Torque on a Spinning Hollow Sphere in a Uniform Magnetic Field

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Summary—The torque on a spinning spherical conducting shell of arbitrary thickness in a uniform magnetic field is calculated in terms of the magnetic vector potential. The torque is found to result from the interaction of eddy currents induced in the shell with the excitation field and is such as to oppose and precess the spinning of the shell.

Introduction

IF A CONDUCTING sphere is rotated in a magnetic field, eddy currents are induced in the sphere. Calculations of these currents have appeared through the literature [1]–[8]. These eddy currents react with the applied field to produce torques which both retard and precess the rotation of the sphere. The earlier authors were interested in this effect as it applied to motional damping and the induction motor principle, and in some cases, calculated the retarding component of the torque from the I^2R heat produced in the sphere.

Recently, interest has reawakened in this problem because of possible applications to earth satellites, electrically suspended gyroscopes and attitude control reaction spheres. In these applications, complete expressions for both retarding and precessing torques are needed. Several authors [9], [10] have obtained such expressions by approximate means; however, the approximations made somewhat limit their utility. In this paper the exact formulas are derived.

Consider a spherical conducting shell of outer and inner radii $r=a$ and $r=b$, respectively, with spin velocity ω in a uniform magnetic field of strength B_0 making an angle α with the spin axis, Fig. 1. The shell material is of resistivity τ and permeability μ_0 which is the same as the permeability of the free space surrounding it. The shell, which sees a changing magnetic field $\boldsymbol{B}$, will be subjected to an electric field $\boldsymbol{E}$ according to Faraday's law,[1]

$$\nabla \times \boldsymbol{E} = -\frac{d\boldsymbol{B}}{dt}. \tag{1}$$

Manuscript received October 1, 1963. The results presented in this paper were obtained under the sponsorship of the Special Projects Office of the U. S. Navy and in part were first reported in Minneapolis-Honeywell Aero Report #1901-QR5, September, 1960.

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[1] Rationalized MKSC units are used throughout this paper. For a discussion of the relations which follow, cf: W. R. Smythe, "*Static and Dynamic Electricity*," McGraw-Hill Book Co. Inc., New York, N. Y. 2nd edition, ch. 11; 1950.

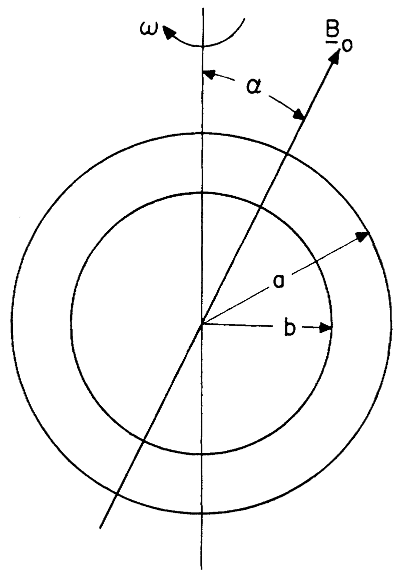

Fig. 1—Spherical shell rotating in a uniform magnetic field.

Introducing a vector potential $\boldsymbol{A}$ by the relations

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad \nabla \cdot \boldsymbol{A} = 0, \tag{2}$$

one can write the electric field as

$$\boldsymbol{E} = -\frac{d\boldsymbol{A}}{dt}. \tag{3}$$

The electric field will give rise to eddy currents J in the conductor according to Ohm's law,

$$\tau J = \boldsymbol{E} \tag{4}$$

The currents J will, in turn, give rise to a magnetic field according to Ampere's law,

$$\nabla \times \boldsymbol{B} = \mu_0 J. \tag{5}$$

Combining (2)–(5), it is seen that the field is completely described by the vector partial differential equations

$$\nabla^2 \boldsymbol{A} = \frac{-\mu_0}{\tau}\frac{d\boldsymbol{A}}{dt} \tag{6}$$

in the shell, and

$$\nabla^2 \boldsymbol{A} = 0 \tag{7}$$

in free space, where the conductivity is zero.

