AMath390

Math & Music **Fall 2014**

Assignment # 4 Due: noon Friday Oct. 21 in dropbox 6 slot 3 (s.1) or slot 4 (s. 2)

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1. Consider a drum consisting of a membrane stretched over a square frame, of length ℓ on each side. Let the tension be T (N/m) and the density ρ (kg/m2) .
   (a) Write down the partial differential equation and boundary conditions describing the motion.
   (b) Solve the equation for arbitrary initial conditions.
   (c) In terms of the physical parameters T and ρ what are the natural frequencies of vibration?
   (d) Sketch the nodal lines of the first 2 modes of vibration.
   (e) Would you expect to be able to hear the difference between the sound of a square drum and a round drum? Explain your answer briefly.
   (f) The *adufe* is a traditional tambourine-like Portugese instrument consisting of a square frame covered with a membrane on both sides. How do you think enclosing the drum changes the sound?

2. The vibrations in a struck bar of length ℓ, such as in a xylophone, mbira, glockenspiel are governed by the equation (assuming constant width and small deflections)

$$\frac{\partial^2 u}{\partial t^2}(x,t) + \underbrace{\frac{EI}{\rho}}_{c^2} \frac{\partial^4 u}{\partial x^4}(x,t) = 0, \quad 0 < x < L \tag{1}$$

   where E, I, ρ are positive physical constants. Since they are constant, define $c^2 = \frac{EI}{\rho}$. Various boundary conditions are possible. For a glockenspiel, the ends are fixed, but free to bend so the boundary conditions are

$$u(0,t) = 0,\ \frac{\partial^2 u}{\partial x^2}(0,t) = 0,\ u(\ell,t) = 0,\ \frac{\partial^2 u}{\partial x^2}(\ell,t) = 0.$$

   (a) Assume that the solution is separable, $u(x,t) = M(x)N(t)$ and show that, for arbitrary A

$$M(x) = A\sin(ax)$$

   solves the spatial differential equation and boundary conditions, for particular values of a. What are the allowable values of a? (Hint: call the separation constant $-a^4$.)
   (b) Write out the general solution. (There will be arbitrary constants, since the initial position and velocity is not specified.
   (c) What is the fundamental frequency? What are the overtones?

(d) What is the fundamental frequency, if the bar is 1.00 m long? The first overtone? Use $c = \sqrt{\frac{EI}{\rho}} = 222m^2/s$.

(e) Using the values from part (d), find the length of a second bar so that the fundamental frequency is twice that of the first bar.