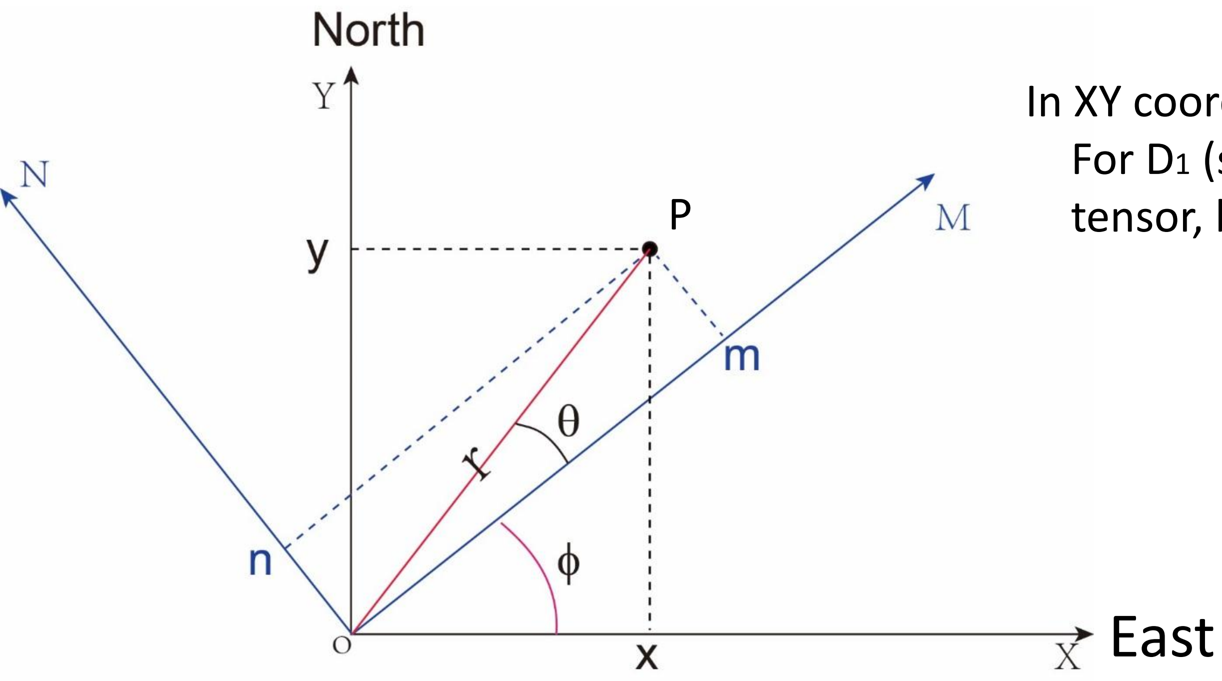
Lab 8 Calculating strain in deformed rocks and determining orientations of foliations

Suppose an area has undergone 2 generations of deformation. D_1 is a dextral simple shear with an east-west striking, vertical shear plane. The shear strain along the shear plane is $2/\sqrt{3}$. D_2 is a pane-strain (no vertical stretch) equal-area pure shear with horizontal maximum principal stretch trending 060°. The maximum principal stretch of the pure shear is 2.

What are the position gradient tensor, principal stretches, and principal strain axes orientation for each deformation?

What are the position gradient tensor, principal stretches, and principal strain axes orientation for the total deformation?

Suppose a foliation developed along the maximum principal axis in D_1. As a material plane, this foliation is deformed by D2. What is the orientation of this foliation at the end of D2?



In XY coordinate system:

For D_1 (simple shear), it is easy to write the position gradient tensor, because the shear direction is along X axis.

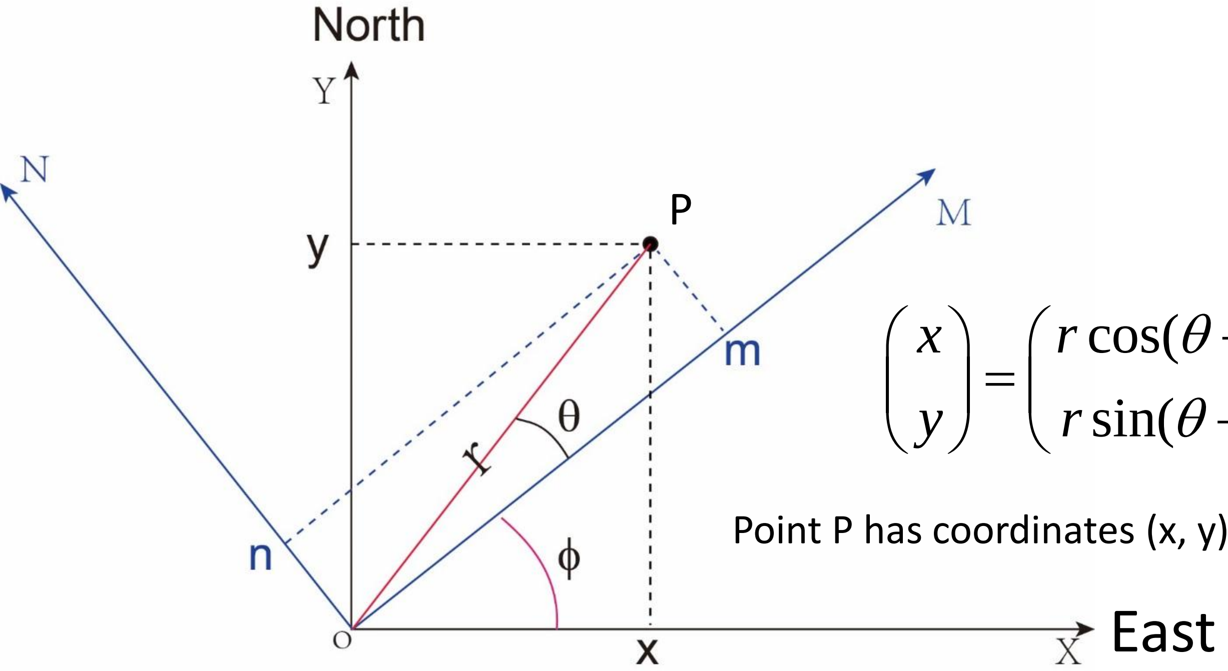
In MN coordinate system:

What is the position gradient tensor F_{MN} for the pure shear when S1 is along M axis and S2 is along N axis?