The solution of the problem then entails solutions of (7) in the regions $r \leq b$ and $r \geq a$, and solution of (6) in the region $b \leq r \leq a$. The terms $\boldsymbol{A}'$, $\boldsymbol{A}$, and $\boldsymbol{A}''$ denote the vector potential in the regions $r \leq b$, $b \leq r \leq a$, and $r \geq a$, respectively. The solution is completed by specifying the necessary boundary conditions.

$$\boldsymbol{A}' \text{ finite at } r = 0$$
$$\boldsymbol{A}'' \to \boldsymbol{A}_0 \text{ as } r \to \infty \tag{8}$$

where $\boldsymbol{A}_0$ corresponds to the excitation field $\boldsymbol{B}_0$, and

$$\boldsymbol{A}' = \boldsymbol{A}, \quad \boldsymbol{r} \times (\nabla \times \boldsymbol{A}') = \boldsymbol{r} \times (\nabla \times \boldsymbol{A}), \quad \text{at } r = b$$
$$\boldsymbol{A}'' = \boldsymbol{A}, \quad \boldsymbol{r} \times (\nabla \times \boldsymbol{A}'') = \boldsymbol{r} \times (\nabla \times \boldsymbol{A}), \quad \text{at } r = a \tag{9}$$

which implies continuity of $\boldsymbol{A}$ and tangential $\boldsymbol{B}$ through the boundaries.

The force per unit volume $d\boldsymbol{F}$ exerted on the current by the magnetic field is given by

$$d\boldsymbol{F} = \boldsymbol{J} \times \boldsymbol{B}, \tag{10}$$

and the torque per unit volume $d\boldsymbol{T}$ is given by

$$d\boldsymbol{T} = \boldsymbol{r} \times d\boldsymbol{F}. \tag{11}$$

Thus, the torque acting on the shell due to its spinning motion is

$$\boldsymbol{T} = \int_V \boldsymbol{r} \times (\boldsymbol{J} \times \boldsymbol{B}) dV \tag{12}$$

where dV is a volume element and the integral is taken over the volume of the shell. In terms of the vector potential, the torque is given by

$$\boldsymbol{T} = \frac{-1}{\tau} \int_V \boldsymbol{r} \times \left[\frac{d\boldsymbol{A}}{dt} \times (\nabla \times \boldsymbol{A})\right] dV. \tag{13}$$

Solution of the Basic Equations

Consider two rectangular cartesian systems, $S(x, y, z)$ and $S'(x', y', z')$, Fig. 2, with S fixed in space and S' fixed in the shell, having their common origin at the center of the shell. Introducing unit orthogonal triads of vectors $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ and $(\boldsymbol{i}', \boldsymbol{j}', \boldsymbol{k}')$ in S and S', respectively, in frame S

$$\boldsymbol{B}_0 = B_0 \sin \alpha \boldsymbol{i} + B_0 \cos \alpha \boldsymbol{k}, \tag{14}$$

where, without loss of generality, $\boldsymbol{B}_0$ has been chosen to lie in the x-z plane. The transformation equations between the vector triads in S and S' are given by

$$\boldsymbol{i} = \boldsymbol{i}' \cos \omega t + \boldsymbol{j}' \sin \omega t$$
$$\boldsymbol{j} = -\boldsymbol{i}' \sin \omega t + \boldsymbol{j}' \cos \omega t \tag{15}$$
$$\boldsymbol{i}' = \boldsymbol{i} \cos \omega t - \boldsymbol{j} \sin \omega t$$
$$\boldsymbol{j}' = \boldsymbol{i} \sin \omega t + \boldsymbol{j} \cos \omega t \tag{16}$$

and, of course, $\boldsymbol{k} = \boldsymbol{k}'$. Thus, in the rotating frame S',

$$\boldsymbol{B}_0 = B_0 \sin \alpha (\boldsymbol{i}' \cos \omega t + \boldsymbol{j}' \sin \omega t) \tag{17}$$

where the constant $\boldsymbol{k}$ component has been dropped from consideration for the present. Hence, the shell "sees" a varying magnetic field and will have currents produced in it.

