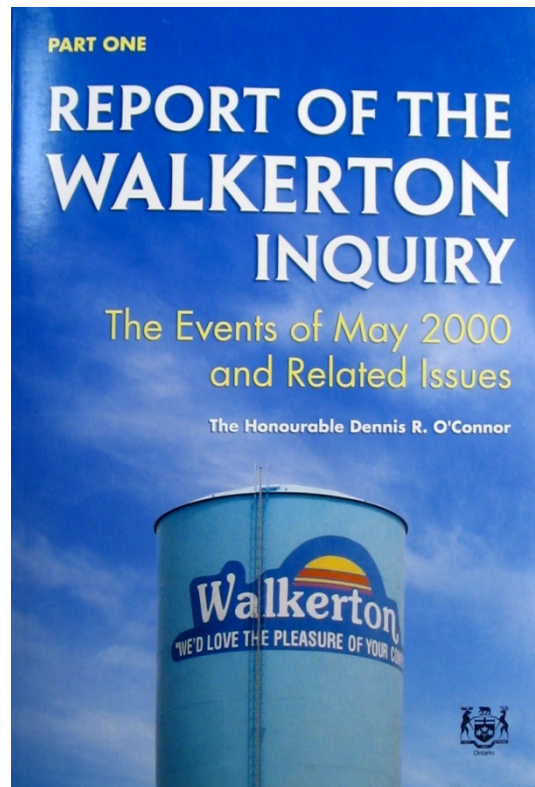


# Groundwater Contamination and Transport Processes

Suggested reading: Schwartz and Zhang Ch. 19 and 22 (plus Ch. 16)

One of the many reasons to study hydrogeology is to evaluate groundwater contamination problems.



(Cey)

# **Sources of Contamination**

## **Point Sources**

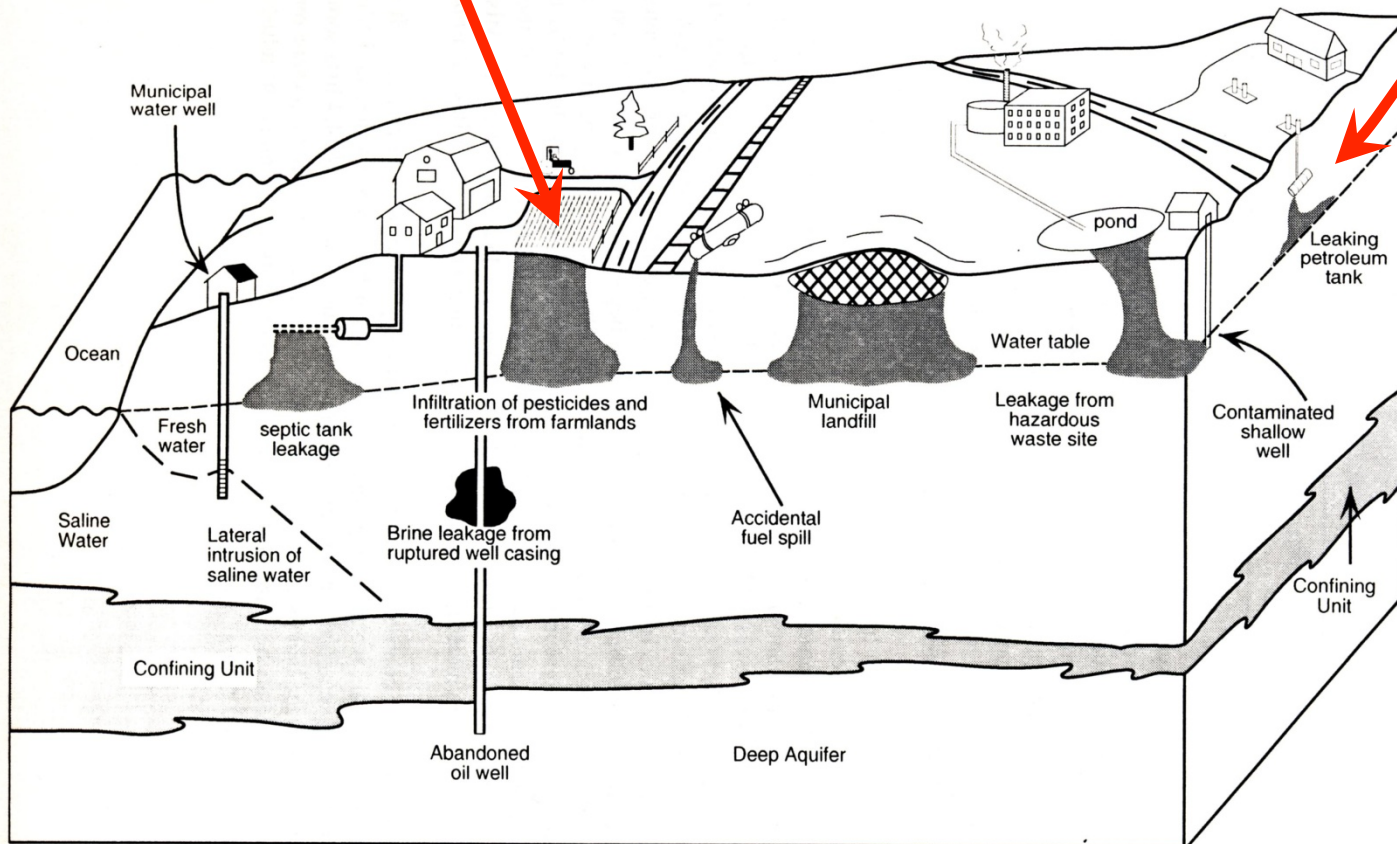
- leaky storage tanks (petroleum or other chemicals)
- accidental spills and historical releases
- waste lagoons, landfills and dumps
- septic systems
- mine tailings/waste rock
- graveyards/animal burial
- injection wells

## Distributed (Non-point) Sources

- land applied manure/sewage/sludge
- fertilizers and pesticides
- air pollution fallout
- urban runoff
- military firing ranges

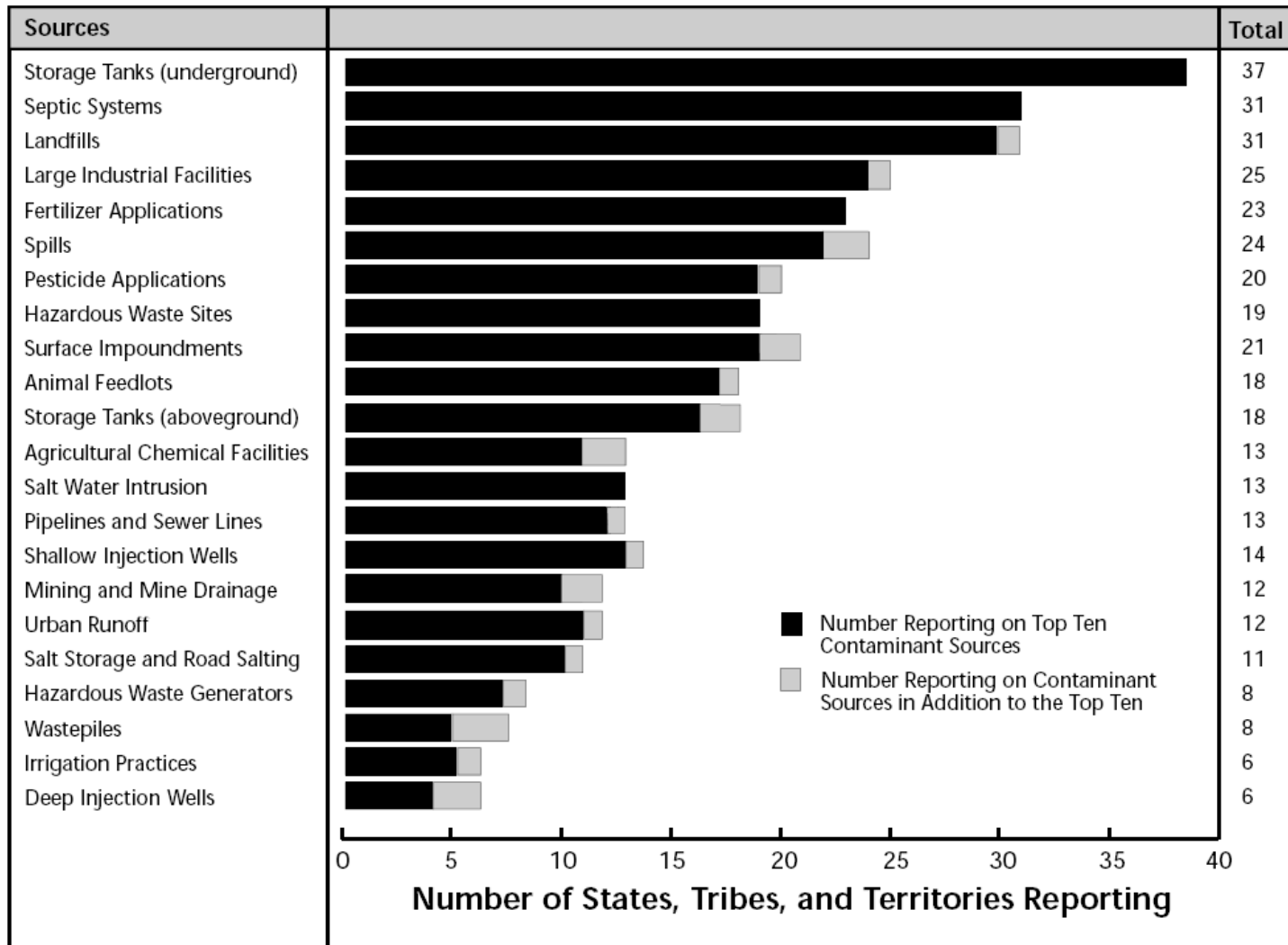
## Linearly Distributed Sources

- pipeline leakage (many leakage points)
- saltwater intrusion (coastal areas)
- road salt
- losing streams
- canal leakage



(Rudolph))

## Major Sources of Ground Water Contamination



Ranking of groundwater contamination sources.

(Source: National Water Quality Inventory, 1998 Report to Congress, USEPA)

## **Contaminant Types**

The type of source will dictate the type of contaminant that is present. Contaminant types are generally divided based on their chemical and physical properties.

### **Chemical:**

Inorganic - nitrate, chloride, phosphorous

Organic - pesticides, petroleum products

Metals - arsenic, lead, chromium, selenium

Radionuclides - radon-222

### **Biological:**

Bacteria - *E. coli*, *Campylobacter jejuni*

Viruses - rotavirus, norwalk, poliovirus

The physical properties of the contaminant can drastically influence the distribution of contaminants in the subsurface.

