AMath390

Math & Music **Fall 2016**

Assignment # 7 Due: noon Dec. 1 in dropbox 6 slot 3 (s.1) or slot 4 (s. 2)

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1. (a) Show that a Pythagorean third is greater than a just third by a frequency ratio of $\frac{81}{80}$. This is known as a *syntonic comma* or sometimes just a *comma*. (A Pythagorean comma is different from a syntonic comma.)

   (b) The mean-tone scale discussed in class was explained in terms of making the (major) thirds just and then using equal interpolation. Mean-tone scales can also be derived from the Pythagorean scale by narrowing the fifths slightly in order to make the thirds better. Each successive fifth is dropped by some fraction of a (syntonic) comma.

   Indicate the fundamental note by C as usual and consider a Pythagorean scale. The fifth (G) is dropped by a fraction of a comma α, so $\alpha\frac{81}{80}$. Moving to the fifth on G, D, drop it by 2α commas, and then drop the next fifth, A, by 3α commas. the fifth on A is E, drop it by 4α commas.

   What value of α makes the third just?

   (c) What is the value of α for the mean-tone scale in the course notes ?

   (d) What fraction of a comma should be used for a mean-tone scale to minimize the squared errors of the fifth and third from their just values?

2. On Learn there is a recording of a piece by Mozart played in 3 different temperaments. Listen to it and then discuss how the piece sounds in different temperaments. Which version do you prefer? Why?

3. Consider a orchestra comprised of instruments that have partials in multiples of 3. That is, if f is the fundamental, the partials are $f, 3f$, $6f$, $9f, 12f\ 15f$, $18f\ldots$. The problem is to construct a set of intervals where different notes work well together, given these partials.

   (a) Suppose 2 notes are played, one with fundamental f and the other with fundamental $2f$. How many partials do the 2 notes have in common?

   (b) What would be the analogy to an "octave" so that all the partials of the higher note are included in the partials of the lower note?

   (c) Construct 3 more notes between the original note and the note in part (b) so that different notes have partials in common. There are several ways to do this, it is important to clearly explain your reasoning.

4. This question considers the possibility of an equal-tempered scale with a different number of notes than the 12 in the usual system. Consider then an arbitrary $n-$tet equal-tempered system. For practical reasons, consider only $n \leq 22$.

(a) Which systems yield fifths with an error of less than 10 cents from a perfect fifth? (You may refer to the chart in Benson.)

(b) For the set of systems in part (a) which, is the closest note to a just fourth? Number the notes from from 1 to n, where n is the number of divisions to an octave.

What is the error in cents? (Hint, for $n = 12$ the scale degree is 5 and the error 2.0 cents.)

Make a chart showing the errors in the just thirds, fourths and fifths in cents (to 2 significant digits) for these systems.

Which yield an error of less than 10 cents in the fourths?

Calculate the sum of squares of the three errors.

(c) Which value of n do you think is best for an equal-tempered scale? Justify your answer.