MCG126

Amath 390 Assignment #1 John Lawson 20466075

① (a) $\ddot{y}(t) + \omega^2 y(t) = 0$

has solutions $y(t) = c\sin(\omega t + \phi)$
for constants c, ϕ

$$y(t) = c\sin(\omega t + \phi)$$
$$\dot{y}(t) = \omega c\cos(\omega t + \phi)$$
$$\ddot{y}(t) = -\omega^2 c\sin(\omega t + \phi)$$

$$\ddot{y}(t) + \omega^2 c\sin(\omega t + \phi) = 0$$
$$\ddot{y}(t) + \omega^2 y(t) = 0$$

(b) $F = ma$
$F = m\ddot{x}$

$F = -kx$ $\qquad m = 0.020\ kg$

$k = \frac{F}{x} \ \left(\frac{N}{m}\right)$ $\qquad k = 8.0\ N/m$

$x(0) = 0.30\ m$

$\dot{x}(0) = 0\ m/s$

$$m\ddot{x} = -kx$$
$$m\ddot{x} + kx = 0$$
$$\ddot{x} + \frac{k}{m}x = 0$$
$$\omega^2 = \frac{k}{m}$$
$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = c\sin(\omega t + \phi)$$
$$\dot{x}(t) = \omega c\cos(\omega t + \phi)$$
$$\ddot{x}(t) = -\omega^2 c\sin(\omega t + \phi)$$
$$\ddot{x}(t) + \omega^2 c\sin(\omega t + \phi) = 0$$
$$\ddot{x}(t) + \omega^2 x(t) = 0$$

↑ is the solution for $\ddot{x} + \frac{k}{m}x = 0$

$$x(0) = 0.30m = c\sin(\phi)$$

$$\dot{x}(0) = 0 = \omega c\cos(\phi)$$

$0.30 = c \sin(\phi)$

$0 = \omega c \cos(\phi)$

$\omega = \sqrt{\frac{k}{m}}$

$\doteq \sqrt{\frac{8.0}{0.020}} = 20$

$\cos(\phi) = 0$

$\phi = \frac{\pi}{2}$

$\downarrow$

$0.30 = c \sin(\phi)$

$c = 0.30$

So $y(t) = 0.30 \sin\left(20t + \frac{\pi}{2}\right)$

with frequency of $f = \frac{\omega}{2\pi}$

$= \frac{20}{2\pi}$

$\doteq 3.18$ Hz

(3)

$f(t) = 3\cos(2t) + 5\sin(2t)$

$\sin(2A) = 2\cos^2 A - 1$

$\cos(2a) = \cos^2 a - \sin^2 a$

$$
\begin{aligned}
f(t) &= 3\cos^2 t - 3\sin^2 t + 10\cos^2 t - 5 \\
&= 13\cos^2 t - 3\sin^2 t - 5 \\
&= \cos^2 t + \sin^2(t) + 12\cos^2 t - 4\sin^2 t - 5 \\
&= 12\cos^2(t) - 4\sin^2(t) - 4 \\
&= 16\cos^2(t) - 4(\sin^2(t) + \cos^2(t)) - 4 \\
&= 16\cos^2(t) - 8 \\
&= 8\left[2\cos^2(t) - 1\right] \qquad \cos(2a) = 2\cos^2(a) - 1 \\
&= 8\cos(2t)
\end{aligned}
$$

∴ the function has a frequency of $\frac{2}{2\pi} = 0.318$ Hz & an amplitude of 8 cm, phase shift of 0

3 (a) $f_{beat} = |f_2 - f_1|$

(b) The pitch of the beat produced seems to increase with larger values of the second tone (220, 249, ...). Not sure if the pitch increase is actually the beat, might be confusing the second, higher tone with it. Beats are all audible, although the 200/220 at times sounds more like a flickering, and less like a tone.

(4)

undamped oscillator forced at natural frequency

$$y''(t) + 4y(t) = \sin(2t)$$

(a)

$$y(t) = bt\cos(2t)$$
$$y'(t) = -2bt\sin(2t) + b\cos(2t)$$
$$y''(t) = -4bt\cos(2t) - 2b\sin(2t) - 2b\sin(2t)$$
$$y''(t) = -4b\left[t\cos(2t) + \sin(2t)\right]$$

$$y''(t) + 4y(t) = -4b\sin(2t)$$

so for $b = -1/4$

$$y''(t) + 4y(t) = \sin(2t)$$

(b)

Show for certain values of A, B

$$y(t) = A\sin(2t) + B\cos(2t) + bt\cos(2t)$$

Solves $y''(t) + 4y(t) = \sin(2t)$

for I.C. $y(0) = 1$
$y'(0) = -1$

$y(t) = A\sin(2t) + B\cos(2t) + bt\cos(2t)$

$\dot{y}(t) = 2A\cos(2t) - 2B\sin(2t) - 2bt\sin(2t) + b\cos(2t)$

$\ddot{y}(t) = -4A\sin(2t) - 4B\cos(2t) - 4bt\cos(2t) - 2b\sin(2t) - 2b\sin(2t)$

$\ddot{y}(t) = -4y(t) - 2b\sin(2t) \quad 2b\sin(2t)$

$\ddot{y}(t) + 4y(t) = -4b\sin(2t)$

For $b = \frac{-1}{4}$ this works as

$\ddot{y}(t) + 4y(t) = \sin(2t)$

given initial conditions $y(0) = 1$

$\dot{y}(0) = -1$

$y(0) = 1 = A\sin(0) + B\cos(0) + 0$

$1 = B\cos(0)$

$B = 1$

$\dot{y}(0) = -1 = 2A\cos(0) - 2B\sin(0) - 0 + b\cos(0)$

$-1 = 2A + b$

$A = \frac{-1-b}{2}$

$= \frac{-4/4 + 1/4}{2}$

$A = -3/8$

$\therefore B = 1, A = -3/8$

5 (a)

