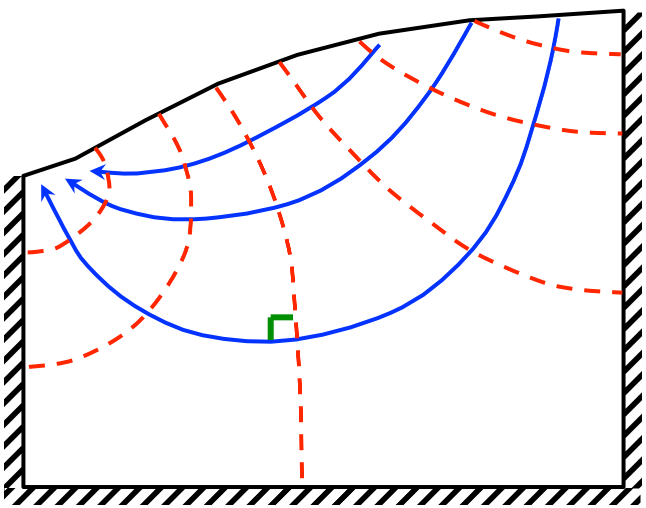
Note: Beware of Vertical Exaggeration

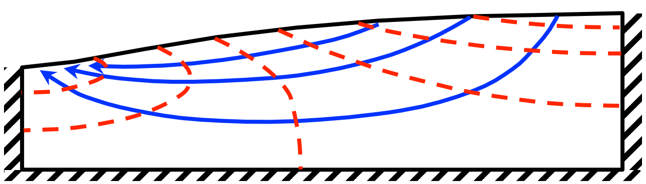
Flow nets are intended to be drawn on figures with a 1:1 ratio for the axes. This is typical for plan view maps, but most cross-sections are drawn with vertical exaggeration.

Flow net on cross-section with vertical exaggeration



3:1 Vert. Exagg.

Same flow net shown with no vertical exaggeration



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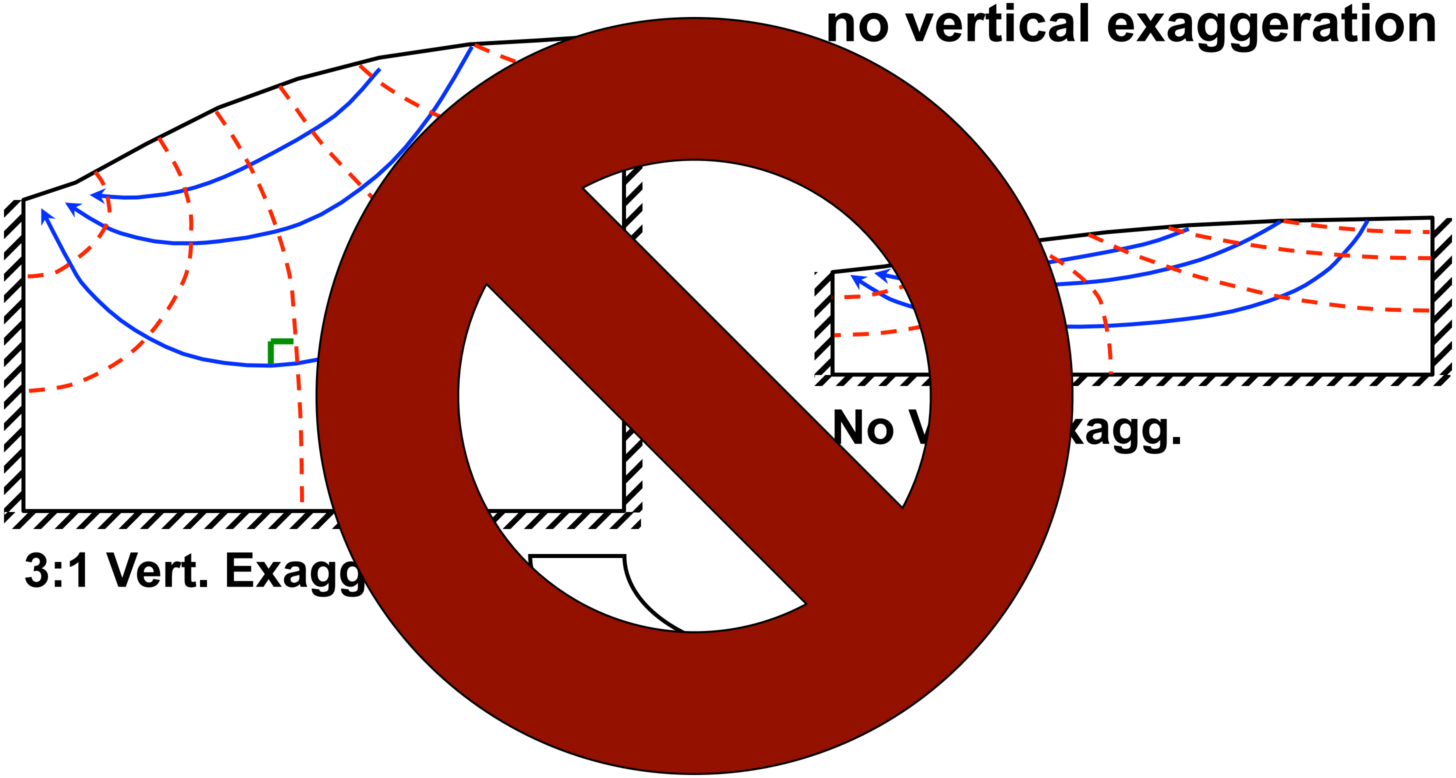
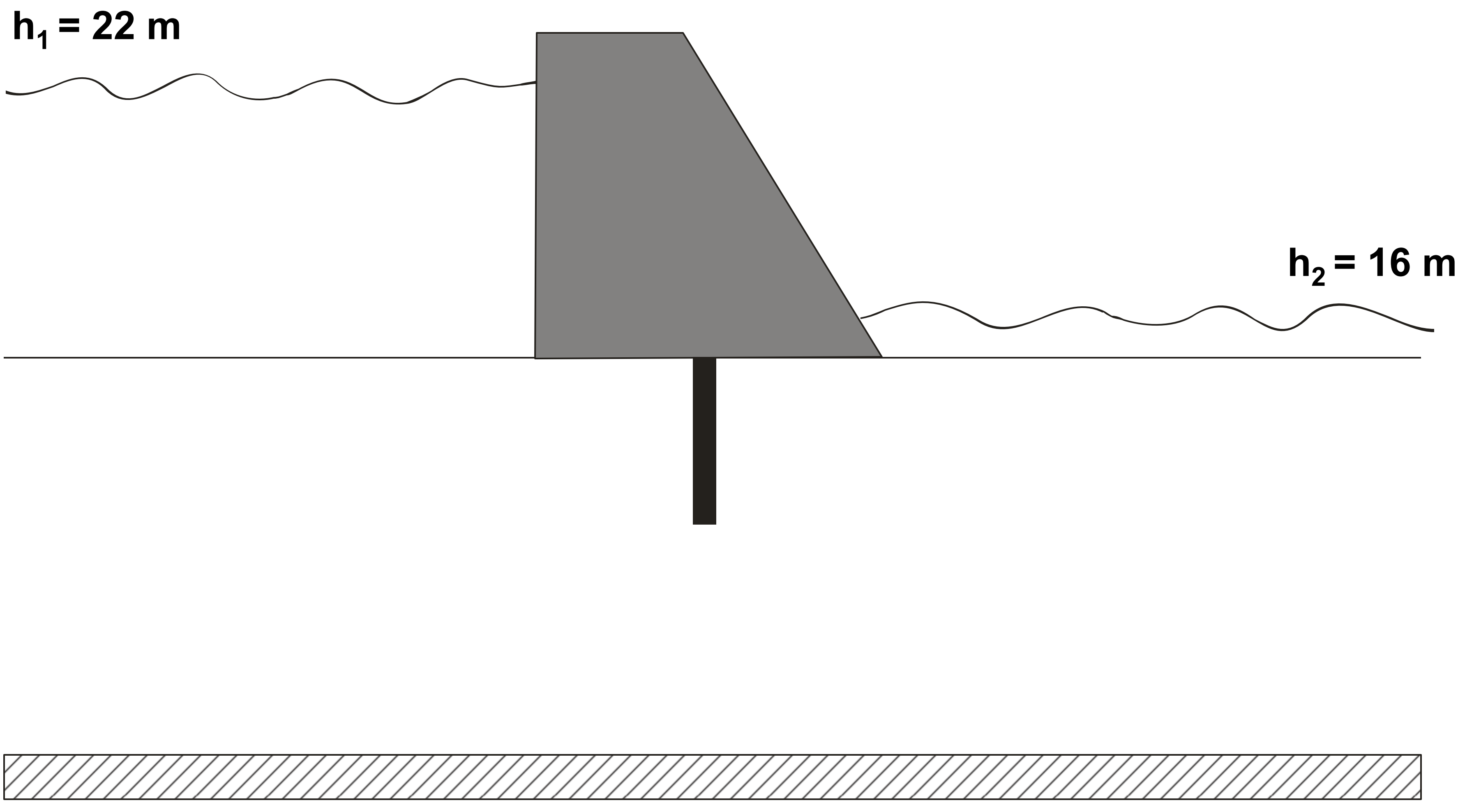
Flow net on cross-section with vertical exaggeration

