

Judge's Commentary: The Outstanding Helix Intersections Papers

Daniel Zwillinger
Zwillinger & Associates
Newton, MA 02165
zwillinger@world.std.com

Introduction

Typical industrial tasks for applied mathematicians are varied, and many require a computational approach to solve a relatively simple problem. The Helix Intersections Problem was representative: The problem statement, solution techniques to be used, interpretation of the result, and techniques for checking the answer were all straightforward. Most of the submissions did, in fact, perform nearly all of the above steps. The judging criteria focused on how well each step was carried out as well as on the overall organization and clarity.

Thorough Analysis of All Cases

A plane and a helix can have no intersections, any finite number of intersections, or an infinite number of intersections (in the case of an infinite helix or a degenerate helix with zero pitch). A computer program asked to find all the intersection points must respond appropriately to each of these cases.

Simplicity of the Resulting Numerical Problem

Given a helix and a plane, it is straightforward to write down the parametric equations for the helix, depending on one variable, and the equation of the plane. Substituting the parametric equations into the equation for the plane results in an equation with a single variable. Finding the roots of this single variable equation is far simpler than finding the roots of a multiple-variable equation (as some teams proposed).

Numerical Solution of the Problem

A bisection method is guaranteed to find a zero if appropriate endpoints are given, but the method is slow. Newton's method is the method of choice for nonlinear problems, since it is so much faster. However, several teams apparently did not know that Newton's method does not always converge (for example, near multiple roots).

Since appropriate numerical bounds can be found, and bisection can be made to work, several teams used it. The judges preferred a bisection technique, with provable bounds on the results, to a Newton iteration with no mention of possible convergence problems. The team from Macalester College used both a Newton iteration and a bisection method (when the Newton method failed).

Testing the Results

There are many ways in which the results of the computer program created to solve this task could be tested. Some teams used graphical methods, while others used the result of a more general-purpose equation solver (such as Mathematica). There often seemed to be confusion about the reliability of the results of programs such as Mathematica.

Responsiveness to the Question

The original problem statement was concerned about the computational speed of locating the intersection points. The judges looked for statements about the computational requirements of the algorithms presented. There are many ways in which this issue could be addressed: the computer time per intersection point, the computer time saved when compared to a more general mathematical solver (such as Mathematica), or the computational complexity of the algorithm.

Conclusion

Of course, really outstanding papers not only solve the problem but also consider possible extensions and possible limitations. Do these make the problem easier or harder? Do they make the problem applicable to another field? Restricting the problem to a finite-length helix (which is more physically reasonable) was considered by several teams, including the teams from Harvey Mudd College and Iowa State University. Using a finite-area sweeping plane was considered by the team from Harvey Mudd

College. Additionally, one team considered more general helices, such as a spiral drawn on a cone.

About the Author

Daniel Zwillinger received an undergraduate degree in mathematics from MIT and a Ph.D. in applied mathematics from Caltech. His Ph.D. research dealt with the focusing of waves as they travel through random media. He taught at Rensselaer Polytechnic Institute for four years, was in industry for several years, and has been managing a consulting group for the last few years. His work areas have included many industrial mathematics needs: radar, sonar, communications, visualization, statistics, and computer-aided design. He is the author of several mathematical reference books.

