# Heterogeneity and Anisotropy

**Natural geologic processes often cause hydraulic properties to vary with location in the subsurface.**

**Fluvial and Alluvial**

**Beach**

**Fractured Rock**

**Glacial**

***Homogeneous***: **a property (say K) is independent of location in the geologic formation**

- **i.e., K(x,y,z) = constant**

***Heterogeneous***: **K varies with location in the geologic formation**

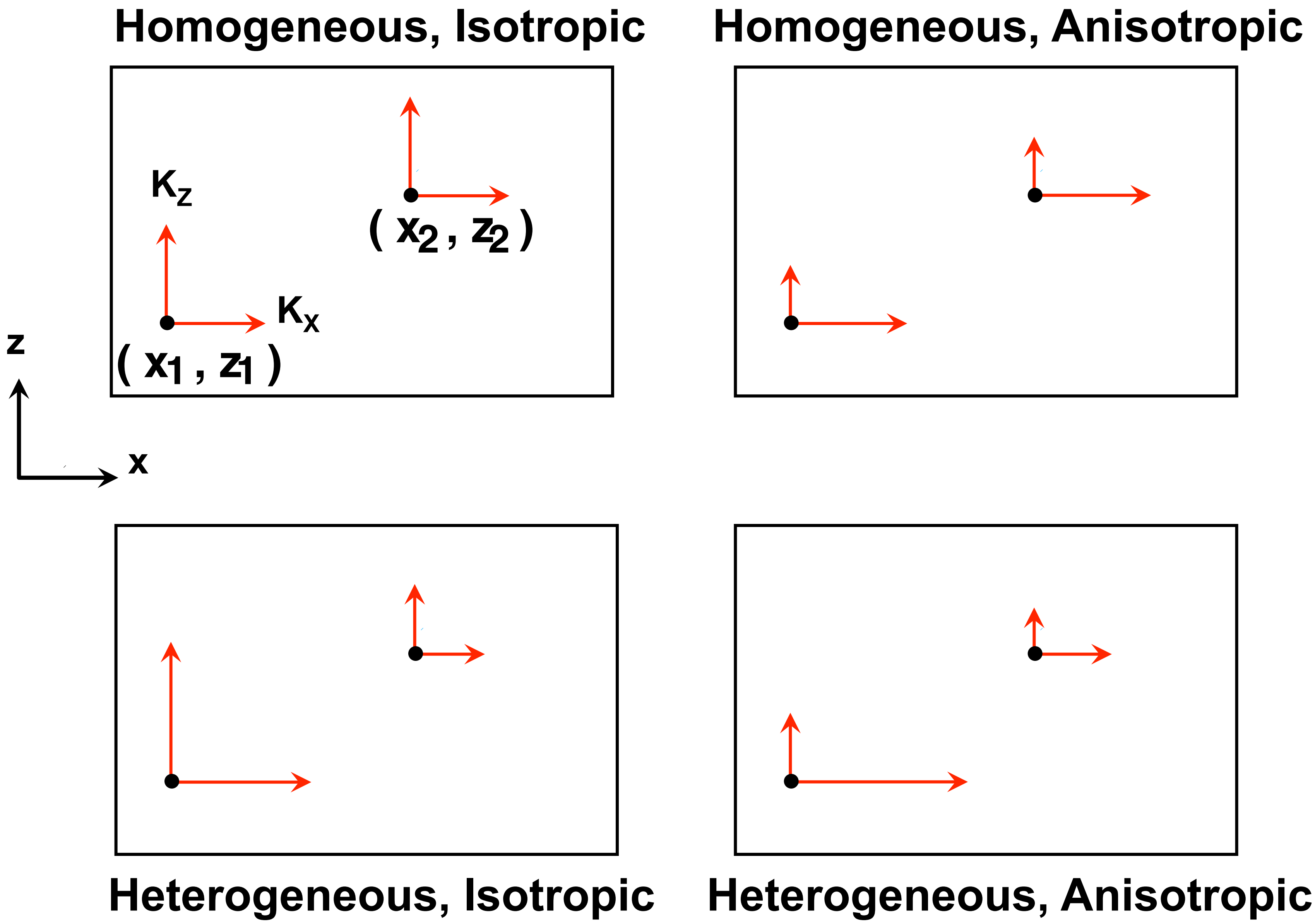
- **i.e., K(x,y,z) ≠ constant**