Same flow net shown with no vertical exaggeration



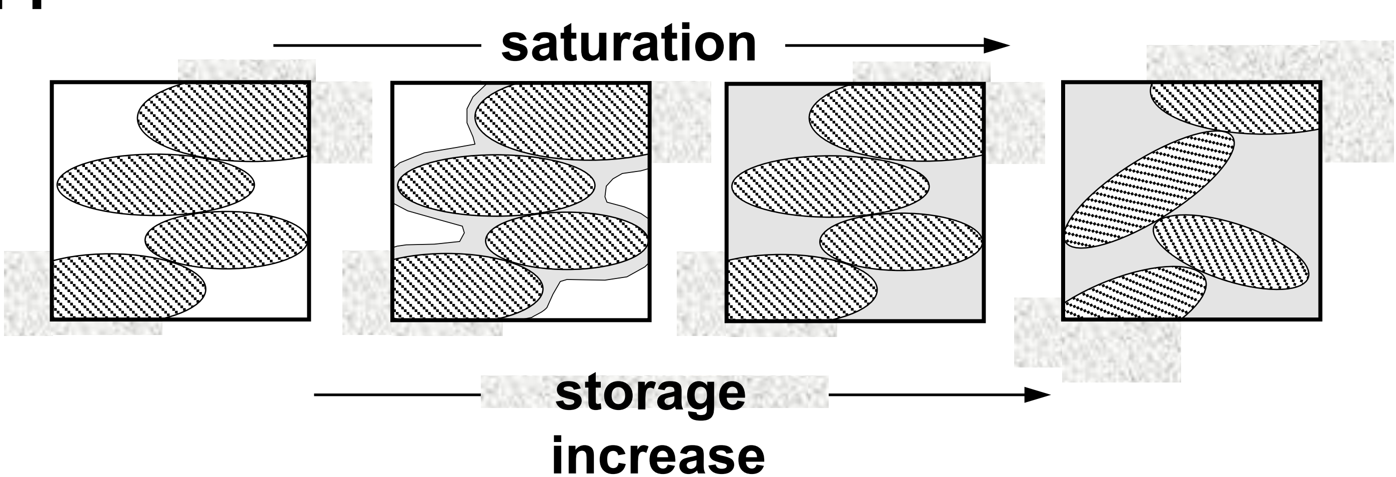


h_1 = 22 m
h_2 = 16 m

Groundwater Storage

Remember how aquifers are units that can store and transmit water? Let's explore storage in more detail.

Consider an REV of dry sediments that we add water to. As water is added, it fills up pore spaces until the sediments become saturated. What happens after that?



Can more water could be added to the REV after it was already saturated?

(1) Water is slightly compressible.
- Higher pressure = more mass per volume; (i.e. higher density of water).

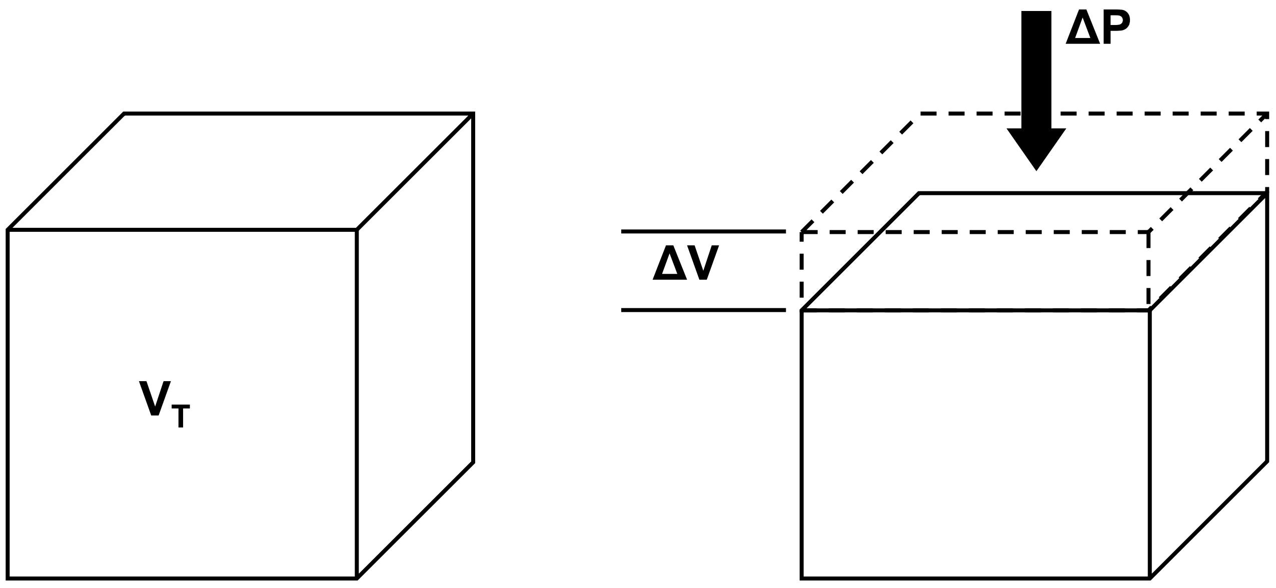
(2) Sediments behave like an elastic body.
- Solid particles rearrange themselves to accommodate more water, resulting in increased porosity.