Miscible/Dissolved: solids/liquids that fully dissolve in water

- mostly cations (K, Na, Ca, Mg) and anions (Cl, SO<sub>4</sub>, HCO<sub>3</sub>), but also organic compounds (e.g., pesticides, benzene)

Immiscible Fluids: oily liquids that have low water solubility and remain as a separate liquid phase in the subsurface

- also called a Non-Aqueous Phase Liquid (NAPL)
- density, viscosity and surface tension are different from water which leads to unique subsurface migration patterns
- e.g., gasoline, creosote, chlorinated solvents

## **Transport Processes in Groundwater**

Transport processes are important in:

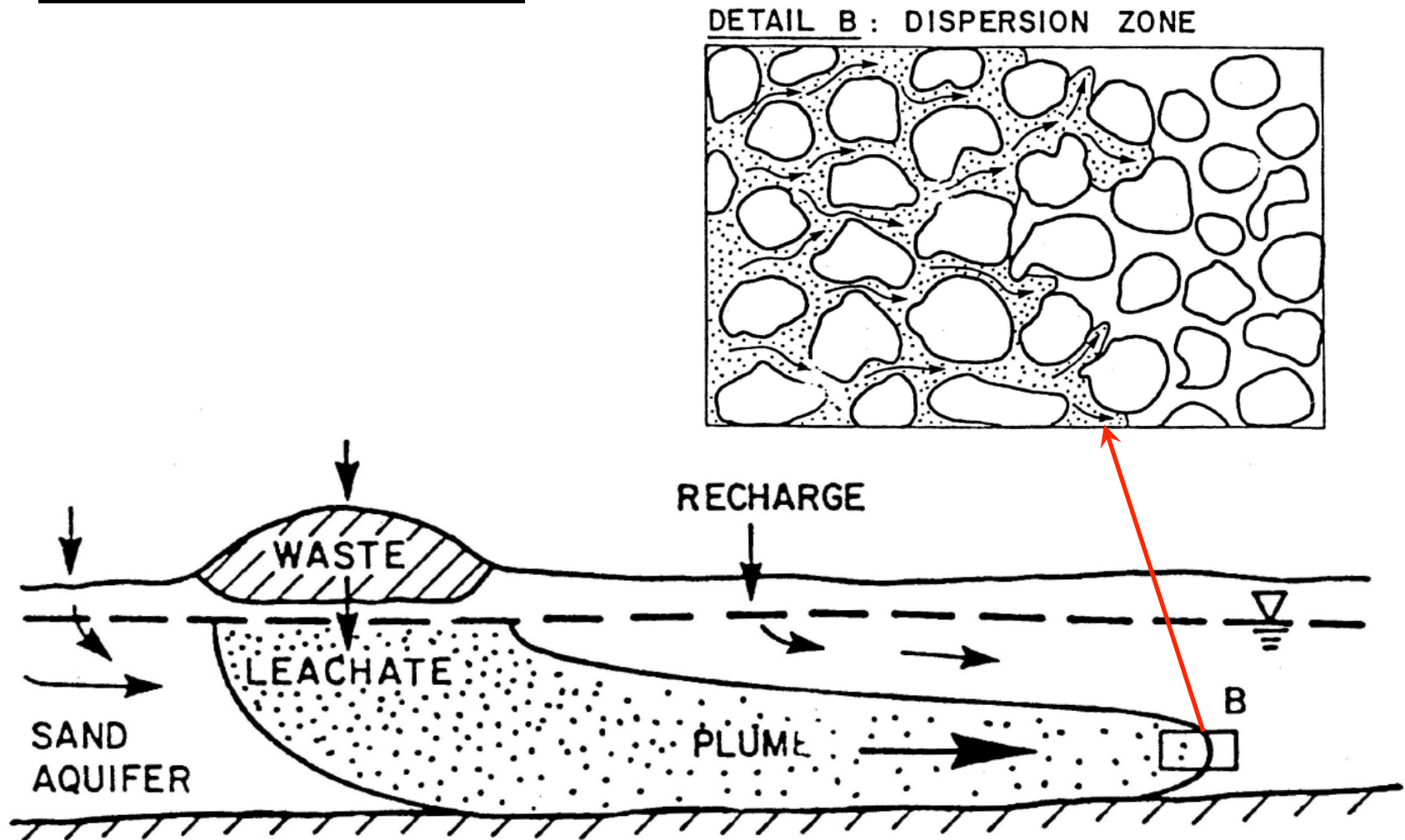
- migration of contaminants
- nutrient cycling in catchments

Most contaminants in groundwater are transported in the dissolved phase (solutes). This can result in plumes of contaminated groundwater traveling long distances in the subsurface.

We will focus on these dissolved contaminants and the physical processes affecting their transport.



# Groundwater Plumes



(Cherry)

# Transport and Fate Processes for Solutes in Groundwater

## Transport

- Advection
- Molecular diffusion
- Mechanical dispersion



## Attenuation

### Chemical

- Sorption
- Precipitation / dissolution
- Volatilization
- Reactions (Redox, acid-base)
- Abiotic degradation

### Biological

- Degradation (biodegradation)



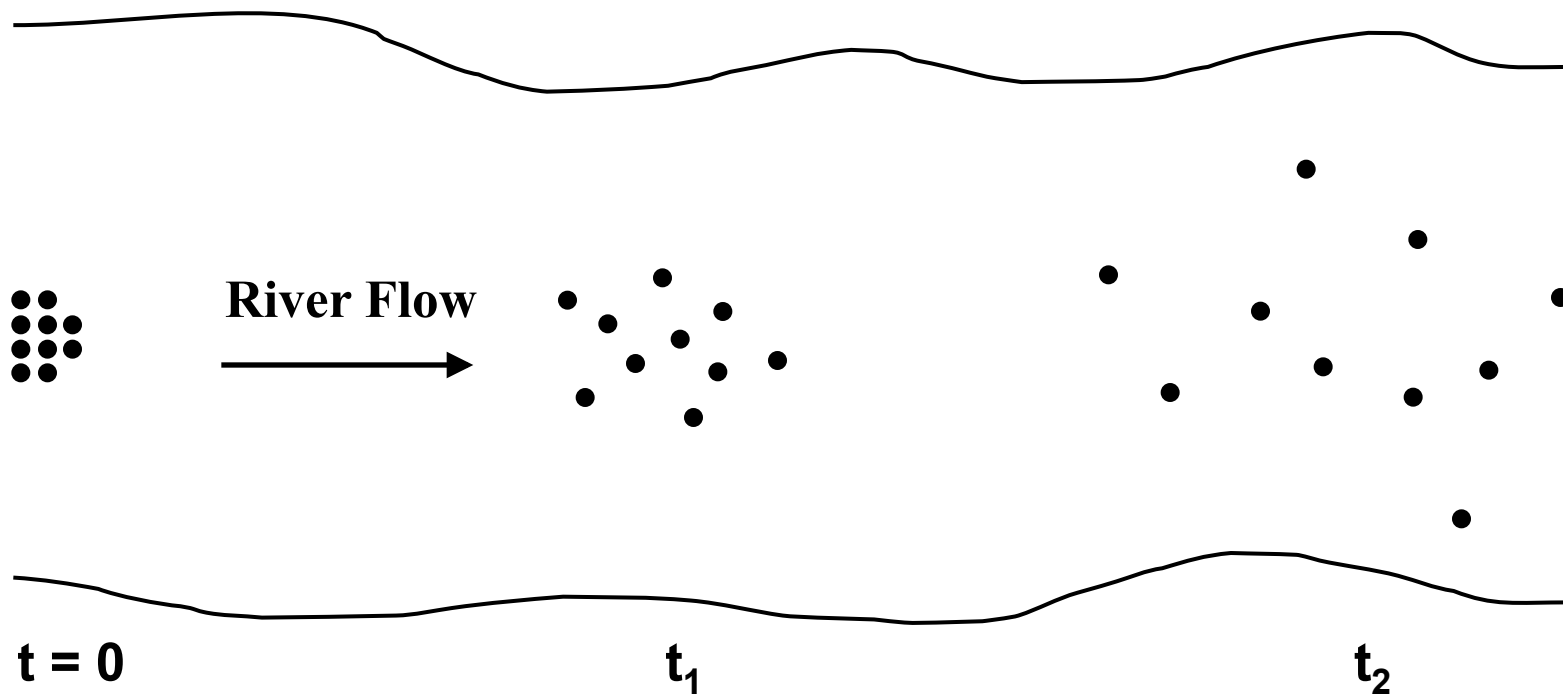
## Analogy of the Day

Solute transport is very much like ducks in water.

Let's consider an experiment where rubber ducks are released into a river or lake. The rubber ducks represent individual contaminant molecules and the water body represents groundwater (a river in the case of flowing groundwater).

Imagine we release 100 ducks simultaneously from the middle of a bridge over a river. What happens to the ducks? How fast do they move? Do they all move at the same velocity? Where will they be after one hour? One day?

## Ducks in a river!



## **Solute Transport Mechanisms**

Three key transport mechanisms:

- 1. Advection** - transport due to bulk groundwater flow
- 2. Diffusion** - molecular transport due to solute concentration gradients (no water movement)
- 3. Mechanical Dispersion** - spreading of solute mass due to groundwater velocity variations at the pore scale (and larger scales)

**Hydrodynamic Dispersion** - lumps diffusion and mechanical dispersion together

## Advection

Solutes move along with the bulk groundwater flow.

Bulk solute velocity = avg. linear groundwater velocity.

Often called plug flow.

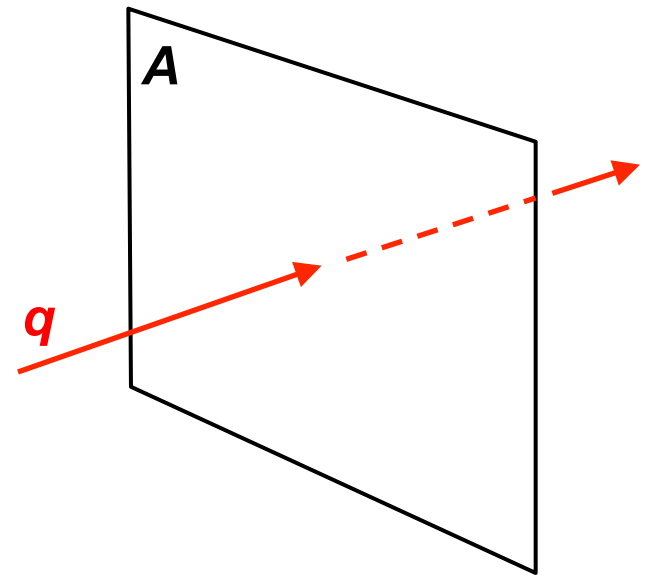
Specific discharge,  $q$ , is the volume of water crossing a unit area per unit time (volumetric flux).

Likewise, the transport flux ( $J$ ) is defined as the *rate of mass transport per unit area* ( $\text{kg s}^{-1} \text{ m}^{-2}$ ).

The advective mass flux of solutes crossing the same plane is:

$$\text{Advective Mass Flux} = J_A = q C$$

where  $C$  is solute concentration ( $\text{M/L}^3$ ).