***Isotropic***: **K is independent of the direction of measurement at a point in the geologic formation**

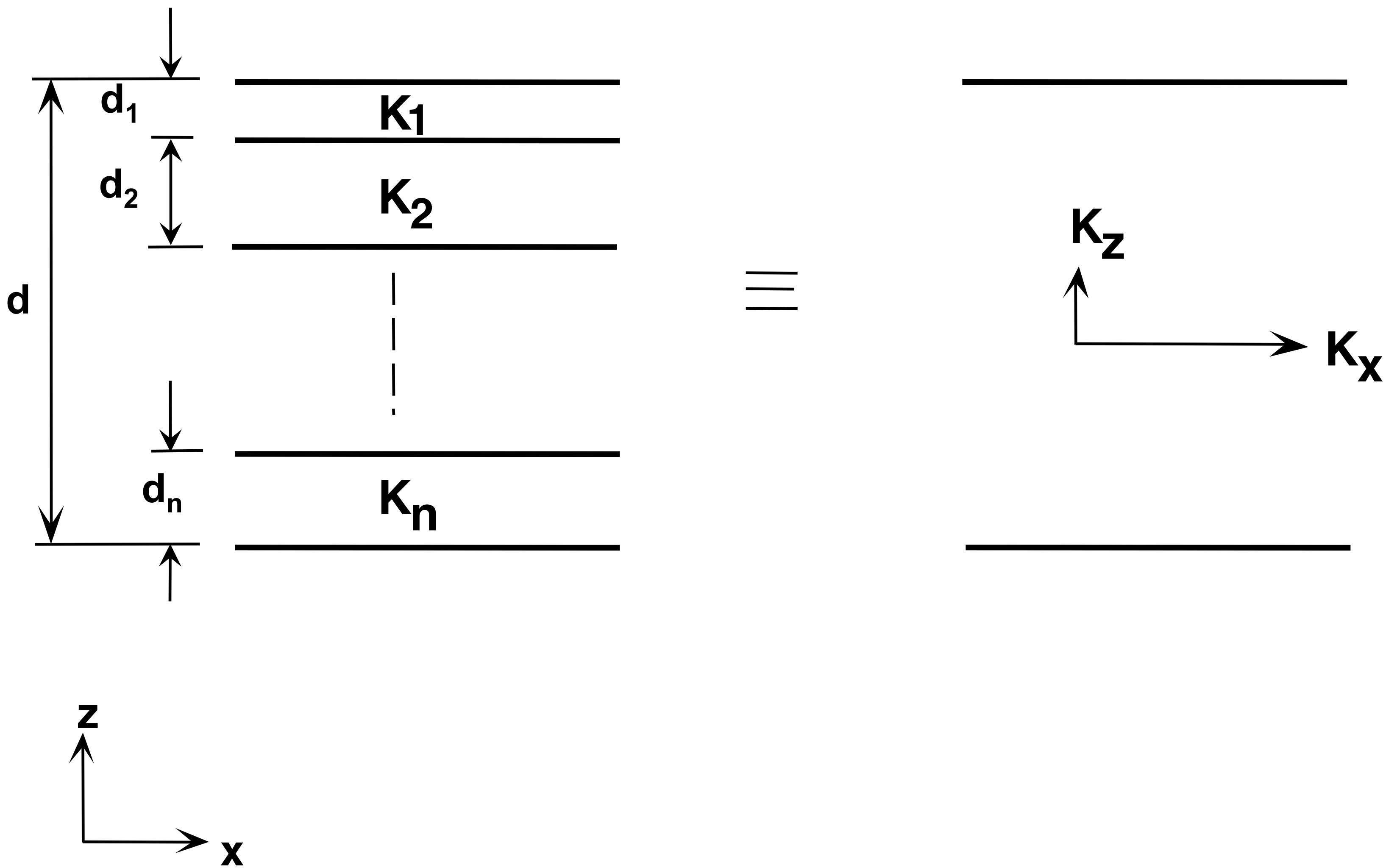
- **i.e., $K_x = K_y = K_z$**

***Anisotropic***: **K varies with the direction of measurement**

- **i.e., $K_x \neq K_y \neq K_z$**
- **K is generally larger in the horizontal direction than the vertical direction**


Homogeneous, Isotropic
Homogeneous, Anisotropic
K_z
K_x
( x_2 , z_2 )
( x_1 , z_1 )
z
x
Heterogeneous, Isotropic
Heterogeneous, Anisotropic

