two possibilities: Either the absolute difference goes to zero (logarithmic core), or the ratio goes to one (linear core).

One might expect a model to have a core that has a horizontal asymptote, so that a faculty member's salary has a clear upper bound. However, as careers are limited to sixty years (Prof. Methuselah does not work at Aluacha Balaclava College), salaries are bounded. As long as the proper rate constants are chosen, no faculty member's salary will get too large for the college to handle.

The first core, the logarithmic, increases more rapidly at the beginning of someone's career than at the end. Larger raises occur early, but by the time one has gained twenty-five years of experience, the salary curve has really flattened out. Here is the logarithmic core:

$$T(i,t) = \begin{cases} d_0 \log_{10}(d(t-t_i)+10), & \text{for } i \text{ an instructor;} \\ c_0 \log_{10}(c(t-t_i)+10), & \text{for } i \text{ an assistant professor;} \\ b_0 \log_{10}(b(t-t_i)+10), & \text{for } i \text{ an associate professor;} \\ a_0 \log_{10}(a(t-t_i)+10), & \text{for } i \text{ a full professor.} \end{cases}$$

The $+10$ term inside the logarithm allows the instructor's starting salary to be the coefficient of the logarithm expression. Indeed, the equation becomes (taking c as an example) $c_0 \log_{10}(0 + 10) = c_0 \cdot 1$, so the factors out front are the initial salaries, with no scaling necessary.

For the linear core, we have:

$$T(i,t) = \begin{cases} c_0 + c(t - t_i - 7), & \text{for } i \text{ an instructor;} \\ c_0 + c(t - t_i), & \text{for } i \text{ an assistant professor;} \\ b_0 + b(t - t_i), & \text{for } i \text{ an associate professor;} \\ a_0 + a(t - t_i), & \text{for } i \text{ a full professor.} \end{cases}$$

The variables a, b, c, and d are determined by Constraints 2 and 4. This core guarantees that a full professor retiring at twenty-five years after the usual promotions will make twice as much (in real dollars) as a new Ph.D. entering as an assistant professor. It also guarantees the equivalent of a seven-year raise for someone who gets promoted. The calculation of these coefficients is a matter of applying Constraints 2 and 4; they depend only on the initial salaries. (See the **Appendix** for a brief derivation.)

Note that in the linear core, the instructor salary is a seven-year time shift of the of the assistant professor salary. Since there is uncertainty about when instructors will receive their Ph.D.s and become assistant professors, we shift the assistant professor salary curve to use it as an instructor salary curve. A seven-year shift makes the initial instructor salary $27,000, as it should be.

In **Figures 1** and **2**, we graph the ideal salaries for all ranks of faculty. Note how the logarithmic core tapers off after twenty-five years, giving more experienced instructors less and less of a raise each year, while the linear core keeps giving them the same raise. It is in this way that the logarithmic core encourages retirement.

The Real World

Inflation

Let γ be the cost-of-living function from one year to the next, so that $\gamma(t + 1)$ is cost-of-living factor from year t to year $t + 1$; a typical value would be 1.03 for 3% inflation. Each person's real target salary for year t is multiplied by the accumulated cost-of-living increases to produce the nominal target salary $N(i, t+1)$. If the new plan started in year t^0, then the nominal target salary is

$$N(i, t+1) = T(i, t+1)\hat{\gamma}(t+1) \prod_{j=t^0}^{t} \gamma(j),$$

where $\hat{\gamma}(t + 1)$ is an estimate of the cost-of-living factor from the current year to the next.

