AMath390 Mathematics & Music Fall 2016

OPTIONAL TAKE HOME QUIZ

Due: 4:00pm Nov. 22, box 6 slot 3 (s.1) or slot 4 (s. 2)

NO. OF EXAM PAGES (incl. this page & rough work)	9 pages
INSTRUCTOR	K. A. Morris
EXAM TYPE	Take-home (books and notes allowed)

Family Name (print): ____________ **Given Name (print)**: ____________

Signature: ________________ **I.D. Number**: ____________

Instructions

1. **You may use the course notes and the text (Benson) but your solutions MUST be obtained independently, not with another person.**
2. **Hand this paper in with the original quiz that you wrote.**
3. Answer all questions in the space provided. Continue on the back of the **PRECEDING** page if necessary.
4. Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions. Identify all variables and constants used. Marks will be deducted for poorly presented answers.
5. Unless otherwise instructed, give all calculated answers to 3 significant digits.

Marking Scheme

Question	Points	
1	7	
2	6	
3	3	
4	2	
5	4	
6	8	
7	3	
8	4	
9	5	
Total:	42	

1. (a) (3 points) Consider a frame drum consisting of a membrane stretched over a square frame, of length ℓ (m) on each side. Let the tension be T (N/m) and the density ρ (kg/m^2). Write down the partial differential equation and boundary conditions modelling this drum.

   (b) (2 points) List two assumptions made in this model.

   (c) (2 points) Typically a round drum is struck with a mallet. Write down appropriate initial conditions, explaining your reasoning. A sketch indicating the the shape of a function can be given.

2. The vibrations of idiophones are produced by striking a bar. The vibrations of a uniform bar of length 1 can be modelled by the equation

$$\frac{\partial^2 u}{\partial t^2}(x,t) + c^2 \frac{\partial^4 u}{\partial x^4}(x,t) = 0, \quad 0 < x < 1 \tag{2}$$

where $c > 0$ is constant. Consider an instrument with both ends clamped so that at each end there is no deflection and also the slope of the bar is zero.

(a) (3 points) Derive the two differential equations that must be solved to calculate $u(x,t)$. DO NOT SOLVE THE EQUATIONS.

(b) (3 points) Derive the equation whose roots yields the natural frequencies of vibration of this instrument. In terms of the roots of this equation, and the bar parameter c, what is the fundamental frequency (in Hz) of the vibrations?

3. Idiophones are an important part of gamelan ensembles. Each idiophone occurs in a pair with another instrument, identical except for a small, but important, differences in tuning introduced in manufacture.

   (a) (2 points) The longest bar on a particular gamelan instrument (A) has a fundamental frequency of 200Hz. What is a good choice for the fundamental frequency of the corresponding bar in the other instrument (B) in the pair so that beats at 5 Hz are heard? Tune instrument B so it is higher in pitch than A.

   (b) (1 point) A shorter bar in each instrument is tuned so the fundamental frequencies are about 400 Hz, and the beats when both instruments are struck are at 5 Hz. However, the fundamental frequency is not exactly twice the frequency of the longer bar on the same instrument; that is they not tuned a perfect octave apart. Explain, in terms of the overtones of idiophones (such as xylophones, mbira etc.) why it makes sense not to tune the shortest bar at exactly twice the pitch of the sound made by the longest bar.

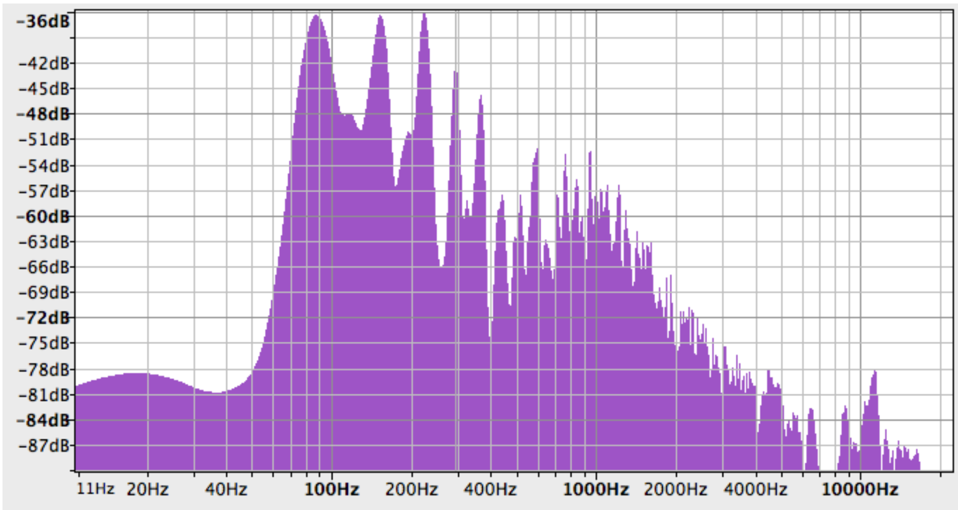

Figure 1: Frequency response of an instrument

4. (2 points) Figure 1 shows the measured frequency response of a instrument heard in class this term. What instrument do you think it is, and why?

