Day 12: Percussion Continued

Friday, October 7, 2016 1:32 PM

Drum be the region $(x,y) \in \Omega$

Solution of Wave equation for Drum:

- Polar coordinates are natural: $x = r\cos\theta$, $y = r\sin\theta$

The wave equation in polar coordinates is:

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

Boundary conditions:

$u(a,\theta,t) = 0$

$u(0,\theta,t) < \infty$

$u(r,0,t) = u(r,2\pi,t)$

$\frac{\partial u}{\partial \theta}(r,0,t) = \frac{\partial u}{\partial \theta}(r,2\pi,t)$

Sub $u(r,\theta,t) = R(r)H(\theta)T(t)$ into wave equation. rearrange:

$$\frac{T''}{c^2 T} = \left[\frac{1}{rR}(rR')' + \frac{1}{r^2}\frac{H''}{H}\right] = -\lambda$$

$\boxed{T}$ $T'' + c^2\lambda T = 0$

$$T(t) = A\cos(c\sqrt{\lambda}\,t) + B\sin(c\sqrt{\lambda}\,t)$$

A and B constants

Rearranging spatial part

$$\frac{r}{R}(rR')' + \lambda r^2 = -\frac{H''}{H} = \mu^2$$

$\boxed{H}$ $H'' + \mu^2 H = 0$

$$H(\theta) = A\cos(\mu\theta) + B\sin(\mu\theta)$$

arbitrary constants

$\mu = n$ $n = 0, 1, 2.$

$n \in \mathbb{Z}_{\geq 0}$

satisfies $H(0) = H(2\pi)$

$H'(0) = H'(2\pi)$ No other values of μ will work.

$\boxed{R}$ $(rR')' + \lambda_n R - \frac{n^2 R}{r} = 0$

The general solution is for arbitrary constants D_n, E_n

- $R(r) = D_n J_n(\sqrt{\lambda}\,r) + E_n Y_n(\sqrt{\lambda}\,r)$

where J_n is nth order Bezel function of first kind

Y_n is ~ ~ ~ ~ of second kind

Y_n is unbounded at $r=0$ so $E_n = 0$

$$R(a) = 0 = D_n J_n(\sqrt{\lambda}\,a)$$

Each J_n has an ∞ # of zeros

Are the zeros of J_n evenly spaced?

- No...

Do J_n, J_m, $n \neq m$ have any common zeros (beside 0) - No...