What is the corresponding position gradient tensor F_{XY} in the XY coordinate system?

$$\mathbf{F}_{MN} = \begin{pmatrix} S1 & 0 \\ 0 & S2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

Relationship between F_{MN} and F_{XY}

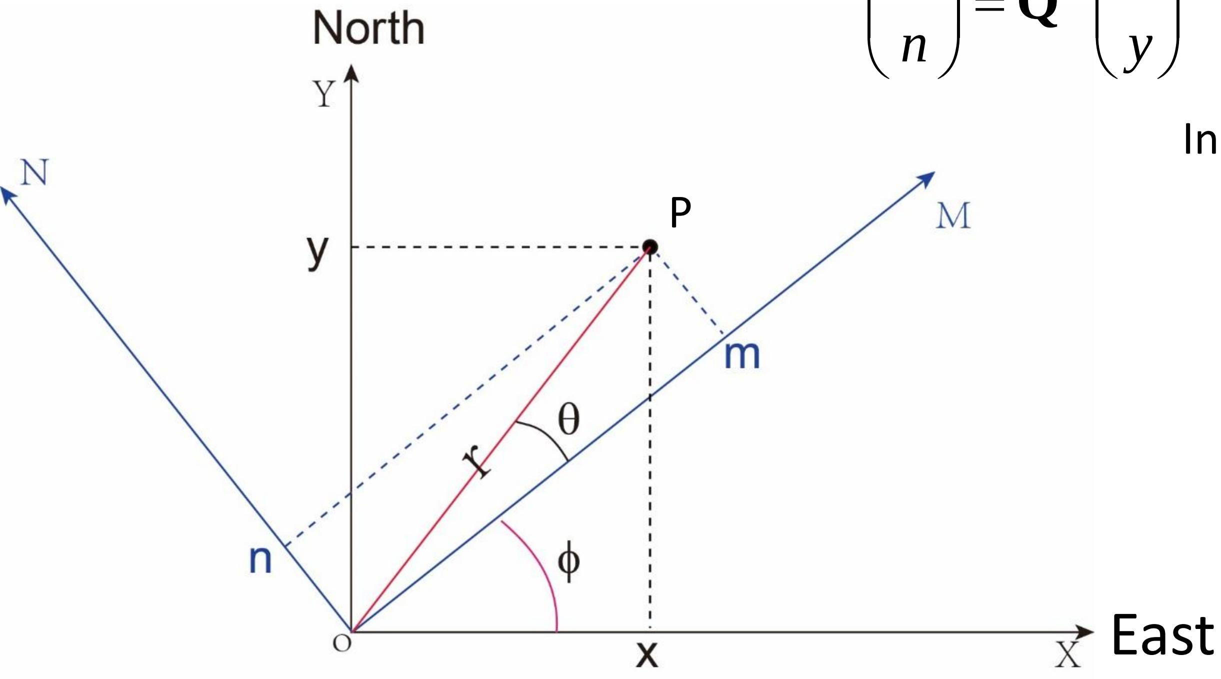


$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos(\theta+\phi) \\ r\sin(\theta+\phi) \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix}$$

Point P has coordinates (x, y) in XY coordinate system and coordinates (m, n) in MN coordinate system

$\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$ is an orthogonal tensor. Let $\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} = \mathbf{Q}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{Q} \begin{pmatrix} m \\ n \end{pmatrix} \Longrightarrow \mathbf{Q}^T \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{Q}^T\mathbf{Q} \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}$$



$$\begin{pmatrix} m \\ n \end{pmatrix} = \mathbf{Q}^T \begin{pmatrix} x \\ y \end{pmatrix}$$

In XY coordinate system:

At undeformed state: P(x0, y0)

At deformed state: P'(x1, y1)

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \mathbf{F}_{\mathbf{XY}} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

In MN coordinate system:

At undeformed state: P(m0, n0)

At deformed state: P'(m1, n1)

$$\begin{pmatrix} m_1 \\ n_1 \end{pmatrix} = \mathbf{F}_{\mathbf{MN}} \begin{pmatrix} m_0 \\ n_0 \end{pmatrix}$$

$$\begin{pmatrix} m_1 \\ n_1 \end{pmatrix} = \mathbf{Q}^T \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \qquad \begin{pmatrix} m_0 \\ n_0 \end{pmatrix} = \mathbf{Q}^T \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$\mathbf{Q}^T \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \mathbf{Q}^T \mathbf{F}_{\mathbf{XY}} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \mathbf{F}_{\mathbf{MN}} \mathbf{Q}^T \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \Longrightarrow \mathbf{Q}^T \mathbf{F}_{\mathbf{XY}} = \mathbf{F}_{\mathbf{MN}} \mathbf{Q}^T$$

$$\mathbf{F}_{\mathbf{XY}} = \mathbf{Q} \mathbf{F}_{\mathbf{MN}} \mathbf{Q}^T$$