Calendar

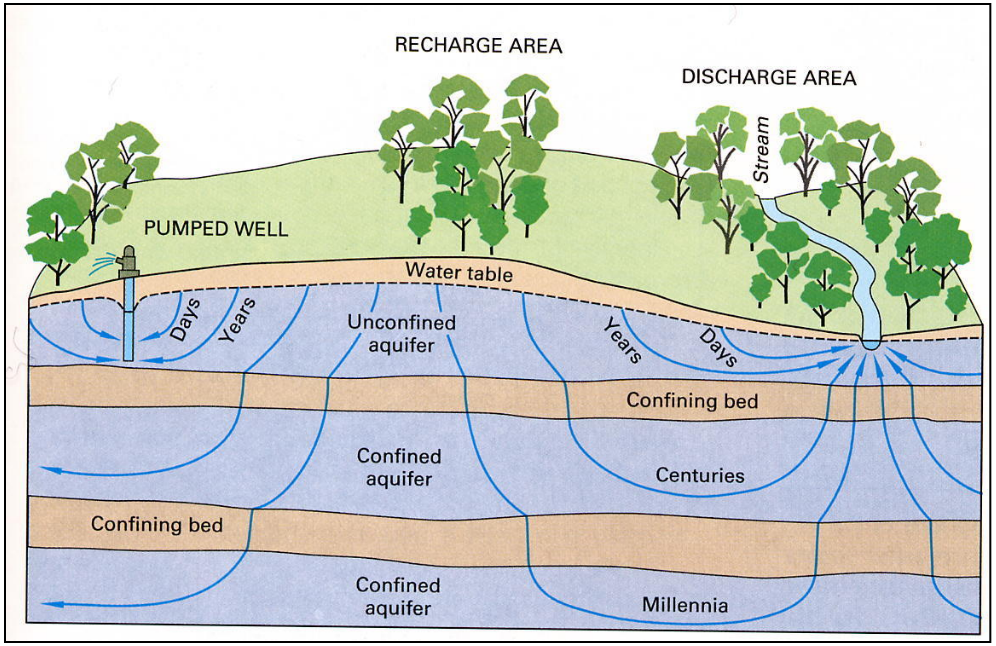
Friday 21st — Farvolden Lecture -- Roger Woeller
2:00 PM Hagey Hall Humanities Theatre
2 bonus points to assignment #2
(half of group must be there)

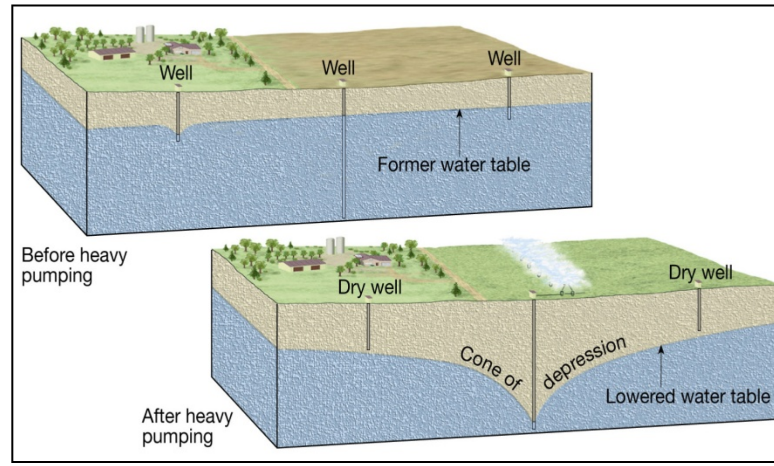
Monday 24th — Assignment #2 due date
Exam Review Guide
Continue with quantifying groundwater flow lecture

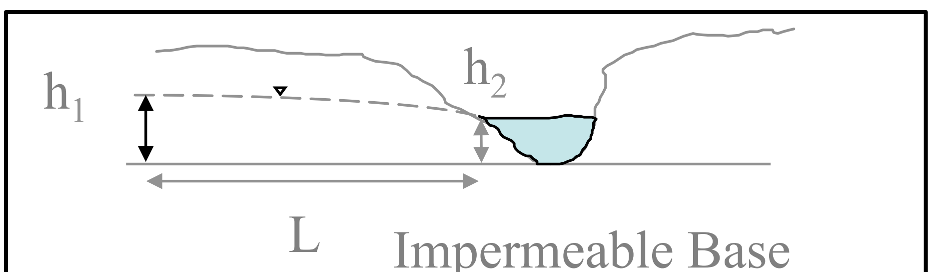
Wed 26th — Exam review day
Assignment #2 Out
Bring questions!!!!!!

Friday 28th — Midterm #1 in class

Quantifying Flow in Regional Groundwater Systems







Recommended Reading: Schwartz and Zhang Ch. 5

Groundwater Flow Equations

A sound mathematical relationship to describe groundwater flow can be developed from the basic hydrogeologic principals we have examined.

- hydraulic potential and gradient
- physical characteristics of the porous media and fluid
    - *porosity*
    - *hydraulic conductivity*
    - *groundwater storage*

Two fundamental relations:

- *Darcy's Law*
- ***mass conservation***

By applying the Law of Mass Conservation, we can derive the groundwater flow equation for an elementary volume.

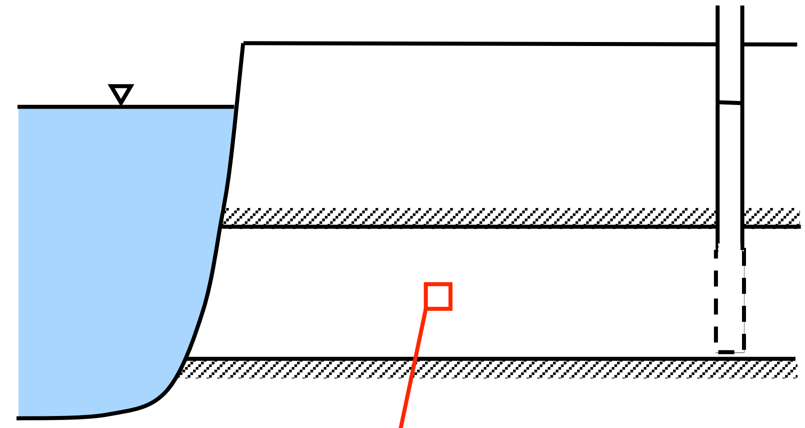
Mass flux in - **Mass flux out** = **Rate of change of mass stored**

$$\underbrace{M_{in} - M_{out}}_{\text{LHS}} = \frac{\Delta M}{\underbrace{\Delta t}_{\text{RHS}}}$$

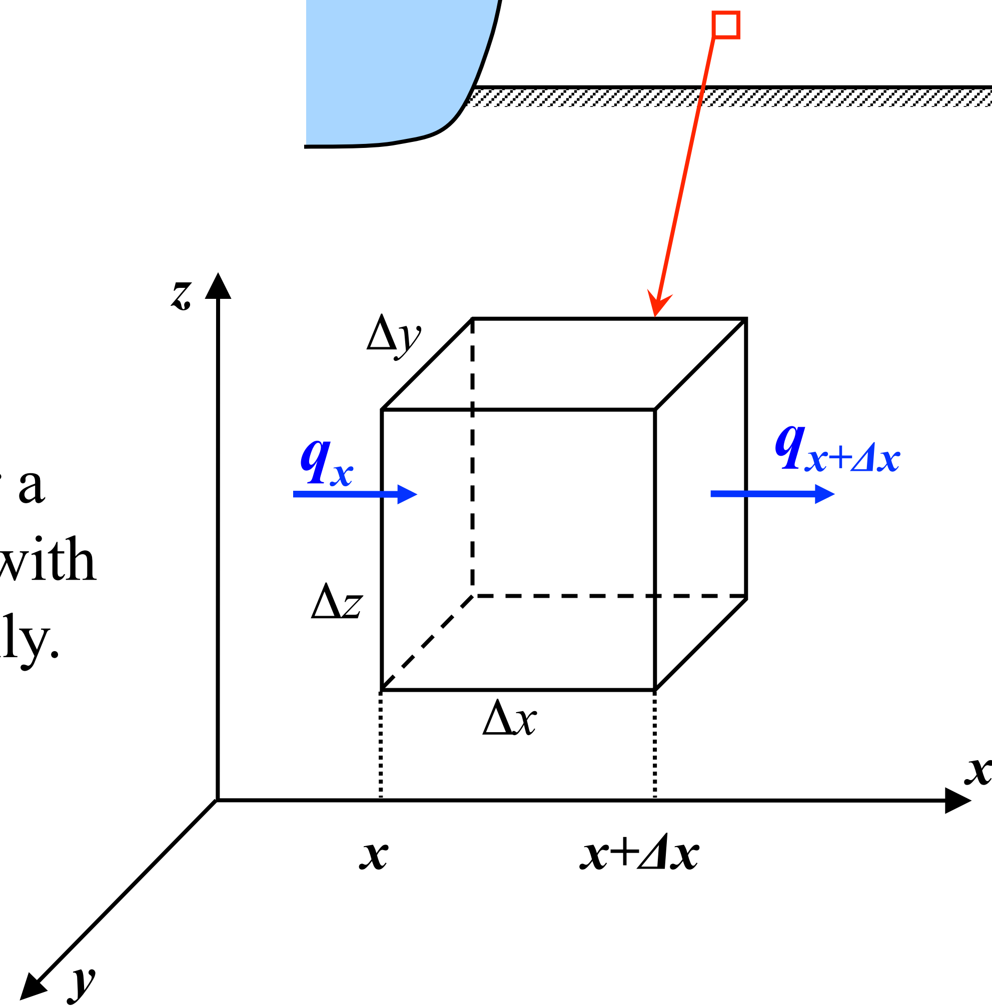
Darcy's Law describes mass fluxes

Groundwater storage term

Consider a confined aquifer connected to a lake. We will write a mass balance equation for a unit volume.



Let's write the mass conservation equation for a one-dimensional system with flow in the x-direction only.



LHS

Mass in at x: $$M_{in} = \text{Density} \times \text{Area} \times \text{Specific Discharge}$$

$$= \rho_w \cdot \Delta y \Delta z \cdot q_x$$

Mass out at x+Δx: $$M_{out} = \text{Density} \times \text{Area} \times \text{Specific Discharge}$$

$$= \rho_w \cdot \Delta y \Delta z \cdot q_{x+\Delta x}$$

$$= \rho_w \cdot \Delta y \Delta z \cdot \left[q_x + \Delta x \frac{\partial q_x}{\partial x} \right]$$

Writing M_{in} - M_{out} and substituting *Darcy's Law* for q_x gives:

$$M_{in} - M_{out} = \rho_w \cdot \Delta y \Delta z \left[q_x - \left(q_x + \Delta x \frac{\partial q_x}{\partial x} \right) \right]$$

LHS

Mass in at x: $$M_{in} = \text{Density} \times \text{Area} \times \text{Specific Discharge}$$
$$= \rho_w \cdot \Delta y \Delta z \cdot q_x$$

Mass out at x+Δx: $$M_{out} = \text{Density} \times \text{Area} \times \text{Specific Discharge}$$
$$= \rho_w \cdot \Delta y \Delta z \cdot q_{x+\Delta x}$$
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Writing M_{in} - M_{out} and substituting Darcy's Law for q_x gives:

$$M_{in} - M_{out} = \rho_w \cdot \Delta y \Delta z \left[\cancel{q_x} - \left(\cancel{q_x} + \Delta x \frac{\partial q_x}{\partial x} \right) \right]$$

LHS

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$$M_{in} = \text{Density} \times \text{Area} \times \text{Specific Discharge}$$
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Writing M_{in} - M_{out} and substituting [Darcy's Law] for q_x gives:

$$M_{in} - M_{out} = \rho_w \cdot \Delta y \Delta z \left[\cancel{q_x} - \left(\cancel{q_x} + \Delta x \frac{\partial q_x}{\partial x} \right) \right]$$
$$= \rho_w \Delta y \Delta z \left(-\Delta x \frac{\partial q_x}{\partial x} \right) = \rho_w \Delta y \Delta z \cdot \Delta x \frac{\partial}{\partial x} \left(K \frac{\partial h}{\partial x} \right)$$

RHS

Now write the storage term. For a confined aquifer we can write the change in mass for our control volume as:

$$\begin{aligned}\Delta M &= \rho_w\left(S_S\, V\Delta h\right) \\ &= \rho_w\left(\Delta x\Delta y\Delta z\right)S_S\,\Delta h\end{aligned}$$

Remembering that specific storage, S_s, is the change in volume of water stored per unit volume per unit change in hydraulic head.

Rewriting for the rate of change in mass per unit time gives:

$$\frac{\Delta M}{\Delta t} = \rho_w \Delta x \Delta y \Delta z\, S_s \frac{\Delta h}{\Delta t}$$

LHS=RHS

Combining the two sets of equations yields:

$$M_{in} - M_{out} = \frac{\Delta M}{\Delta t}$$

$$\rho_w \Delta y \Delta z \cdot \Delta x \frac{\partial}{\partial x}\left(K \frac{\partial h}{\partial x} \right) = \rho_w \Delta y \Delta z \Delta x \cdot S_s \frac{\Delta h}{\Delta t}$$

Eliminating common terms gives:

$$\frac{\partial}{\partial x}\left(K \frac{\partial h}{\partial x} \right) = S_s \frac{\Delta h}{\Delta t}$$

Writing in derivative form (i.e., making Δt very small).

$$\frac{\partial}{\partial x}\left(K \frac{\partial h}{\partial x} \right) = S_s \frac{\partial h}{\partial t}$$

General 1-D form of GW Flow Equation

3-D Groundwater Flow Equation

The same procedure can be used to easily derive the general groundwater flow equation in three dimensions.

$$\boxed{\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_S \frac{\partial h}{\partial t}}$$

The above equation can be applied to transient, 3-D flow in a heterogeneous, anisotropic porous medium.

This equation can of course be simplified depending on the application.

For a medium that is **isotropic**, $K_x = K_y = K_z = K$, and **homogeneous**, $K(x, y, z) = \text{constant}$.

$$K\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right] = S_S \frac{\partial h}{\partial t}$$

For **steady-state** flow conditions, there is no change in hydraulic head with time. Hence, the right side of the equation becomes zero.

