CO 250 Online - Spring 2018

Assignment 1

Due date : Friday, May 11, 2018, by 4:00pm sharp

Submission Guidelines:

- Please submit your solutions to Crowdmark. Late assignments will not be accepted, and will receive a mark of zero. It is the responsibility of the students to make sure that the pdf file they submit is clearly readable. Illegible submissions will receive a mark of zero, and hard to read submissions may receive deductions.
- You answers **need to be fully justified**, unless specified otherwise. Always remember the WHAT-WHY-HOW rule, namely explain in full detail what you are doing, why are you doing it, and how are you doing it. Dry yes/no or numerical answers will get 0 marks.
- In some questions you are asked to *formulate* the problem. You are *not* asked to actually solve the formulation, e.g., compute optimal solutions. Your formulations should be easy to modify if we change the data and constants defining the problems. Clearly define all your variables (including their units) and any other new notation you use in all your answers. Your solutions must also contain a brief justification of all the constraints (explain the relation between each of the constraints and the requirements stated in the problem) and the objective function.

Assignment policies: While it is acceptable to discuss the course material and the assignments, you are expected to do the assignments on your own. Copying or paraphrasing a solution from some fellow student or old solutions from previous offerings qualifies as cheating and we will instruct the TAs to actively look for suspicious similarities and evidence of academic offenses when grading. Students found to be cheating will be given a mark of 0 on the assignment. In addition, all academic offenses will be reported to the Math Academic Integrity Officer (which may lead to further penalties) and recorded in the student's file.
Re-marking policies: If you have any complaints about the marking of assignments, then you should first check your solutions against the posted solutions. After that, if you see any marking error, then write a letter detailing clearly the marking errors, and submit this to one of the head TAs within one week from the date the graded assignment is returned. If you still have concerns after the final decision of the TA, then please contact your instructor communicating all the correspondence with the TA and the original petition.

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[1] It is an academic offense to post this assignment or solutions to any web page.

Question 1. Linear Algebra Review **(20 marks)**

For each of the following statements either prove that the statement is correct, or provide a counterexample or a suitable proof that shows that the statement is incorrect.

Each part below is worth 4 marks.

(a) Let A be an $m \times n$ matrix, where $n, m \geq 2$, and $y \in \mathbb{R}^n$ such that $Ay = 0$. Then the columns of A are linearly dependent.

(b) The following matrix has four linearly independent columns:

$$\begin{bmatrix} -1 & 2 & 5 & 0 & -3 & -2 & 9 \\ 3 & 4 & 1 & 9 & -3 & 0 & 7 \\ 7 & -6 & 5 & 4 & -3 & 8 & -6 \end{bmatrix}.$$

(c) Let A be a 8×4 matrix with four linearly independent rows, and $x \in \mathbb{R}^4$. If $Ax = 0$, then $x = 0$.

(d) Let A be an $m \times n$ matrix and $b \in \mathbb{R}^m$ such that the system of equations $Ax = b$ over variables $x \in \mathbb{R}^n$ has a solution. Then there is a solution $x = a \in \mathbb{R}^n$ with at most m non-zero coordinates.

(e) Let A be an $m \times n$ matrix such that that the system of equations $Ax = 0$ over variables $x \in \mathbb{R}^n$ has n linearly independent solutions. Then $A = 0$.

Question 2. LP Modeling I **(30 marks)**

The CNO textile company produces five kinds of cloth, each with a different composition of fabric (linen, cotton, wool, and silk). The composition of cloth is summarized in the following table, along with the profit the company makes on each type of cloth.

Type of cloth	Profit ($\$/m^2$)	Linen (g/m^2)	Cotton (g/m^2)	Wool (g/m^2)	Silk (g/m^2)
Litton	5	120	150	–	–
Cool	7	–	200	75	–
Cilk	10	50	100	–	20
Silen	12	100	80	20	50
Wolk	25	–	–	100	100

The company has an inventory of 75 kg of linen, 120 kg of cotton, 60 kg of wool, and 8 kg of silk. CNO would like to determine how many m^2 of each kind of cloth it should produce so as to maximize profit.

Formulate the above optimization problem as a linear programming problem.

Question 3. LP Modeling II **(30 marks)**

A company that produces pasta in bulk has a factory in each of the cities $i \in \{1, 2, \ldots, m\}$. It has won a government bid to meet the yearly demands $d_1, d_2, \ldots, d_n$ (in tons of pasta) in the regions $1, 2, \ldots, n$, respectively. Now the company has to decide how much pasta to produce at each of its factories and how much of each factory's output will be shipped to each of the n regions.

The cost of producing pasta at the factory at location i is p_i in dollars per ton per year, and the yearly capacity of the factory is w_i tons. The cost of transporting one ton of the product from city i to region j is c_{ij}.

Construct a linear programming problem whose solution indicates how many tons of pasta to produce and ship from each factory to each demand region so that the demand is met and the total yearly cost of the company is minimized.

Question 4. IP Modeling **(20 marks)**

Suppose you went to the Bahamas during reading week, and purchased a number of items. The following table lists the items, their weights, and their value:

Items	A	B	C	D	E	F
Weight (kg)	6	7	4	9	3	8
Value ($)	60	70	40	70	16	100

The day before your return, you realize that your luggage allowance is only 30 kg. The luggage that you brought to the Bahamas, which weighed 7 kg, and the items you bought weigh much more than the allowance. You would like to decide which of the new items to pack along with your original luggage so that you maximize the total value of the items selected, without exceeding the luggage allowance.

(a) Formulate this problem as an IP. (10 marks)

(b) Suppose that you only wish to pack item D when item A is selected, but it is OK to pack A without selecting D. Add one or more constraints to your formulation that impose this additional condition. (5 marks)

(c) Suppose that the airline allows you to carry luggage over the 30 kg limit at a cost of $15 per additional kg. For instance, you could select items A, B, D, and E for a total value of $216, and total weight of 32 kg (including the original luggage). You would then pay $30 for the 2 kg over your allowance.

Modify your formulation so that you are allowed to go over the 30 kg allowance, and such that you maximize the total value of the items selected minus the airline fee for the extra weight. (5 marks)