Overview of Compressibility and Stress

A material property relating the change in volume per unit volume to the applied pressure (or stress σ).

As pressure increases, volume decreases proportionally. Hence the negative sign in the equation.

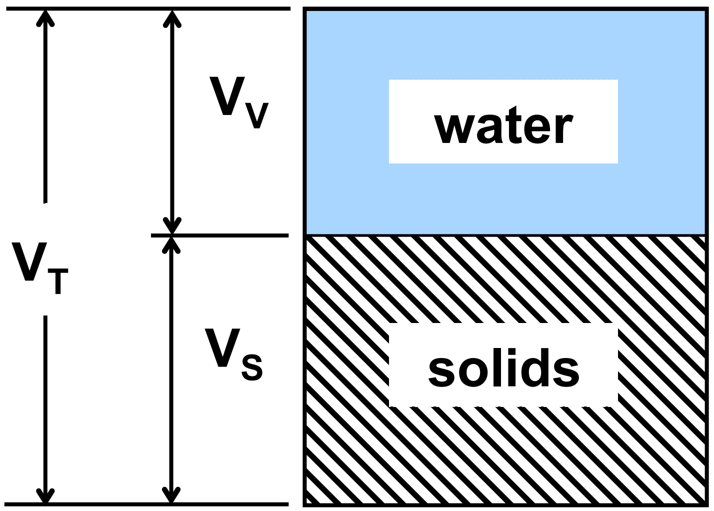
$$\beta = -\frac{\Delta V / V_T}{\Delta P} \text{ or } -\frac{\Delta V / V_T}{\Delta \sigma}$$



Compressibility in Groundwater Systems

Changes in the volume of the porous medium can occur as a result of compression or expansion of the following:

1. **Water or air in the voids**
2. **Solid grains**
3. **Aquifer matrix (solids-voids framework)**



For now we will deal with saturated systems, so we can assume only water in the voids. Let's examine the compressibility of each component in detail.

1. Compressibility of Water (β_w)

We typically assume water is incompressible, but really it is not. The compressibility of water, β_w, is expressed as:

$$\beta_W = -\frac{\Delta V_W / V_W}{\Delta P} = -\frac{1}{V_W}\frac{dV_W}{dP}$$

β_w is:

- **approximately 4.7×10^{-10} Pa^{-1}**
- **assumed to be a constant**
- **the slope of the line relating strain per unit stress**
- **valid for typical groundwater pressures, even in vadose zone**
- **not influenced by temperature (typical range for gw)**

2. Compressibility of Solids (β_S)

Individual solid grains or rock matrix are highly incompressible for the most part: i.e., $\beta_s << \beta_w$

Typical values: $\beta_s \approx 10^{-12}$ to 10^{-11} Pa^{-1}

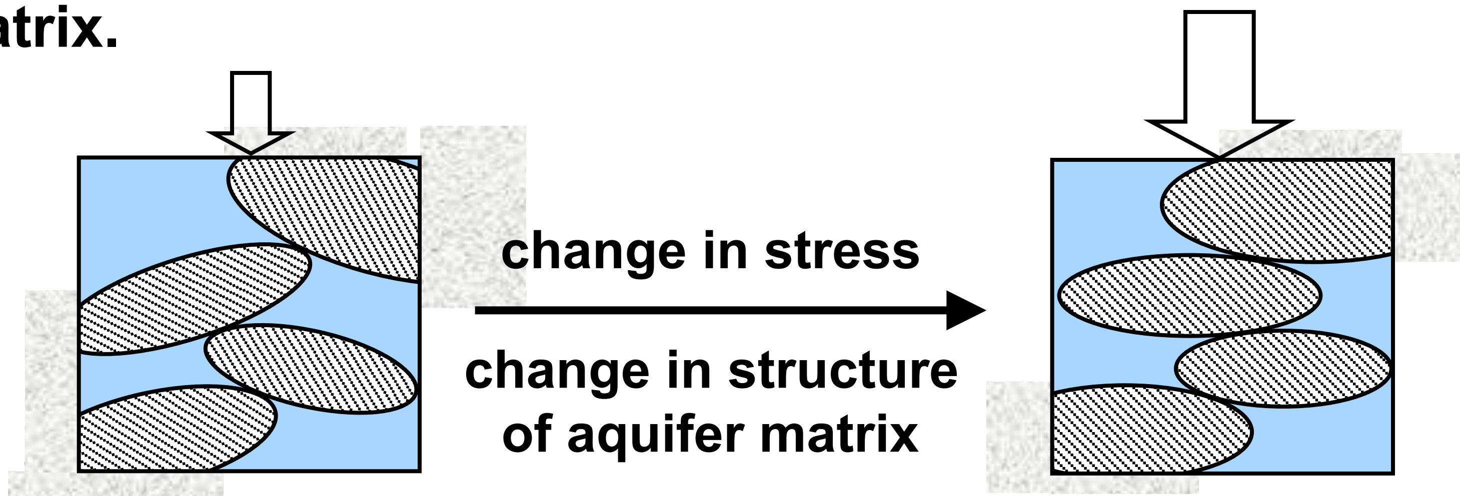
Compression of rock or solid grains is usually considered *negligible* relative to changes in water storage.