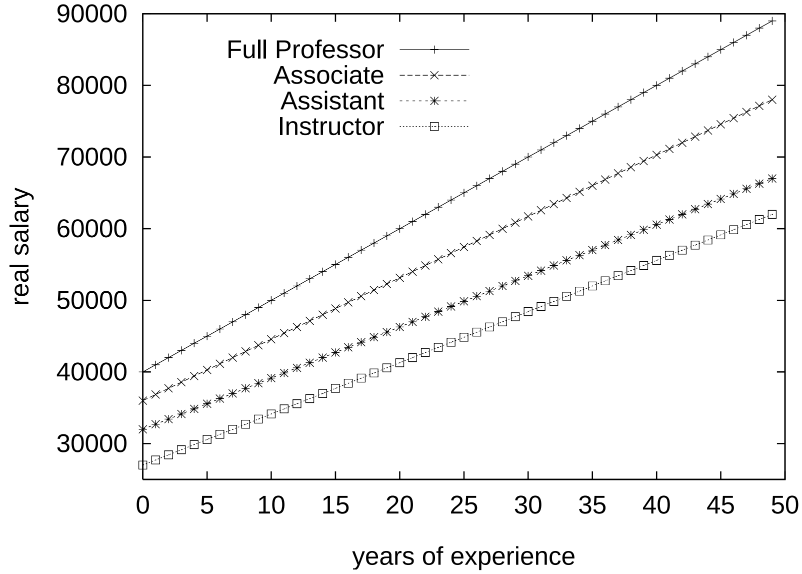

Figure 1. The ideal salaries using the linear core.

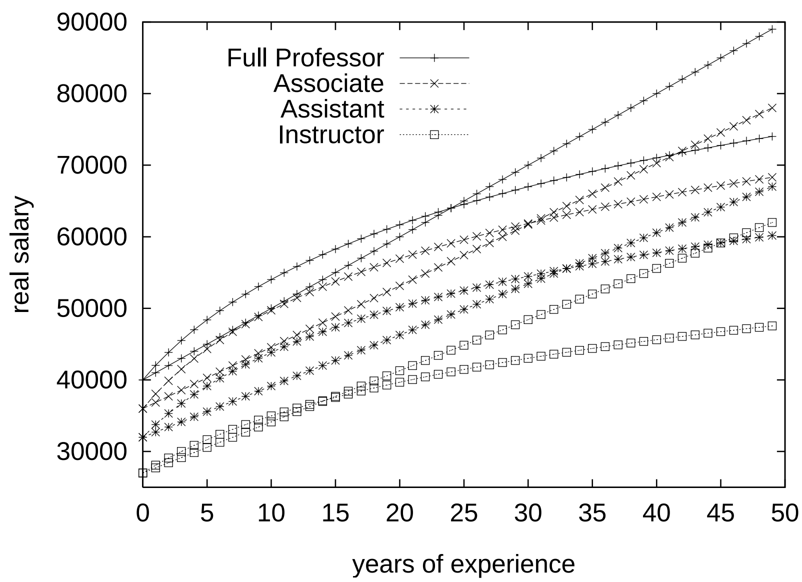

Figure 2. The ideal salaries using the logarithmic core, with linear core salaries shown for comparison.

Finite Budgets

Of course, the college will not always have enough money to give each faculty member the full raise each year. This calls for some way to portion out the raises in accordance with who deserves them the most. If two faculty members each make $40,000 in 1996, and (according to one of the salary plans) the first has a target $41,000 in 1997 while the second's target is only $40,100, then the first should get a larger raise.

Let $n(i, t)$ be the amount in nominal dollars that faculty member i gets paid in year t. Since budgets are limited, this will be at most $N(i, t)$ if nobody is overpaid. We have only some small amount of money, $M_r(t+1)$, for raises next year. This number is given to us by the outside world (the college's treasury). Let $M_n(t + 1)$ be the amount needed next year for raises if all targets are to be met. That is,

$$M_n(t+1) = \sum_i \left[N(i, t+1) - n(i, t)\right],$$

where all Ms are measured in nominal dollars. Usually, $M_n > M_r$: There isn't enough to give the faculty the raises that they deserve.

A fair way to give raises is to give each person a raise in proportion to how much one is needed, where the proportion is the amount available for raises over the amount needed for raises. Thus, next year's salary for instructor i is

$$n(i, t+1) = n(i, t) + \text{raise}(i, t),$$

where i's raise from year t to year $t + 1$ is

$$\text{raise}(i, t) = \frac{M_r(t+1)}{M_n(t+1)} \left[N(i, t+1) - n(i, t)\right].$$

In this manner, instructors who are getting paid far less than their target will get a larger portion of the raises, bringing them closer to their target.

Bruised Egos

The former salary plan has given some faculty members more money than they deserve under the new system. Thus, some salaries need to be reduced. We can't actually cut salaries; indeed, if there is money available, everyone has to get a raise. However, raises can be unequal. To make things simple, we could give an overpaid faculty member an ϵ-dollar raise each year until the target salary catches up with the actual salary. However, this is likely to bruise a few (overpaid) egos.