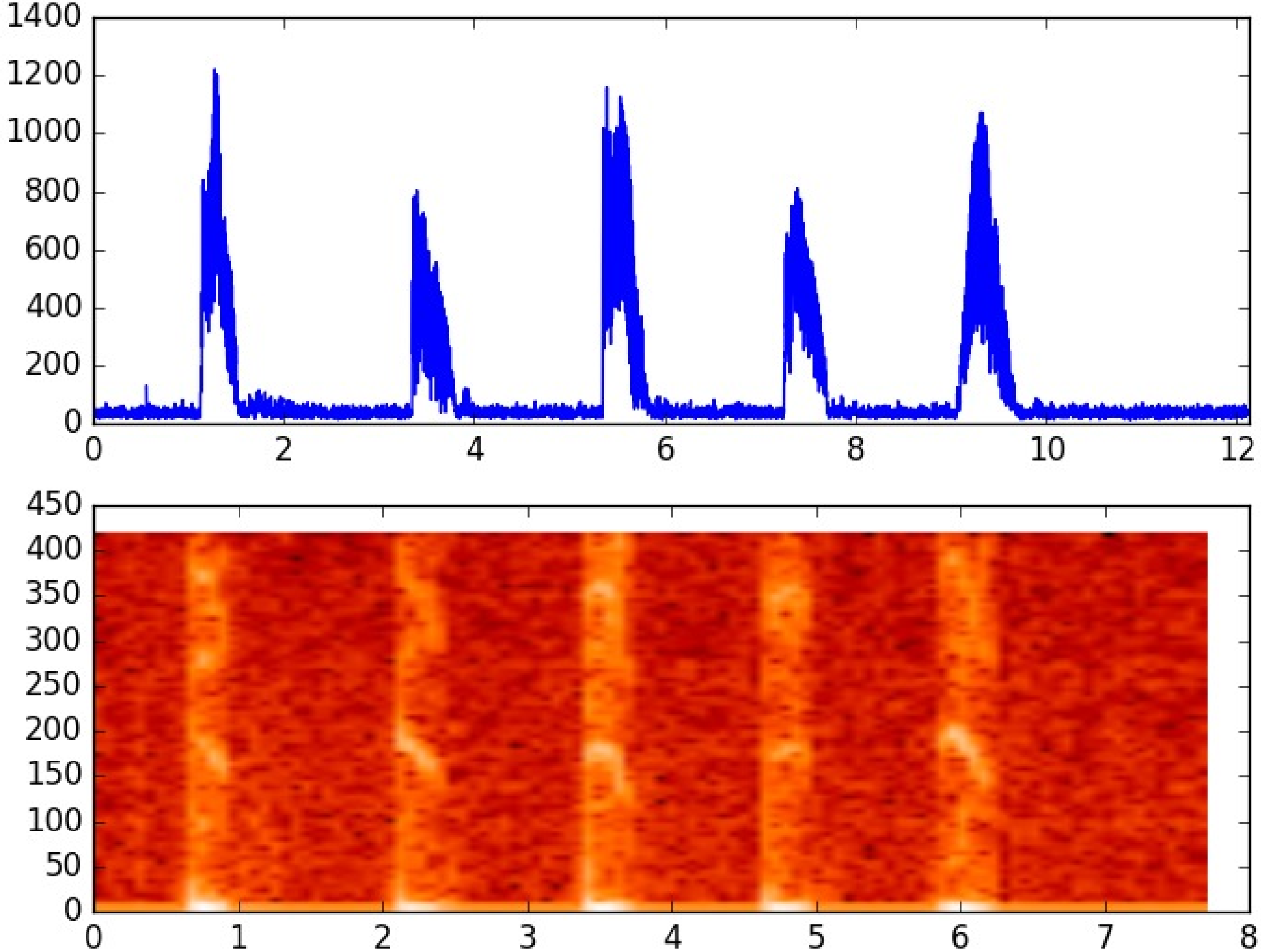
(c) The pitch of 'a', 'e', 'i', and 'u' at the 100-200 Hz range declines over the course of the sound, while the 'o' slightly increases or flatlines in pitch over the course of the sound.

All of the vowels appear to be strongest in roughly 3 "bands", 0-20 Hz, 125-200 Hz, and 300-400 Hz, with the most noticeable being the 125-200 Hz range. The 'i' appears to have the strongest high (300-400) Hz component, along with the 'a' to a lesser degree

(d)

(d) Given a pitch for F1 of ~300 Hz, and F_2 at ~900 Hz the sound is most likely [u]. Given that the F1 is at least 300 Hz, the voice is likely female.

(e) likely a clap, or some type of other short, percussive sound


1400
1200
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0 2 4 6 8 10 12
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