3. Compressibility of Aquifer Matrix (α)

The aquifer matrix represents the interconnected individual grains that form the sediment skeleton. The compressibility of this solid matrix, α, is expressed as:

$$\alpha = -\frac{\Delta V_T / V_T}{\Delta \sigma_e} = -\frac{1}{V_T}\frac{dV_T}{d\sigma_e}$$

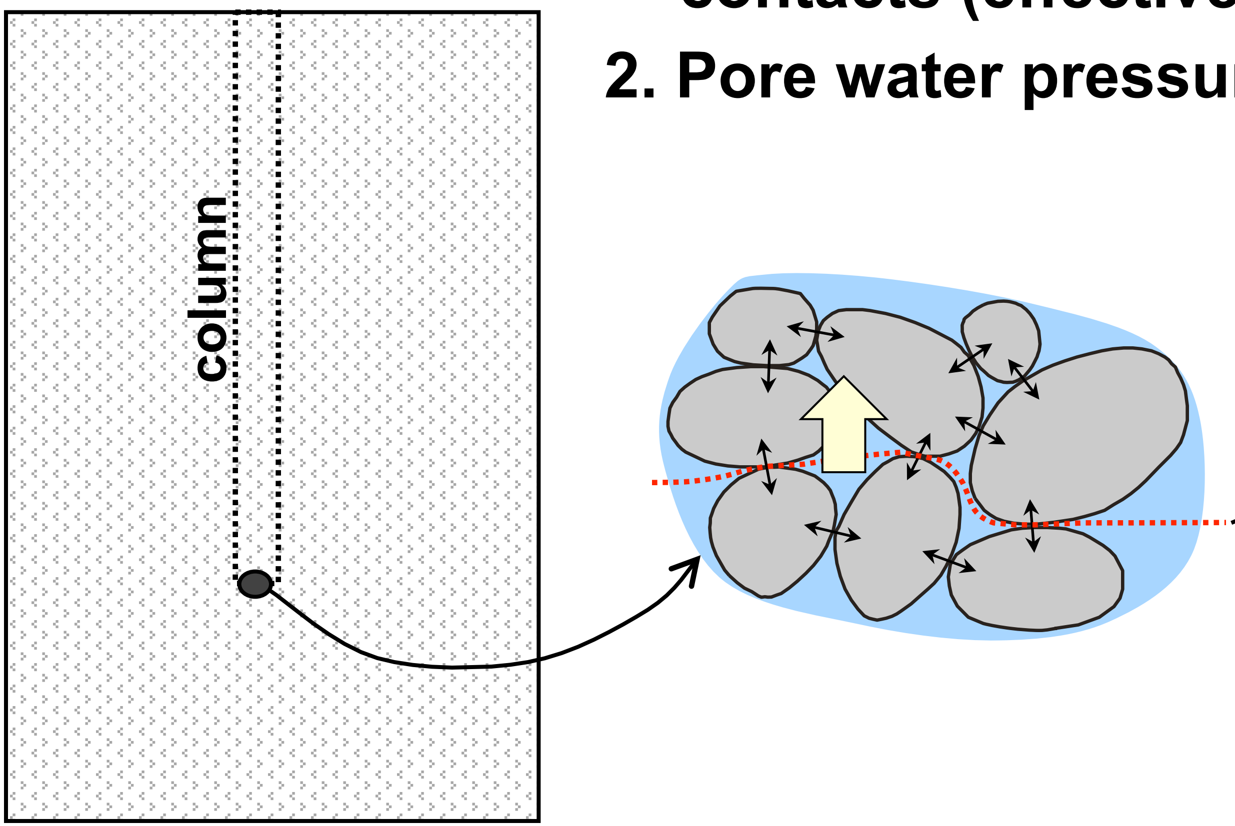
where σ_e is the pressure or effective stress acting on the matrix.



Effective Stress

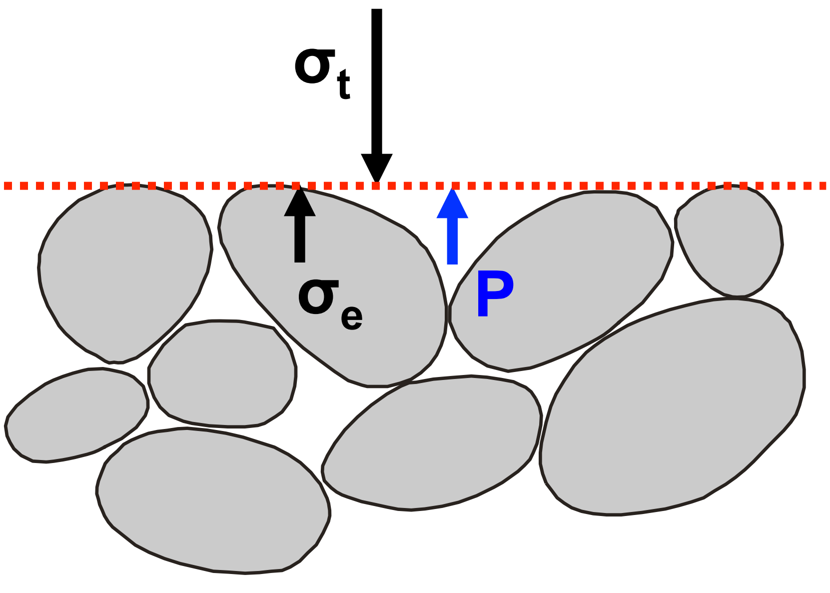
Two things support the weight of the column of soil and water at the point shown:

1. **Forces across grain-to-grain contacts (effective stress)**
2. **Pore water pressure**



Imagine a roughly planar surface at the base of the column

Terzaghi (1925) proposed the concept that total vertical stress, σ_t, is the weight per unit area of the overlying column (soil + water). Acting against σ_t are the fluid pressure, P, and the effective stress, σ_e, borne on the solid matrix.



$$\sigma_t = \sigma_e + P$$

or

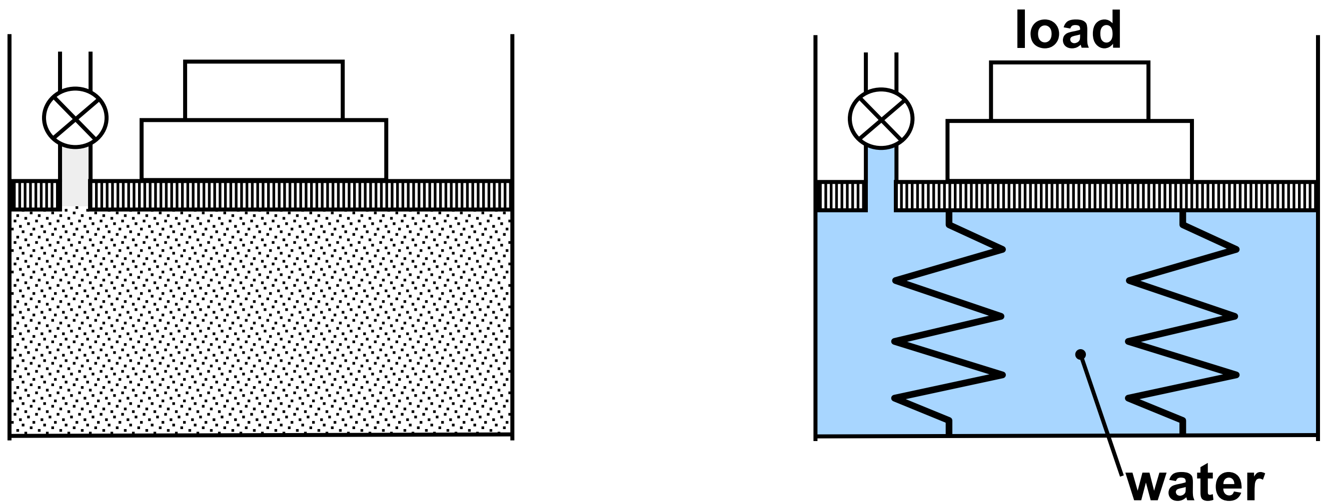
$$\sigma_e = \sigma_t - P$$

Assuming the total load or σ_t is constant, then:

$P\uparrow \;\rightarrow\; \sigma_e\downarrow$ **or**

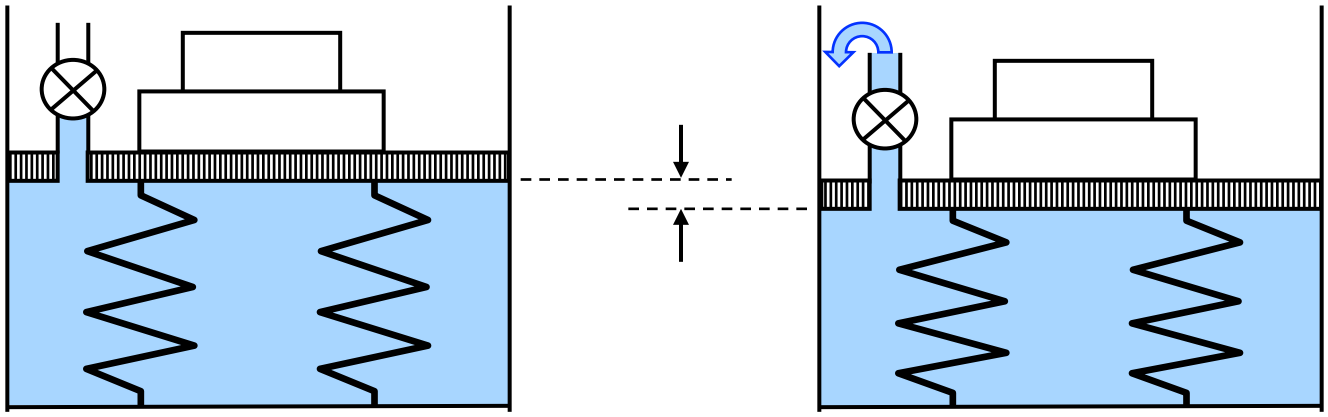
$$\Delta\sigma_e = -\Delta P$$

Think of it a different way. Consider saturated sand in a container with a movable top plate. The water-sand mixture may be viewed as a water-spring system. The spring represents the solid matrix.



Suppose we extract a small volume of water while <u>keeping the load constant</u>. What happens to the top plate? What happens to the water and the spring?

The top plate will fall slightly. Since the spring is compressed slightly, the force supported by the spring will increase. What happens then to the water pressure?



Will the volume of solid grains change with water pressure?

The fall of the top plate is due to a change in the porosity of the granular matrix. The grains change orientation to support the increased stress.

This collective reorientation of soil grains and reduction in pore volume is called <u>consolidation</u>.

Land subsidence in San Joaquin Valley caused by groundwater pumping. The former position of the land surface is illustrated on the power pole.

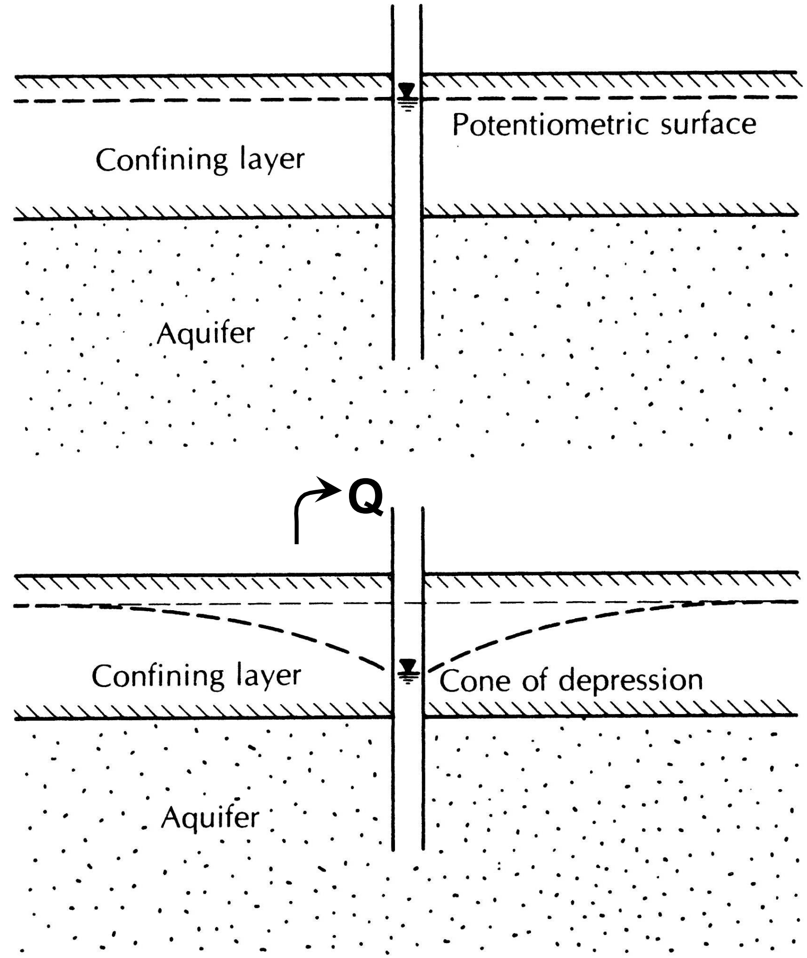
Typical values of vertical compressibility, α

Material	**α (Pa^{-1})**
Plastic clays	10^{-7} - 10^{-6}
Stiff clays	10^{-8} - 10^{-7}
Sands	10^{-9} - 10^{-7}
Sandy gravels	10^{-9} - 10^{-8}
Sedimentary rocks	10^{-11} - 10^{-9}
Crystalline rocks	10^{-12} - 10^{-10}
Fractured crystalline rocks	10^{-11} - 10^{-9}
Water	4.7×10^{-10}

Storage in Confined Aquifers

How much water will be released when we lower the potentiometric surface a specified amount?

Consider pumping from a well. Pumping reduces the pore water pressure and lowers the potentiometric surface. This results in an increase in *effective stress* since the total stress has not changed (we have not dewatered the aquifer).