# At a larger scale, heterogeneity leads to anisotropy.

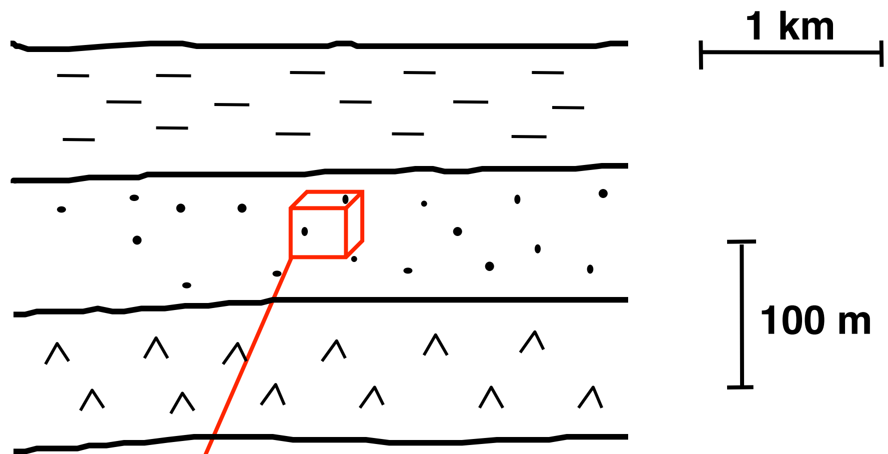


# Types of Heterogeneity

## 1. Layering

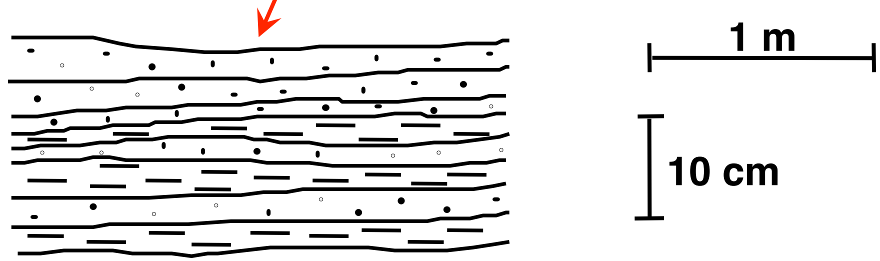
### (a) can result from large scale depositional processes

