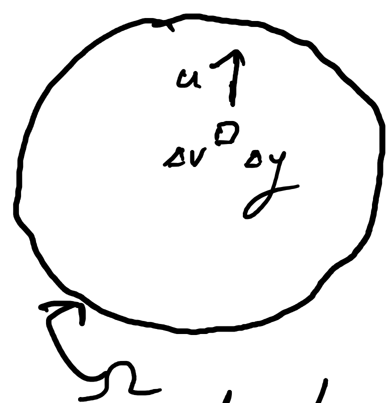
Day 11: Percussion

Wednesday, October 5, 2016 1:59 PM

Math Model for Drum Vibrations
-membrane stretched over a frame and attached to the frame

Newton's Law: $\rho \Delta x \Delta y \dfrac{\partial^2 u}{\partial t^2} = \text{Force}$



ρ density kg/m^2

-constant density
-flexible
-no resistance, friction, their dissipative forces
-small deflections so nonlinearities neglected

Using the same techniques as for vibration string and vector Calculus:

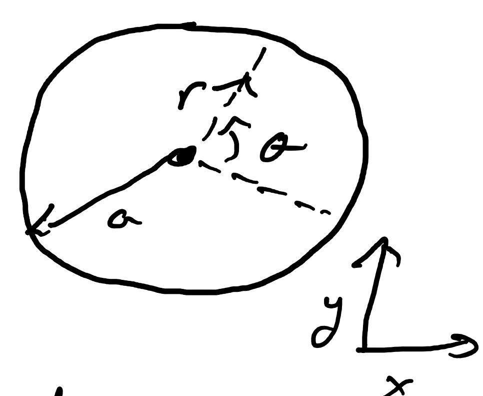
$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (x,y) \in \Omega$$

$c^2 = T/\rho$ (T, tension, N/m)

$u(x,y,t) = 0$, $(x,y) \in \text{bdy}\,\Omega$.

$u(x,y,0) = f(x,y)$

$\dot{u}(x,y,0) = g(x)$

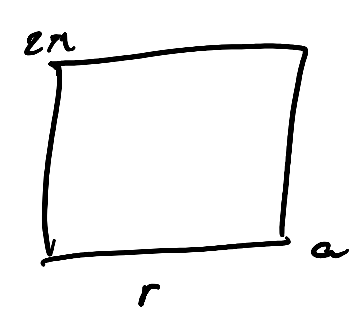


For Ω a disk, as is usual for a drum,
use polar coordinates

$x = r\cos\theta$, $y = r\sin\theta$

Using chain rule and trig. identities,

$$\text{①}\quad \frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$



$u(a,\theta,t) = 0 \qquad u(0,\theta,t) = ?$

$$\left.\begin{array}{l} u(r,0,t) = u(r,2\pi,t) \\ \dfrac{\partial u}{\partial \theta}(r,0,t) = \dfrac{\partial u}{\partial \theta}(r,2\pi,t) \end{array}\right\} \text{periodic}$$

Solution: Try $\to u(r,\theta,t) = R(r)\,H(\theta)\,T(t)$.

Sub into ①:

$$\frac{T''(t)}{c^2 T(t)} = \left[\frac{1}{rR}(rR')' + \frac{1}{r^2}\frac{H''}{H}\right] = -\lambda$$

$T''(t) + c^2\lambda T(t) = 0$

$T(t) = A\cos(\sqrt{\lambda}\,t) + B\sin(\sqrt{\lambda}\,t)$