CO 250 Online- Spring 2018

Assignment 2

Due date : Friday, May 18, 2018, by 4:00pm on Crowdmark

Submission Guidelines:

- Please submit your solutions to Crowdmark. Late assignments will not be accepted, and will receive a mark of zero. It is the responsibility of the students to make sure that the file they submit is clearly readable. Illegible submissions will receive a mark of zero, and hard to read submissions may receive deductions.
- You answers **need to be fully justified**, unless specified otherwise. Always remember the WHAT-WHY-HOW rule, namely explain in full detail what you are doing, why are you doing it, and how are you doing it. Dry yes/no or numerical answers will get 0 marks.
- In some questions you are asked to *formulate* the problem. You are *not* asked to actually solve the formulation, e.g., compute optimal solutions. Your formulations should be easy to modify if we change the data and constants defining the problems. Clearly define all your variables (including their units) and any other new notation you use in all your answers. Your solutions must also contain a brief justification of all the constraints (explain the relation between each of the constraints and the requirements stated in the problem) and the objective function.

Assignment policies: While it is acceptable to discuss the course material and the assignments, you are expected to do the assignments on your own. Copying or paraphrasing a solution from some fellow student or old solutions from previous offerings qualifies as cheating and we will instruct the TAs to actively look for suspicious similarities and evidence of academic offenses when grading. Students found to be cheating will be given a mark of 0 on the assignment. In addition, all academic offenses will be reported to the Math Academic Integrity Officer (which may lead to further penalties) and recorded in the student's file.
Re-marking policies: If you have any complaints about the marking of assignments, then you should first check your solutions against the posted solutions. After that, if you see any marking error, then write a letter detailing clearly the marking errors, and submit this to one of the TAs within one week from the date the graded assignment is returned. If you still have concerns after the final decision of the TA, then please contact your instructor communicating all the correspondence with the TA and the original petition.

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[1] It is an academic offense to post this assignment or solutions to any web page.

Question 1. Integer Programming Modeling I **(15 marks)**

Hiromi is organizing a party. She will be ordering fish, rice, tea and sake to serve to her guests. Each piece of fish that she orders costs 1$ and weighs 0.1 kilograms, and the maximum available is 80. The rice is ordered by weight at 3$ per kilogram. The tea costs for 2$ per liter, and each liter of tea weighs 1 kilogram. The sake is sold in .5 liter bottles for 15$ each, with a bottle weighing 1 kilogram. The following constraints must be met:

(i) The total weight of food (fish and rice) should be at least 10 kilograms.

(ii) The weight of the rice must at least match the weight of the fish.

(iii) There must be 0.2 liters of tea available for each kilogram of food.

(iv) If fewer than 70 pieces of fish were ordered, then at least one bottle of sake must be served.

(v) If fewer than 50 pieces of fish were ordered, then at least two bottles of sake must be served.

(vi) If the total weight of food and drink exceeds 20kg, there is a 10$ delivery surcharge.

Formulate the problem of minimizing Hiromi's costs for food and drink at her party, as an Integer Program.

Question 2. Integer Programming Modeling II **(15 marks)**

You have been tasked to decide on a convenient room allocation for the final exam of the following 3 sections of a course at UW. The enrollment in the 3 sections is as follows.

Section	A	B	C
Enrollment	40	20	30

Students can be accommodated in any of 4 different classrooms whose capacities can be seen below.

Classroom	1	2	3	4
# of seats	60	90	100	50
Required TA's	1/20	1/40	1/30	1/15

In particular, students of a section may be split across multiple classrooms. The maximum number of students each classroom can accommodate is only half the number of its seats. At the same time, UW administration requires that for every section, no more than half its enrolled students are allocated to the same classroom. Finally, the table above summarizes the number of TAs required to invigilate each classroom (per students). For example, classroom 1 requires at least 1 TA for every 20 students that will be allocated in classroom. This means that if the number of students allocated in classroom 1 is anything between 1 to 20, then the number of TAs required is at least 1, while if the number of students allocated to the same classroom is anything between 21 to 40, the number of required TAs is at least 2, etc.

You need to decide how many students of each section to allocate to each classroom (complying with the above specifications) so as to minimize the number of TAs that will invigilate the exam. Formulate this problem as an Integer Program, and prove its correctness.