(Fetter, Applied Hydrogeology, 2001)

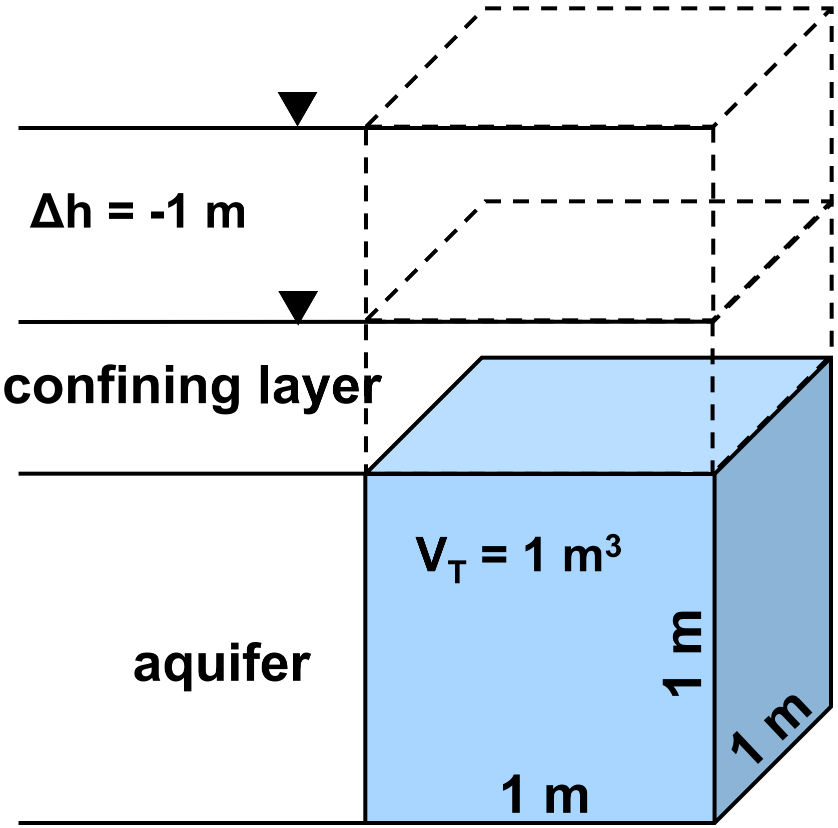
Specific Storage (S_s)

Volume of water released from storage per unit volume of porous medium per unit decline in hydraulic head.

Water is released by the:

1. **Expansion of water (β_w)**
2. **Consolidation of the matrix (α)**

Remember compression of soil grains (β_s) is considered negligible.



Consider a unit volume of porous medium, and calculate the volume of water released by water expansion ΔV_w.

Recalling:

$$\beta_W = -\frac{\Delta V_W / V_W}{\Delta P}$$

and

$$\Delta P = \rho g \Delta\psi$$

$$\Delta h = \Delta\psi + \cancel{\Delta z}^{\,0}$$

will give,

$$\beta_W = -\frac{\Delta V_W}{V_w \rho g \Delta h}$$

Given,

$$V_w = V_T \cdot n$$

We can rewrite:

$$\Delta V_W = -\beta_W V_W \rho g \Delta h$$

$$\Delta V_W = -\beta_W n V_T \rho g \Delta h$$

Similarly, the volume of water released by compression of the aquifer matrix can be determined.

Recalling:

$$\alpha = -\frac{1}{V_T}\frac{\Delta V_T}{\Delta \sigma_e}$$

From equation on pg. 39, $\Delta \sigma_e = -\Delta P$ **and** $\Delta P = \rho g \Delta h$

Gives,

$$\alpha = \frac{1}{V_T}\frac{\Delta V_T}{\rho g \Delta h} \qquad \therefore \; \Delta V_T = \alpha V_T \rho g \Delta h$$

The change in total volume is assumed to be due to changes in void space only.

$$\Delta V_T = \Delta V_V + \cancelto{0}{\Delta V_S} \approx \Delta V_V$$

$$\Delta V_W = -\Delta V_T = -\alpha V_T \rho g \Delta h$$

Note that for a decrease in head (Δh = negative), ΔV_T and ΔV_V are also negative. This means that the release of water ΔV_W is considered positive.

The equation for the total volume of water released per unit volume of porous medium per unit decline in head is written:

$$S_S = \underbrace{\Delta V_W}_{\substack{\text{Expansion} \\ \text{of } H_2O}} + \underbrace{\Delta V_W}_{\substack{\text{Compression} \\ \text{of matrix}}}$$

Knowing $V_T = 1$ and $\Delta h = -1$ we can substitute our values of ΔV_w into this equation:

$$S_S = -\beta_W n V_T \rho g \Delta h - \alpha V_T \rho g \Delta h$$

Rearranging gives:

$$\boxed{S_S = \rho g (\beta_W n + \alpha)}$$

Specific storage ranges between 10^{-5} to 10^{-2} m^{-1} for most unconsolidated sediments. Generally higher for clays (aquitards) and lower for sands and gravel (aquifers). In most cases, S_s is dominated by the α term.

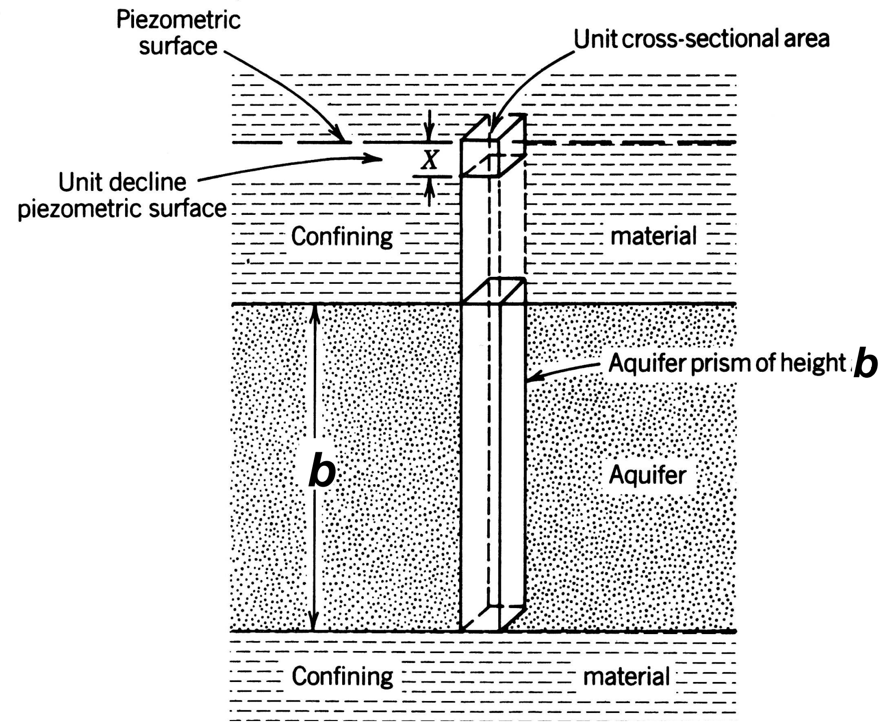
<u>Note on Elasticity</u>

It is typically assumed that both components of S_s are elastic. That is, they are linear and reversible. This holds true for the compression/expansion of water. The consolidation of sediments, on the other hand, is often not completely reversible. (persistent land subsidence effects)

Storativity (S)

A more useful storage parameter in practice is to integrate over the vertical thickness of an aquifer (b). Hence, storativity is defined as:

$$\boxed{S = S_s \cdot b}$$



It represents the volume of water released from storage per unit surface area per unit decline in head.