A good way to placate the overpaid would be to give them a new nominal target $O(i, t)$ that corresponds only to the projected cost of living increase:

$$O(i, t+1) = \hat{\gamma}(t+1)n(i, t).$$

We then treat the overpaid who are underneath their new target O just like those who are underneath their original target N. If there is no positive inflation that year, just give the overpaid instructors some small amount each, to make sure they get a raise. Now, recompute the amount of money needed for raises of both types, and portion out the money that we have according to who needs it the most:

$$M_n(t+1) = \sum_i \begin{cases} N(i,t+1) - n(i,t), & \text{if } i \text{ would be underpaid;} \\ O(i,t+1) - n(i,t), & \text{if } i \text{ would be overpaid.} \end{cases}$$

The new salaries are then computed as

$$n(i,t+1) = n(i,t) + \text{raise}(i,t),$$

where the raise from year t to year $t+1$ is

$$\text{raise}(i,t) = \frac{M_r(t+1)}{M_n(t+1)} \times \begin{cases} N(i,t+1) - n(i,t), & \text{if } i \text{ would be underpaid;} \\ O(i,t+1) - n(i,t), & \text{if } i \text{ would be overpaid.} \end{cases}$$

Excess Funds

Perhaps this belongs under an "Unreal World" section, but on the off-chance that there is more money available than is needed to put everyone on target, there are several options:

- *Raise everyone's salary.* This has a negative consequence: it could put people over their targets for the next year, if the excess is very large. It could put the college into dire financial straits in the future, when the faculty members' salaries cannot be cut. However, if the excess is not very large, instructors will still be below target for the year after the excess, and not much harm is done.
- *Give everyone bonuses.* This would take care of the excess without raising the faculty's expectations for years to come. This is a better option from the college's point of view than raising salaries, and it is a common practice in industrial settings.
- *Give it to the General Fund,* perhaps to caffeine grants for sleep-starved students.

Model Verification

We have projected the performance of the proposed models over the next fifty years. We analyzed the model both with and without such influences as limited budgets, inflation, hiring of new faculty, promotion of faculty members, and retirement. We have also analyzed the effect on this performance of changes in the chosen constants a_0 and b_0.

Figures 3 and **4** show the long-term effects on existing faculty of our model, using the linear and logarithmic cores. These figures assume no hiring, promotion, or retirement, and they ignore cost-of-living increases and possible monetary constraints.

From these two figures, we can see how our model will move the faculty toward a uniform salary system over time. Faculty members with current salaries below the model's target are given raises to bring them up to target. Faculty members with current salaries above target are held to a constant salary (since there is no inflation) until the target catches up to them.

Figures 5 and **6** show how our model, using the linear and logarithmic cores, behaves in the presence of 3% annual inflation. In these graphs, the faculty retire according to the schedule described later, which explains why the graphs become more sparse at the left side. The graphs assume that the college has unlimited funds. Note that these two graphs are essentially identical for large t, when the inflation terms dominates the model.

Figures 7 and **8** show the behavior of the model, with the linear and logarithmic cores, when faculty are promoted, eventually retire, and are replaced by new hires who also are promoted and eventually retire. As before, the model brings faculty into a coherent salary structure over time.

Figures 9 and **10** show the effect of budgetary constraints on the salary of an individual faculty member over time, for each of the cores. These graphs do not include promotions. In **Figure 9**, under the linear core, an early difference between the actual and target salaries gets magnified, because the yearly raise never decreases under the linear model but there is never enough money to give a full raise. In **Figure 10**, under the logarithmic core, an early difference between the actual and target salaries is eliminated as the yearly raises get smaller. These graphs demonstrate the model's ability to cope with limited-money situations.

Sensitivity Analysis

To analyze the sensitivity of our model to changes in a_0 and b_0, we varied these constants and examined the effect on the salaries of faculty members at each rank with fifty years of service. We held c_0 and d_0 constant because they were provided in the problem statement. **Tables 1** and **2** show our results.

Table 1 shows that the fluctuation in salary is higher with the linear core, as is to be expected. It is still only 8%, though. **Table 2** shows that in spite of 10% variation in a_0 and b_0, the fifty-year salaries of all four levels of faculty fluctuate by at most 2% under the logarithmic core. We conclude that our model, especially with the logarithmic core, is relatively insensitive to its initial parameters.