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Homogeneous & Isotropic
(*Laplace equation*)

Methods of Solution

1. Analytical
   - *Give exact solutions to the equations*
   - *Restricted to simple systems with regular geometry*
     - *i.e., homogeneous, isotropic, 1D or 2D*

2. Numerical
   - *Computers give approximate solutions to more complex problems by breaking the domain into small regions*
   - *Many different codes (e.g., Modflow, Feflow, Seep/W) ranging from simple to very complex*

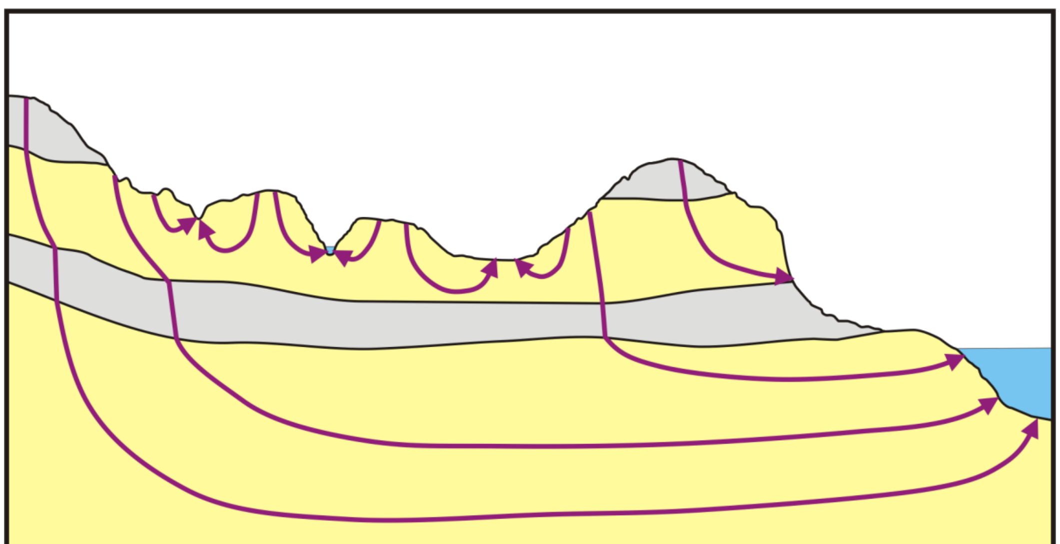
3. Graphical
   - *Flow nets for steady state, 2D problems*

4. Analog
   - *Electrical analogs for water flow (old technology)*

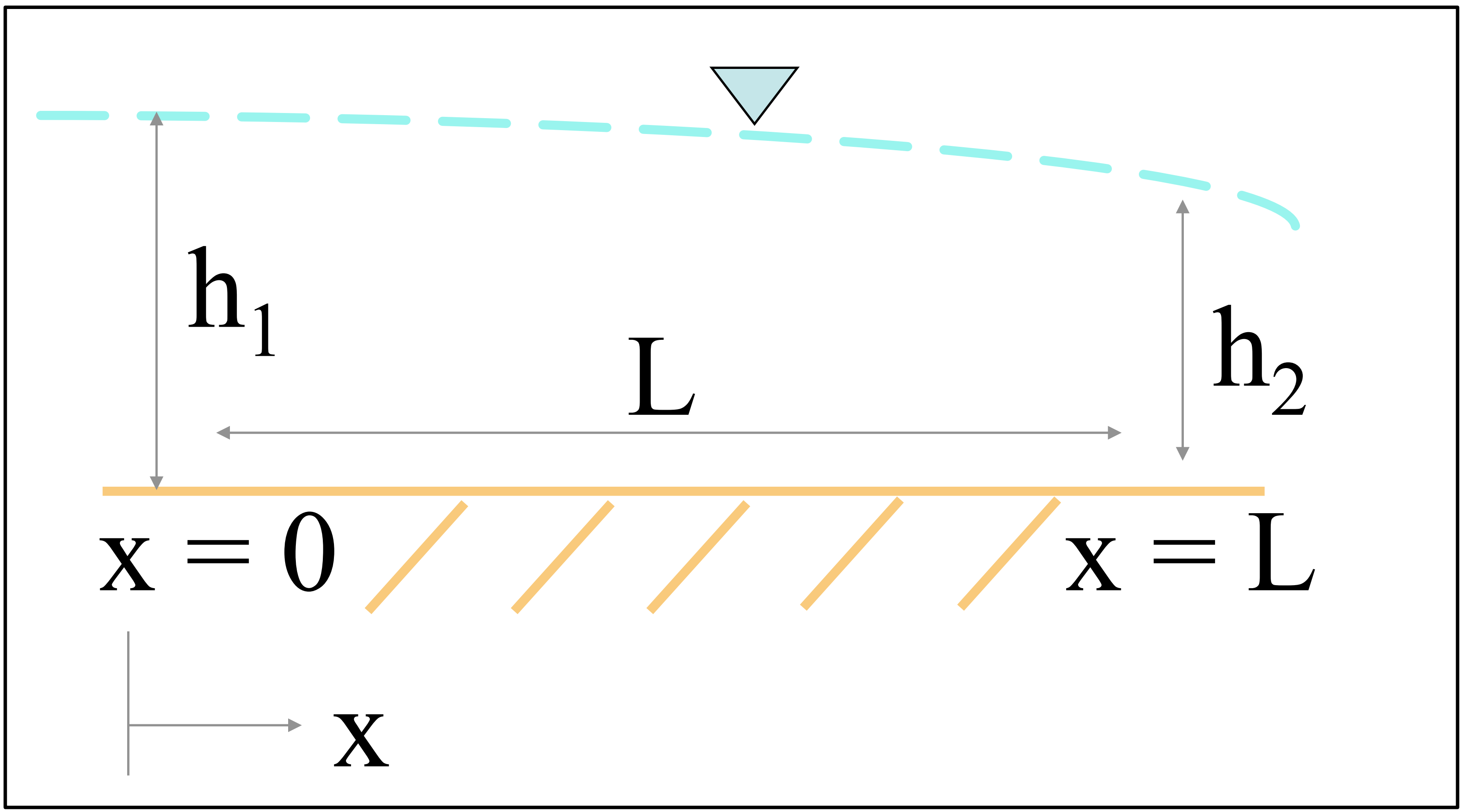
Regional Groundwater Flow

We will start by examining regional flow patterns under steady state conditions.
We are trying to solve:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$



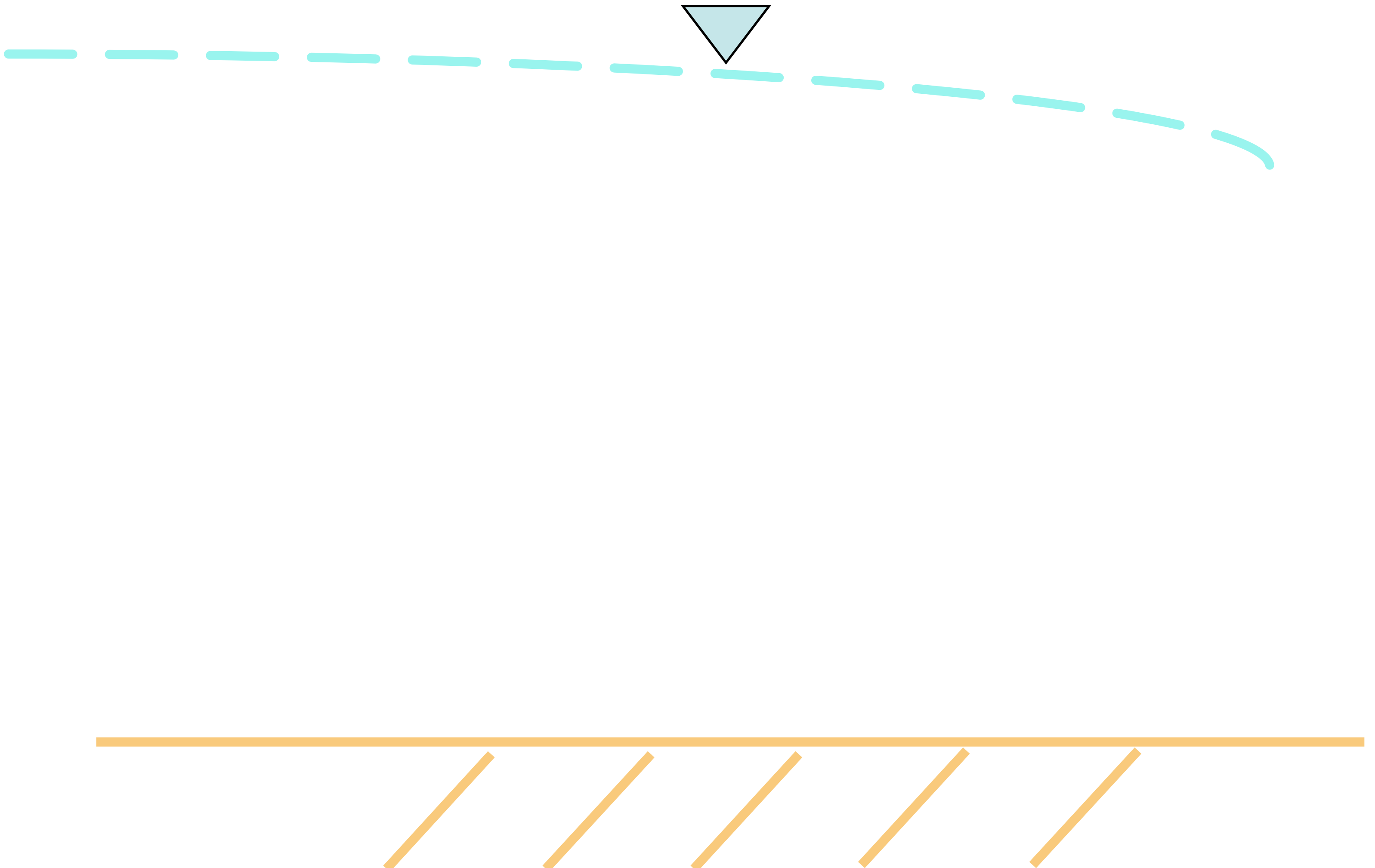
Steady Flow in an Unconfined Aquifer: Dupuit-Forchheimer Approach