5. (4 points) Consider $f(t) = t$ on $[-\pi, \pi]$. Calculate the Fourier series (complex form) for the 2π periodic extension.

6. (a) (5 points) Find the complex Fourier series of the function with period T defined by

$$f(t) = \begin{cases} 1 & |t| \leq \frac{T}{4} \\ 0 & \frac{T}{4} < |t| \leq \frac{T}{2} \end{cases}. \tag{5}$$

(b) (3 points) Write the sine/cosine form of the series in part (a).

7. (3 points) Let $f(t)$ be a Fourier transformable function with Fourier transform $\hat{f}(\nu)$. Show that the function delayed by a seconds, $f(t-a)$, has Fourier transform $\hat{f}(\nu)e^{-\imath 2\pi\nu a}$.

8. (4 points) Suppose that f is sampled at a rate N at least as great as the Nyquist rate, for a period of time so that, defining $T = \frac{1}{N}$, M samples $f(nT)$, $n = 0 \ldots M-1$ are obtained. Consider $f(t)$ to be 0 outside this interval of time. Assume ideal sampling so that the sampled signal f_s can be regarded as a series of impulses and show that

$$\widehat{(f_s)}(\nu) = \sum_{n=0}^{M-1} f(nT)e^{-\imath 2\pi nT\nu}.$$

9. (a) (3 points) The Fourier transform $\hat{f}(\nu)$ of a function f of time is shown in Figure 2. Suppose that the function f is sampled once per second. Sketch the Fourier transform of the sampled function on the graph.

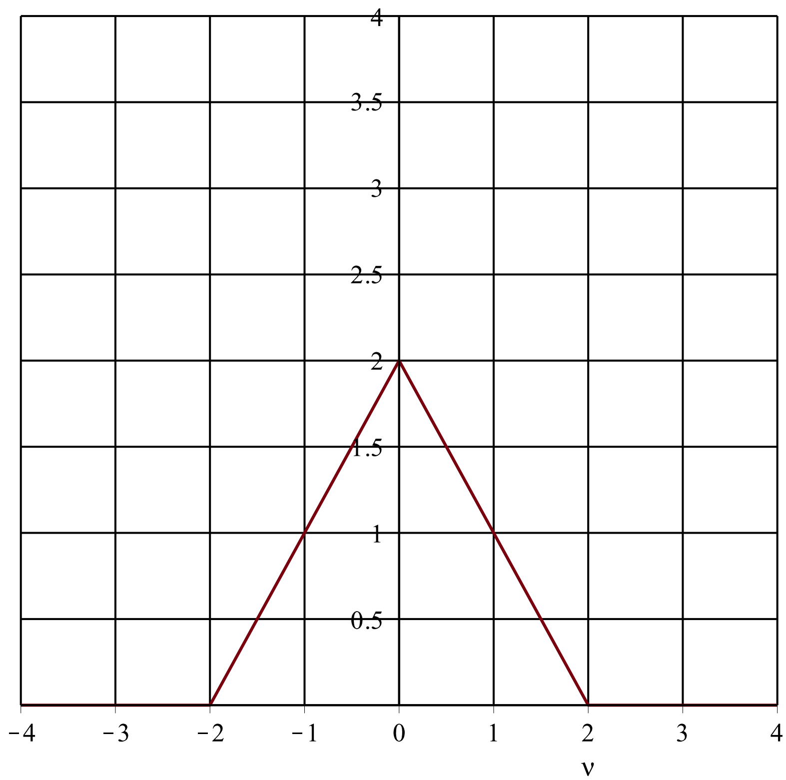

Figure 2: Fourier transform for question 9

(b) (2 points) What should the sample rate be so that the original function can be recovered from the samples with no error? Assume the units for frequency are Hertz. Briefly justify your answer.

ROUGH WORK - Will NOT be graded