AMath390

Math & Music **Fall 2014**

Assignment # 2 Due: noon Sept. 29 in dropbox 6 slot 3 (s.1) or slot 4 (s. 2)

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1. Draw graphs of the functions

$$\sin(220\pi t) + \frac{1}{2}\sin(440\pi t)$$

and

$$\sin(220\pi t) + \frac{1}{2}\cos(440\pi t)$$

Explain why these sound the same, even though the graphs look quite different.

2. Piano wire is manufactured from steel of density $7.86 \times 10^3 kg/m^3$. The diameter of a string is $0.750mm$ and it is $50.1cm$ long.

   (a) What should the tension be so that string sounds at middle C on the piano; that is 262 Hz? Give the answer in Newtons (kgm/s^2).

   (b) How long should a string be to obtain the lowest C on the piano, which has a frequency of about 33Hz? Use the same tension as in (a).

   (c) The length obtained in part (b) is too long for most rooms, so thicker string is needed to obtain the low frequency notes. What should the diameter be to obtain the lowest C, if the length and tension are the same as in part (a).

3. For a vibrating string of length ℓ with fixed ends, each mode of vibration can be written as

$$u_k(x,t) = M_k \sin(\omega_k t + \phi_k)\sin(\frac{\omega_k}{c}x)$$

where $\omega_k = \frac{k\pi c}{\ell}$ and M_k, ϕ_k are determined by initial conditions. For all $k > 1$, $\sin(\frac{\omega_k}{c}x)$ has zeros in $[0, \ell]$, at say points $x = z$, so $u_k(z,t) = 0$ for all t. Points where the deflection u of a mode is zero ($u(z,t) = 0$) are called nodes.

   (a) What is the distance between nodes for each mode $k > 1$?

   (b) Where should you place your finger so that it is at a node of the second mode? This will prevent any mode that doesn't have a node at this point, such as the first mode, from vibrating. Express your answer in terms of fraction of the distance from the bridge, so that an answer 1 would be an open string, 0 would be no string at all.

   (c) Where should you place your finger so that it is at a node of the third mode? This will prevent any mode that doesn't have a node at this point, such as the first and second modes, from vibrating. Again give your distance in terms of a fraction of the distance from the bridge.

4. Suppose one stringed instrument has a non-zero initial position with $u_p(x,0) = F(x)$, zero initial velocity, while another initially has a zero position and non-zero velocity, $\dot{u}_v(x,0) = F(x)$.The length of the string, tension, cross-sectional area and density are all the same for both instruments. Compare the coefficients of each mode against frequency k. For which instrument are the high frequency modes more audible?

5. Listen to the cover of Cheap Thrills by Kina Grannis & Kurt Schneider made using a bicycle for the instruments. (It's on youtube.)

   List all the different ways a sound was made using the bicycle. For which sounds, was the pitch adjustable and how was the pitch adjusted?