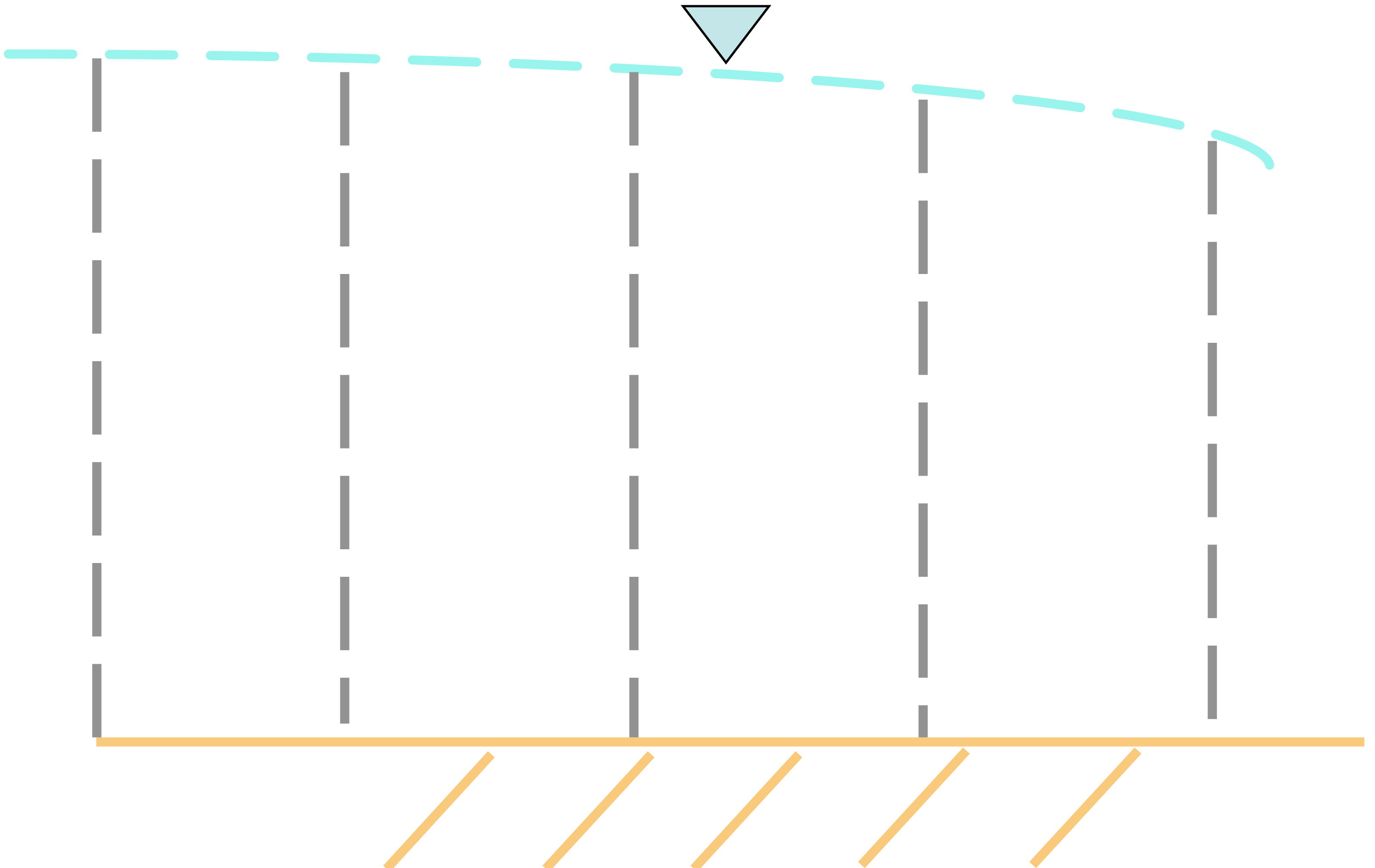


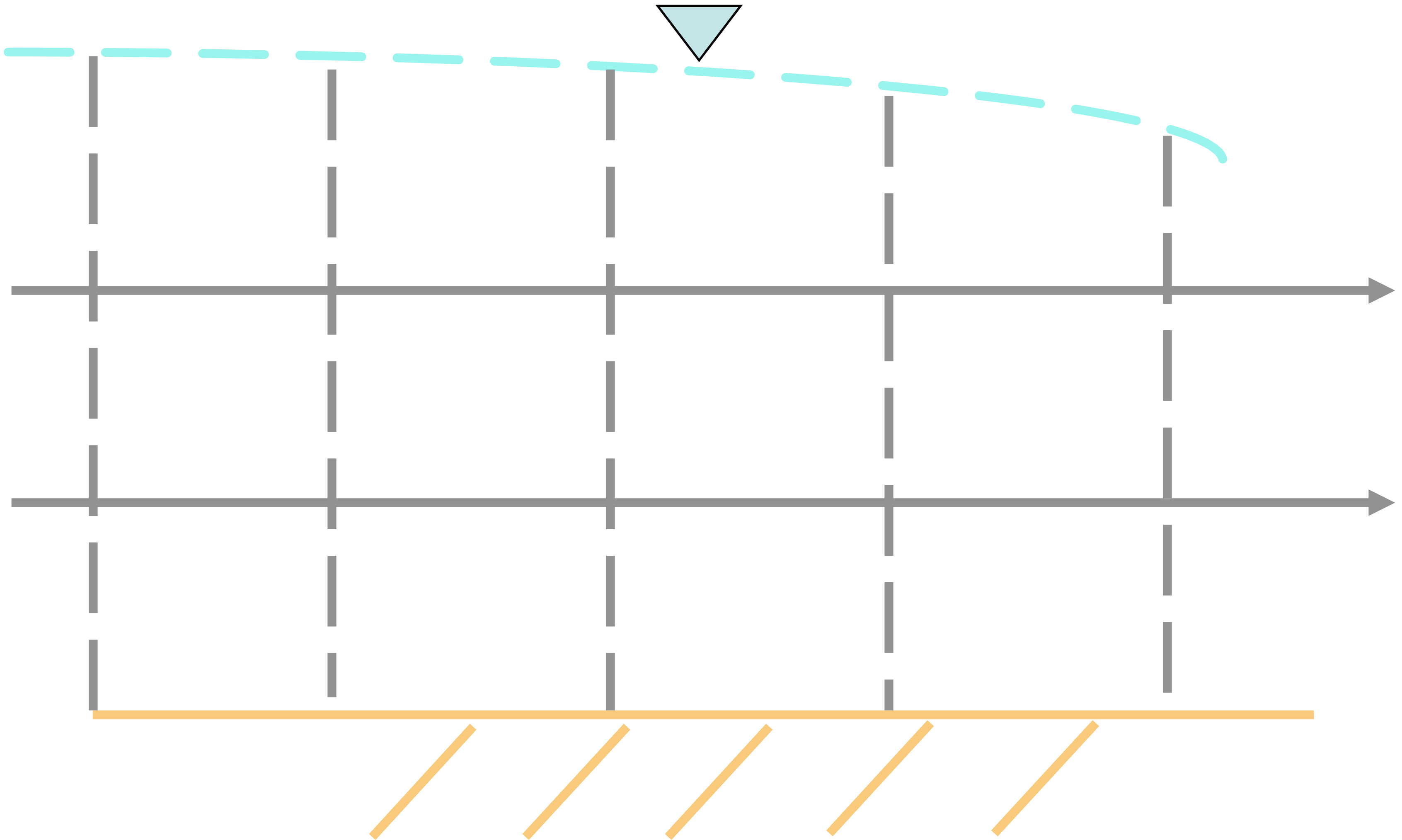
Dupuit Assumptions

1. Hydraulic gradient throughout a vertical panel in the aquifer = slope of water table.

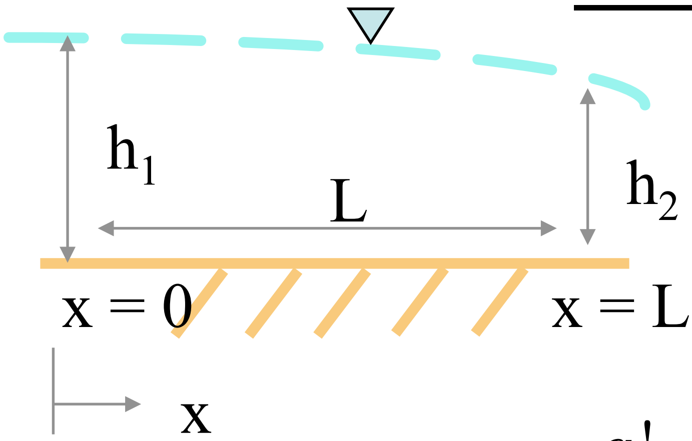
2. For small hydraulic gradients, flow lines are essentially horizontal and equipotential lines are vertical. (i.e. The vertical flow component is very small)







Dupuit Equation



$$q' = -Kh\frac{dh}{dx}$$

Specific Discharge over the depth of the aquifer (h)

q' = m²/s

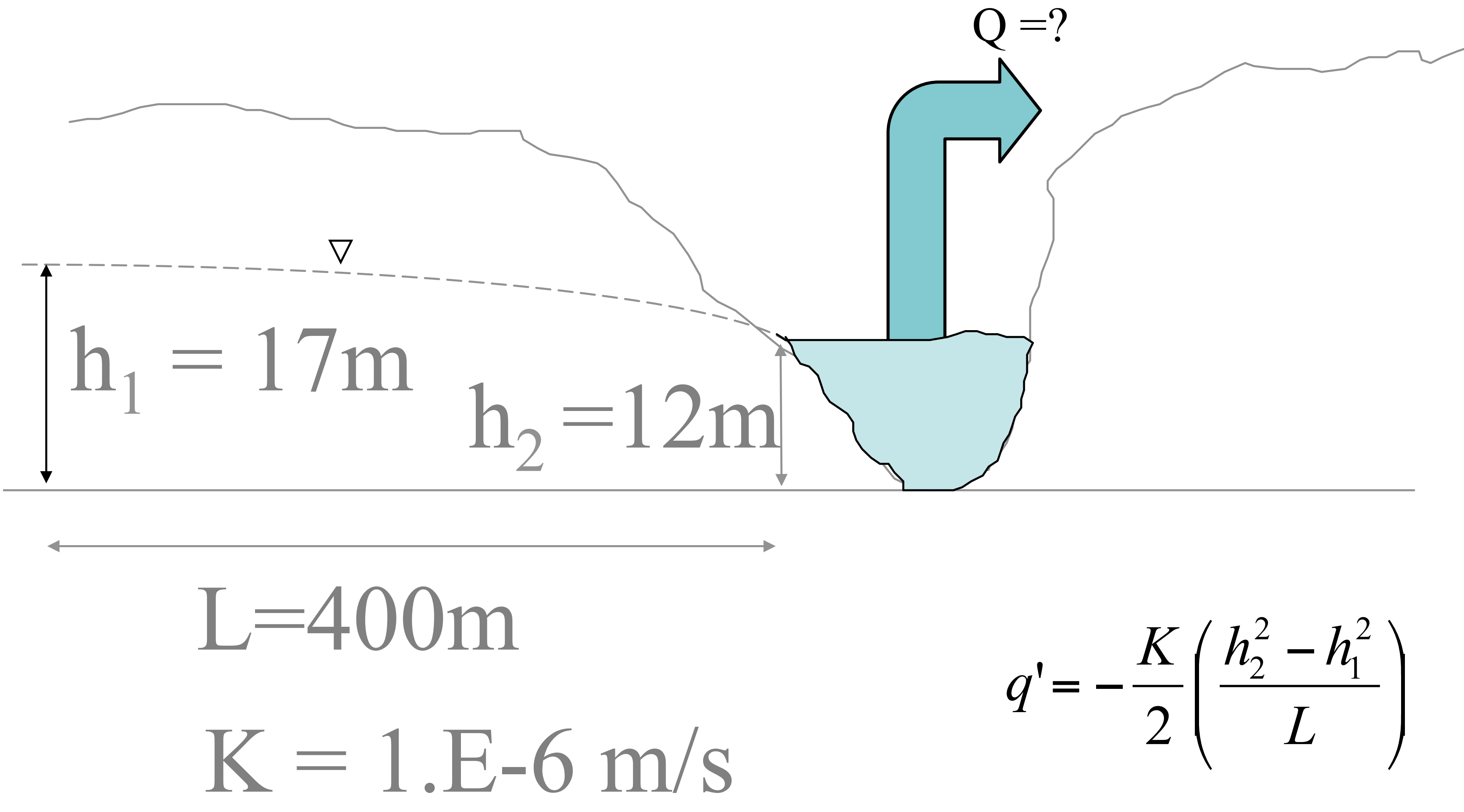
$$\int_0^L q'\,dx = -K\int_{h_1}^{h_2} h\,\partial h$$

Combine variables and use Boundary Conditions:
$x=0$, $h=h_1$; $x=L$, $h=h_2$

$$q'L = -K\left(\frac{h_2^2}{2} - \frac{h_1^2}{2}\right) \longrightarrow \boxed{q' = -\frac{K}{2}\left(\frac{h_2^2 - h_1^2}{L}\right)}$$

The canal is 300 m wide.

How much water enters the canal (m^3/day)?



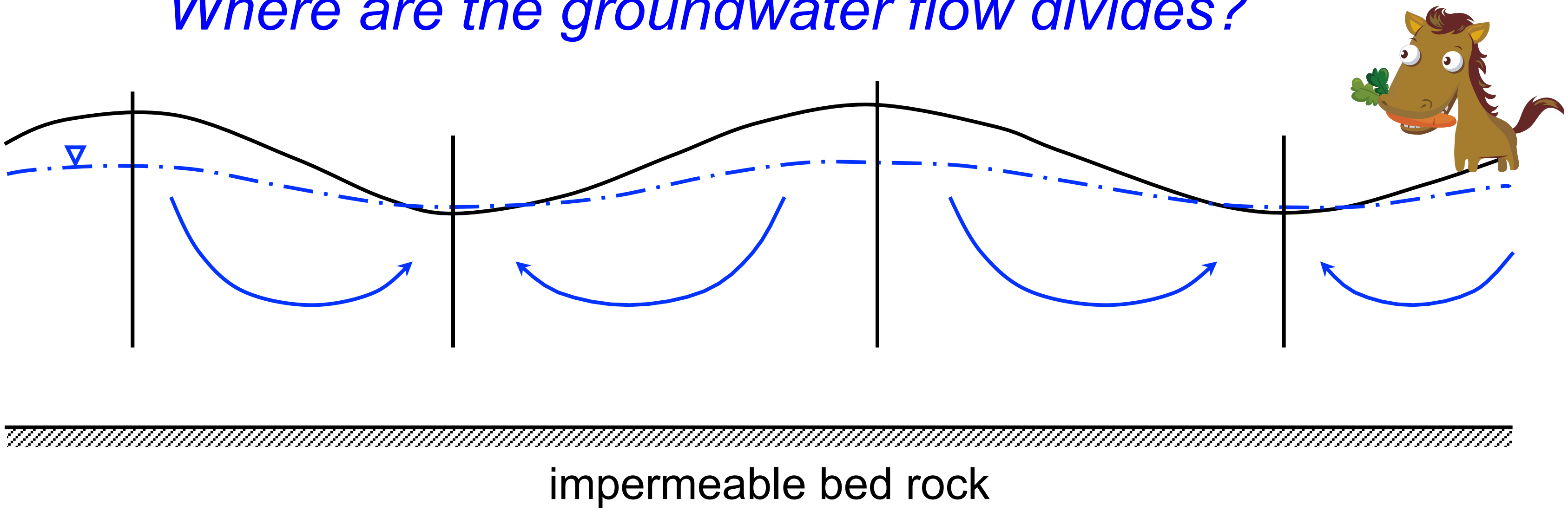
$$q' = -\frac{K}{2}\left(\frac{h_2^2 - h_1^2}{L}\right)$$

Groundwater Divides

Recommended Reading: Schwartz and Zhang Ch. 8

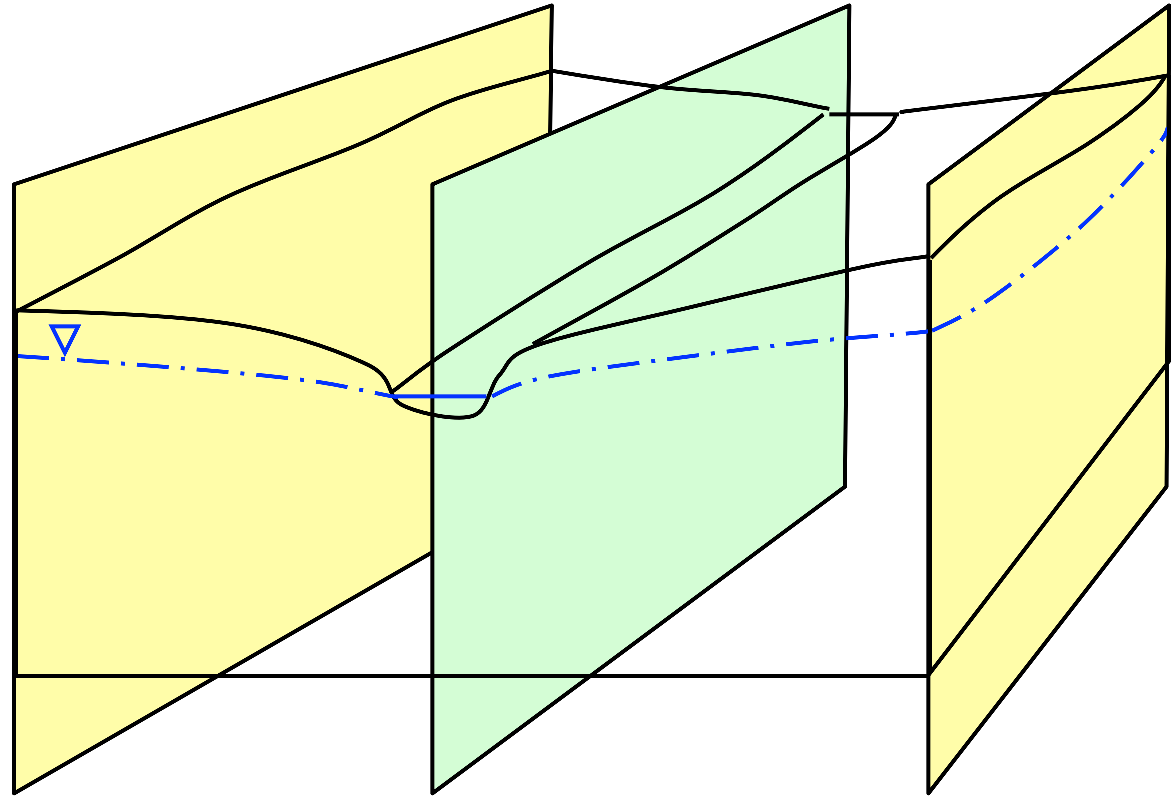
Just like surface watersheds, groundwater flow systems have divides. In humid regions, the water table is a subdued replica of topography, so flow divides coincide with topographic divides.