major
stratigraphic
units



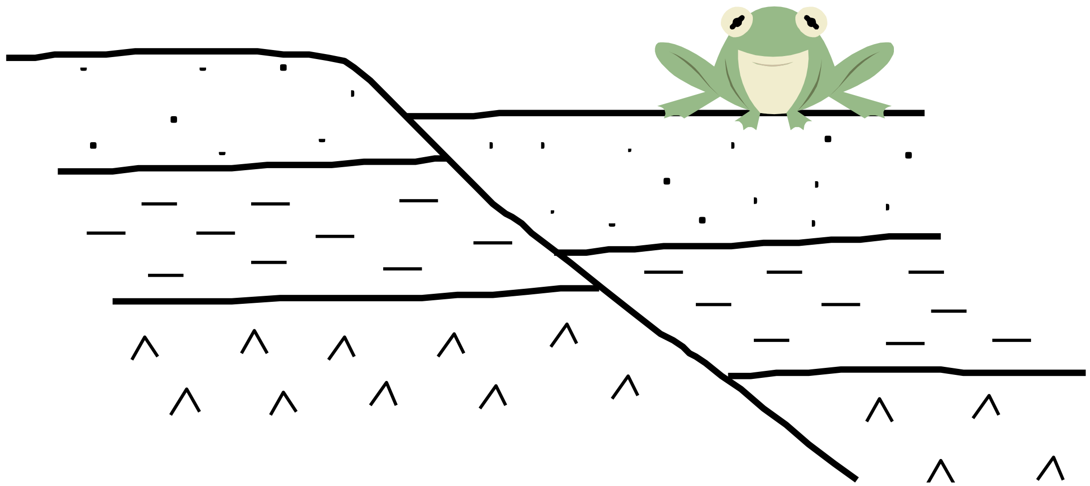

### (b) or as small-scale variations within a larger system

small
scale
bedding

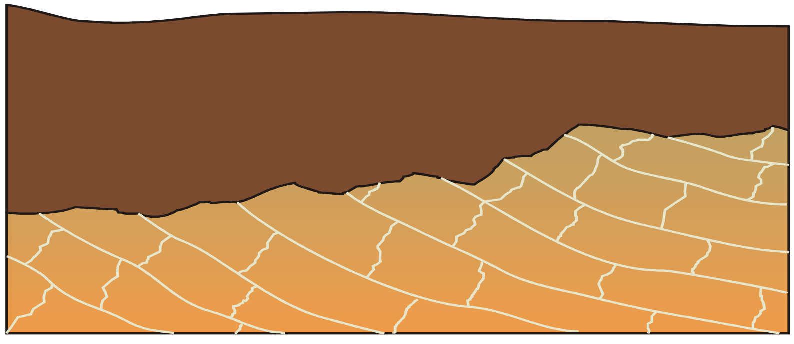


## 2. Discontinuities

### (a) Fold, Faults, etc.



### (b) Overburden-Bedrock Contacts, Unconformities

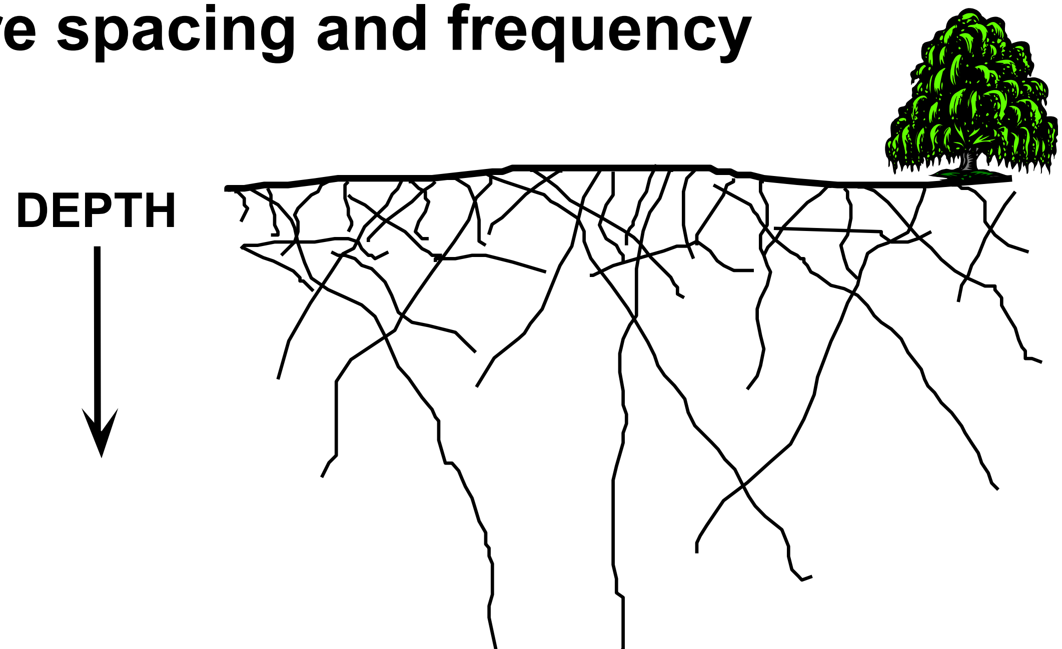


## 3. Spatial Trends

### (a) depositional trends

- e.g., deltas, glacial outwash, and some fluvial deposits

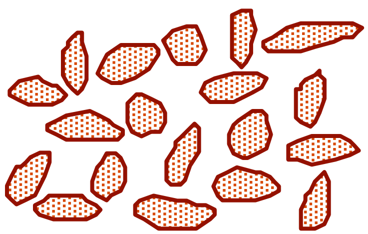
### (b) fracture spacing and frequency



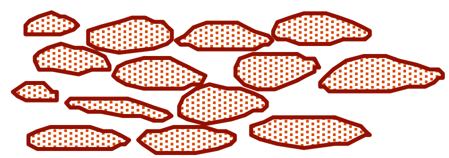
# Causes of Anisotropy

## 1. Grain Orientation

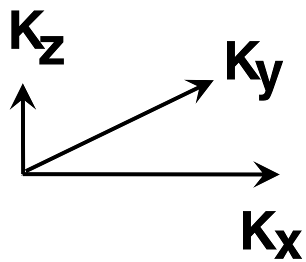
- particularly prevalent in clays



Kx = Ky = Kz
Random Orientation
*Isotropic*



Kx = Ky > Kz
Directional Orientation
*Anisotropic*




- Anisotropy caused by grain orientation is generally no greater than 10

•i.e.,

$$1 < \frac{K_X}{K_Z} < 10$$

## 2. Layering

- **anisotropy results from layering at both the local and regional scales**
- **anistropy ratios can exceed 100 (i.e., $K_x/K_z$ > 100)**