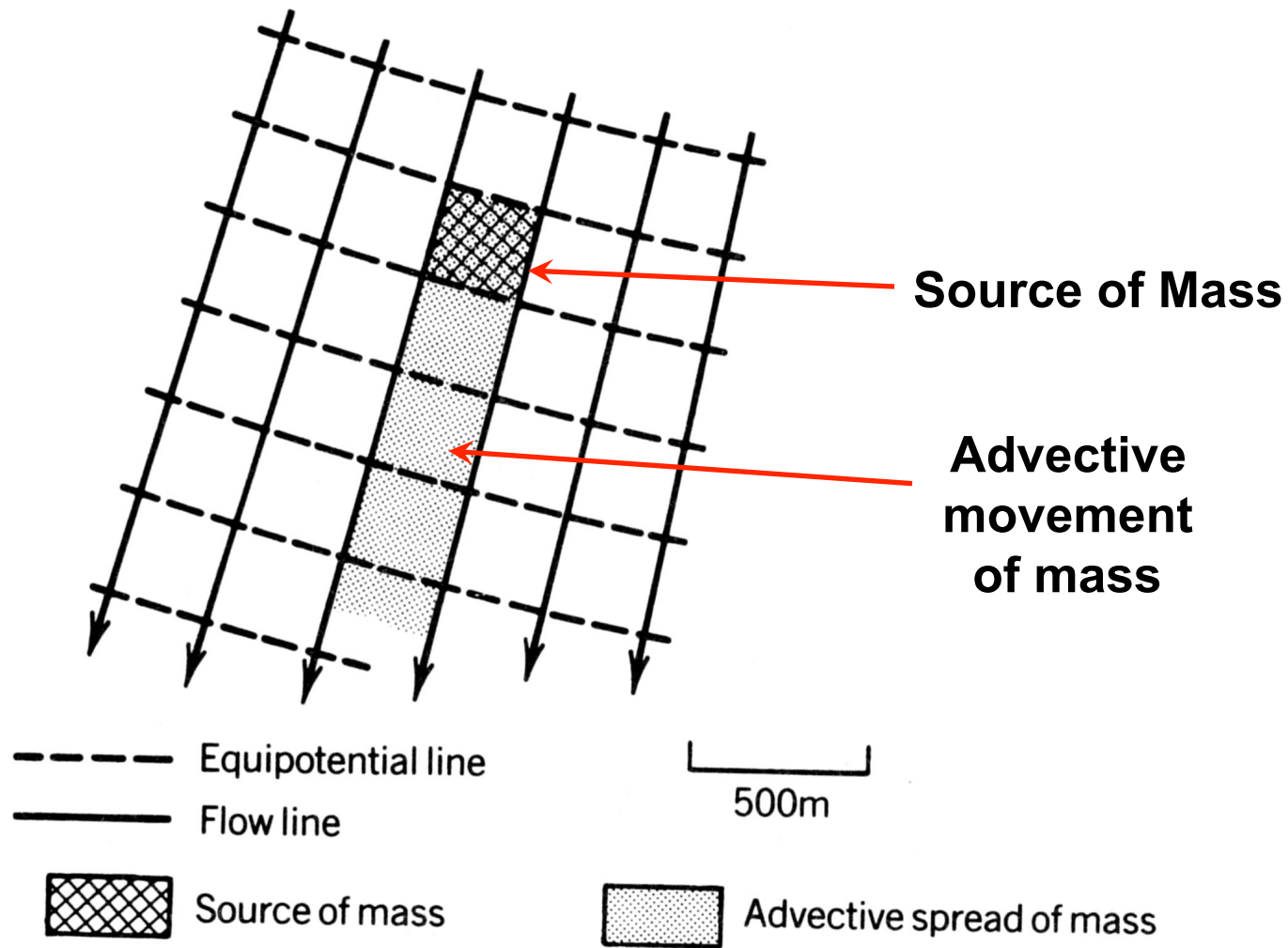


What are the units of flux?

When we express the flux in terms of groundwater flow velocity, we must remember to account for effective soil porosity using  $v=q/n_e$ , because solute only moves through the open pore spaces.

The advective flux in a porous medium becomes:

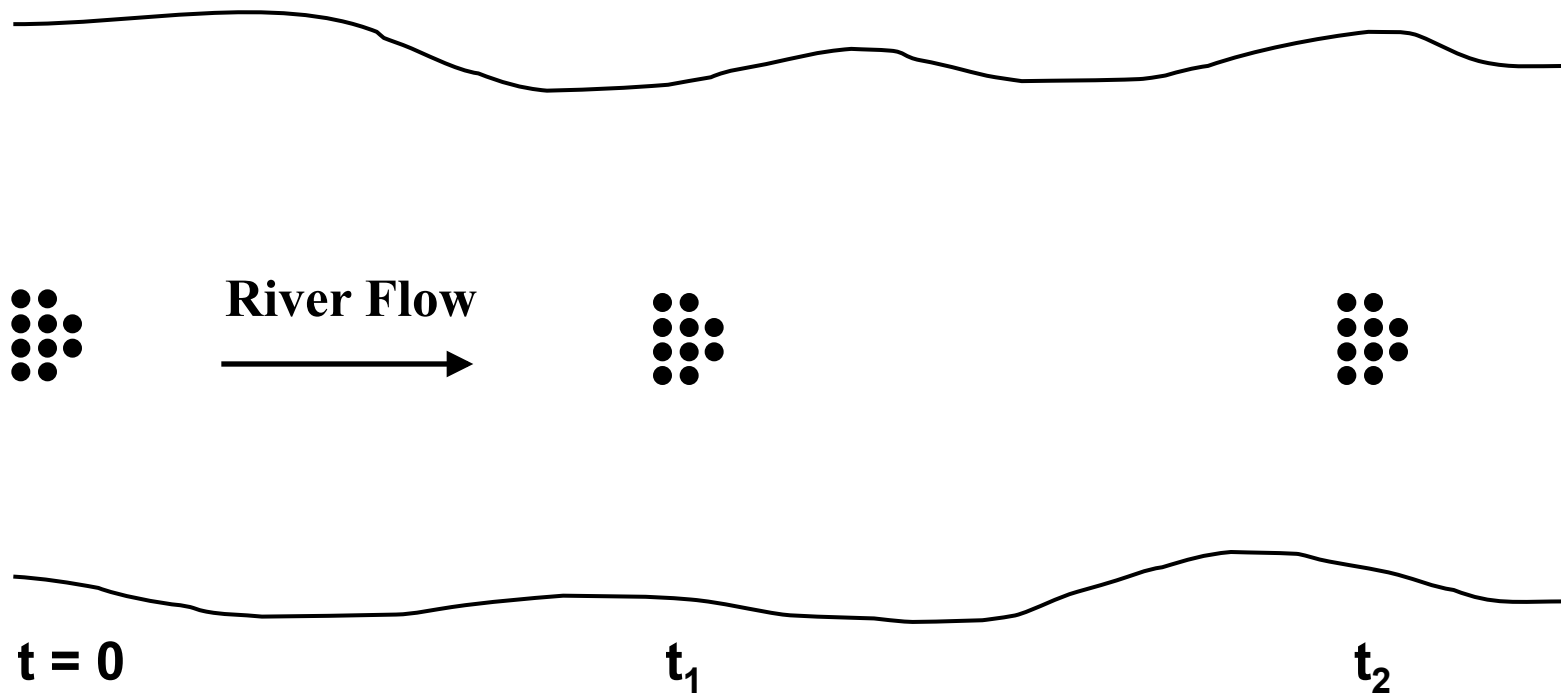
$$\begin{aligned} \text{Advective Mass Flux} &= J_A = q C \\ &= n_e v C \end{aligned}$$



Plan view of solute mass spreading by advection alone  
(Modified from Domenico and Schwartz, 1990)



# Duck Advection



Is this realistic?

## Molecular Diffusion

Solutes can move through a stationary fluid due to random “Brownian” motion. This molecular motion, referred to as diffusion, causes solutes to move from areas of high concentration to areas of low concentration.

The diffusive mass flux is given by Fick’s first law:

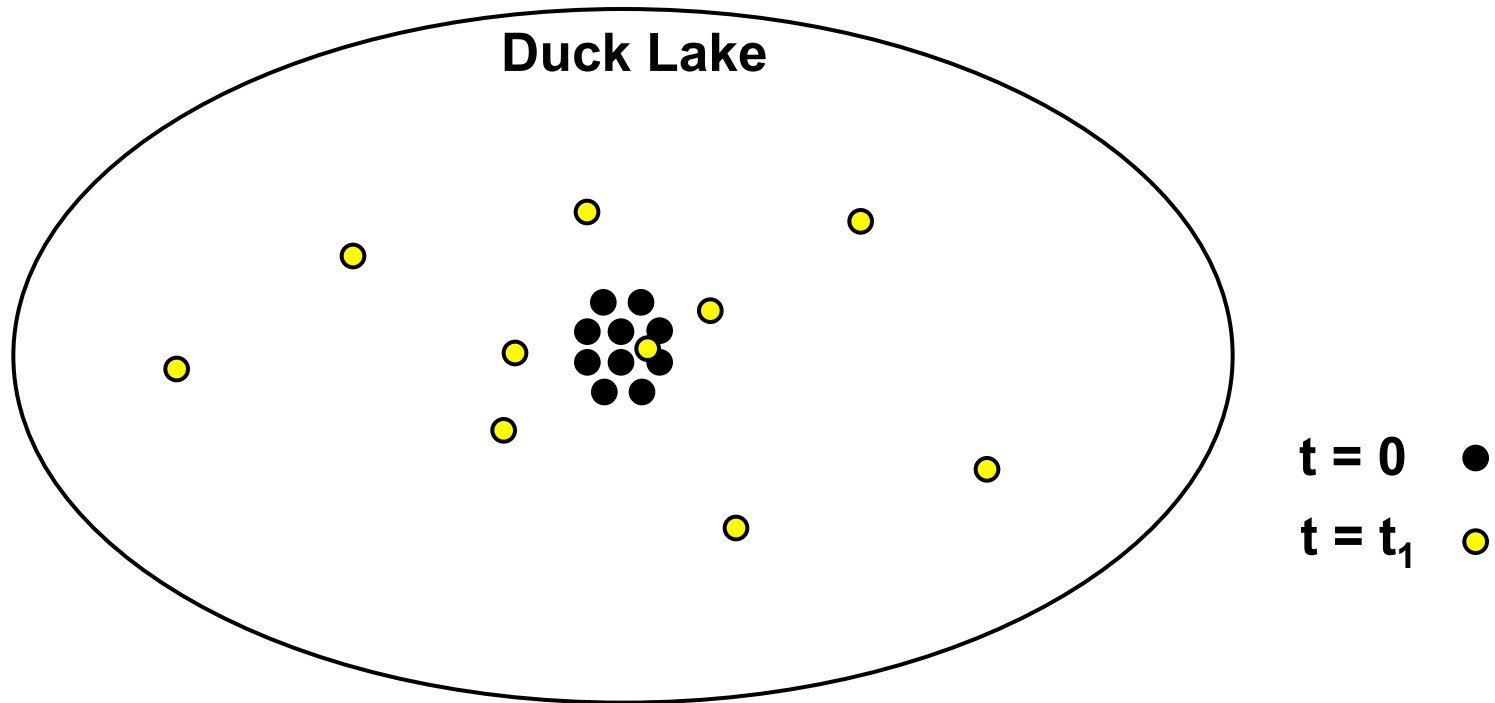
$$\text{Diffusive Mass Flux} = J_D = -D_d \frac{dC}{dx} \quad \left. \vphantom{\frac{dC}{dx}} \right\} \begin{array}{l} \text{Free} \\ \text{Water} \end{array}$$

where:  $J_D$  = diffusive mass flux (units?)