Where are the groundwater flow divides?



Streams and drainage divides are common examples of symmetry. Groundwater does not flow across the symmetry planes if climate and geology are homogeneous.

What boundary conditions can we use for these planes?



Recharge and Discharge Areas

Recharge areas

- flow directed downward at the water table
- occurs under topographic highs (hills)
- generally larger than discharge areas

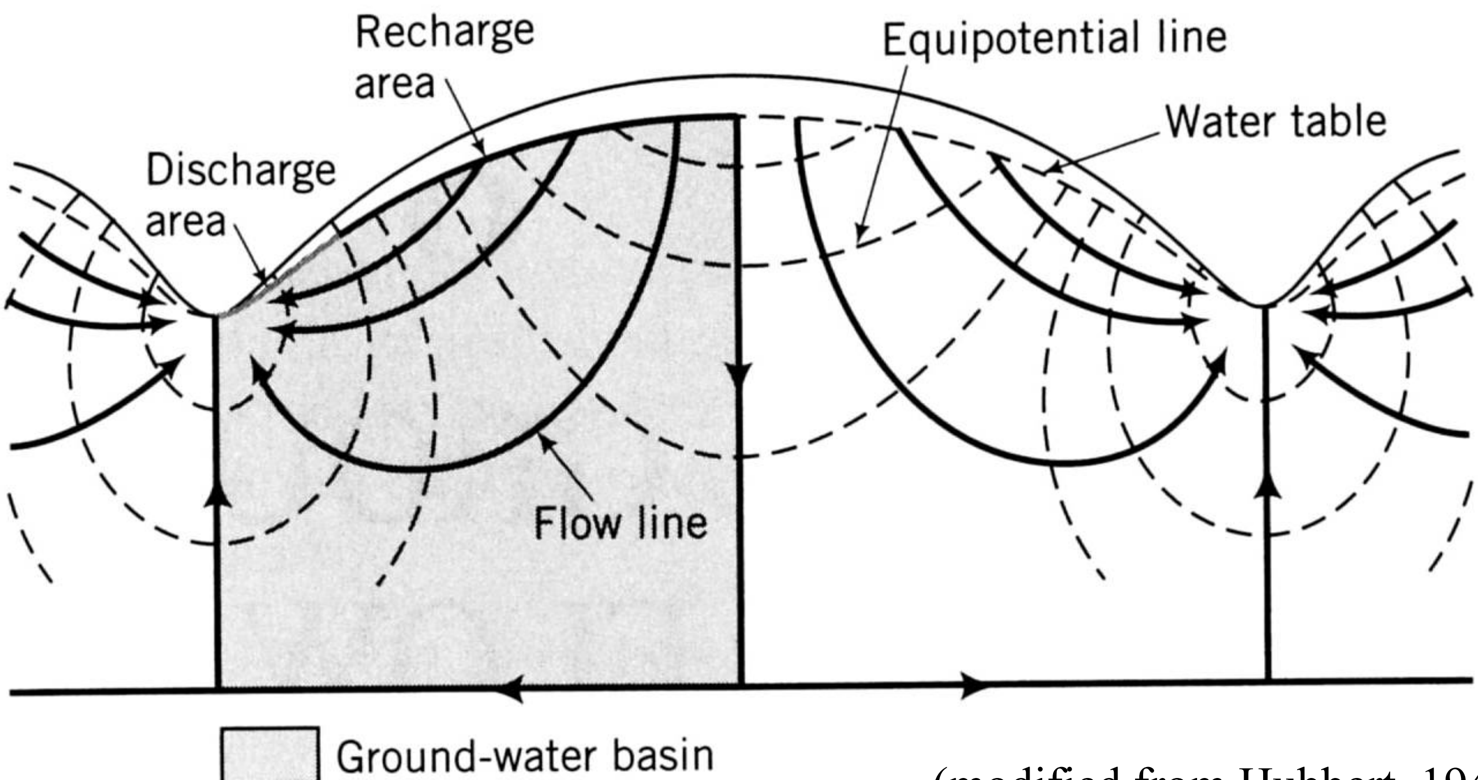
Discharge areas

- flow directed upward at the water table
- generally higher flux than recharge areas
- occurs under topographic lows (valleys)
    - e.g., springs, streams, lakes, saline sloughs

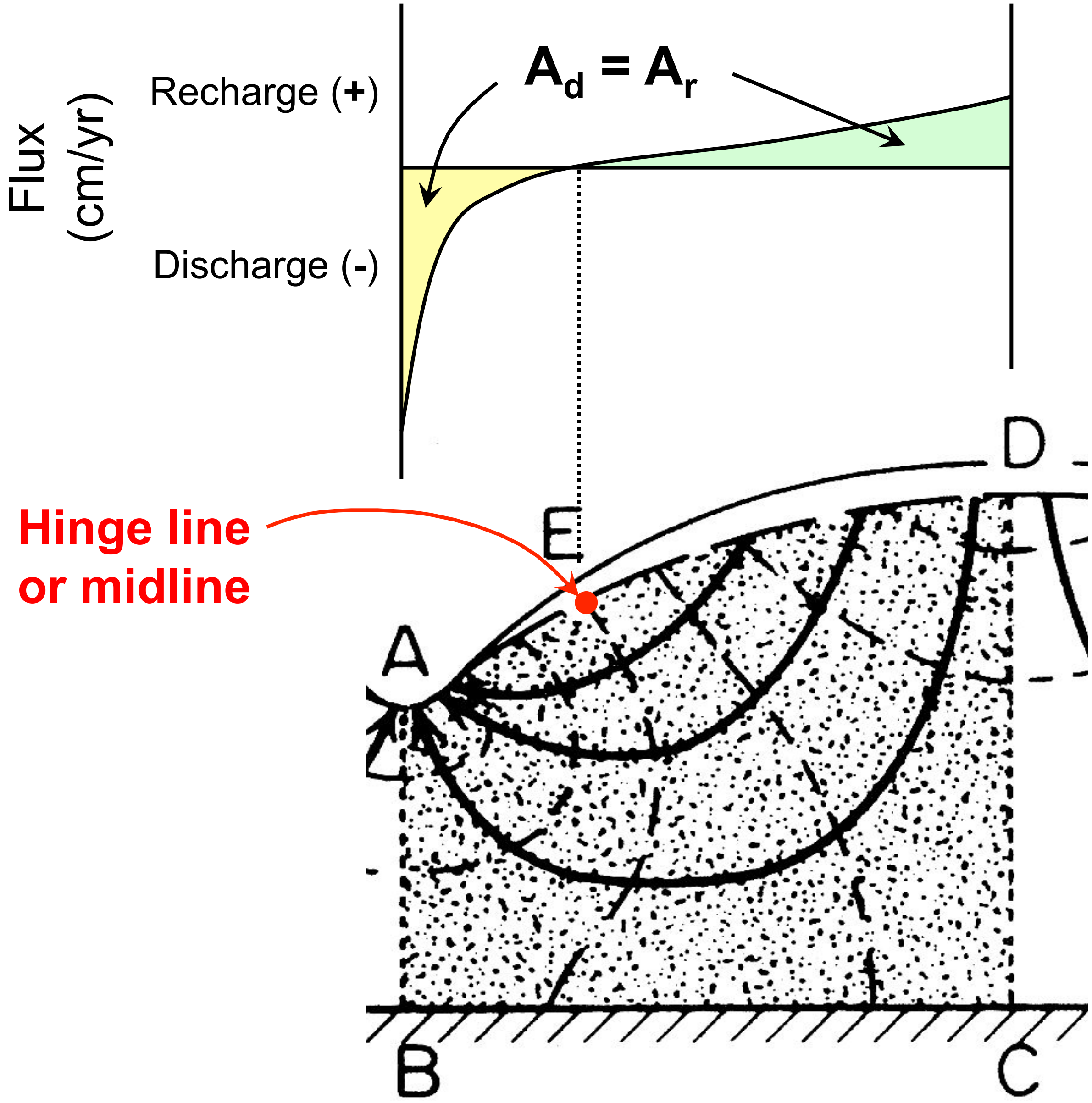
Hinge line - separates recharge areas from discharge areas

M. K. Hubbert was a pioneer of hydrogeology. He used symmetry and flow nets to publish the first descriptions of regional groundwater flow.

- this is called a Hubbert section



(modified from Hubbert, 1940)



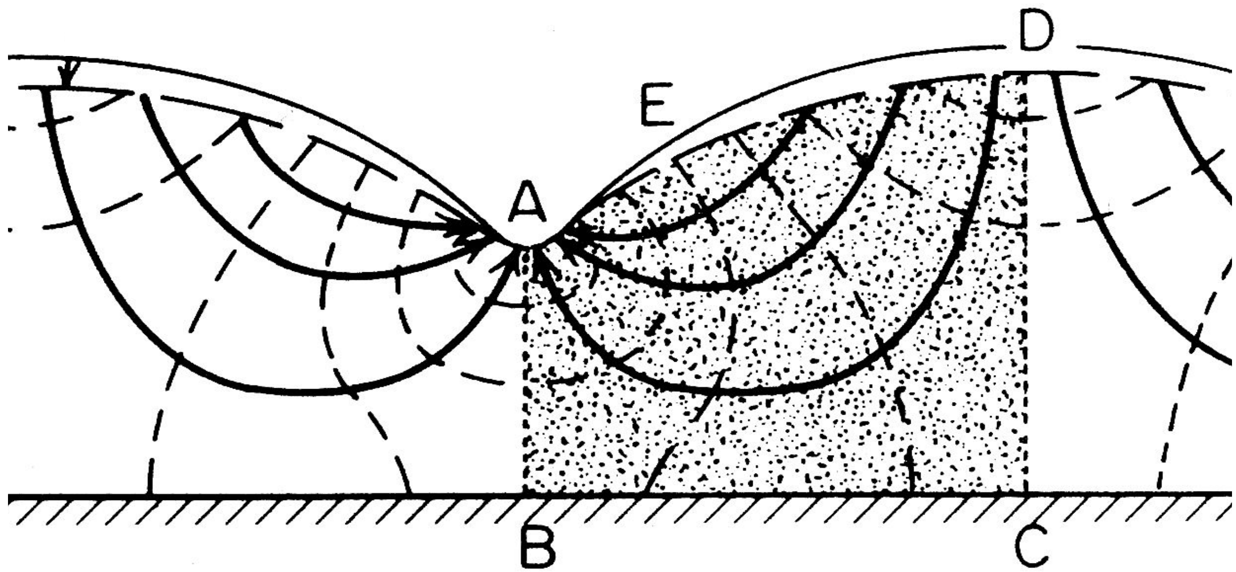
Flux (cm/yr)
Recharge (+)
Discharge (-)
$A_d = A_r$
Hinge line or midline
D
E
A
B
C

$\overline{AB}$ $\overline{DE}$

$\overline{BC}$ $\overline{AD}$

$\overline{CD}$ $\overline{AE}$

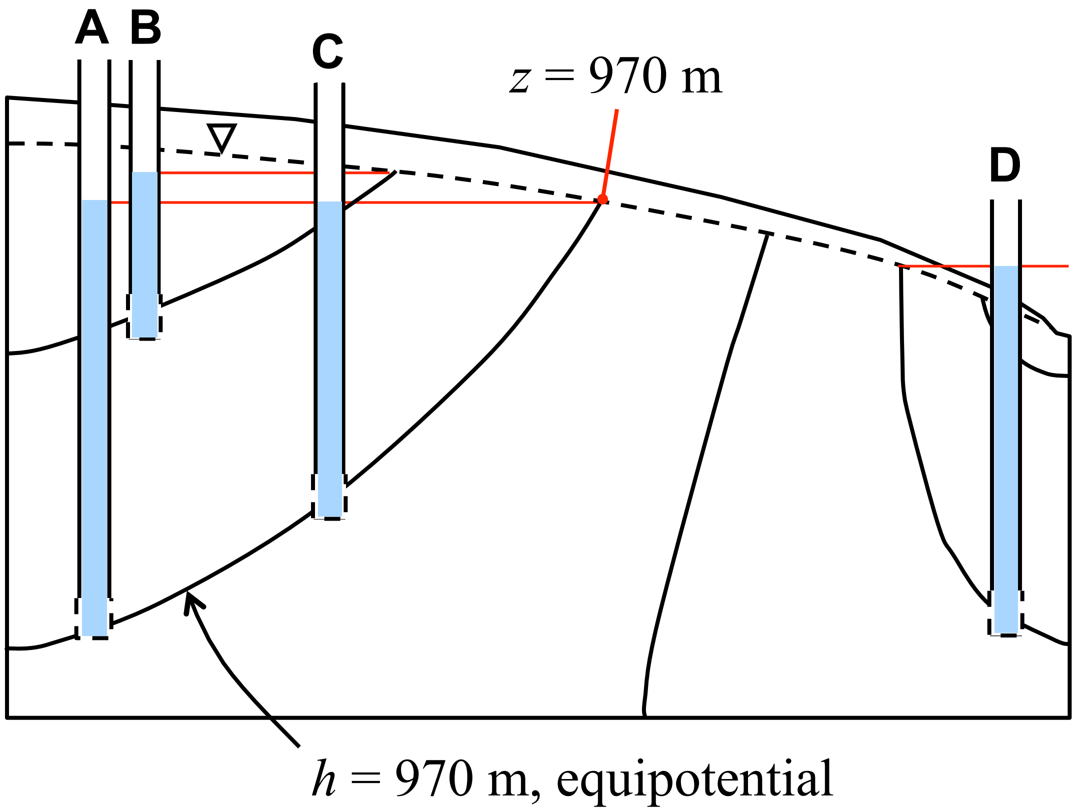
1. Recharge area
2. Discharge area
3. Water table boundary
4. No flow boundary
5. Groundwater divide



Groundwater flow in a 2D, homogeneous, isotropic cross-section.
(Freeze and Cherry, Groundwater, 1979)

In the field, equipotentials are inferred based on data from a few piezometers. Hydrogeologists need to develop a good sense of flow systems because field data are usually very limited.

Note that the piezometer at D is under "flowing" conditions. This is commonly observed near rivers and lakes.