What are the units?

(Domenico and Schwartz, 1990)

How much water comes out of the ground?

To calculate the volume of water removed from an aquifer we must know the cross-sectional area (A), aquifer thickness (B), and the change in head (h).

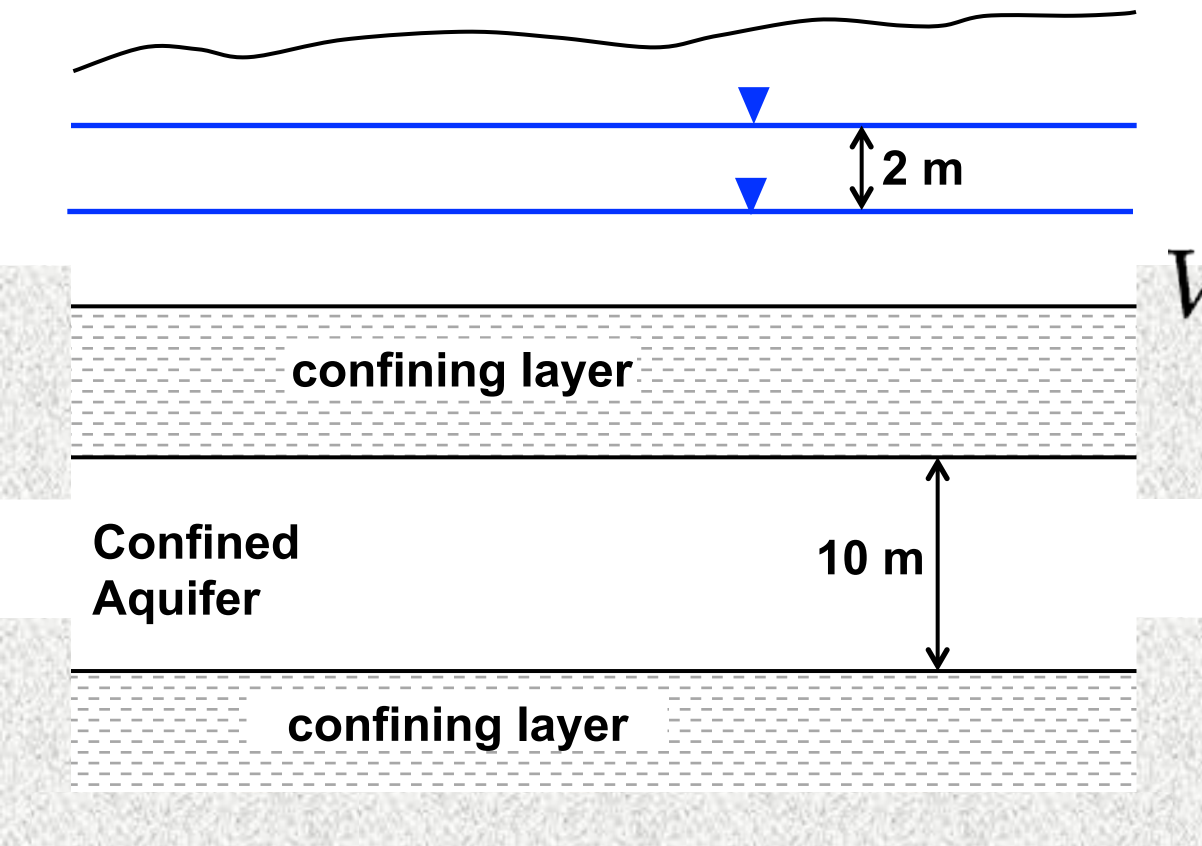
$$V = S_s \cdot b \cdot A \cdot \Delta h$$

$$S = S_s \cdot b$$

$$V = S \cdot A \cdot \Delta h$$

Example

How much water would be released by a head decline of 2 m over an area 200 × 200 m, for the aquifer shown below? Assume n = 0.30, $\alpha = 1\times10^{-8}$ Pa^{-1} and $\beta_w = 4.7\times10^{-10}$ Pa^{-1}.