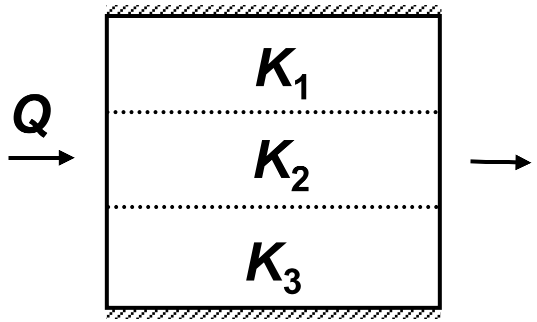
## 3. Fracture Orientation

- **fractures often have a preferred orientation, resulting in preferential flow in one direction**

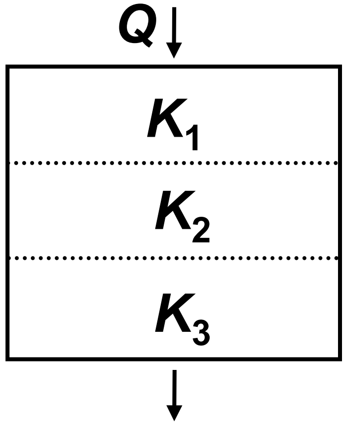
# Estimating K in Layered Media

**We would like to define a bulk average K value for layered media. Consider two particular cases.**

**1. Flow Parallel to Layering**



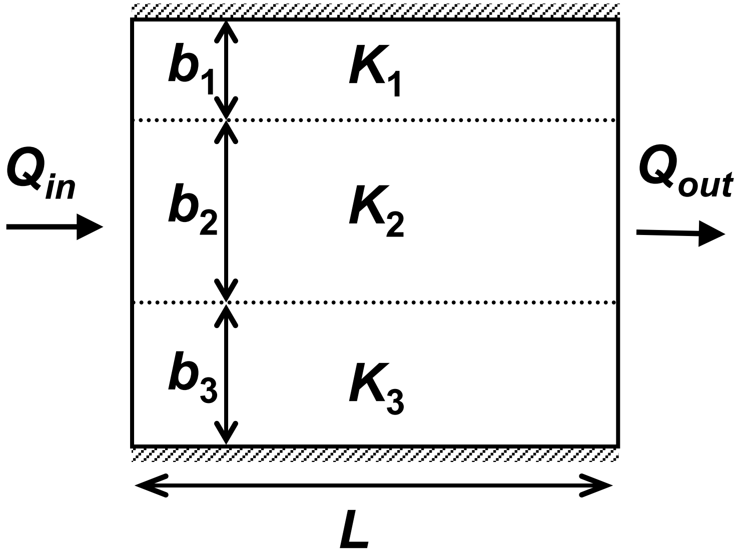

**2. Flow Perpendicular to Layering**



# Flow Parallel to Layering

Find bulk $K_x$

The total volumetric flow is the sum in all layers.




$$Q = Q_1 + Q_2 + Q_3$$

$$= -A_1 K_1 \frac{\Delta h}{L} - A_2 K_2 \frac{\Delta h}{L} - A_3 K_3 \frac{\Delta h}{L}$$

For a unit thickness, $A_n = b_n \times 1$

$$Q = -\frac{\Delta h}{L}\left(b_1 K_1 + b_2 K_2 + b_3 K_3\right)$$

If $b = b_1 + b_2 + b_3$, the bulk horizontal $K_x$ can be written as:

$$Q = -\frac{\Delta h}{L} b \cdot K_X$$

Therefore, $K_x$ becomes:

$$K_X = \frac{b_1 K_1 + b_2 K_2 + b_3 K_3}{b}$$

In more general form:

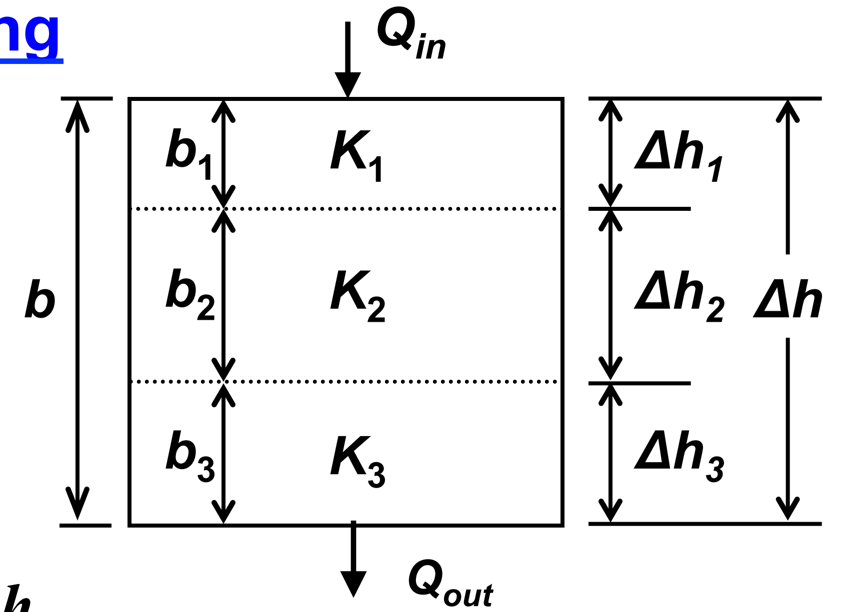
$$\boxed{K_X = \frac{\sum_{i=1}^{n} b_i K_i}{b}}$$

Weighted Arithmetic Mean

*Key points:*

- Effective K is controlled by the *most* conductive layer
- Layer thickness serves as a weighting factor

# Flow Perpendicular to Layering




**Find bulk $K_z$**

**In this case we have the same Q or q in all layers.**

$$q = q_1 = q_2 = q_3$$

$$q = -K_1 \frac{\Delta h_1}{b_1} = -K_2 \frac{\Delta h_2}{b_2} = -K_3 \frac{\Delta h_3}{b_3}$$

**The sum of head losses in each layer must equal the total head loss, *Δh*.**

$$\Delta h_1 = -\frac{q_1 b_1}{K_1}$$

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3$$

$$= -\frac{q_1 b_1}{K_1} - \frac{q_2 b_2}{K_2} - \frac{q_3 b_3}{K_3} = -q\left(\frac{b_1}{K_1} + \frac{b_2}{K_2} + \frac{b_3}{K_3}\right)$$