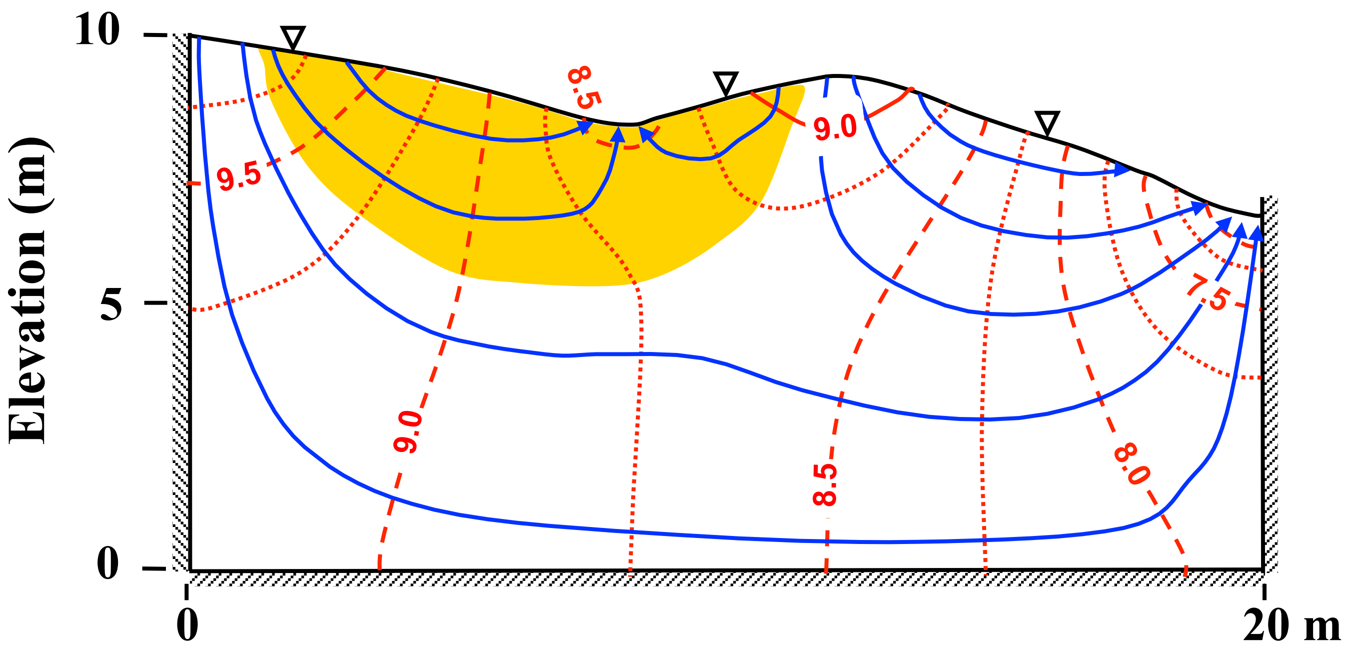
Regional Flow Influences

There are four major factors that influence regional groundwater flow patterns:

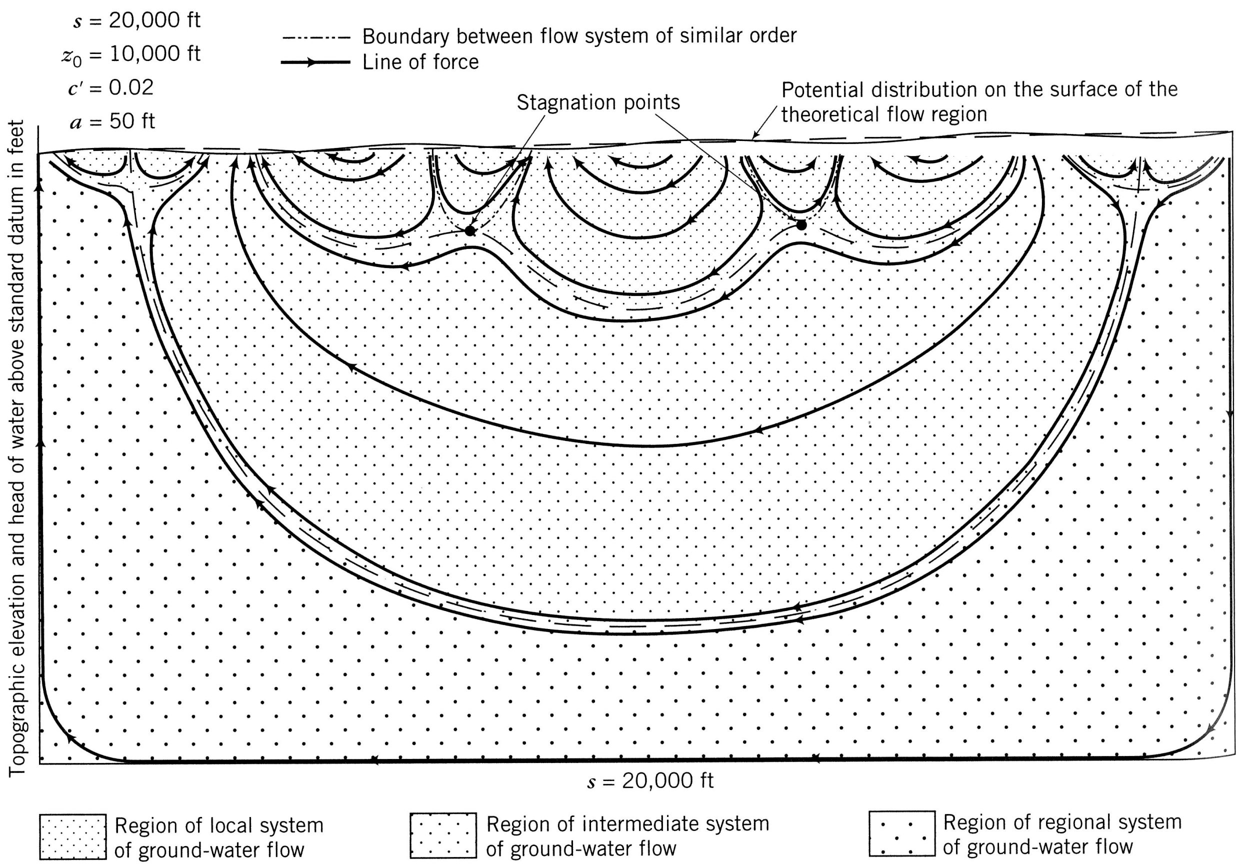
- Topography
- Basin Geometry
- Heterogeneity
- Anisotropy

Influence of Topography

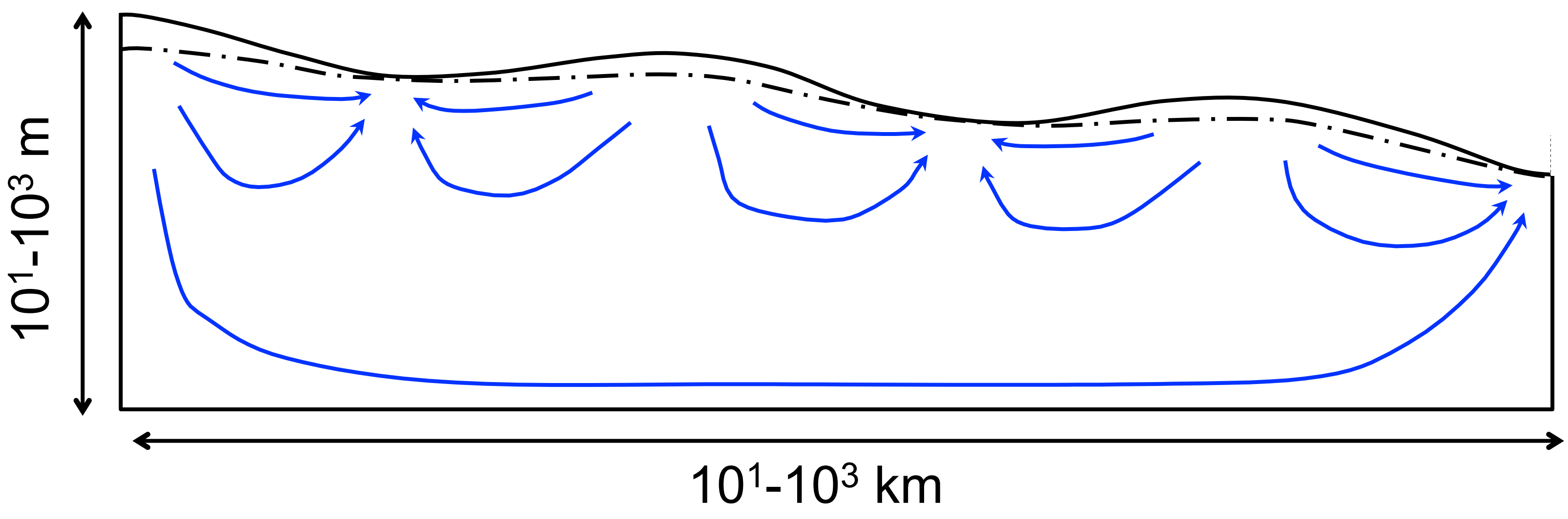
An undulating water table results in complex flow patterns having multiple recharge and discharge areas. A short-range flow system (shaded region) is superimposed on top of a longer-range flow system.

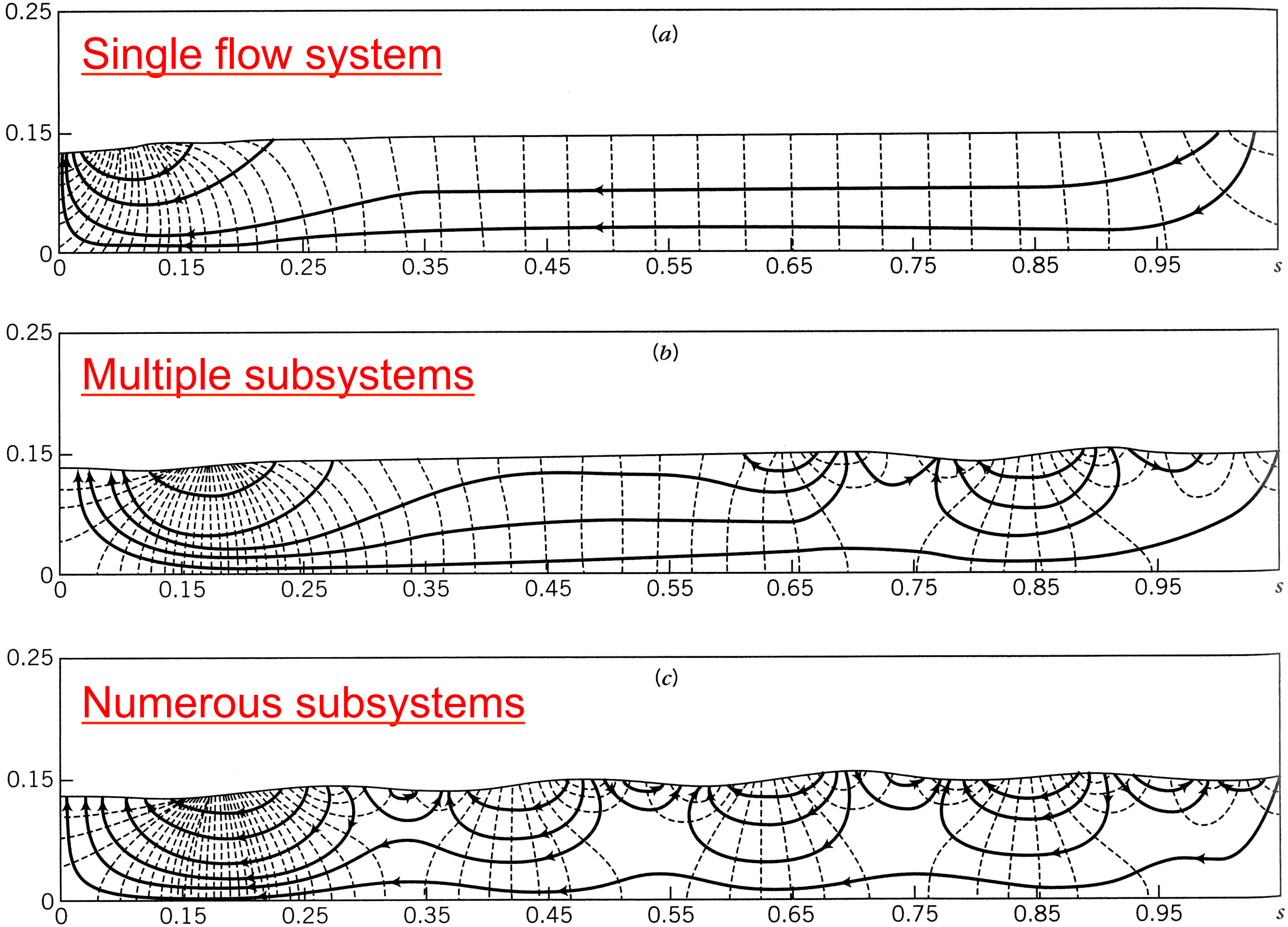


This idea was first proposed by a famous Albertan hydrogeologist, J. Tóth, in 1963. Based on his observations of groundwater flow near Red Deer.



An undulating water table occurs below undulating terrain. In larger scale systems this causes <u>local flow systems</u> to be superimposed on top of <u>intermediate and regional flow systems</u>.