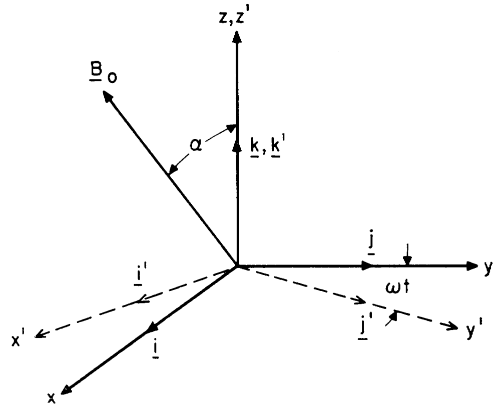

Fig. 2—Coordinate reference frames.

For convenience, a complex phasor notation is introduced.

$$\hat{\boldsymbol{B}}_0 = B_0 e^{i\omega t}(\boldsymbol{i}' - i\boldsymbol{j}') \sin \alpha \tag{18}$$

where

$$i^2 = -1$$
$$\boldsymbol{B}_0 = \operatorname{Re} \hat{\boldsymbol{B}}_0$$

and $e^{i\omega t}$ is carried along with the phasor for convenience in the later calculation of the torque. The vector potential $\hat{\boldsymbol{A}}_0$ corresponding to the excitation field $\hat{\boldsymbol{B}}_0$ is given by

$$\hat{\boldsymbol{A}}_0 = \tfrac{1}{2} B_0 \sin \alpha e^{i\omega t}[-z'(i\boldsymbol{i}' + \boldsymbol{j}') + (y' + ix')\boldsymbol{k}'] \tag{19}$$

as can be verified directly from (2). Introducing the usual spherical polar coordinates (r, θ, ϕ) in S', one can write (19) as

$$\hat{\boldsymbol{A}}_0 = \tfrac{1}{2} B_0 \sin \alpha e^{i\omega t} r \cdot [-\cos \theta (i\boldsymbol{i}' + \boldsymbol{j}') + i \sin \theta e^{-i\phi} \boldsymbol{k}']. \tag{20}$$

This suggests a solution (6) and (7) of the form

$$\hat{\boldsymbol{A}} = e^{i\omega t} \hat{A}(r) \boldsymbol{\Phi} \tag{21}$$

where $\hat{A}(r)$ is a complex scalar function of r and

$$\boldsymbol{\Phi} = \tfrac{1}{2} B_0 \sin \alpha (-i \cos \theta \boldsymbol{i}' - \cos \theta \boldsymbol{j}' + i \sin \theta e^{-i\phi} \boldsymbol{k}). \tag{22}$$

By substitution of (21) into (6) and (7), $\hat{A}(r)$ must satisfy

$$\frac{d^2 A}{dr^2} + \frac{2}{r} \frac{dA}{dr} - \left(ip + \frac{2}{r^2}\right) A = 0 \tag{23}$$

in the shell, where $p = \mu_0 \omega / \tau$, and

$$\frac{d^2 A}{dr^2} + \frac{2}{r} \frac{dA}{dr} - \frac{2A}{r^2} = 0 \tag{24}$$

in free space.