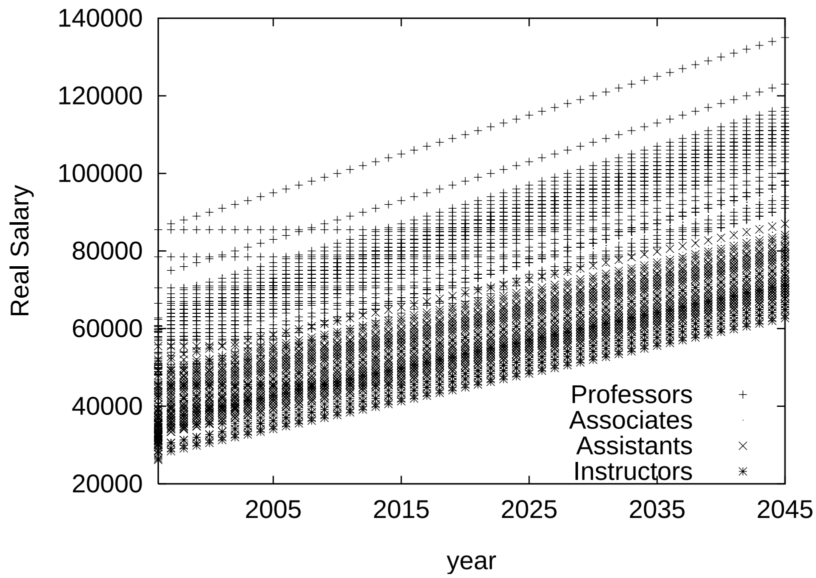

Figure 3. Long-term transition with linear core.

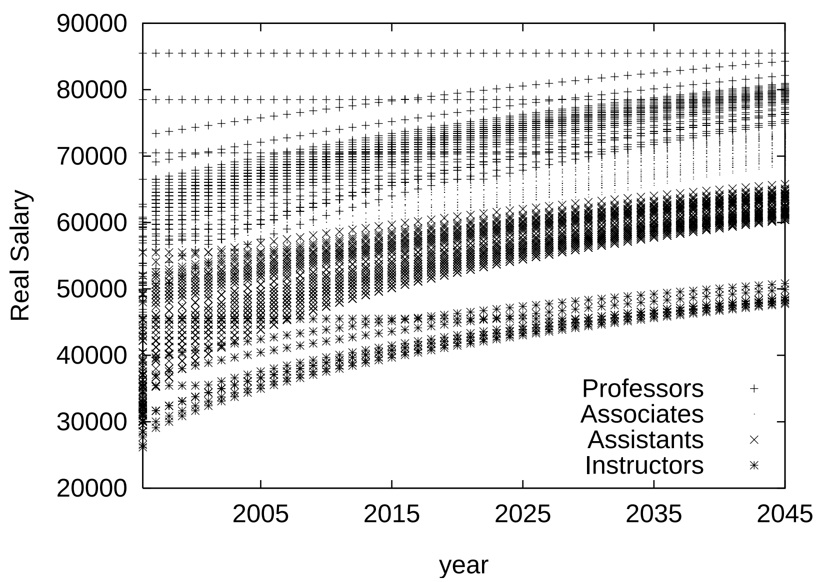

Figure 4. Long-term transition with logarithmic core.

Table 1.

Effect of variations in initial conditions on salary after fifty years of service, using the linear core.

a_0	b_0	Professor	Associate Prof.	Assistant Prof.	Instructor
40,000	36,000	89,000	78,000	67,000	62,000
42,000	36,000	86,917	79,944	67,972	62,833
40,000	38,000	89,000	75,333	71,667	66,000
42,000	38,000	86,917	77,278	72,639	66,833
38,000	34,000	91,083	78,722	61,361	57,166