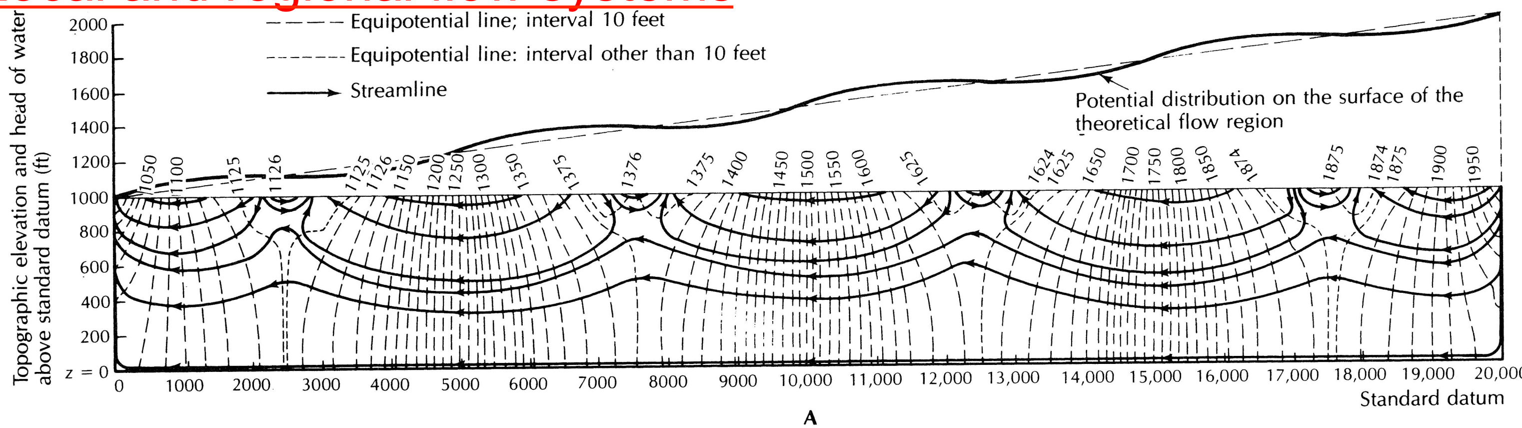




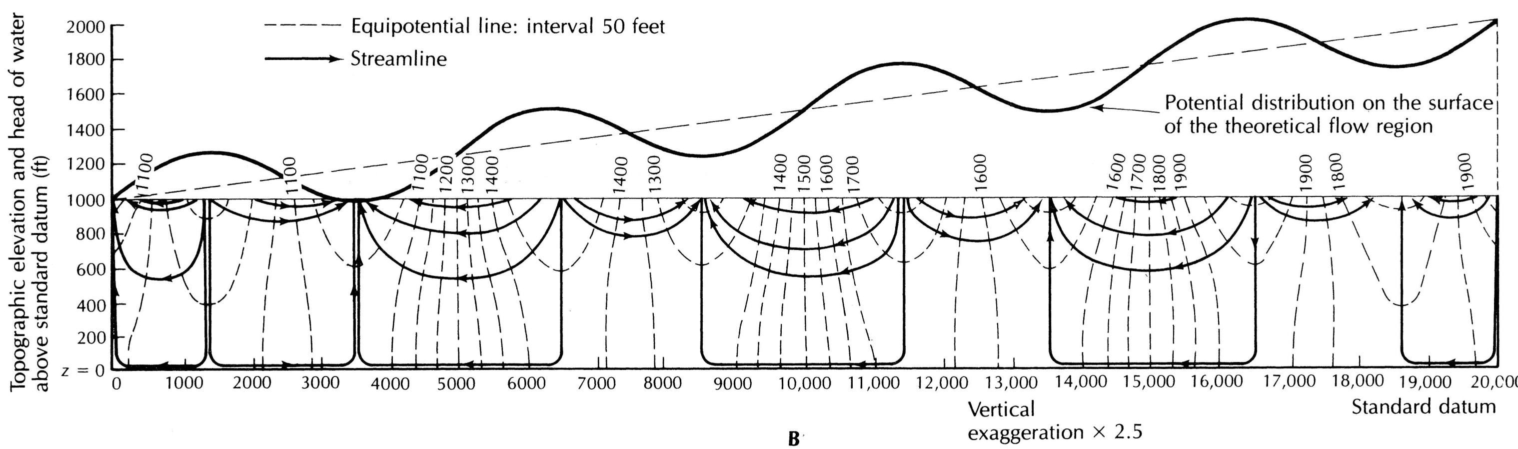
(Schwartz and Zhang, 2003)

The amplitude of water table undulations control the depth of local flow systems.

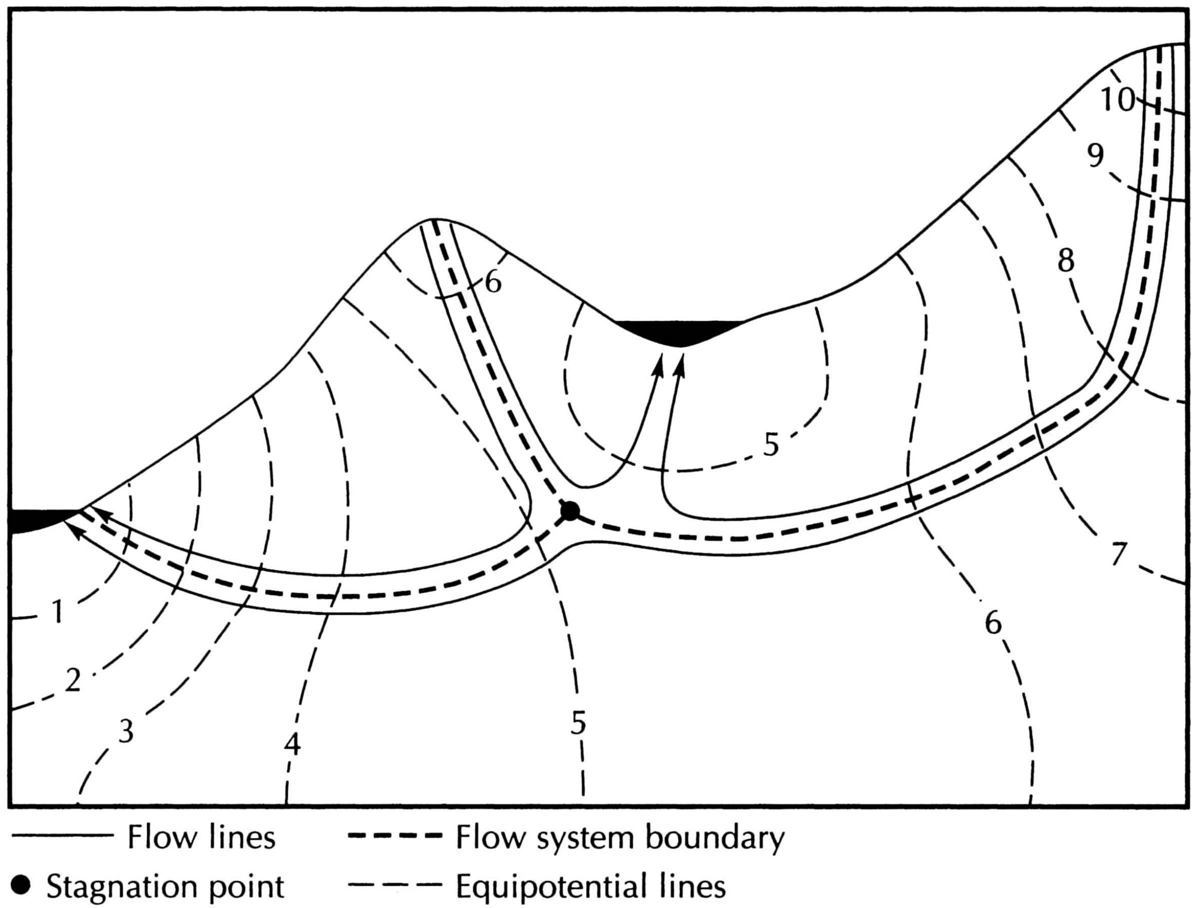
Local and regional flow systems



Local flow systems only



The flow systems that develop can result in regions of very low flow. A ***stagnation point*** will develop where three flow systems intersect.

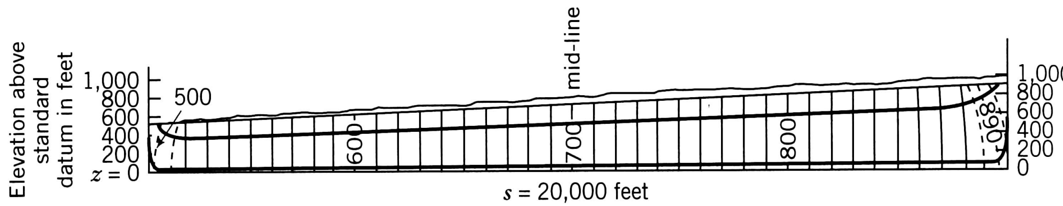


(Fetter, Applied Hydrogeology, 2001)

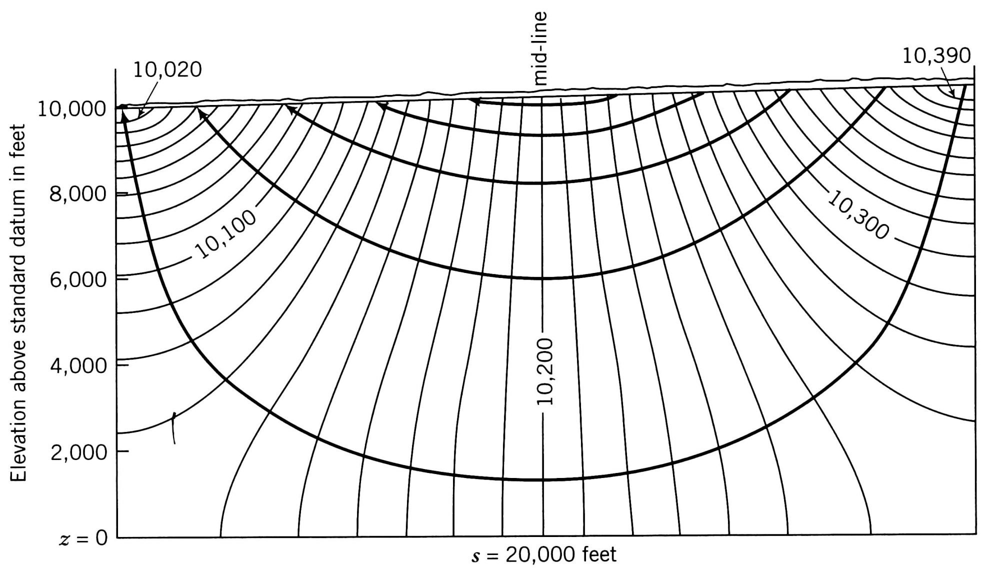
Influence of Basin Geometry

The ratio of groundwater basin depth to lateral extent affects regional flow patterns.

Shallow Basin



Deep Basin



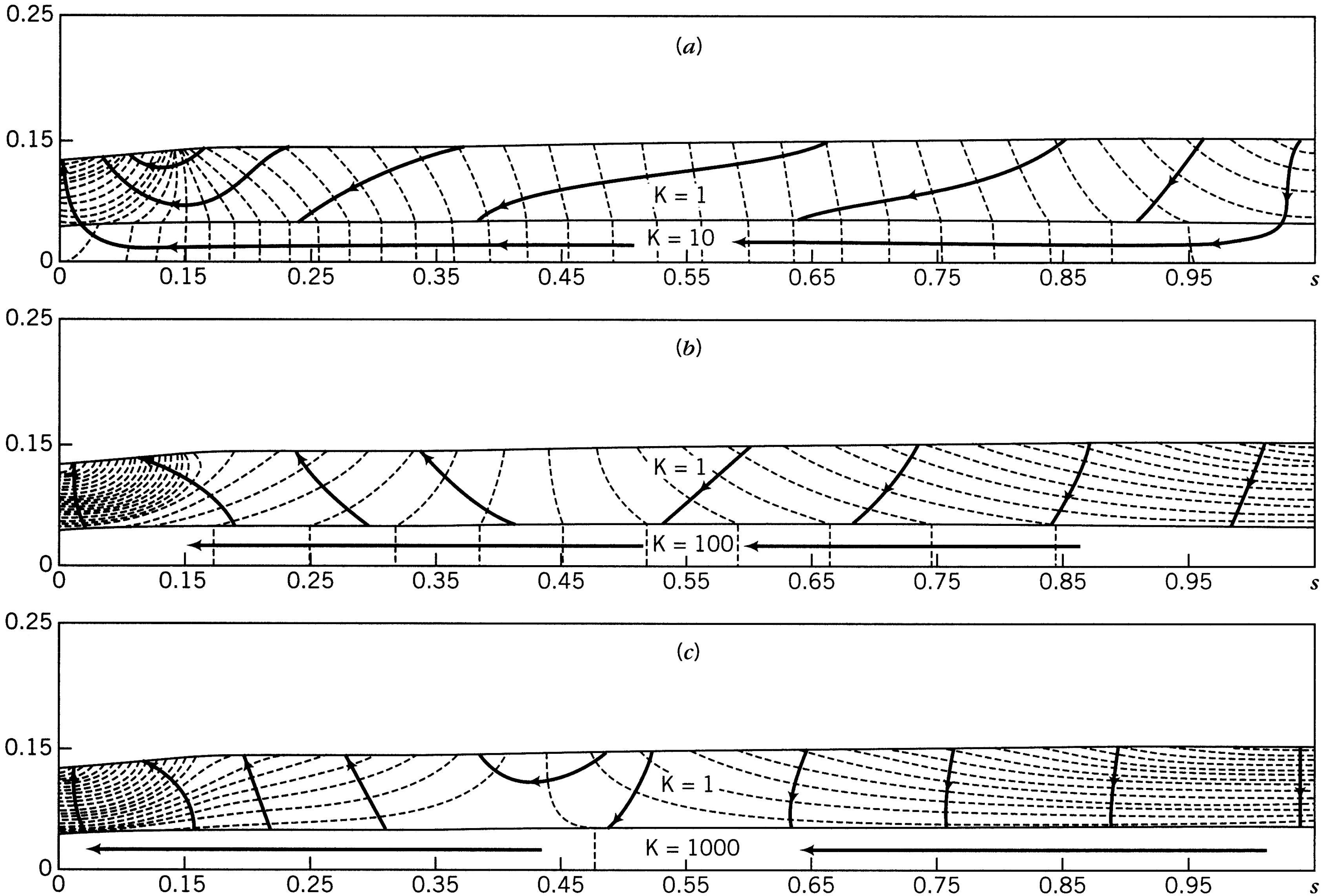
(Schwartz and Zhang, 2003)

Influence of Heterogeneity

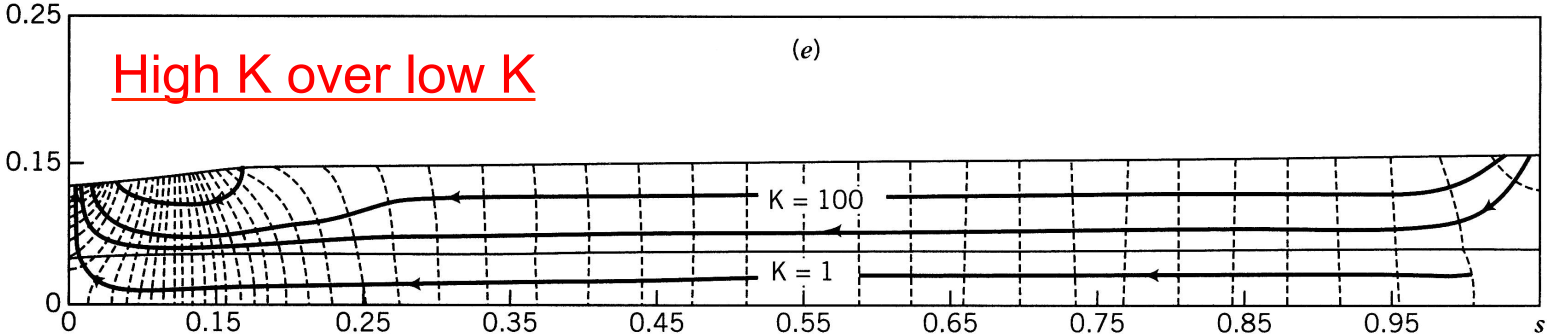
Groundwater flow systems are typically comprised of horizontal or slightly sloping hydrostratigraphic layers. The different layers have varying K values, resulting in a wide range of flow patterns.