**We can transform Darcy's Law for bulk vertical $K_z$ to get:**

$$q = -K_Z \frac{\Delta h}{b} \quad \text{or} \quad \Delta h = -\frac{qb}{K_Z}$$

**Equating the previous two expressions, $K_z$ becomes:**

$$K_Z = \frac{b}{b_1/K_1 + b_2/K_2 + b_3/K_3}$$

**More generally:**

$$\boxed{K_Z = \frac{b}{\sum_{i=1}^{n} b_i/K_i}}$$

**Weighted Harmonic Mean**

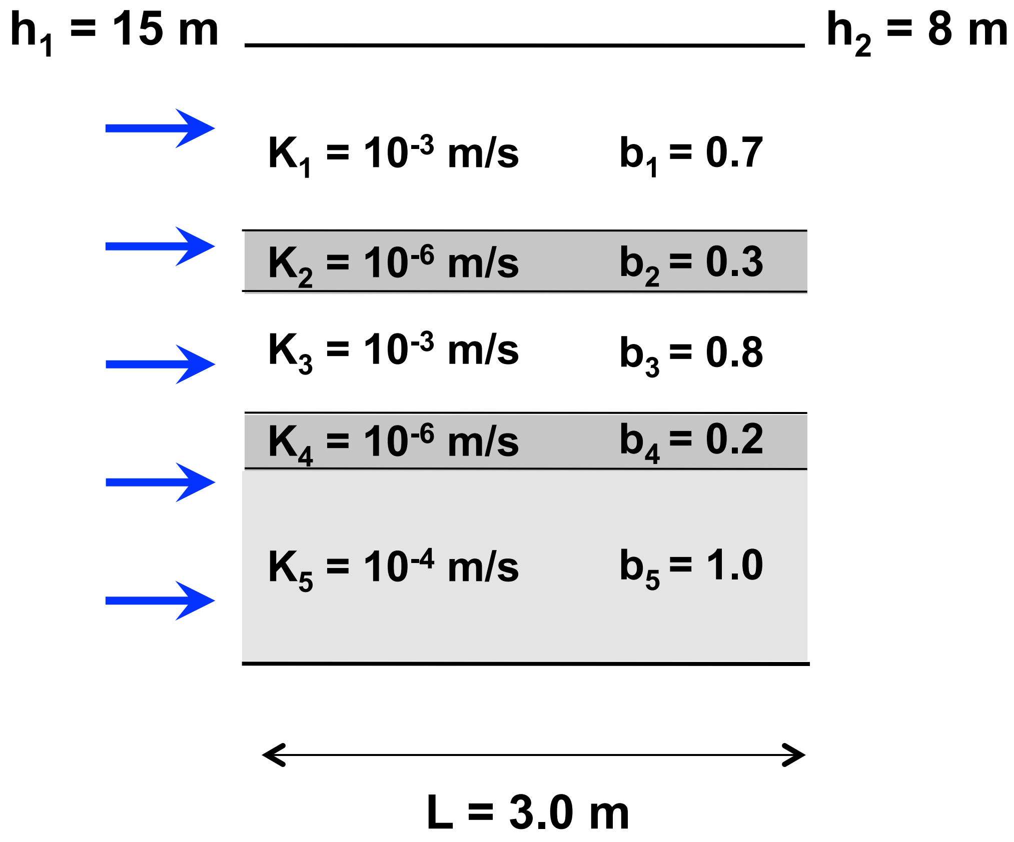
***Key points:***

- **Effective K is controlled by the *least* conductive layer**
- **Layer thickness serves as a weighting factor**