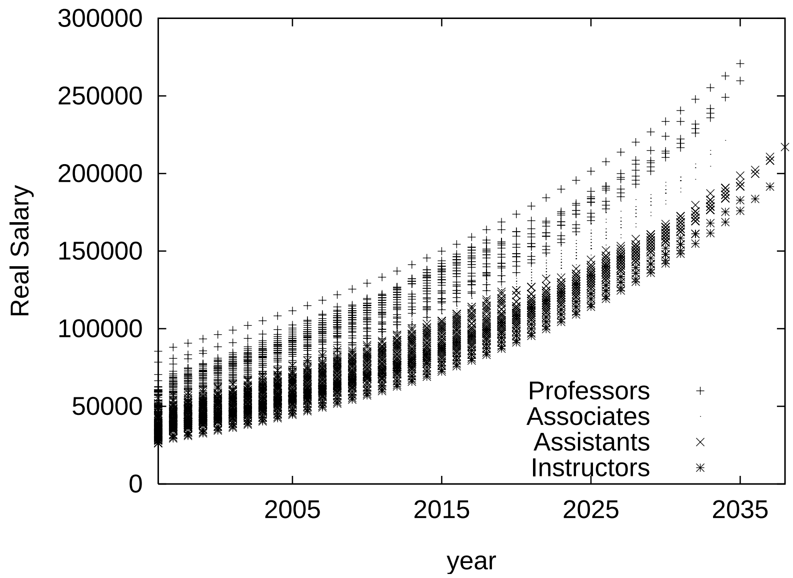

Figure 5. Linear core with retirement and inflation.

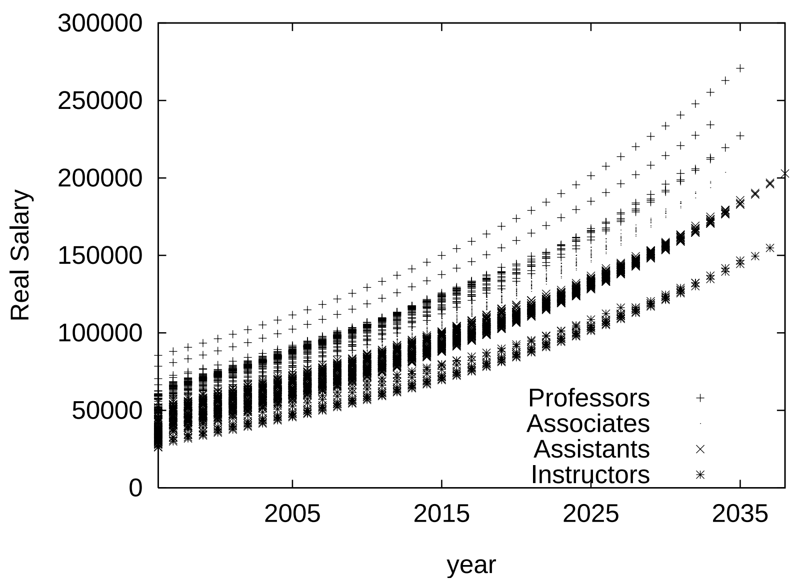

Figure 6. Logarithmic core with retirement and inflation.

Table 2.
Effect of variations in initial conditions on salary after fifty years of service, using the logarithmic core.

a_0	b_0	Professor	Associate Prof.	Assistant Prof.	Instructor
40,000	36,000	74,107	68,312	60,180	47,556
42,000	36,000	73,996	68,545	60,365	47,601
40,000	38,000	74,017	68,306	60,898	47,744
42,000	38,000	73,996	68,548	61,061	47,788
38,000	34,000	73,938	68,048	59,530	47,391