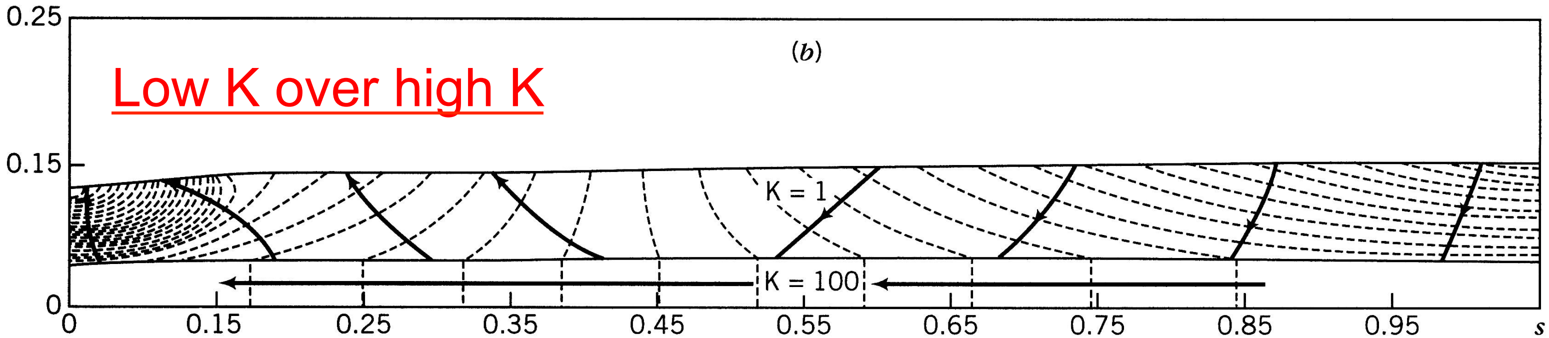
Generally, groundwater wants to preferentially flow through high K layers (because there is less resistance to flow) and across low K layers.

Systems with low K layers over high K layers.

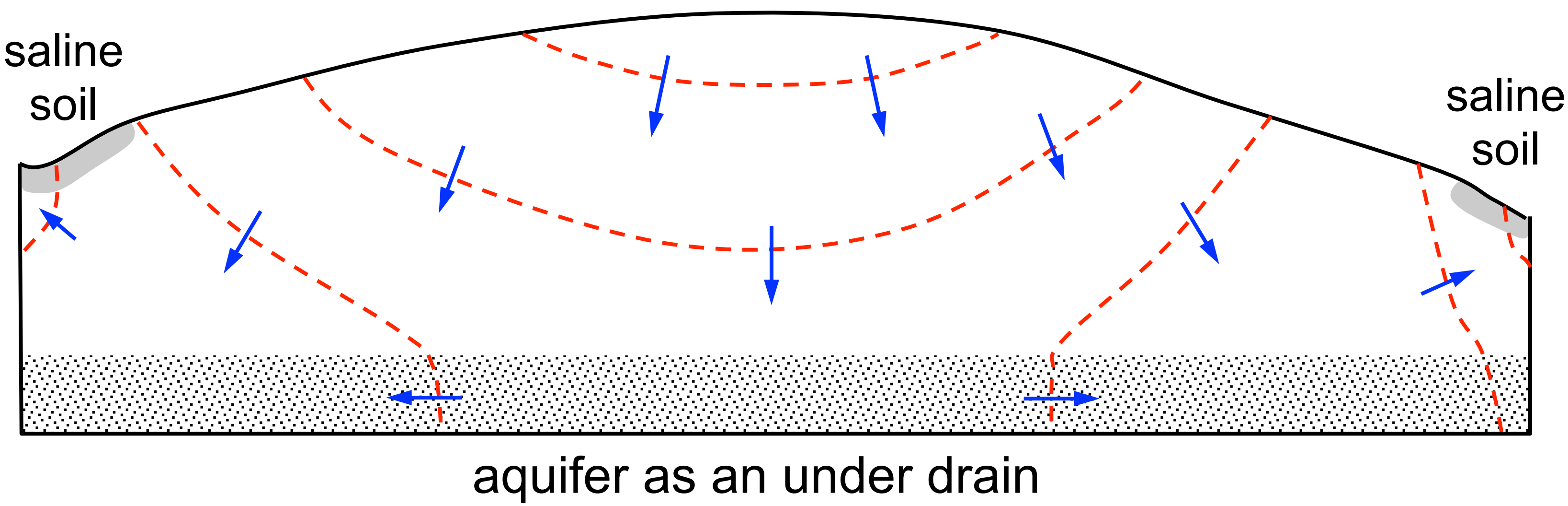


The order of layering has a pronounced role in the resulting flow patterns.



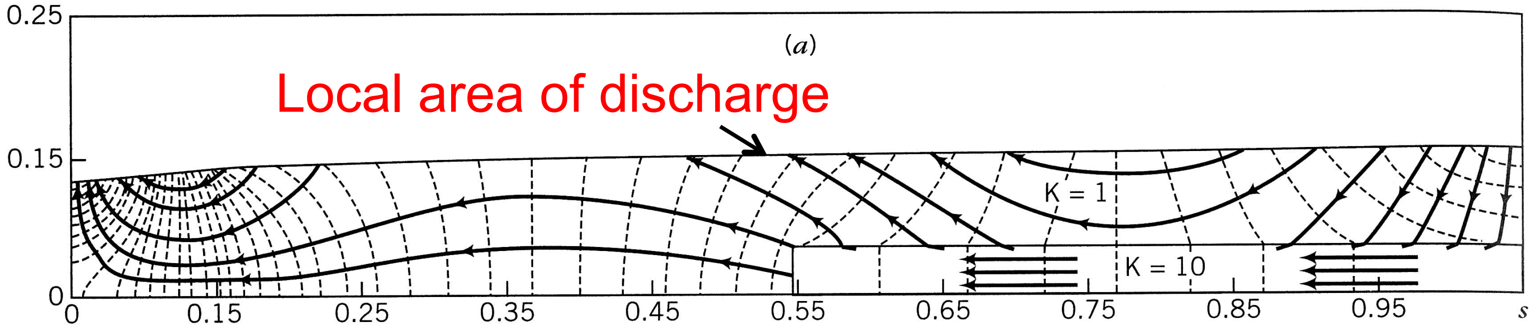


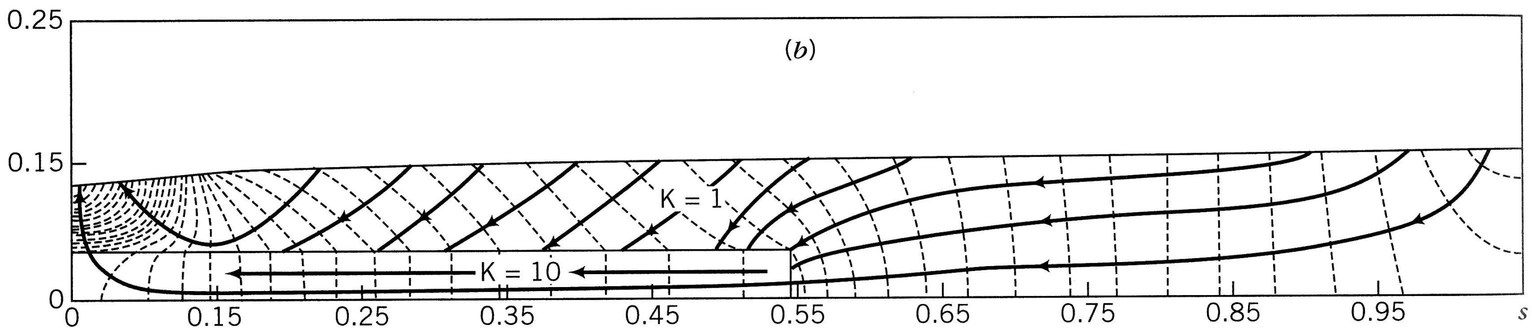
The situation of higher K layers at depth is commonly encountered in the prairies, where the clay-rich glacial till is underlain by inter-till aquifers. This is called a "prairie profile" after Meyboom.



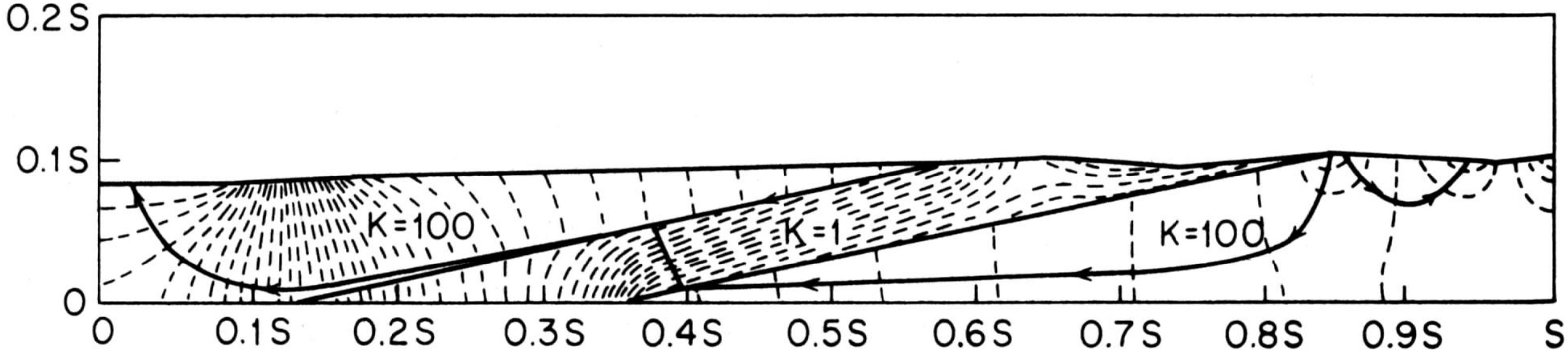
Why does saline soil occur in the discharge areas?

Even discontinuous layers of high K material drastically alter flow paths. Note how we again have both local and regional flow systems.

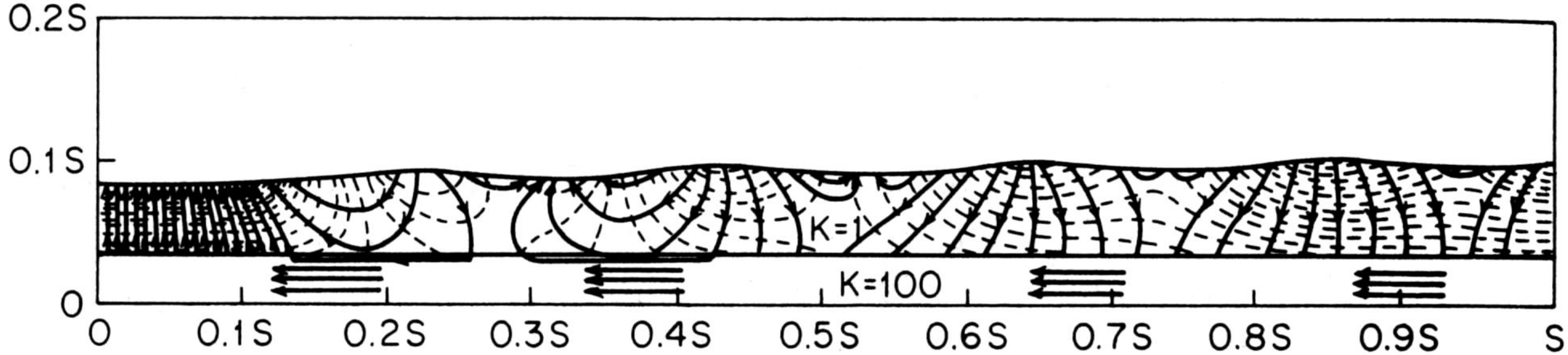


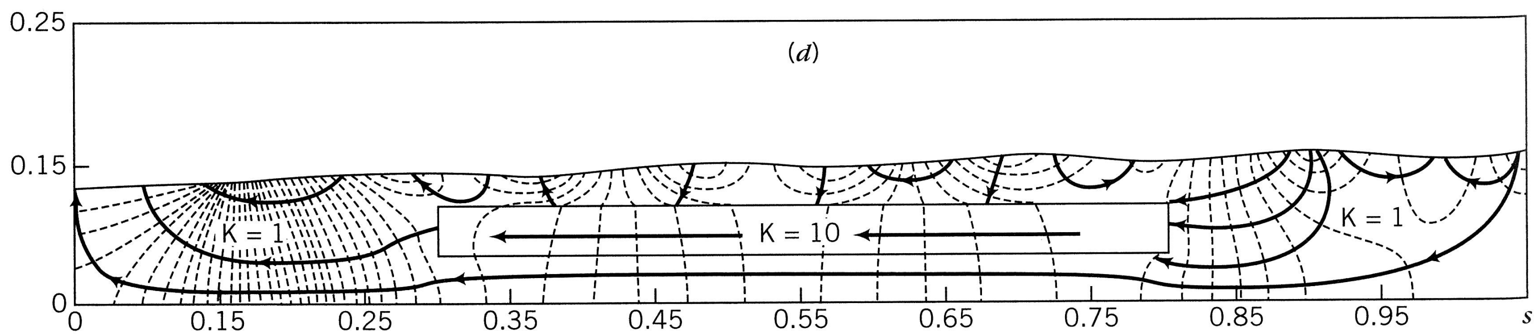


The same is true for systems with sloping layers.



