AMath390

Math & Music **Fall 2014**

Assignment # 3 **SOLUTIONS**

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1. The length of many flutes is 66cm and the diameter is 2cm. The speed of sound in air $c = \sqrt{P'(\rho_0)}$ depends strongly on temperature and slightly on humidity. For temperatures near 20C the speed c in m/s is approximated by, letting T indicate the temperature in Celsius,

$$c = 331 + (0.60\frac{m}{sC})T. \tag{1}$$

   (a) Calculate the lowest pitch (in Hz) of a flute at 25 C.
   Look at the table of *scientific pitch notation* in the coursewares notes. Which note is it closest to?

   (b) The note obtained is affected by the blowing. Different blowing speeds and angles cause different resonant modes to be excited. What note is obtained if the flautist blows so as to excite the second harmonic of the note in part (a)?

   (c) How much length should be added to a flute to obtain 247 Hz (the B below middle C).

   (d) Calculate the lowest pitch of a flute at 10C. Use 66cm for the length.

Solution: The fundamental frequency yields the pitch. The formula is

$$f = \frac{c}{2\ell}. \tag{2}$$

(a) Formula (1) yields that at 25C, the speed of sound $c = 346$m/s . Then (2) predicts a fundamental frequency of 262Hz. This is close to middle C (or C4) which is listed as 261 Hz.

(b) The second harmonic is double the fundamental frequency, so 524Hz. This is the C above middle C (C5) .

(c) Again using the formula (2) with $c = 346$m/s , and solving for ℓ, the length should 0.7m or 70cm, so we add 4cm.

(d) At 10C, $c = 337$m/s and the lowest frequency is 255Hz (a flat C4; the difference between this and 261Hz is clearly audible).

2. The length of the strings on a grand piano are chosen to obtain the required pitch (or fundamental frequency). Modern pianos are usually tuned so that, letting n be the n^{th} key from the left, and f_0 is the lowest pitch, the pitch f is

$$f(n) = (2)^{\frac{n}{12}} f_0.$$

(This will be discussed later in this course.) Let ℓ_0 be the length of the string associated with the lowest pitch f_0. Show that for some α, the lengths ℓ of the strings have the pattern

$$\ell(n) = e^{\alpha n}\ell_0.$$

What is α?

Solution: Since

$$\frac{f(n)}{f_0} = 2^{\frac{n}{12}}$$

and for a vibrating string the pitch (in Hz) is

$$f = \frac{c}{2\ell},$$

$$\frac{\ell_0}{\ell(n)} = 2^{\frac{n}{12}}.$$

Rearranging,

$$\begin{aligned} \ell(n) &= 2^{-(n/12)}\ell_0 \\ &= e^{\ln(2^{-(n/12)})}\ell_0 \\ &= e^{-(\frac{n}{12}\ln 2)}\ell_0. \end{aligned}$$

Defining $\alpha = -\frac{\ln 2}{12}$,

$$\ell(n) = e^{\alpha n}\ell_0.$$

Thus, the back of a grand piano is close to an exponential curve.

3. The material used in a wind instrument affects the timbre. For example, clarinets and oboes made from hardwood typically have a richer sound than those made of plastic, and piccolos made from wood are preferable to those made of metal. Speculate on reasons for this.

Solution: This must have something to do with multi-dimensional effects neglected in the model we are using. It appears that sound waves reflect differently off different materials and this will affect the strength of various overtones.