$D_d$  = diffusion coefficient in free water ( $L^2/T$ )

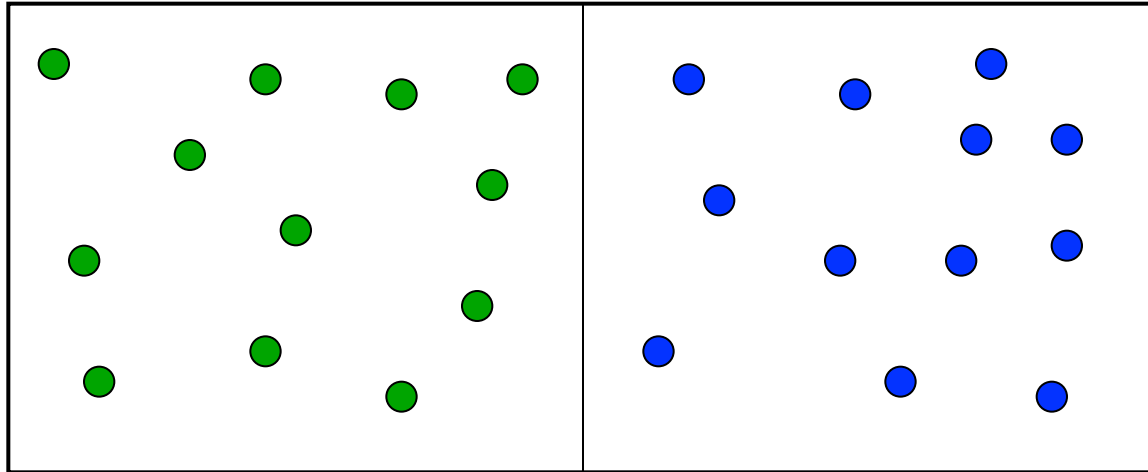
$dC/dx$  = concentration gradient ( $M/L^3/L$ )

## Duck Diffusion (in a lake with no flow)



# Molecular Diffusion

Gas movement driven by concentration gradient



Note that  $D_d$  is closely related to the random motion of molecules, and hence is temperature dependent.

**TABLE 2.1** Diffusion coefficients in water at 25°C.

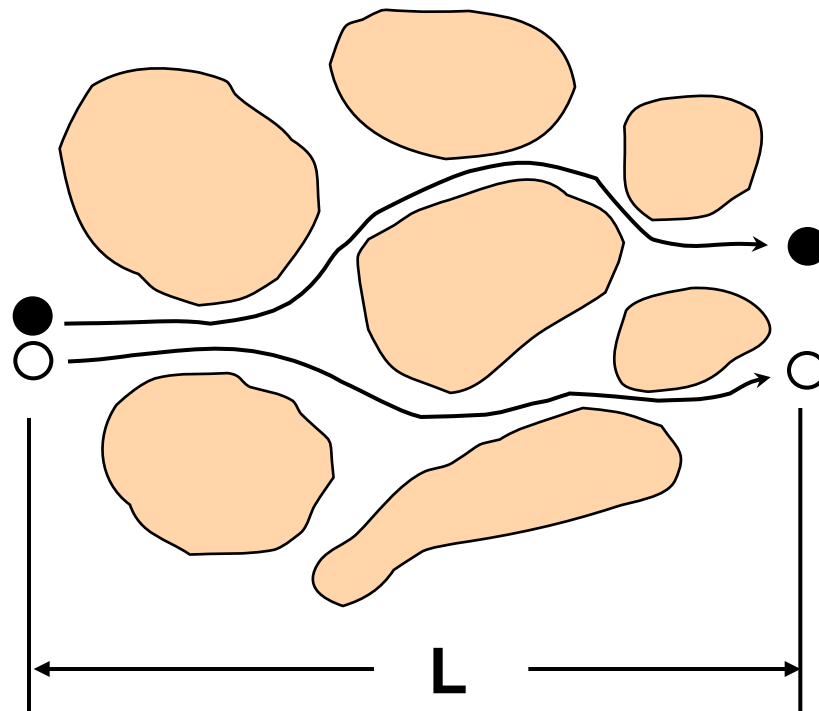
Cations	
H <sup>+</sup>	$9.31 \times 10^{-9} \text{ m}^2/\text{sec}$
Na <sup>+</sup>	$1.33 \times 10^{-9} \text{ m}^2/\text{sec}$
K <sup>+</sup>	$1.96 \times 10^{-9} \text{ m}^2/\text{sec}$
Rb <sup>+</sup>	$2.06 \times 10^{-9} \text{ m}^2/\text{sec}$
Cs <sup>+</sup>	$2.07 \times 10^{-9} \text{ m}^2/\text{sec}$
Mg <sup>2+</sup>	$7.05 \times 10^{-10} \text{ m}^2/\text{sec}$
Ca <sup>2+</sup>	$7.93 \times 10^{-10} \text{ m}^2/\text{sec}$
Sr <sup>2+</sup>	$7.94 \times 10^{-10} \text{ m}^2/\text{sec}$
Ba <sup>2+</sup>	$8.48 \times 10^{-10} \text{ m}^2/\text{sec}$
Ra <sup>2+</sup>	$8.89 \times 10^{-10} \text{ m}^2/\text{sec}$
Mn <sup>2+</sup>	$6.88 \times 10^{-10} \text{ m}^2/\text{sec}$
Fe <sup>2+</sup>	$7.19 \times 10^{-10} \text{ m}^2/\text{sec}$
Cr <sup>3+</sup>	$5.94 \times 10^{-10} \text{ m}^2/\text{sec}$
Fe <sup>3+</sup>	$6.07 \times 10^{-10} \text{ m}^2/\text{sec}$
Anions	
OH <sup>-</sup>	$5.27 \times 10^{-9} \text{ m}^2/\text{sec}$
F <sup>-</sup>	$1.46 \times 10^{-9} \text{ m}^2/\text{sec}$
Cl <sup>-</sup>	$2.03 \times 10^{-9} \text{ m}^2/\text{sec}$
Br <sup>-</sup>	$2.01 \times 10^{-9} \text{ m}^2/\text{sec}$
HS <sup>-</sup>	$1.73 \times 10^{-9} \text{ m}^2/\text{sec}$
HCO <sub>3</sub> <sup>-</sup>	$1.18 \times 10^{-9} \text{ m}^2/\text{sec}$
SO <sub>4</sub> <sup>2-</sup>	$1.07 \times 10^{-9} \text{ m}^2/\text{sec}$
CO <sub>3</sub> <sup>2-</sup>	$9.55 \times 10^{-10} \text{ m}^2/\text{sec}$

Source: Y. -H. Li and S. Gregory, 1974. Diffusion of ions in sea water and in deep-sea sediments. *Geochemica et Cosmochemica Acta*, Vol. 38. © 1974, with the kind permission of Elsevier Science.

(Fetter)

## Diffusion in Porous Media

Diffusion conditions in porous media differ from those in free water. The value of the diffusion coefficient is usually smaller than  $D_d$  because of the tortuous flow paths that molecules have to travel.



We will apply an effective diffusion coefficient for porous media and denote it by  $D^*$ .

$$D^* = D_d \omega$$

where  $\omega$  (always  $< 1$ ) is an empirical coefficient that accounts for tortuosity.  $\omega$  is determined experimentally and generally ranges from 0.01 to 0.8 (average 0.6-0.7).

We again consider the reduction in diffusion area due to the solid grains by introducing effective porosity,  $n_e$ , into the diffusive flux equation for porous media.

$$\text{Diffusive Mass Flux} = J_D = -n_e D^* \frac{dC}{dx}$$

**Porous  
Media**

## **Mechanical Dispersion**

It has long been known that solutes in groundwater will spread out further than diffusion can account for. This spreading is termed mechanical dispersion and is the result of our “incomplete” understanding of the flow system.

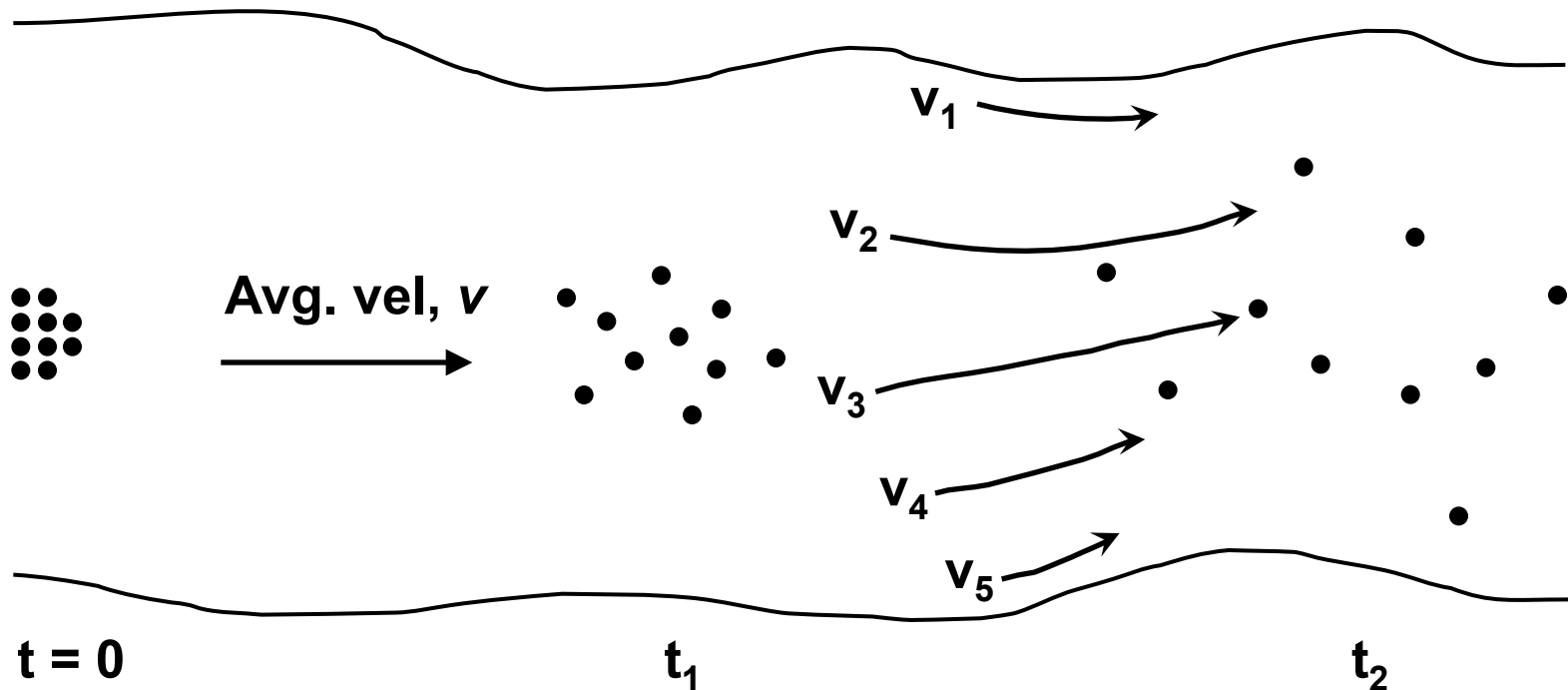
We have viewed groundwater flow using the REV concept where groundwater can be described with an average linear velocity,  $v$ .

In reality, the velocity of each particle or packet of groundwater (and solute it carries) in the subsurface is different. These velocity variations about the mean velocity cause an apparent spreading or dispersing of the solute mass.



# Duck Dispersion

In the duck analogy, the ducks disperse due to variations in stream velocity (eddies, boulders, drag along banks, etc.).