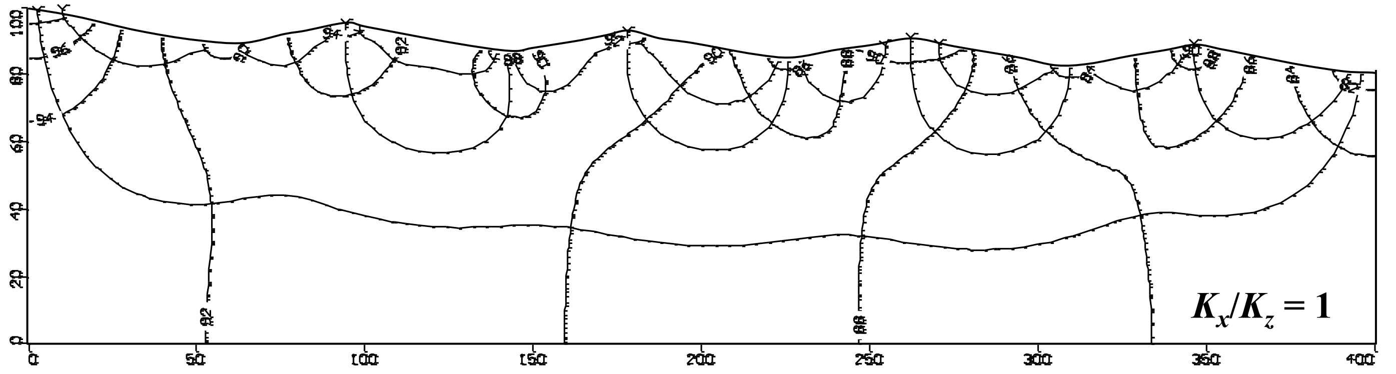
Combining heterogeneity with an undulating water table creates highly complex flow patterns with local, intermediate, and regional flow systems.

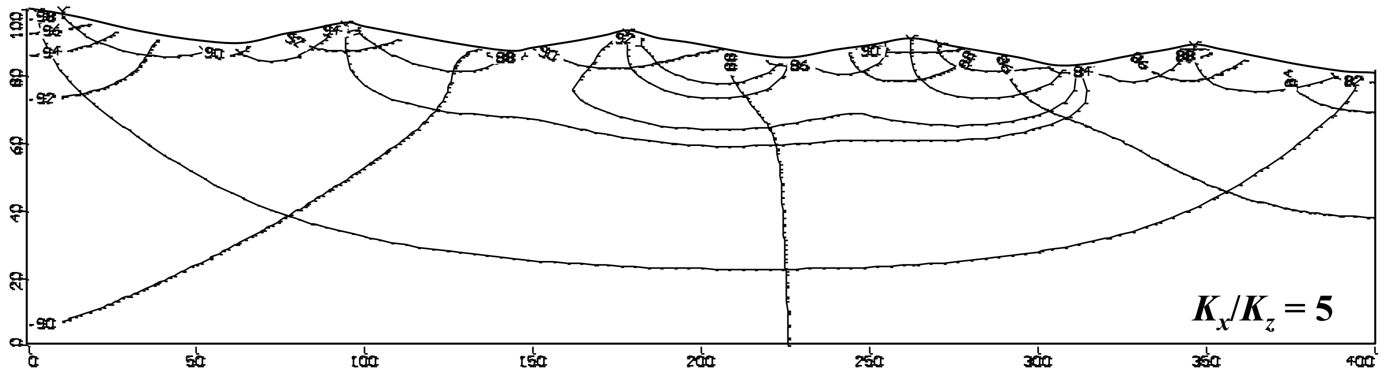




Influence of Anisotropy

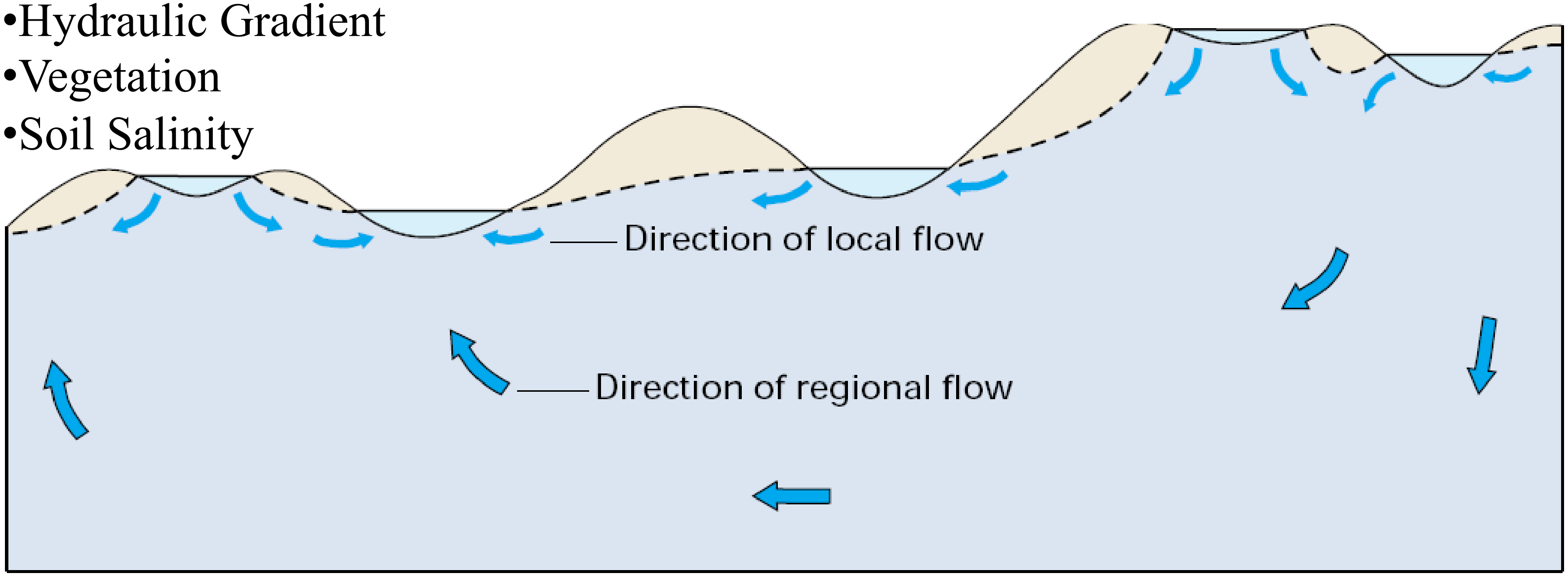
In anisotropic materials flow lines are generally not perpendicular to equipotentials. When $K_x >> K_z$, the flow paths tend to be shallower.





How might you be able to distinguish recharge areas from discharge areas?

- Hydraulic Gradient
- Vegetation
- Soil Salinity



(Winter et al., USGS, 1998)

In what season is depression focused recharge most likely? Why?

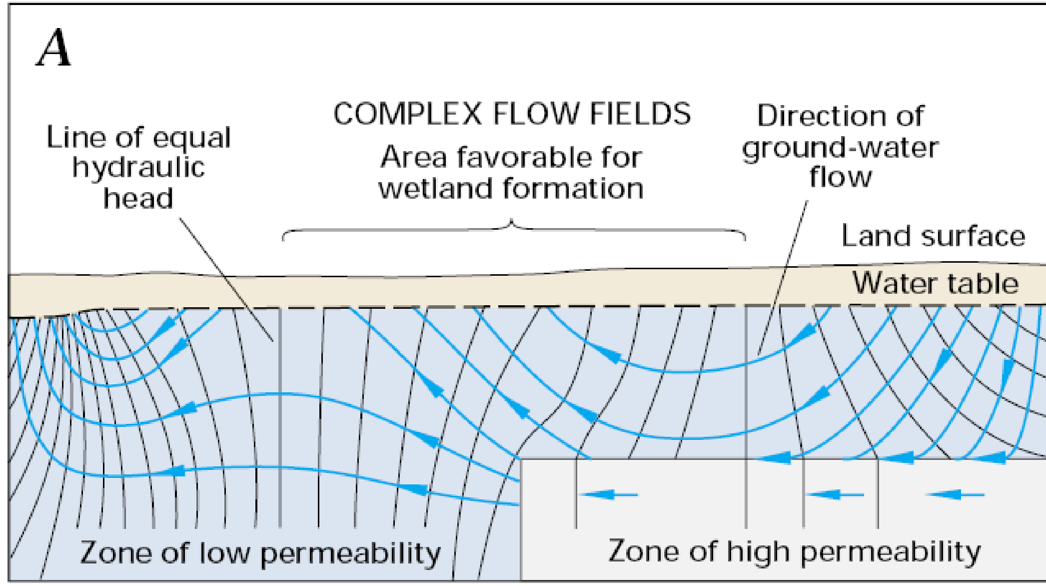
Groundwater-Surface Water Interactions

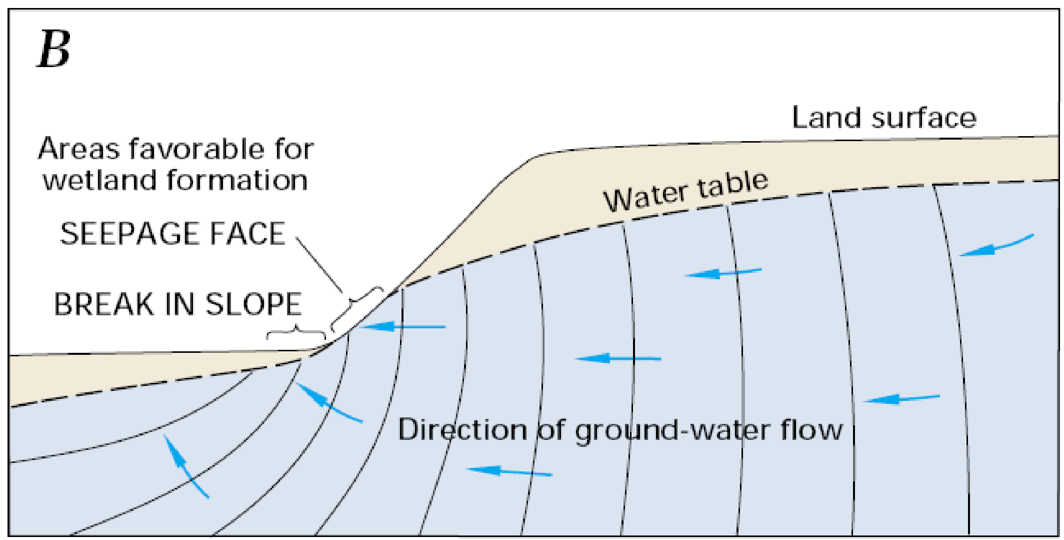
Most streams, rivers, wetlands, and lakes are connected to groundwater, with groundwater either entering into or seeping out of the surface water body.

You could say that groundwater "outcrops" as surface water.

Wetlands form in areas of groundwater discharge.

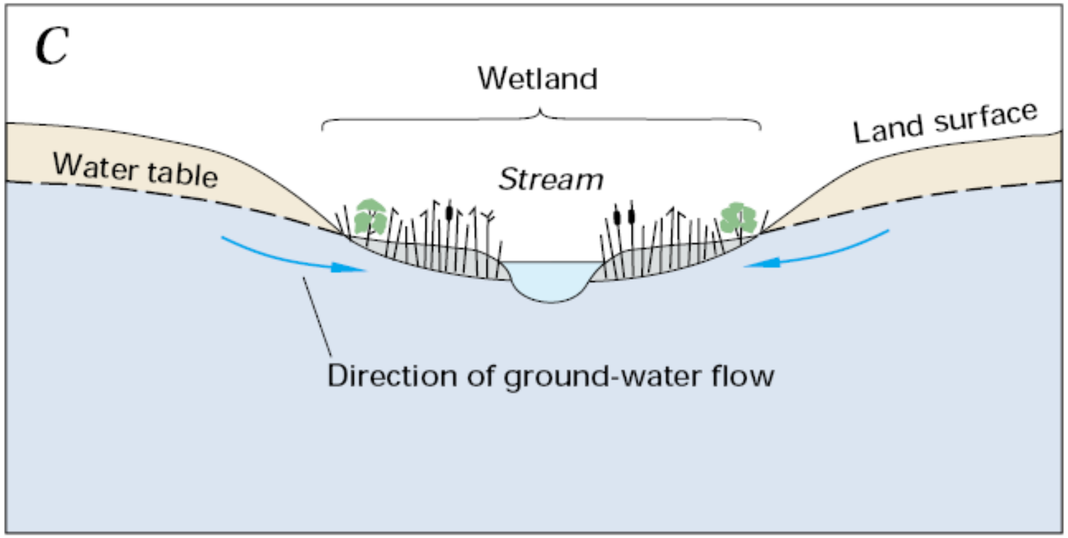
This can occur due to complex local and regional flow systems (A) or along groundwater springs that form near topographic breaks (B).

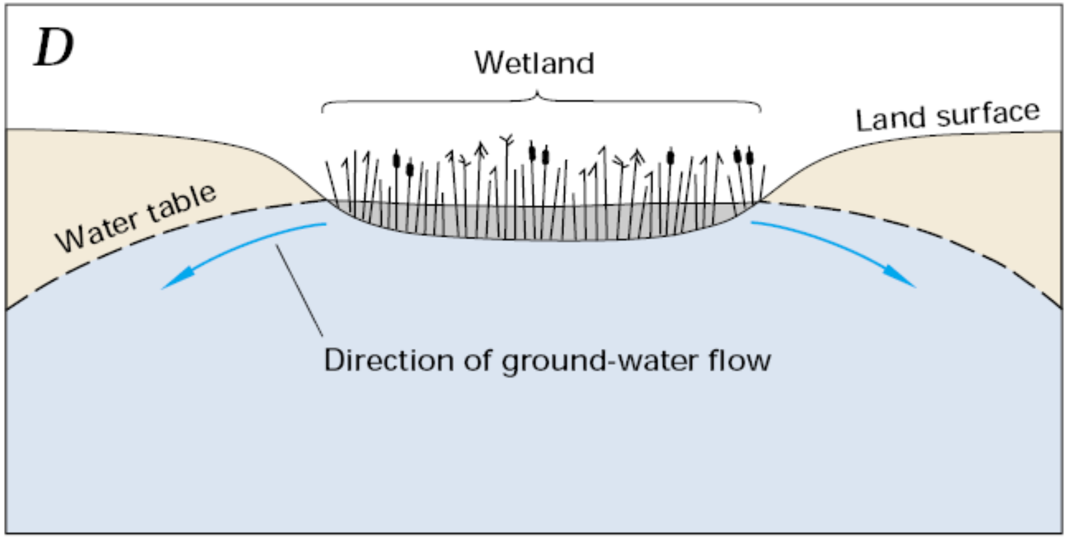




(Winter et al., USGS, 1998)

Wetlands also form adjacent to streams (C) or as a result of precipitation and runoff in closed topographic depressions (D).





(Winter et al., USGS, 1998)