AMath390

Math & Music **Fall 2016**

Extra Questions on Frequency Response and Sampling

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1. Find the period T, the complex coefficients in the Fourier series and the energy in the interval ($\int_0^T f(t)^2 dt$) of
$$f(t) = 2 + 5\sin 6t + 4\cos 8t.$$
(Hint: You do not to compute any integrals to find the Fourier series.)

2. Find the Fourier series for
$$f(t) = t^2, \quad t \in [-\pi, \pi]$$
where f is extended to all values of t as a 2π periodic function. Calculate both the real and complex forms of the Fourier series.

3. Let f be a Fourier-transformable function with Fourier transform $\hat{f}$. Let a be any non-zero constant.
   (a) Show that the Fourier transform of $f(at)$ is $\frac{1}{|a|}\hat{f}(\frac{\nu}{a})$.
   (b) Show that the Fourier transform of $f(t-a)$ is $e^{-\imath 2\pi\nu a}\hat{f}(\nu)$.
   (c) Suppose that f is differentiable and f' is also Fourier transformable. Show that $\widehat{(f')}(\nu) = (\imath 2\pi\nu)\hat{f}(\nu)$.

4. Any Fourier transform $F(\nu)$ of a function $f(t)$ can be decomposed into real and imaginary parts:
$$F(\nu) = R(\nu) + \imath X(\nu)$$
where R, X are real-valued functions.
Show that if $f(t)$ is a real-valued function then R is an even function and X is an odd function. Then show that
$$F(-\nu) = R(\nu) - \imath X(\nu).$$
In other words, if f is real-valued, its Fourier transform F satisfies
$$F(-\nu) = \overline{F(\nu)}.$$
This also implies that $|F(-\nu)| = |F(\nu)|$.

5. A function $f(t) = 0$ outside of the interval $[0, A]$ and its Fourier transform $\hat{f}(\nu)$ is negligible for $|\nu| > \sigma = 10^3$Hz. What is the minimum rate of samples/s at which the signal f must be sampled so it can be reconstructed from its samples? (This is the Nyquist rate.)

6. Write out the proofs for Theorems 8,9,10,11. (These proofs are all in the course notes.)