Eq. (23) has solutions $r^{-1/2} I_{3/2}(v_r)$ and $r^{-1/2} I_{-3/2}(v_r)$ where $I_{3/2}$ and $I_{-3/2}$ are the modified Bessel functions of orders $3/2$ and $-3/2$ respectively and

$$v_r = (ip)^{1/2} r \tag{25}$$

It follows that

$$\hat{E} = 3\left(\frac{\pi}{2v_a}\right)^{1/2} a^{3/2}\left[\frac{(3+v_b^2)\cosh v_b - 3v_b \sinh v_b}{3v_b\cosh(v_a - v_b) + (3+v_b^2)\sinh(v_a - v_b)}\right] \tag{29}$$

$$\hat{F} = 3\left(\frac{\pi}{2v_a}\right)^{1/2} a^{3/2}\left[\frac{3v_b\cosh v_b - (3+v_b^2)\sinh v_b}{3v_b\cosh(v_a - v_b) + (3+v_b^2)\sinh(v_a - v_b)}\right] \tag{30}$$

where $v_a = (ip)^{1/2}a$, and $v_b = (ip)^{1/2}b$. Hence, in the shell, the vector potential $\hat{\boldsymbol{A}}$ is given by the real part of (21) with

$$\hat{A}(r) = \frac{3a}{v_r^2}\left\{\frac{[v_r(3+v_b^2) - 3v_b]\cosh(v_r - v_b) + [3v_bv_r - (3+v_b^2)]\sinh(v_r - v_b)}{3v_b\cosh(v_a - v_b) + (3+v_b^2)\sinh(v_a - v_b)}\right\}. \tag{31}$$

Eq. (24) has solutions r and r^{-2}. Thus, the phasor vector potential is given at all points of space by

$$\begin{aligned} \hat{\boldsymbol{A}}' &= e^{i\omega t}(\hat{C}r + \hat{D}r^{-2})\boldsymbol{\Phi} && r \le b \\ \hat{\boldsymbol{A}} &= e^{i\omega t} r^{-1/2}[\hat{E}I_{3/2}(v_r) + \hat{F}I_{-3/2}(v_r)]\boldsymbol{\Phi} && b \le r \le a \\ \hat{\boldsymbol{A}}'' &= e^{i\omega t}(\hat{G}r + \hat{H}r^{-2})\boldsymbol{\Phi} && r \ge a \end{aligned} \tag{26}$$

where $\hat{C}$, $\hat{D}$, $\hat{E}$, $\hat{F}$, $\hat{G}$, and $\hat{H}$ are complex constants to be evaluated.

Conditions on A at $r=0$ and as $r\rightarrow\infty$ require that $\hat{C}=0$ and $\hat{G}=1$. The continuity conditions (9) require

$$\begin{aligned} A'(b) &= A(b) & \frac{dA'}{dr}(b) &= \frac{dA}{dr}(b) \\ A''(a) &= A(a) & \frac{dA''}{dr}(a) &= \frac{dA}{dr}(a) \end{aligned} \tag{27}$$

where

$$\begin{aligned} A' &= \hat{D}r^{-2} \\ A &= r^{-1/2}[\hat{E}I_{3/2}(v_r) + \hat{F}I_{-3/2}(v_r)]. \\ A'' &= r + \hat{H}r^{-2} \end{aligned} \tag{28}$$

Upon using the differentiation formulas, [11]

$$\frac{dI_{3/2}}{dx}(x) = I_{1/2}(x) - \frac{3}{2x}I_{3/2}(x)$$

$$\frac{dI_{-3/2}}{dx}(x) = I_{-1/2}(x) - \frac{3}{2x}I_{-3/2}(x)$$

and the expressions

$$I_{1/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2}\sinh x$$

$$I_{-1/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2}\cosh x$$

$$I_{3/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2}\left(\cosh x - \frac{1}{x}\sinh x\right)$$

$$I_{-3/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2}\left(\sinh x - \frac{1}{x}\cosh x\right).$$