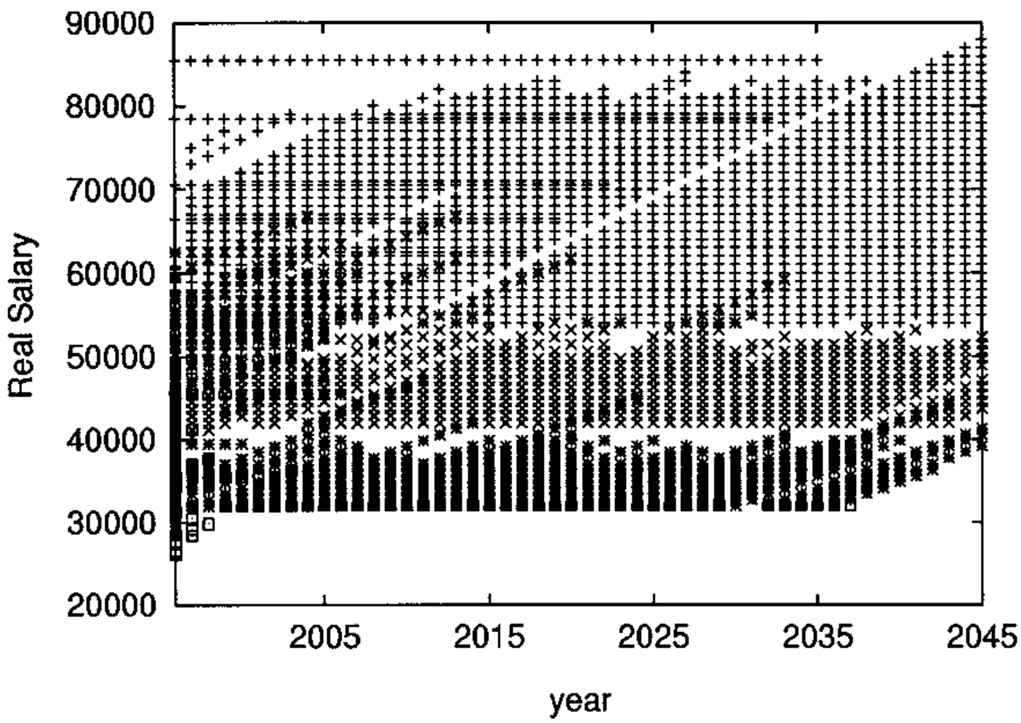

Figure 7. Hiring, promotion, and retirement, under the linear core.

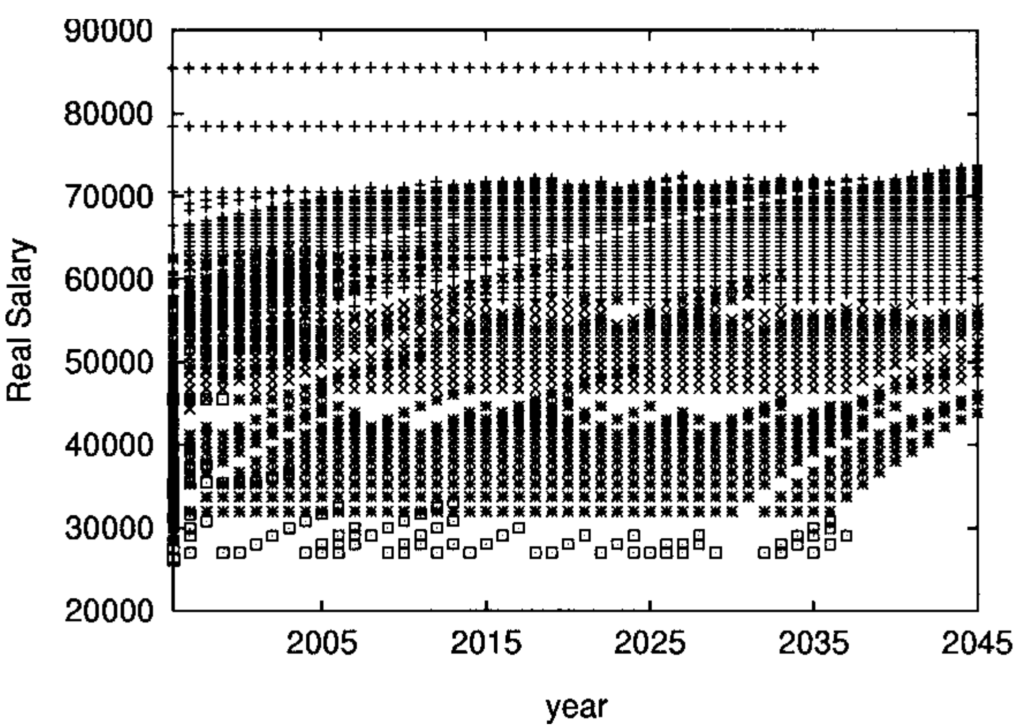

Figure 8. Hiring, promotion, and retirement, under the logarithmic core.