Question 3. Integer Programming Modeling III **(15 marks)**

Let n be a fixed integer and let $c \in \mathbb{R}^n$ be a fixed vector. Consider the following Integer Programming (IP) problem (P):

$$\begin{array}{rl} \max & c^\intercal x \\ \text{subject to} & \\ & x_i \in \{0,1\} \text{ for } i \in \{1,\dots,n\} \end{array}$$

(a) Provide new constraints (and possibly new variables), which one needs to add to (P) to model the additional constraint that $x_1 + x_2 + \ldots + x_n$ is a multiple of 3. The resulting problem should be an Integer Programming (IP) problem. (3 marks)

(b) Provide new constraints (and possibly new variables), which one needs to add to (P) to model the additional constraint that $x_1 + x_2 + \ldots + x_n$ is either larger than 20 or less than 10. The resulting problem should be an Integer Programming (IP) problem. (4 marks)

(c) Provide new constraints (and possibly new variables), which one needs to add to (P) to model the additional constraint that $x \neq \mathbb{1}$, where $\mathbb{1} \in \mathbb{R}^n$ is the vector with all coordinates equal to 1. (4 marks)

(*Note: if x, y are two vectors in $\mathbb{R}^n$, $x \neq y$ holds if and only if there is some coordinate i such that $x_i \neq y_i$.*)

(d) Provide new constraints (and possibly new variables), which one needs to add to (P) to model the additional constraint that x has an odd number of coordinates i with $x_i = 0$. (4 marks)

Question 4. Graph Optimization **(15 marks)**

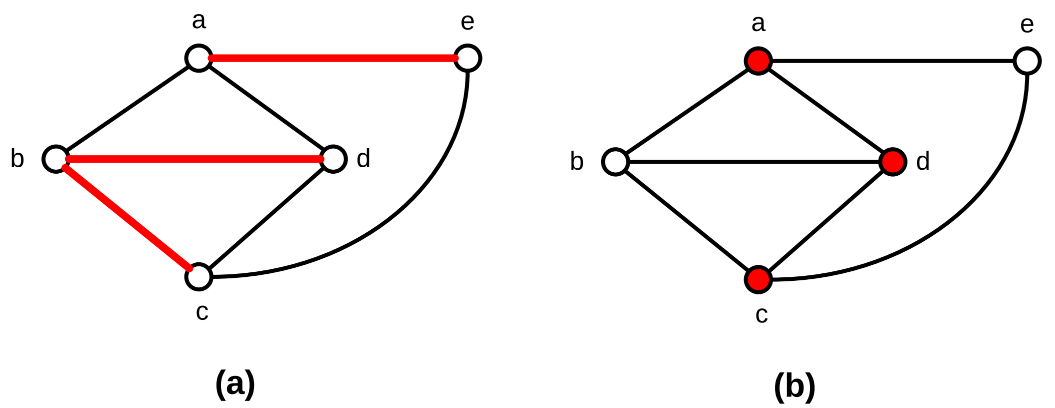

Figure 1: (a) the red edges depict an edge cover; (b) the red nodes depict a vertex cover.

(a) Let $G = (V, E)$ be a graph. A subset $C \subseteq E$ of edges is called an *edge cover* of G if every vertex $v \in V$ is incident to *at least one* edge in C. For example, in Fig. 1(a), we have a graph on 5 vertices and the thick red edges depict an edge cover.

Formulate an integer program to find an edge cover of a graph with the fewest number of edges. (6 marks)

(b) Let $G = (V, E)$ be a graph. A subset $D \subseteq V$ of vertices is called a *vertex cover* of G if for every edge $e \in E$, *at least one endpoint of* e is in D. For example, in Fig. 1(b), the red vertices depict a vertex cover for the same graph.

Write an integer program to find a vertex cover of a graph with the fewest number of vertices. (6 marks)

(c) Suppose we denote the constraints of the IP in part (a) by

$$\{Ax \geq b,\ x \geq 0,\ x \text{ integer}\} \tag{1}$$

and the constraints of the IP in part (b) by

$$\{Bx \geq d,\ x \geq 0,\ x \text{ integer}\}. \tag{2}$$

What is the relation between the matrices A and B? (3 marks)