CALCULATION OF THE TORQUE

The torque acting on the shell can be written

$$\boldsymbol{T} = \boldsymbol{T}_1 + \boldsymbol{T}_2, \tag{32}$$

where from (13),

$$\boldsymbol{T}_1 = -\frac{1}{\tau}\int_V \boldsymbol{r}\times\left[\mathrm{Re}\left(\frac{d\hat{\boldsymbol{A}}}{dt}\right)\times \mathrm{Re}\,(\nabla\times\hat{\boldsymbol{A}})\right]dV \tag{33}$$

and

$$\boldsymbol{T}_2 = -\frac{1}{\tau}\,\mathrm{Re}\int_V \boldsymbol{r}\times\left(\frac{d\hat{\boldsymbol{A}}}{dt}\times B_0\cos\alpha\boldsymbol{k}\right)dV. \tag{34}$$

The term $\boldsymbol{T}_2$ is seen to arise from the interaction of the induced currents with the constant field component in the $\boldsymbol{k}$ direction, which had previously been dropped from consideration.

Using (21) and (22), after considerable algebraic manipulation and transformation back to the fixed reference frame S by (16), one obtains

$$\boldsymbol{T}_1 = -\frac{B_0^2\sin^2\alpha}{2\tau}\int_{r=b}^{r=a}\int_{\theta=0}^{\theta=\pi}\int_{\phi=0}^{\phi=2\pi}\mathrm{Re}\,(Ae^{-i\phi}\sin\theta)$$

$$\cdot\,\mathrm{Re}\left\{i\omega A[-\cos\theta(\boldsymbol{j}+i\boldsymbol{i}) + ie^{-i\phi}\sin\theta\boldsymbol{k}]\right\}r^2\sin\theta\,dr\,d\theta\,d\phi \tag{35}$$

$$\boldsymbol{T}_2 = -\frac{B_0^2\sin 2\alpha}{4\tau}\,\mathrm{Re}\int_{r=b}^{r=a}\int_{\theta=0}^{\theta=\pi}\int_{\phi=0}^{\phi=2\pi} i\omega Ar\cos\theta$$

$$\cdot\,[-\cos\theta(\boldsymbol{j}+i\boldsymbol{i}) + ie^{-i\phi}\sin\theta\boldsymbol{k}]r^2\sin\theta\,dr\,d\theta\,d\phi. \tag{36}$$

Eqs. (35) and (36) can be reduced to

$$\boldsymbol{T}_1 = \boldsymbol{k}\frac{2}{3}\frac{\omega B_0^2\sin^2\alpha}{\tau}\int_b^a\int_0^{2\pi} r^2[\mathrm{Re}\,(Ae^{-i\phi})]^2\,dr\,d\phi \tag{37}$$

$$\boldsymbol{T}_2 = \frac{\pi\omega B_0^2\sin 2\alpha}{3\tau}\,\mathrm{Re}\int_b^a r^3A(-\boldsymbol{i}+i\boldsymbol{j})\,dr. \tag{38}$$

If A_r and A_i denote the real and imaginary parts of $\hat{A}$, respectively,

$$A = A_r + iA_i, \tag{39}$$

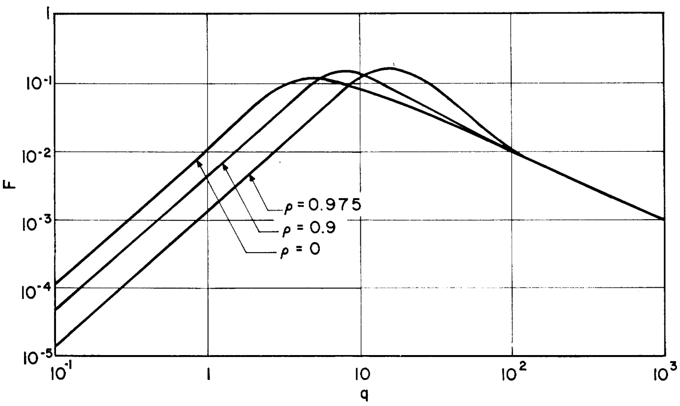

Fig. 3—Nondimensional rundown torque vs q.