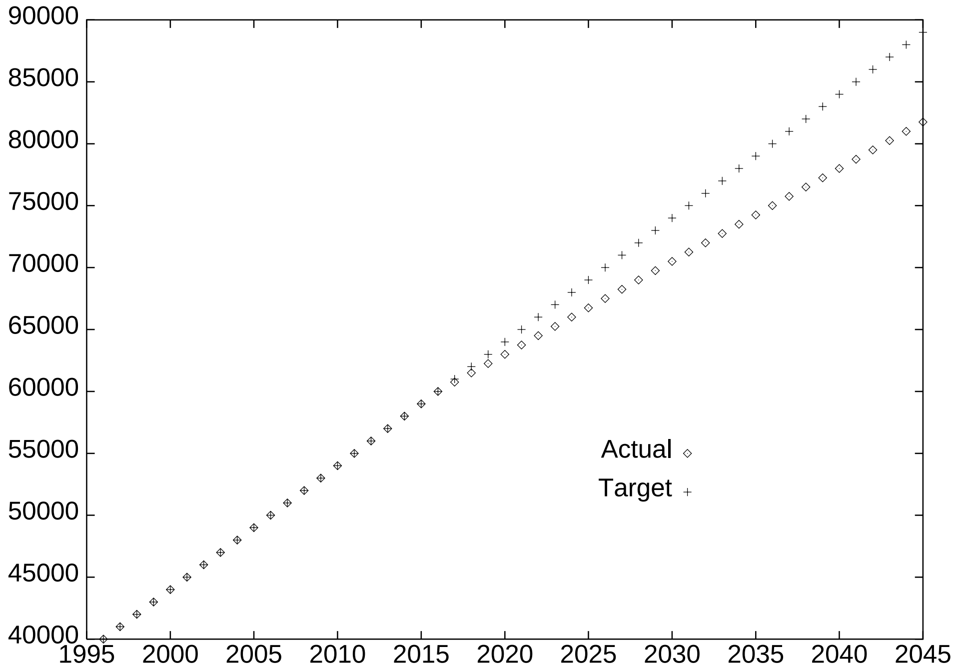

Figure 9. Effect of monetary pressure on an individual faculty member, under the linear core.

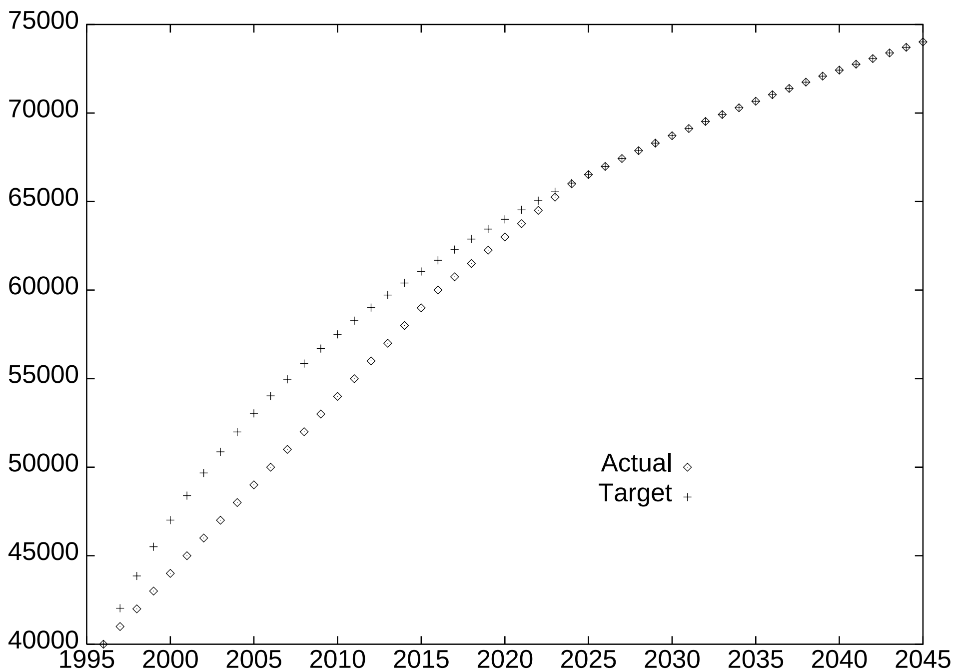

Figure 10. Effect of monetary pressure on an individual faculty member, under the logarithmic core.

Long-Term Performance

In making long-term predictions, many real-world influences on our model had to be simulated. Specifically, we had to decide how many faculty members would change status each year, including new hires, promotions, and retirements. Also, we had to simulate the money available for promotion and arrive at a reasonable cost of living factor for each year.

By examining the initial data, we concluded that that the college hires a mean of 9 faculty per year, with a standard deviation of 5 on a discretized normal distribution. We decided more or less arbitrarily that faculty will retire after working for 40 years, with a standard deviation of 2 years, again on a discretized normal distribution. For promotions, we decided that 50% of assistant professors would become associate professors in 7 years, 25% would become associate professors in 8 years, and so on with continued halving. We used the same probability distributions for associate professors being promoted to full professor.

