CO 250 Online- Spring 2018

Assignment 7

Due date : Saturday, June 30th, 2018, by 4:00pm

Submission Guidelines:

- Please submit your solutions to Crowdmark. Late assignments will not be accepted, and will receive a mark of zero. It is the responsibility of the students to make sure that the file they submit is clearly readable. Illegible submissions will receive a mark of zero.
- You answers **need to be fully justified**, unless specified otherwise. Always remember the WHAT-WHY-HOW rule, namely explain in full detail what you are doing, why are you doing it, and how are you doing it. Dry yes/no or numerical answers will get 0 marks.
- In some questions you are asked to *formulate* the problem. You are *not* asked to actually solve the formulation, e.g., compute optimal solutions. Your formulations should be easy to modify if we change the data and constants defining the problems. Clearly define all your variables (including their units) and any other new notation you use in all your answers. Your solutions must also contain a brief justification of all the constraints (explain the relation between each of the constraints and the requirements stated in the problem) and the objective function.

Assignment policies: While it is acceptable to discuss the course material and the assignments, you are expected to do the assignments on your own. Copying or paraphrasing a solution from some fellow student or old solutions from previous offerings qualifies as cheating and we will instruct the TAs to actively look for suspicious similarities and evidence of academic offenses when grading. Students found to be cheating will be given a mark of 0 on the assignment. In addition, all academic offenses will be reported to the Math Academic Integrity Officer (which may lead to further penalties) and recorded in the student's file.

Re-marking policies: If you have any complaints about the marking of assignments, then you should first check your solutions against the posted solutions. After that, if you see any marking error, then write a letter detailing clearly the marking errors, and submit this to one of the TAs within one week from the date the graded assignment is returned. If you still have concerns after the final decision of the TA, then please contact your instructor communicating all the correspondence with the TA and the original petition.

[1] It is an academic offense to post this assignment or solutions to any web page.

Question 1. Extreme Points and Basic Feasible Solutions **(20 marks)**

Consider the feasible region S of an LP in SEF:

$$\max \quad \{c^\top x : Ax = b, x \geq \mathbb{0}, x \in \mathbb{R}^n\} \ ,$$

where $c \in \mathbb{R}^n, b \in \mathbb{R}^m$, and $A \in \mathbb{R}^{m\times n}$ with row rank m. Recall that S is a convex set.

(a) Prove that if $d \in \mathbb{R}^n$ is a basic feasible solution of the LP, then it is an extreme point of S.

(b) Suppose a feasible solution d is not basic, and column A_j with j in the support of d is in the span of the columns $\{A_i : i \neq j,\ d_i > 0\}$. Using this, express d as a convex combination of two feasible solutions e, f with $e_j < d_j < f_j$. (This proves that if $d \in \mathbb{R}^n$ is not a basic feasible solution of the LP, it is not an extreme point of S.)

Question 2. Extreme Points Computation **(20 marks)**

(a) Determine all extreme points of C defined as follows:

$$C = \left\{ x \in \mathbb{R}^2 \ : \ \begin{pmatrix} 1 & 2 \\ -2 & 1 \\ 1 & 0 \end{pmatrix} x \leq \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \right\}$$

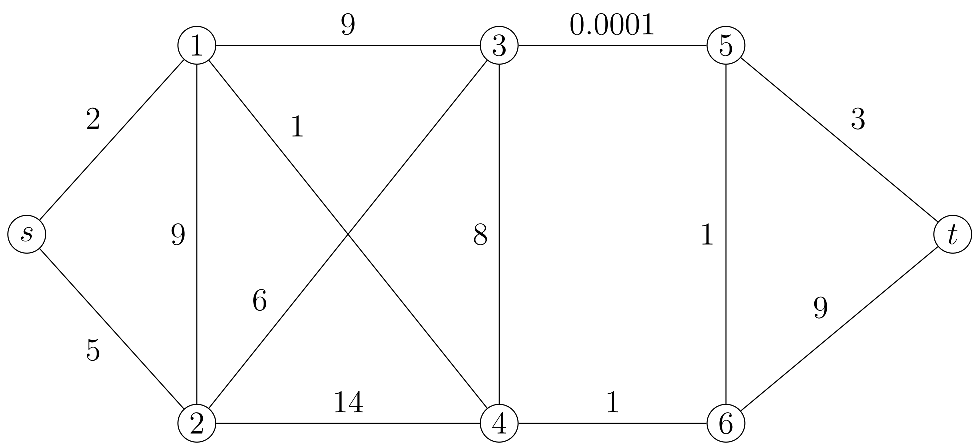
(b) Determine all extreme points of C' defined as follows:

$$C' = \left\{ x \in \mathbb{R}^4 \ : \ \begin{pmatrix} 1 & 2 & 0 & -1 \\ 1 & 3 & -2 & 0 \end{pmatrix} x = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \ x \geq \mathbb{0} \right\}$$

Justify your answers.

Question 3. Shortest Path Computation **(25 marks)**

Consider the graph below with edge lengths given:



Find a shortest s,t-path in G using our Shortest Path Algorithm. In your solution, you must list all arcs generated throughout the algorithm and all y values that are generated by the algorithm. Together with the shortest s,t-path you computed, provide a set of feasible cut widths whose total equals the length of the shortest s,t-path you provide.

In each iteration, whenever there is a tie in the choice of the new arc (i.e., ithere are many candidates for the new arc) you must choose the arc ab that makes the largest two digit number. For example, if arcs ab and dc are the two arcs attaining the smallest slack, with $a = 1, b = 9, c = 3, d = 2$, then we choose dc as the new arc, since ab is $19 < 23$ (dc is 23). Treast $s := 0$ and $t := 7$.