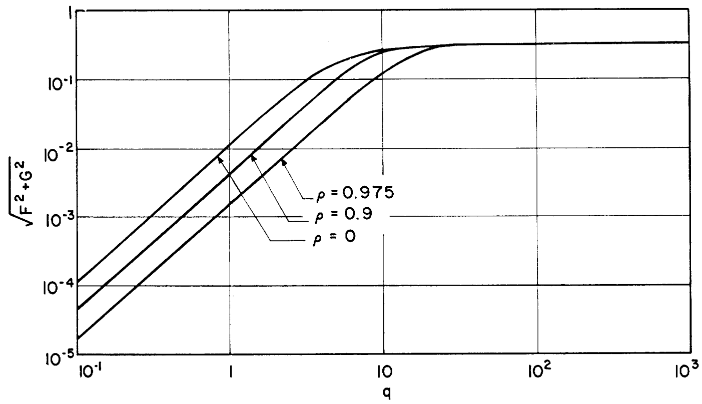

Fig. 4—Nondimensional precession torque vs q.

then substituting (39) into (37) and (38) reduces the torque expressions to

$$\boldsymbol{T}_1 = \boldsymbol{k}\frac{2\pi\omega B_0^2 \sin^2\alpha}{3\tau}\int_b^a r^2(A_r^2 + A_i^2)dr \tag{40}$$

$$\boldsymbol{T}_2 = -\frac{\pi\omega B_0^2 \sin 2\alpha}{3\tau}\left(\boldsymbol{i}\int_b^a r^3 A_r dr + \boldsymbol{j}\int_b^a r^3 A_i dr\right). \tag{41}$$

The torque terms can be interpreted as follows. $\boldsymbol{T}_1$ is a run-down torque which is produced by the interaction of the currents with the field producing them, while $\boldsymbol{T}_2$ is a precessional torque which is produced by the interaction of the induced currents with the constant field component along the spin axis. This interaction will cause the spin axis to cone into the direction of the magnetic field.

Eqs. (40) and (41) with the use of (31) yield for the total torque in newton meters

$$\boldsymbol{T} = \frac{3\pi a^3 B_0^2}{\mu_0}[(-\boldsymbol{i}F + \boldsymbol{j}G)\sin 2\alpha + \boldsymbol{k}F2\sin^2\alpha] \tag{42}$$

where

$$F = \frac{1}{q}\left\{\frac{36q\rho C + 6q^3\rho^3\overline{C} + 18q^2\rho^2\overline{S} + (36+q^4\rho^4)S}{36q\rho S + 6q^3\rho^3\overline{S} + 18q^2\rho^2 C + (36+q^4\rho^4)\overline{C}}\right\} - \frac{2}{q^2}$$

$$G = \frac{1}{3} - \frac{1}{q}\left\{\frac{36q\rho\overline{C} + 6q^3\rho^3 C + 18q^2\rho^2 S + (36+\rho^4 q^4)\overline{S}}{36q\rho S + 6q^3\rho^3\overline{S} + 18q^2\rho^2 C + (36+q^4\rho^4)\overline{C}}\right\}$$

$$\left.\begin{matrix} S \\ \overline{S} \end{matrix}\right\} = \sinh q(1-\rho) \pm \sin q(1-\rho)$$

$$\left.\begin{matrix} C \\ \overline{C} \end{matrix}\right\} = \cosh q(1-\rho) \pm \cos q(1-\rho)$$

$$q = \sqrt{2p}\, a, \quad p = \mu_0\omega/\tau$$

$$\rho = b/a. \tag{43}$$

From (42), it is apparent that there is no component of torque along the direction of the B_0 field and hence the sphere will tend to a steady-state condition in which it is spinning about an axis in the direction of the applied field with angular velocity equal to its initial component in this direction. For the special case of a perfect conductor, the spin axis executes steady precession about the direction of the field.