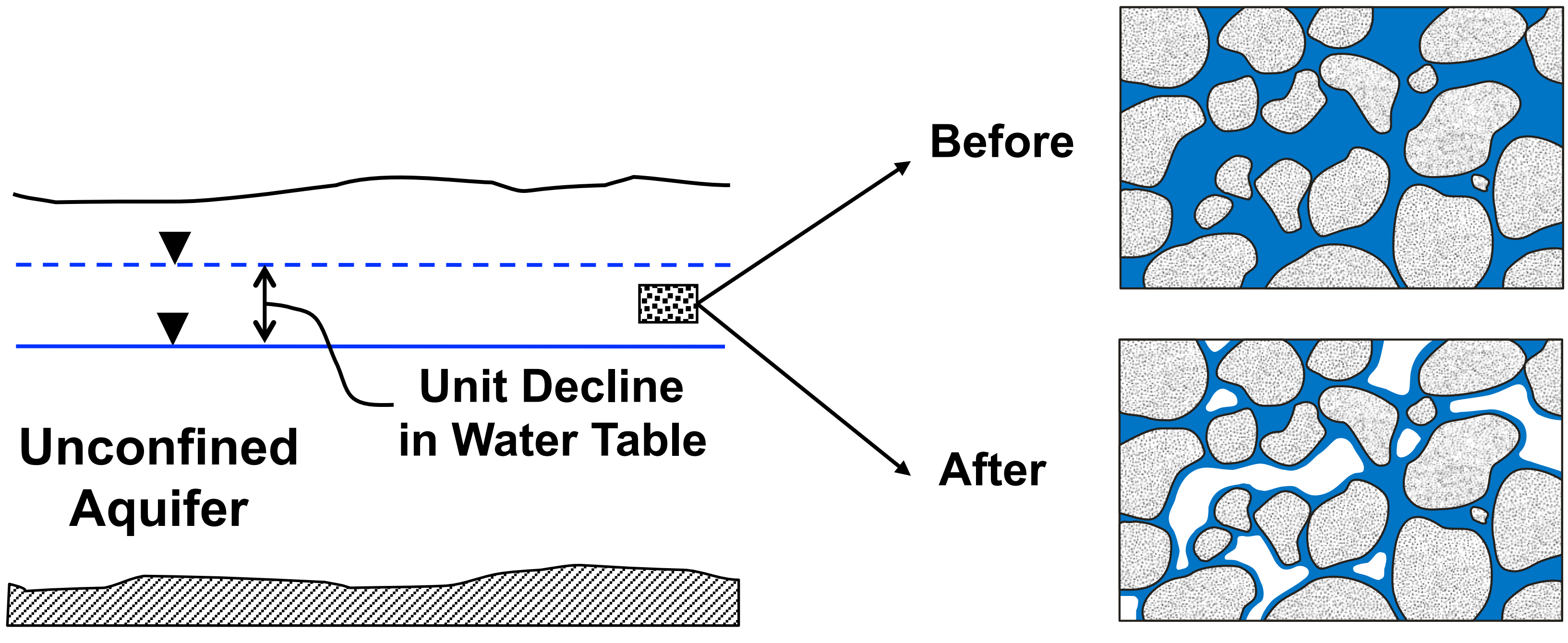


$$S_S = \rho g(\beta_W n + \alpha)$$

$$V = S_s \cdot b \cdot A \cdot \Delta h$$

Storage in Unconfined Aquifers

In unconfined aquifers, when the head declines so does the water table. Water is released by fluid expansion and matrix compression (as in the saturated case), but the pores themselves also drain. This drainage gives rise to a concept known as <u>specific yield</u> (S_y).



Specific Yield, S_y

Defined as the volume of water released by gravity drainage per unit surface area per unit decline in the water table. It can be expressed as:

$$\boxed{S_y = n - S_r}$$

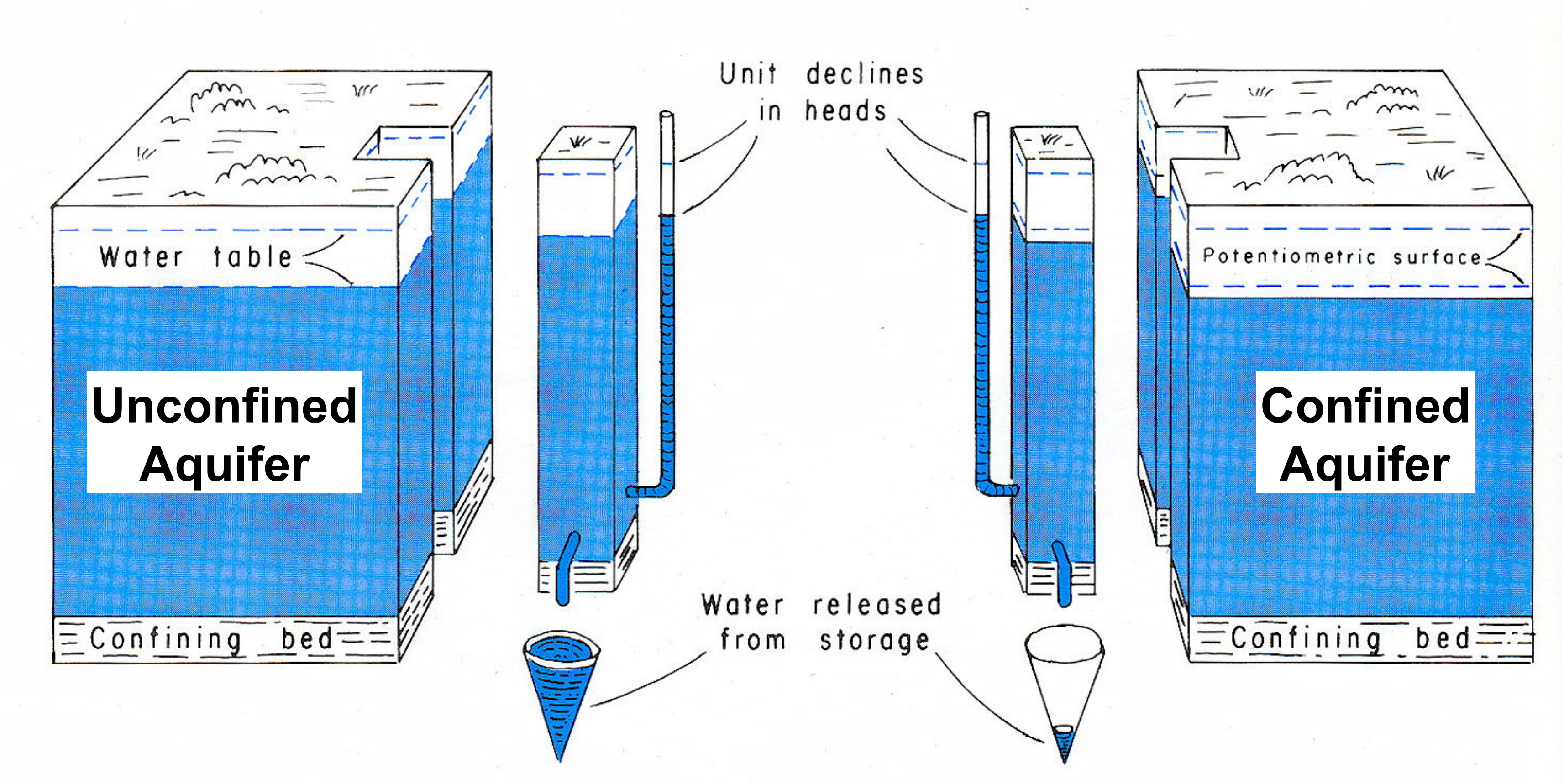
where *n* is total porosity and S_r is specific retention, which is the ratio of the volume of water a soil retains after gravity drainage to the total volume of soil ($S_r = V_r/V_T$).

Note that S_y is unitless. The amount of water that is retained after drainage is a function of the capillary forces. Thus, the coarser the material, the more closely S_y approaches n.

Typical values of specific yield, S_y

Material	S_y
Clay	0.0 - 0.05
Silt	0.03 - 0.19
Fine sand	0.10 - 0.32
Medium sand	0.15 - 0.32
Coarse sand	0.20 - 0.35
Gravel	0.14 - 0.30
Sandstone	0.05 - 0.10
Soil	0.20 - 0.40

Confined vs. Unconfined Storage



$S = S_y + b \cdot S_s$ ← **Storativity** → $S = b \cdot S_s$

Example

How much water would be released by a head decline of 2 m over an area 200 × 200 m, for the aquifer shown below? Assume n = 0.30, $\alpha = 1\times10^{-8}$ Pa^{-1} and $\beta_w = 4.7\times10^{-10}$ Pa^{-1}, Sy = 0.27.