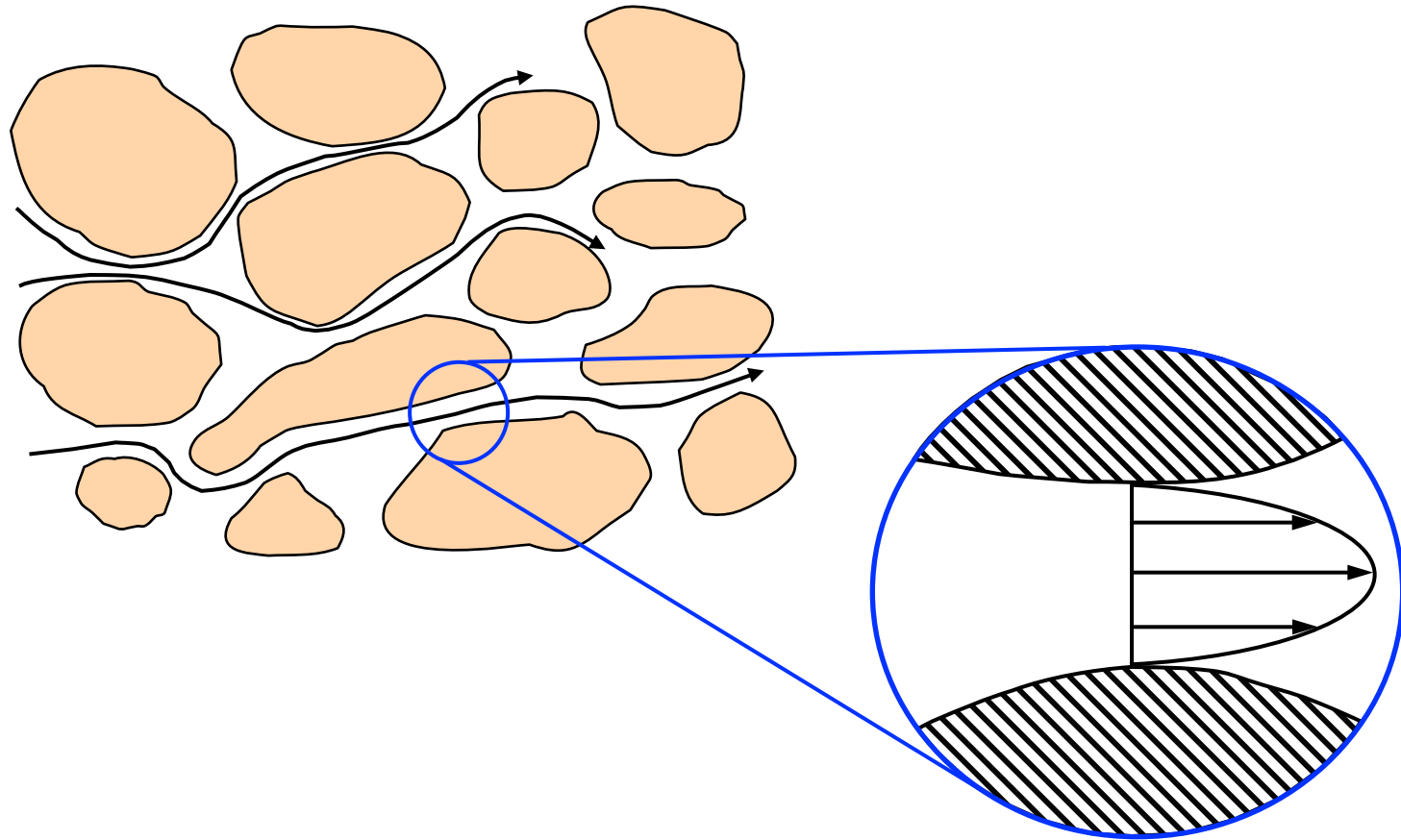
Mechanical dispersion is really an advective process that results from velocity variations we cannot account for.

Note that some velocities ( $v_2$  and  $v_3$ ) in the stream were greater than  $v$  and some velocities ( $v_1$ ,  $v_4$ , and  $v_5$ ) were less than  $v$ .

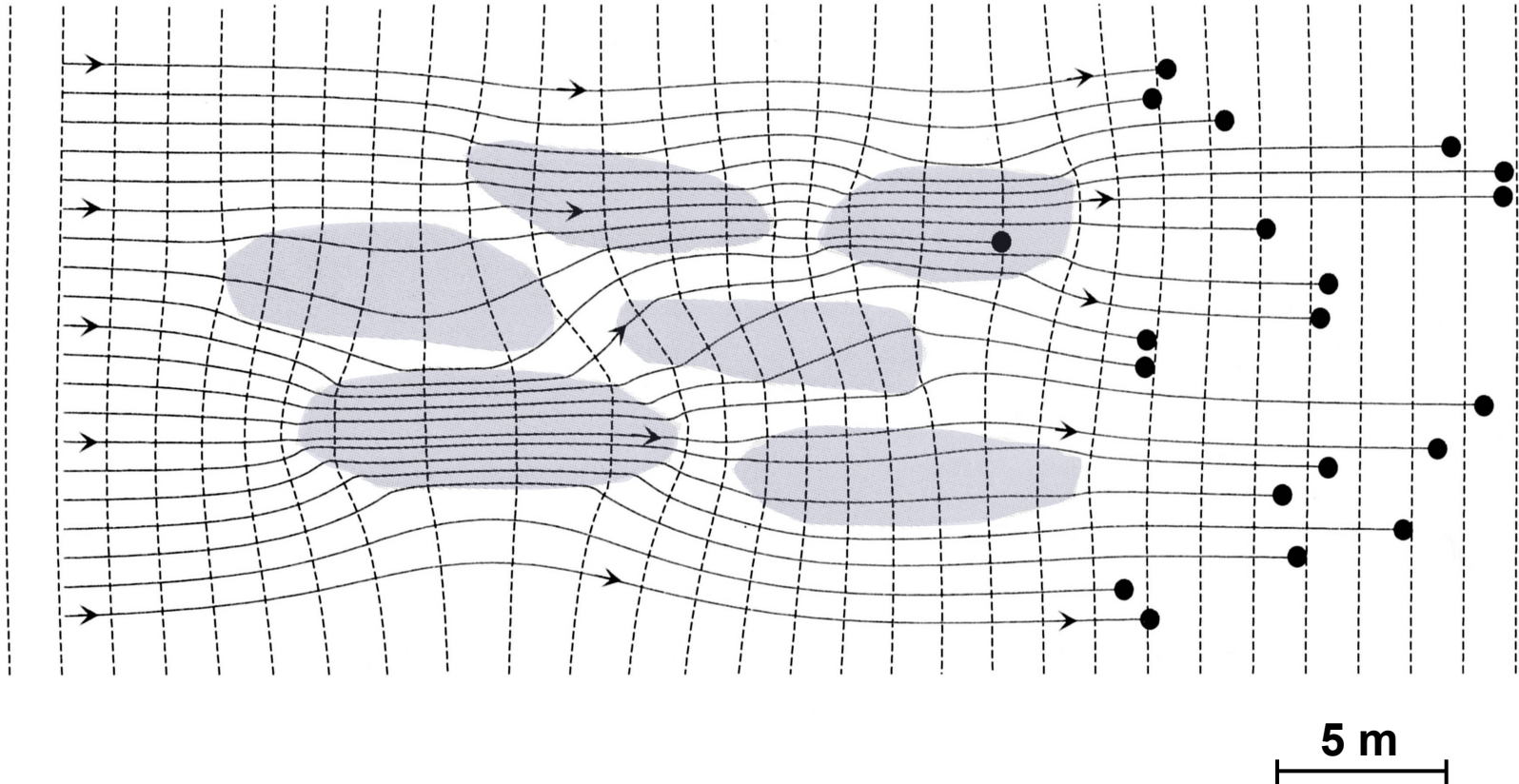
The end result is that part of the solute mass moves farther than the predicted center of mass and part of it does not move as far. The center of solute mass moves a distance  $d=vt$ .

The variability in flow velocity occurs at a variety of scales.

### Pore-scale Velocity Variations



## Field-scale Velocity Variations Caused by Heterogeneity



Experiments show that the mechanical dispersion flux can be expressed in much the same way as Fickian diffusion.

$$J_M = -n_e D_M \frac{dC}{dx}$$

where  $J_M$  is the dispersive flux in the direction of groundwater flow and  $D_M$  is the mechanical dispersion coefficient.

Again, we employ the effective porosity to account for flow only in the pore spaces. Hereafter, we will revert to using  $n$  instead of  $n_e$ . It is assumed that we are referring to effective porosity when evaluating transport equations.

The mechanical dispersion coefficient has been evaluated experimentally and is proportional to the average groundwater velocity. The equation is given by:

$$D_M = \alpha v$$

where  $\alpha$  is referred to as the dispersivity.

Some researchers have suggested that  $D_M$  is not linearly proportional to  $v$ , but the relation seems to hold well for many granular porous media.

The mechanical dispersion flux becomes:

$$J_M = -n_e \alpha v \frac{dC}{dx}$$

**Mechanical Dispersion  
Mass Flux**

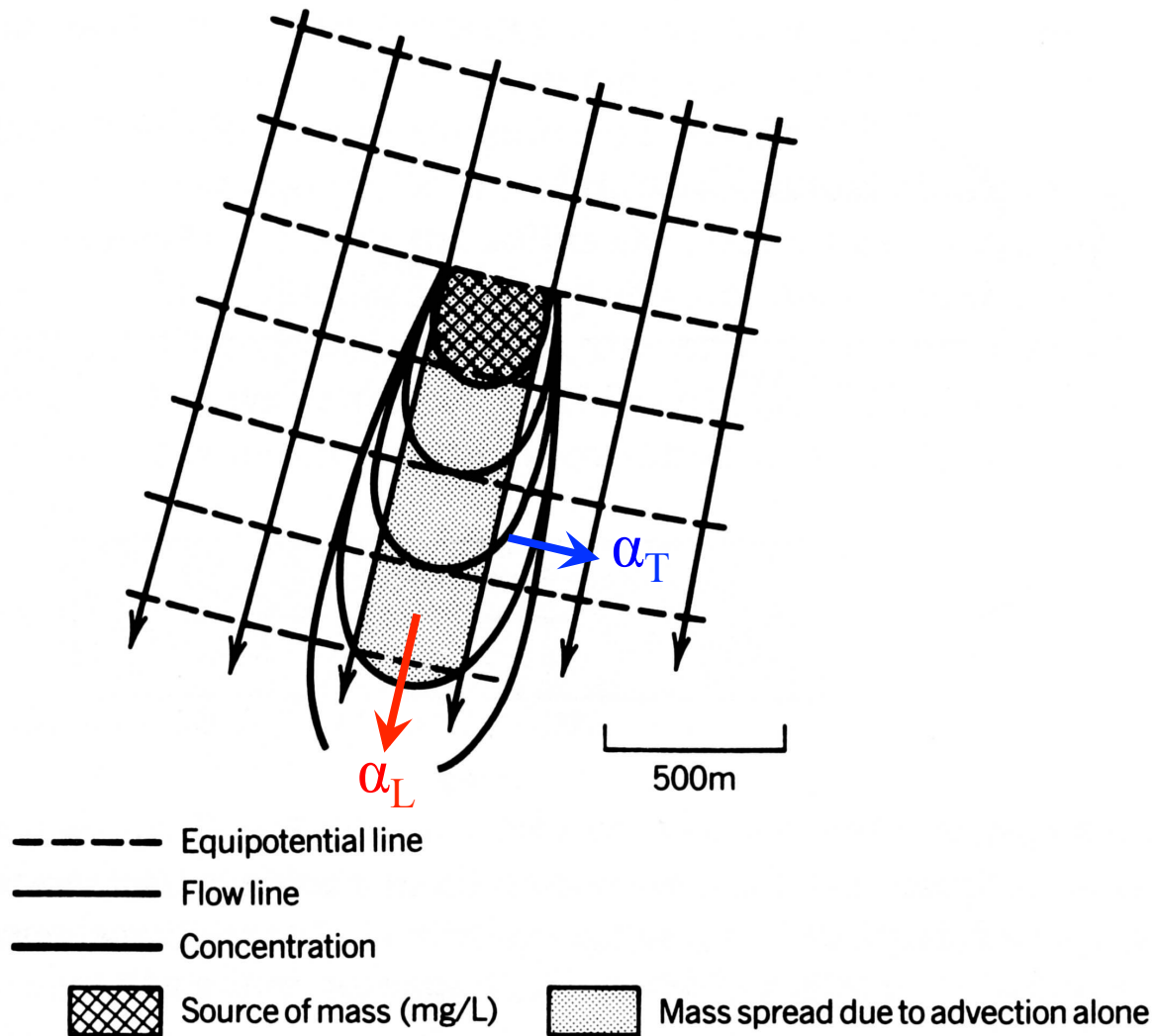
Mechanical dispersion is a hydraulic mixing process that causes solute spreading in three dimensions. If we align our coordinate system with the direction of groundwater flow we can define:

### **Longitudinal dispersion**

- spreading parallel to the direction of groundwater flow

### **Transverse dispersion**

- spreading in the direction normal to groundwater flow
- much smaller than longitudinal dispersion
- can be different horizontally than vertically



Plan view of solute mass spreading by advection and dispersion  
 (Modified from Domenico and Schwartz, 1990)



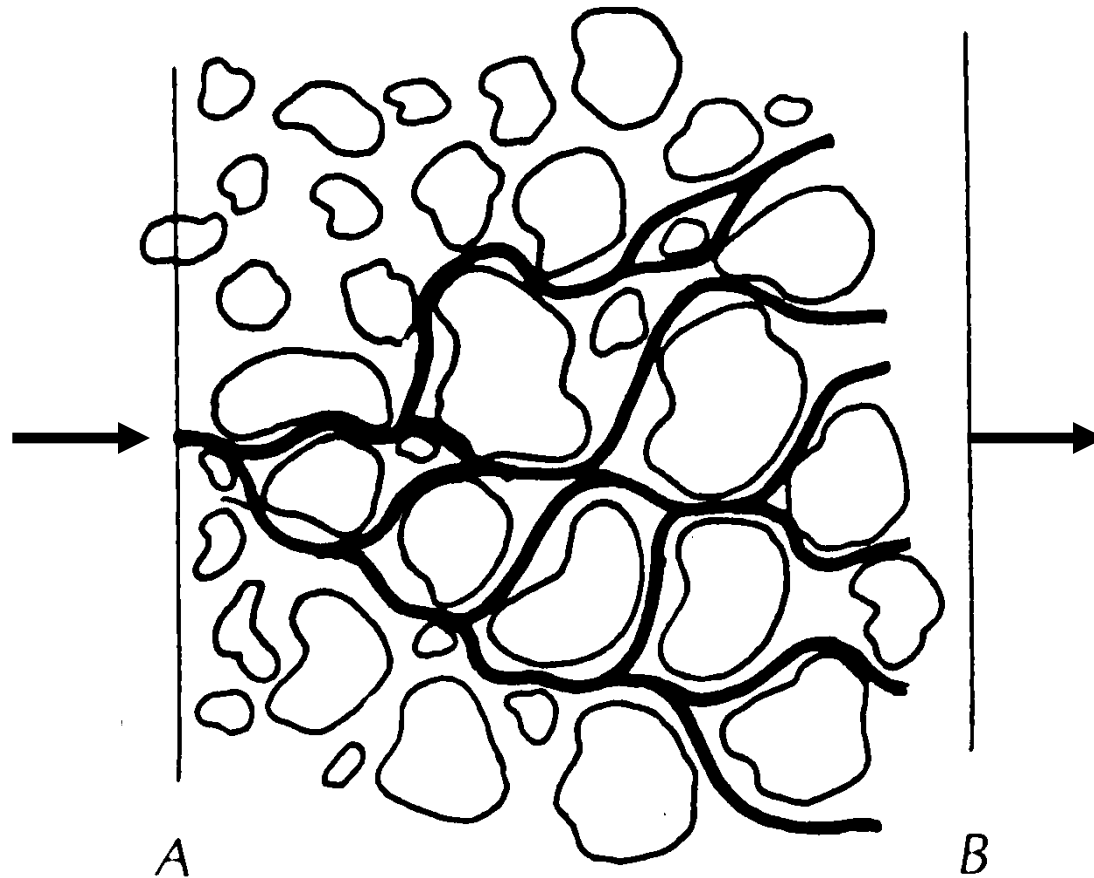
Since mechanical dispersion is directional, so is dispersivity.

- Longitudinal dispersivity -  $\alpha_L$
- Horizontal transverse dispersivity -  $\alpha_{TH}$
- Vertical transverse dispersivity -  $\alpha_{TV}$

\*As a rule of thumb, transverse dispersivity is much smaller than longitudinal, typically by a factor of 10 or 100.

However, few accurate studies of transverse dispersivity have been conducted and it is still not well understood.

Here is a classic representation of transverse dispersion. Does this make sense based on what we know about groundwater flow?

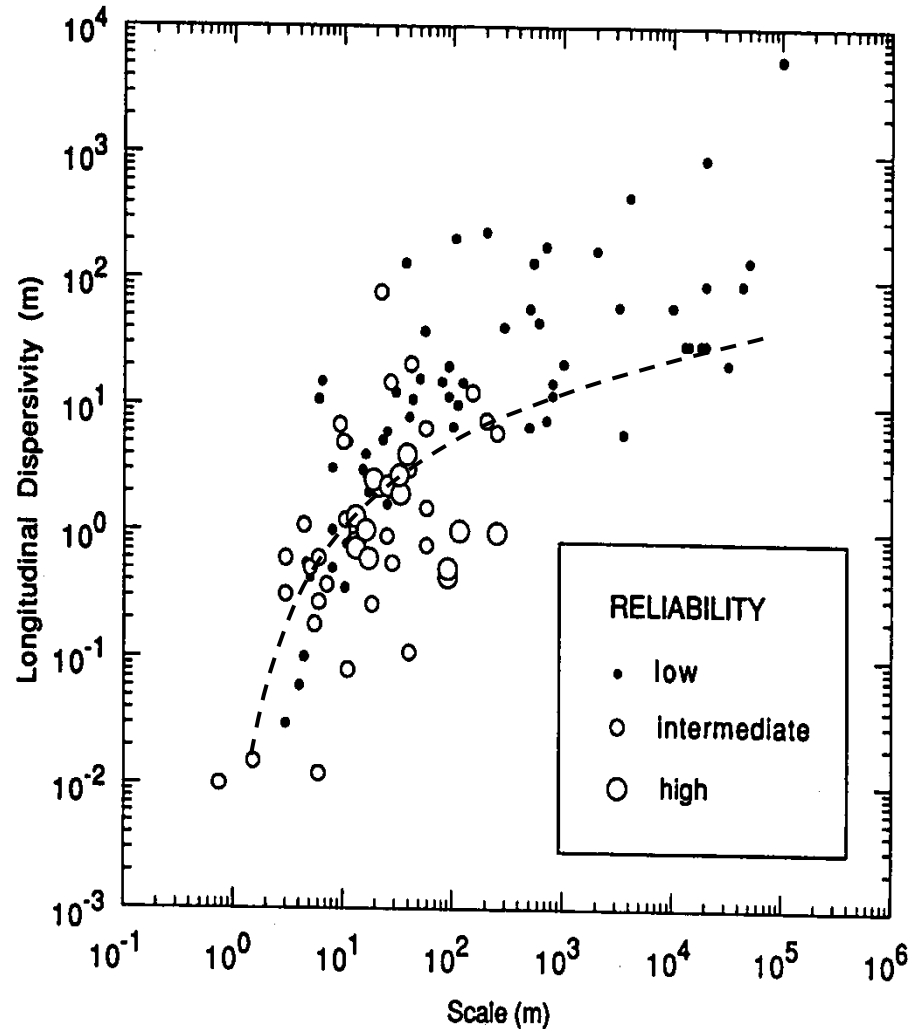


## Scale Effects of Dispersion

Studies have shown that dispersivity is scale dependent.

The diagram shows  $\alpha_L$ , but  $\alpha_T$  is also scale dependent.

(Fetter)



This scale dependence is not surprising given the underlying mechanisms that cause dispersion. Again, what is the main physical cause of dispersion?

The key implication is that geologic heterogeneity has a major influence on solute dispersion. As the size of the flow system under investigation increases, more and more geologic heterogeneity (i.e., velocity variations) are encountered, resulting in an apparent increase in dispersion. This is termed macrodispersion.

## Hydrodynamic Dispersion

In practical situations, it is nearly impossible to distinguish molecular diffusion from mechanical dispersion. Both are expressed as gradient laws and are generally combined together and referred to as hydrodynamic dispersion.

The equations for the longitudinal and transverse hydrodynamic dispersion coefficient are:

$$D_L = \alpha_L v + D^*$$

Longitudinal

$$D_T = \alpha_T v + D^*$$

Transverse

Hydrodynamic   =   Mechanical   +   Diffusion  
Dispersion        Dispersion

The resulting hydrodynamic dispersive mass flux (or just dispersive mass flux),  $J_H$ , can be written:

### **Hydrodynamic Dispersion Mass Flux**

$$J_H = -nD_i \frac{dC}{dx} = -n(\alpha_i v + D^*) \frac{dC}{dx}$$

where subscript ‘i’ refers to the direction in which the mass flux is being calculated (i.e., longitudinal or transverse).

## **Diffusion vs. Mechanical Dispersion**

When would you expect mechanical dispersion or diffusion to dominate the dispersive flux?

Under low flow velocities:

$$D_H = \alpha v + D^*$$

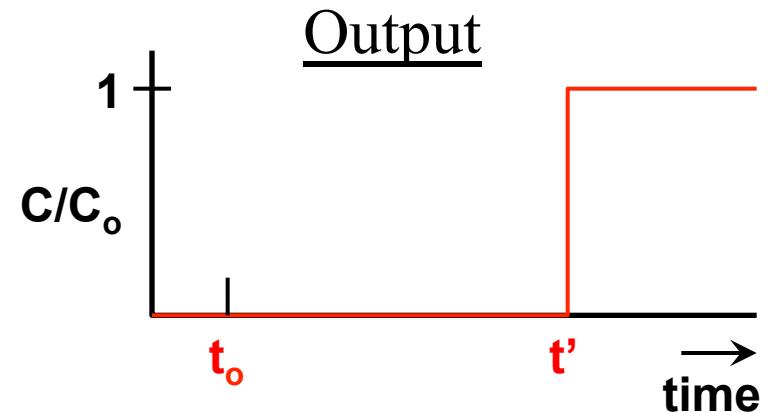
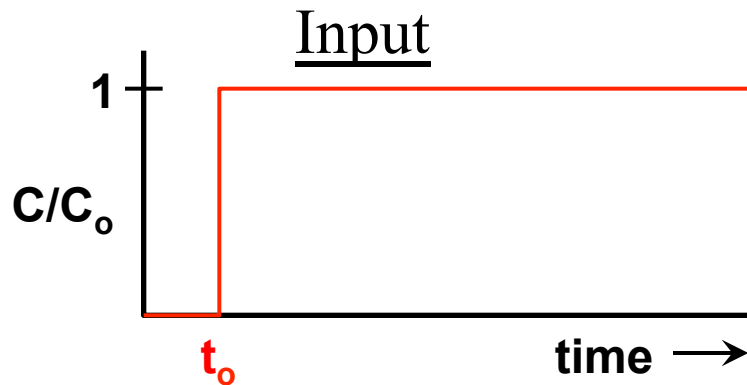
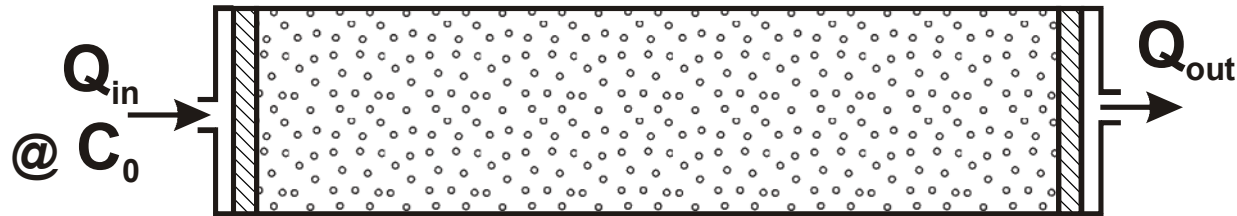
Under high flow velocities:

$$D_H = \alpha v + D^*$$

Larger  $D_H$  results in more mixing. In what situations would you expect to see more dispersion? Less dispersion?