From (42) and (43), the following useful asymptotic expressions for F and G may be obtained.

as $q \to 0$

$$F \to \frac{q^2}{90}(1-\rho^5), \qquad G \to \frac{q^4}{108}\left(\frac{2}{35} - \frac{\rho^5}{5} + \frac{\rho^7}{7}\right) \tag{44}$$

as $q \to \infty$

$$F \to \frac{1}{q} - \frac{2}{q^2}, \qquad G \to \frac{1}{3} - \frac{1}{q}. \tag{45}$$

The normalized run-down torque, F, and the normalized total precessional torque, $\sqrt{F^2+G^2}$, are plotted in Figs. 3 and 4, respectively, vs the parameter q, defined in (43). Multiplication of the values obtained from the curves by the appropriate constant and trigonometric function from (42) will give actual torque values in newton meters for the three ratios of inner to outer radius ρ shown. It is observed that the use of the exact expressions (42) and (43) is required only in the approximate range, $1 < q < 100$; for q values outside this range, the asymptotic expressions (44) or (45) are very accurate.

For a thin shell, the torque integrals must be evaluated by a proper limiting process in which the shell thickness ($\delta = a - b$) goes to zero, and the surface resistivity ($\zeta = \tau/\delta$) remains finite. Taking the shell to be of radius a, one obtains from (31) in the limit

$$A = 3a\zeta\left[\frac{3\zeta - i\mu_0\omega a}{9\zeta^2 + \mu_0^2\omega^2 a^2}\right] \tag{46}$$

and

$$A_r = \frac{9a\zeta^2}{9\zeta^2 + \mu_0^2\omega^2 a^2} \qquad A_i = \frac{3a^2\mu_0\omega\zeta}{9\zeta^2 + \mu_0^2\omega^2 a^2}. \tag{47}$$

Substituting (47) into (40) and (41) and writing $dr=\delta$, one finds that

$$T = \frac{\pi\omega a^4 B_0^2}{9\zeta^2 + \mu_0^2\omega^2 a^2} \cdot [(-i3\zeta + j\mu_0\omega a)\sin 2\alpha + k6\zeta \sin^2\alpha], \quad (48)$$

which agrees with the result of performing the same limiting operation on the general torque expression given by (42).

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Torque on a Spinning Hollow Sphere in a Uniform Alternating Magnetic Field

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Summary—The calculation of torque acting on a spinning spherical shell in a uniform stationary magnetic field first presented by Halverson and Cohen[1] is extended to include the alternating magnetic field case. The torque is found under certain conditions to be positive and will increase the sphere's angular velocity up to a stable equilibrium condition at which point the torque changes sign. Differential equations are solved in terms of the magnetic vector potential which is used to calculate the induced eddy currents and resultant magnetic field. A complete solution is given for the solid sphere while approximations are used to obtain a thin shell solution.

Introduction

Solution of the torques produced on a rotating spherical conductor by an alternating magnetic field is useful for such applications as torquing and stabilizing electrostatic gyros,[1] attitude control reaction sphere,[2,3] field supported ultra-high-speed centrifuges[1,4] and other similar devices. Torques are derived from the fundamental field equations in terms of the magnetic vector potential.

The alternating field, like the stationary field case presented by Halverson and Cohen,[1] induces eddy currents which react with the applied field to produce precessional torques. In the alternating field case, however, the induced eddy currents may react to produce an acceleration torque instead of a retarding torque. Change-over from a retarding to an acceleration torque occurs at a stable equilibrium point as shown in Figs. 1 and 2. The equilibrium point depends upon the alternating field frequency, sphere angular velocity, sphere thickness and sphere resistivity.

Geometry and Equations

The geometry for determining the torques experienced by a rotating spherical conducting shell placed in a uniform alternating magnetic field is illustrated in Fig. 3. The shell has an outer and an inner radius, $r=a$ and $r=b$, respectively, and is rotating with an angular velocity about the k axis. The uniform alternating magnetic field is inclined at an angle α to the spin axis in the x-z plane defined by the first and last members of the unit orthogonal trial (i, j, k). Resistivity of the shell is represented by τ and has the same permeability as free space.

Manuscript received October 11, 1963.

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