We used a constant 3% inflation per year, or $\gamma_i = 1.03$ for all i. We also assumed that the college would always be able to predict this value accurately for the next year. Small fluctuations in any γ_i will have negligible effects on the model's performance, as will small inaccuracies in the college's yearly predictions. Finally, we analyzed the model in the presence of both limited and unlimited money supplies. For limited money, we made a constant amount of money available for raises and chose this constant such that it would be inadequate soon after the initial year of the simulation.

Strengths and Weaknesses

The logarithmic option encourages instructors to retire earlier than the linear model, since the logarithm curve flattens out at higher values. This means that the raise for a professor with forty years of experience is small relative to the raise for someone with twenty-six years of experience. This gives the Provost a way to encourage or discourage retirement, based on the needs of the college.

If the college wishes to adjust the real values of the starting salaries, everyone's salaries will change, not just those hired after the change is made.

The faculty must be willing to settle for a higher potential salary, instead of a guaranteed higher salary, for promotions and retirement. As long as the college cannot guarantee enough money for everyone's raises, this will remain.

Salary increases were calculated according to an faculty member on a track with the minimum years between promotions. This causes late promotions to receive the equivalent of an eight- or nine-year raise at their current rank. For example, an instructor promoted in eight years will receive a raise greater than the raise awarded for a promotion in seven years.

Furthermore, all future promotions (if any) will have larger raises. Likewise, a professor retiring at twenty-six years experience, instead of twenty-five years will receive a salary greater than twice the salary of a beginning assistant professor (see the **Appendix**).

Appendix

The two cores allow the initial salaries to be set by the outside world; however, the rate at which salary increases depends on these initial salaries. The constraints apply in this manner:

Constraint 2: Consider a new Ph.D. with no prior experience. If promotions occur on time, then years 0 through 6 are spent as assistant professor, years 7 through 13 as associate professor, and years 14 and on as full professor. Thus, the Ph.D.'s first year as associate professor is year 7; it should correspond in salary to the year 14 as assistant professor:

$$T(\text{associate}, 7) = T(\text{assistant}, 14).$$

Similarly, the Ph.D. becomes a full professor at year 14; that should correspond to the year 21 as associate professor:

$$T(\text{professor}, 14) = T(\text{associate}, 21).$$

We don't have to track inflation and budget constraints through the actual year because we are dealing in real dollars, and the salary curves don't change: Ideally, Associate Professor X with 15 years of experience in 1997 makes the same real amount as Associate Professor Y with 15 years of experience in 2010.

Constraint 4: Similar to the previous constraint:

$$2T(\text{assistant}, 0) = T(\text{professor}, 25).$$

Since one's salary increases with time, those who retire after year 25 receive more than twice $T(\text{assistant}, 0)$, which fits the constraint.

Earlier, we discussed the reasoning behind choosing the instructor salary curve in the linear core as a time shift: There is no explicit method of solving for a linear rate constant for the Instructor salary. We run into the same problem for the logarithmic model, except that a time shift would not produce a suitable result for the given starting salaries of an assistant professor and an instructor. Thus, we need to choose an average year in which to promote instructors to assistant professors. An instructor may be promoted after exactly one year; if we base the salary curve on a longer period, instructors who take longer than expected to earn their Ph.D.s would receive less than a seven-year raise upon promotion. Therefore, we promote instructors after one year.

Substitution and simplifying produce the following rate constants:

Logarithmic Core:

$$\begin{aligned} a &= \left[10^{2c_0/a_0} - 10\right]/24 \\ b &= \left[(14a+10)^{a_0/b_0} - 10\right]/21 \\ c &= \left[(7b+10)^{b_0/c_0} - 10\right]/14 \\ d &= \left[(c+10)^{c_0/d_0} - 10\right]/8 \end{aligned}$$

Linear Core:

$$\begin{aligned} a &= (2c_0 - a_0)/24 \\ b &= (14a + a_0 - b_0)/21 \\ c &= (7b + b_0 - c_0)/14 \end{aligned}$$

How do the starting salaries $a_0 = \$40{,}000$ and $b_0 = \$36{,}000$ work out so well? For the linear core, look at the initial salary for instructors:

$$c(0-7) + c_0 = -7c + c_0.$$

The quantity c is determined by b and b_0, while b is determined by a and a_0. So, a_0 and b_0 affect the instructor's initial salary. With a_0 and b_0 chosen as they were, the starting instructor's salary comes out to \$27,000, exactly as specified in the problem statement.