## Transport in Uniform Media

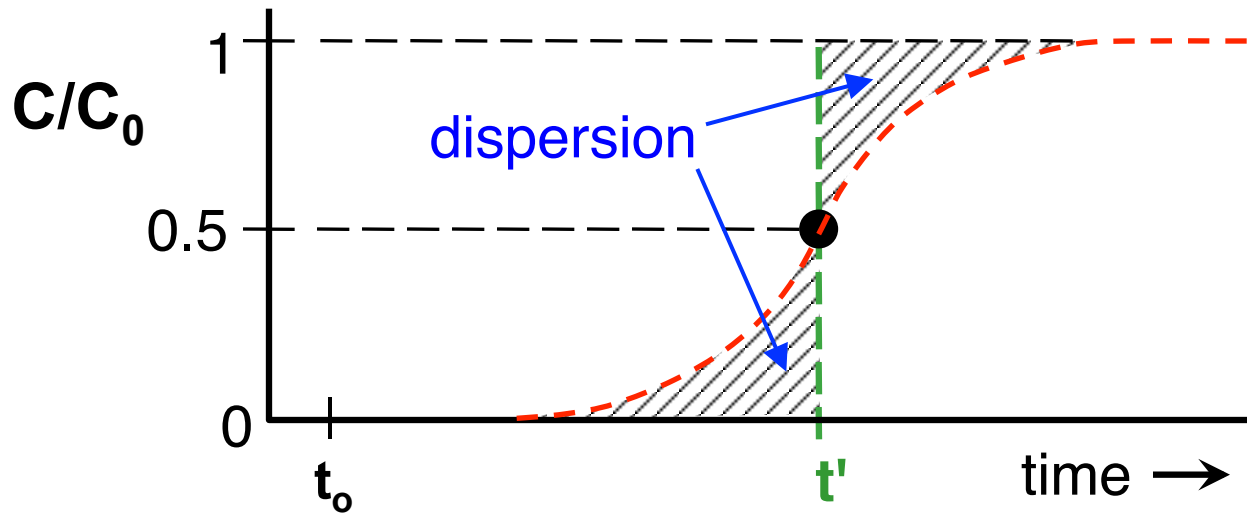
Consider 1-D transport in a homogeneous soil column with a continuous source input (similar to Darcy's experiment)



What is this called?



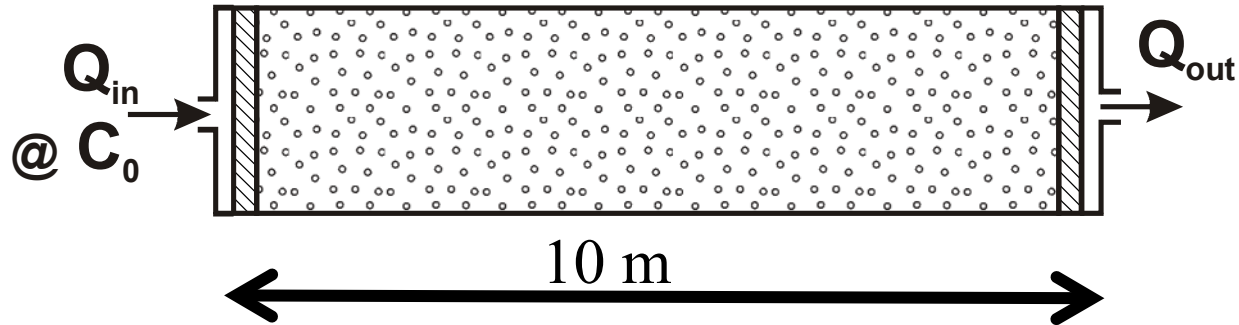
In reality, we observe a breakthrough curve at the outlet.



$C/C_0=0.5$  represents the center of solute mass. From it, we calculate average groundwater velocity.

$$v = \frac{d}{t} = \frac{\text{length of column}}{t' - t_0}$$

# Lets Model This

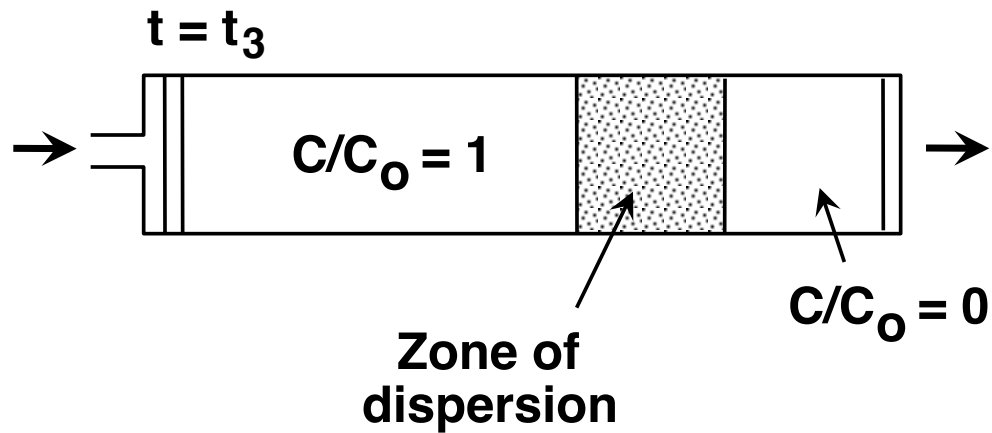
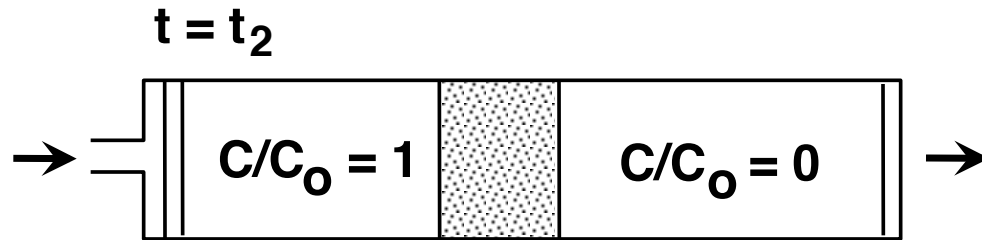
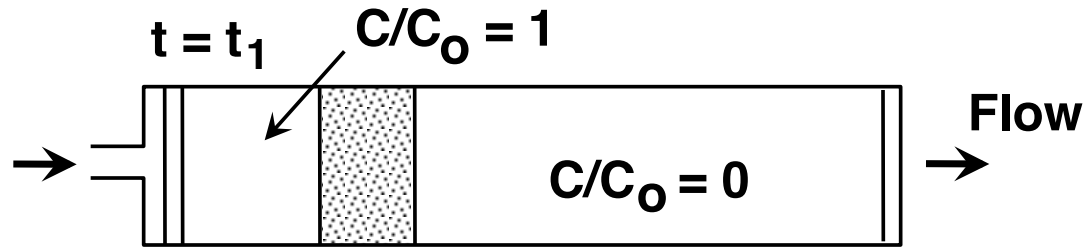


$$q = 0.5 \text{ m/s}$$

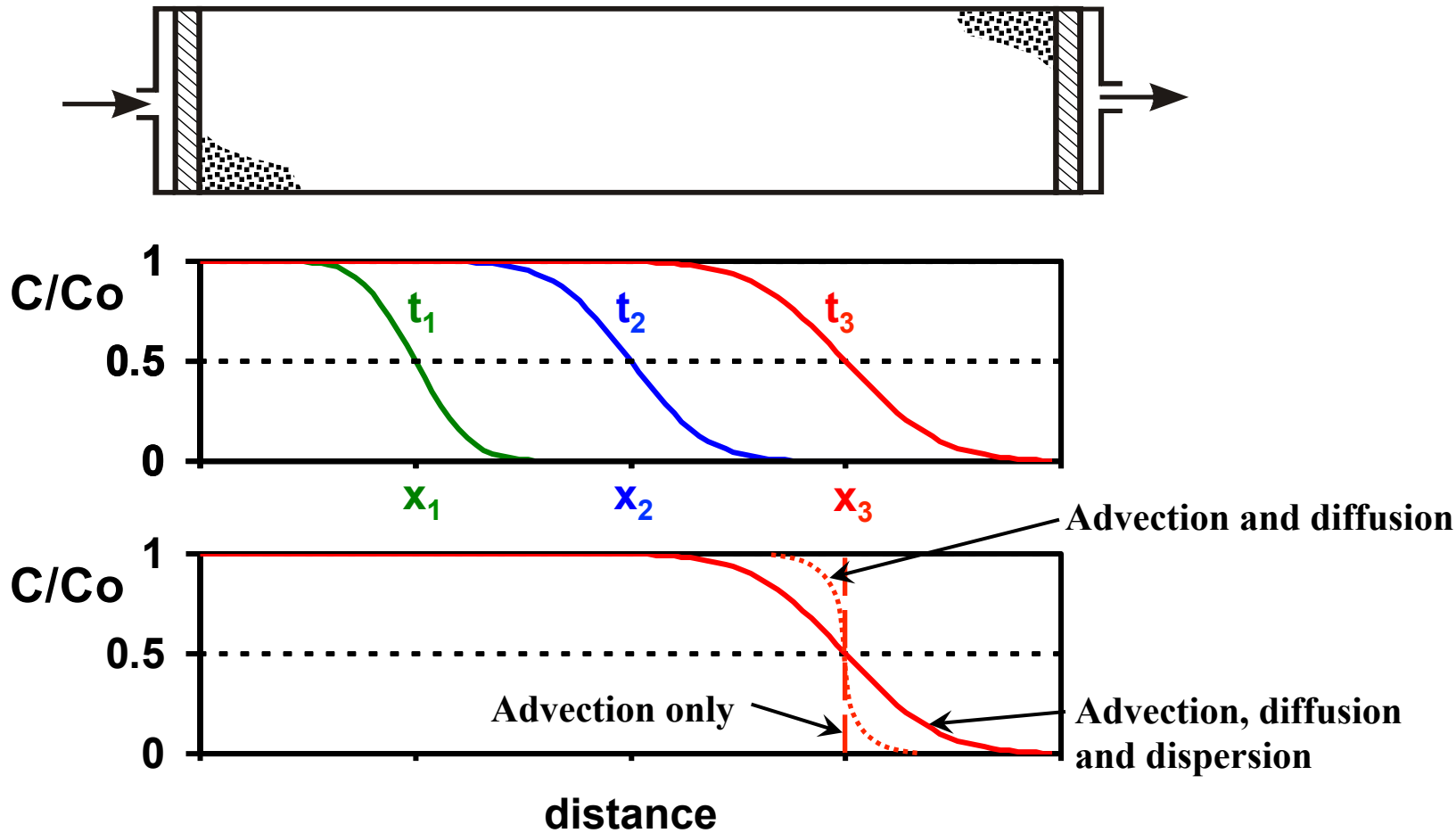
$$D = 0.1 \text{ m}^2/\text{s}$$

What BC's should we use?