## Example

**Calculate K**

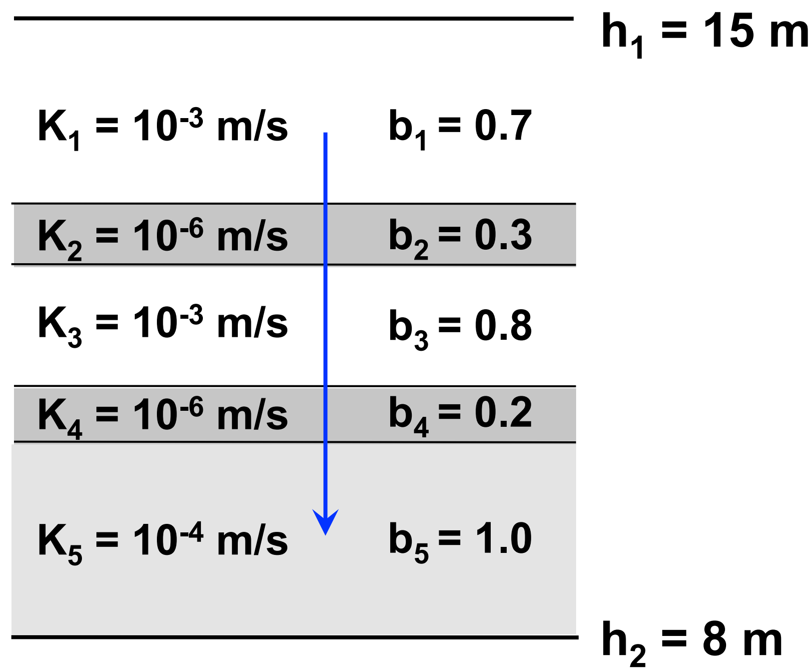
**Calculate q**



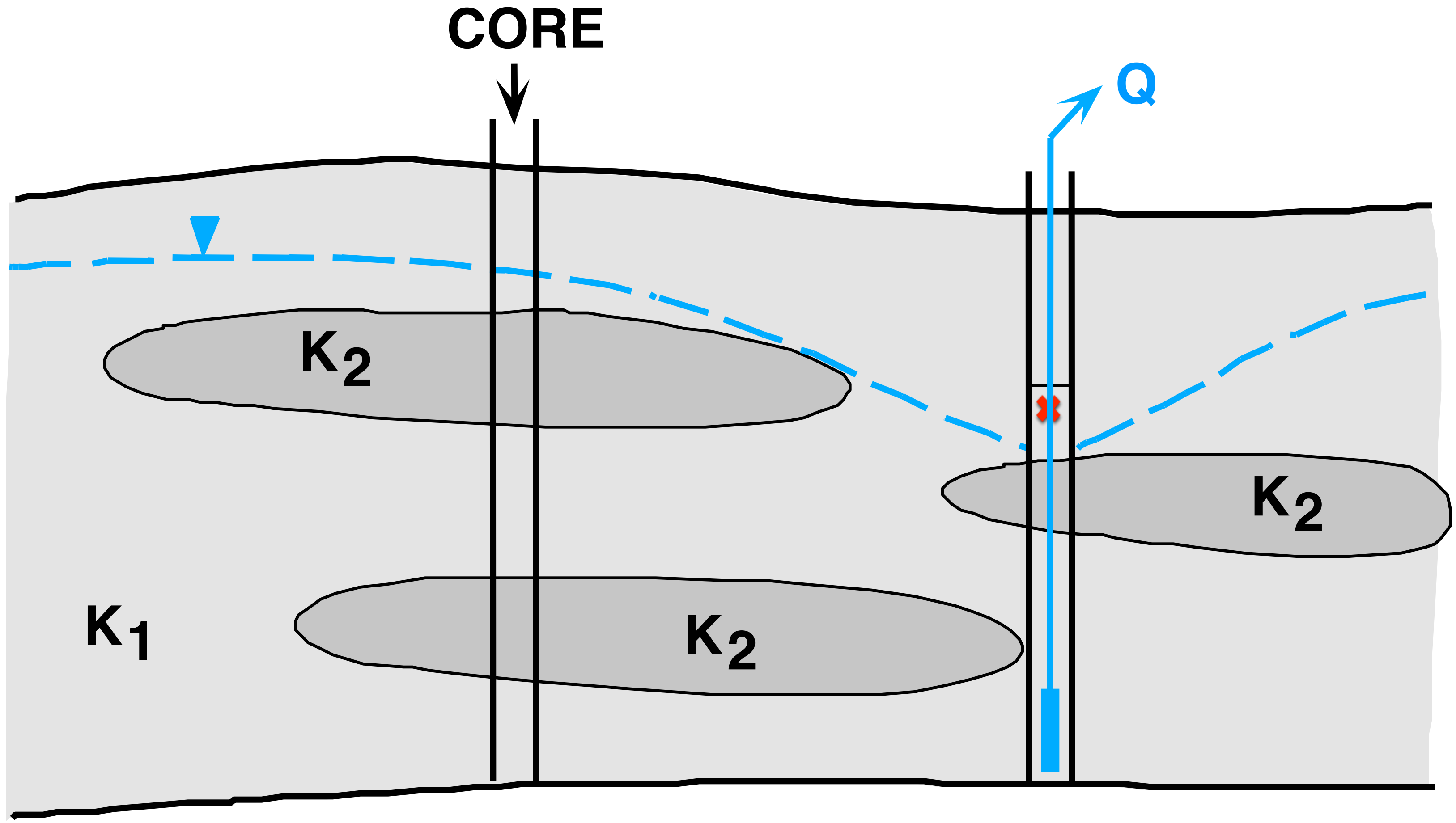
## Example

**Calculate K**

**Calculate q**

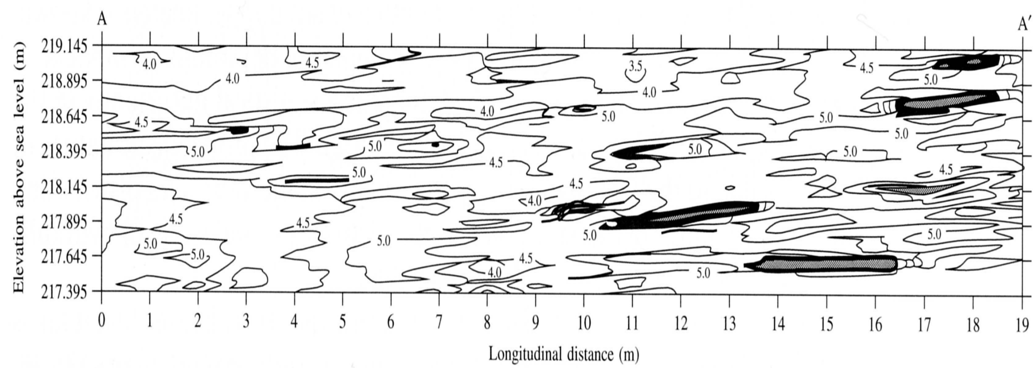


# How do we define this layering or anisotropy?



# We even have layering in “homogeneous” materials. Is the Borden sand really homogeneous? How would this influence flow and transport?

**Contours are given as -log K**
**(e.g., K=1×10$^{-4}$ m/s is 4.0 contour)**




**Hydraulic conductivity distribution of the Borden aquifer in cross-section.**
**(Fetter, Contaminant Hydrogeology, 1999)**