How does the solute progress along the column?



Concentration profile of  $C/C_0$  vs distance.



## **Summary of Solute Transport Processes**

- Advection moves the contaminants at the bulk average groundwater velocity
- Hydrodynamic dispersion spreads the contaminants in all directions
- Spreading is greatest parallel to flow (longitudinal)
- Dispersion is due to variations in flow velocity

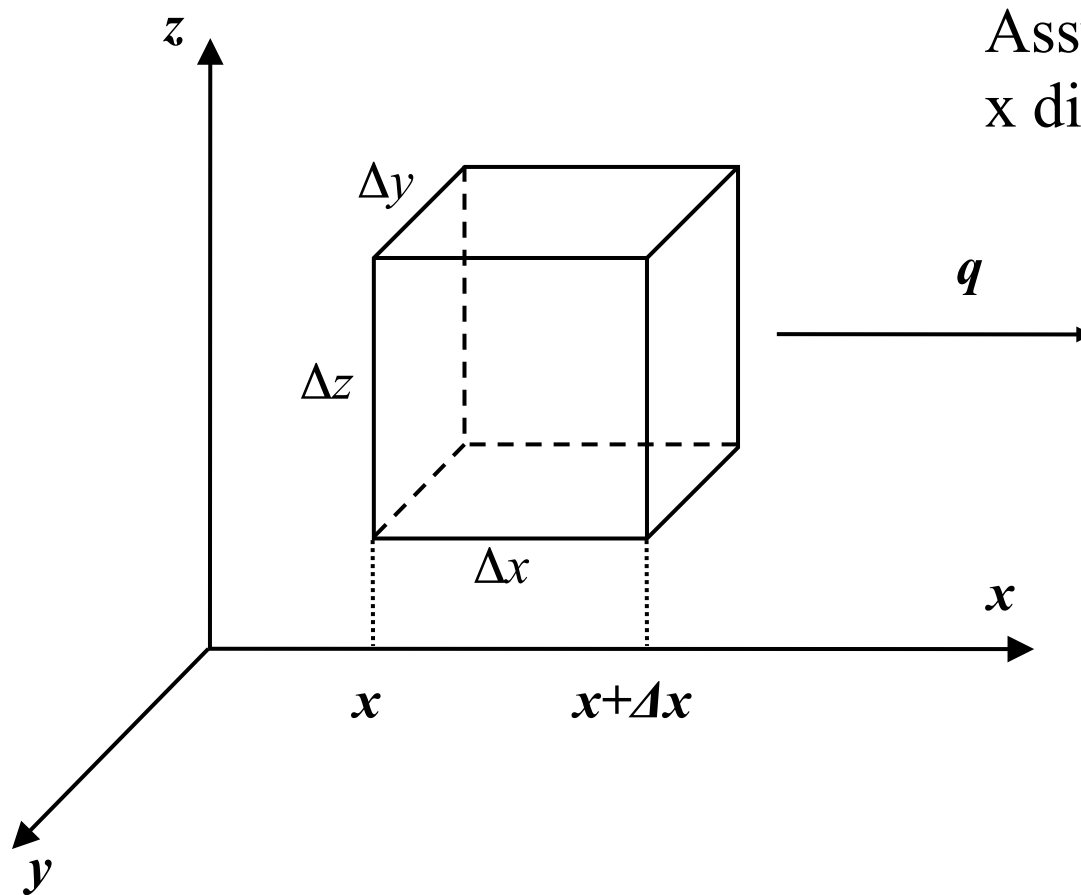
Remember, we have only considered steady-state flow. Transport is a transient process occurring within a steady groundwater flow system.

## Advection-Dispersion Equation

(Suggest reading Freeze and Cherry, p.549-553 for more details)

By applying the Law of Mass Conservation, we can derive the advection dispersion equation for an elementary volume.

$$\begin{array}{ccccccc} \text{Mass} & & \text{Mass} & & \text{Loss or} & & \text{Net rate of} \\ \text{flux in} & - & \text{flux out} & \pm & \text{gain of} & = & \text{change} \\ & & & & \text{solute} & & \text{of solute} \\ & & & & \text{mass} & & \text{mass} \end{array}$$



Assume flow is in the  $x$  direction only.

Let's write the mass conservation equation for a one-dimensional system ( $x$ -direction).

$$\begin{aligned}\text{Mass entering element at } x: &= \textit{Solute flux} \times \textit{Area} \\ &= J_x \Delta y \Delta z\end{aligned}$$

$$\begin{aligned}\text{Mass leaving element at } x+\Delta x: &= J_{x+\Delta x} \Delta y \Delta z \\ &= \left( J_x + \frac{\partial J_x}{\partial x} \Delta x \right) \Delta y \Delta z\end{aligned}$$

$$\begin{aligned}\text{Net rate of change of mass in element with time:} \\ &= \frac{\partial}{\partial t} (nC \Delta x \Delta y \Delta z)\end{aligned}$$

$$\begin{aligned}\text{Mass produced or consumed in the element:} \\ &= \pm G \cdot \Delta x \Delta y \Delta z\end{aligned}$$

$$\text{where } G = \frac{\text{mass produced or consumed by reactions}}{\text{unit volume p.m.} \times \text{time}}$$



Now substitute into the mass conservation equation and divide through by  $\Delta x \Delta y \Delta z$ .

$$J_x \Delta y \Delta z - \left( J_x + \frac{\partial J_x}{\partial x} \Delta x \right) \Delta y \Delta z \pm G \Delta x \Delta y \Delta z = \frac{\partial}{\partial t} (nC \Delta x \Delta y \Delta z)$$

$$- \frac{\partial J_x}{\partial x} \pm G = n \frac{\partial C}{\partial t}$$

Because n is constant: $\frac{\partial}{\partial t} (nC) = n \frac{\partial C}{\partial t}$
--

Remember our earlier flux equations.

$$J_x = \text{Advective flux} + \text{Dispersive flux}$$

$$= n v C - n D_L \frac{\partial C}{\partial x}$$

We can now substitute  $J_x$  into the equation.

$$-\frac{\partial}{\partial x} \left( n\nu C - nD_L \frac{\partial C}{\partial x} \right) \pm G = n \frac{\partial C}{\partial t}$$

If saturated and  $n$  is constant, divide both sides by  $n$ :

$$-\frac{\partial}{\partial x} \left( \nu C - D_L \frac{\partial C}{\partial x} \right) \pm \frac{G}{n} = \frac{\partial C}{\partial t}$$

$$-\frac{\partial}{\partial x} (\nu C) + \frac{\partial}{\partial x} \left( D_L \frac{\partial C}{\partial x} \right) \pm \frac{G}{n} = \frac{\partial C}{\partial t}$$

In homogeneous medium where  $\nu$  and  $D_L$  are uniform in space:

$$\boxed{-\nu \frac{\partial C}{\partial x} + D_L \frac{\partial^2 C}{\partial x^2} \pm \frac{G}{n} = \frac{\partial C}{\partial t}}$$

**1-D Advection  
Dispersion Equation**

If we keep flow parallel to the x-axis:

$$\boxed{D_L \frac{\partial^2 C}{\partial x^2} + D_T \frac{\partial^2 C}{\partial y^2} - v \frac{\partial C}{\partial x} \pm \frac{G}{n} = \frac{\partial C}{\partial t}}$$

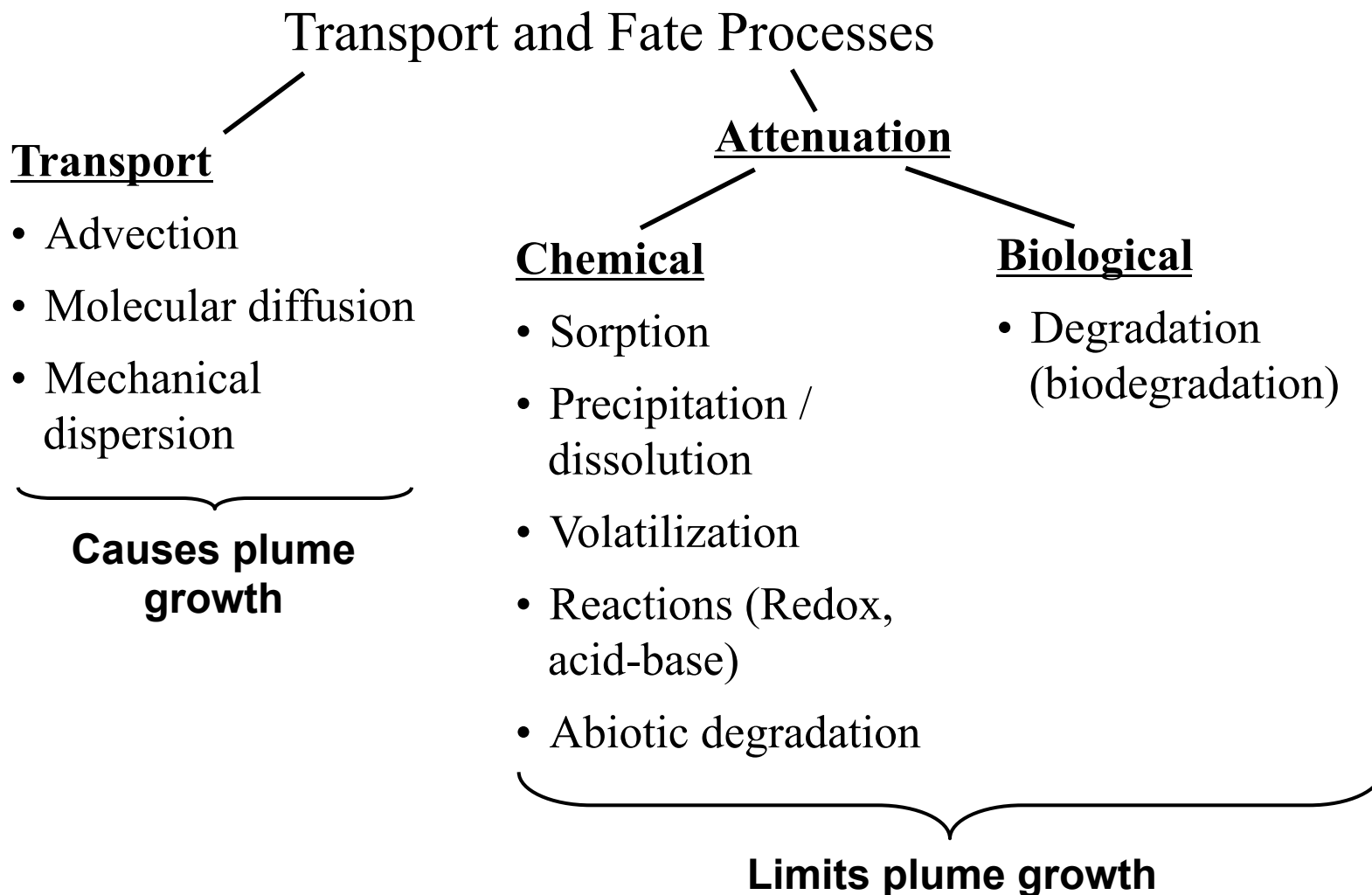
**2-D Advection  
Dispersion Equation**

Assumptions implicit in equations as written:

- saturated porous medium
- groundwater velocity is uniform and steady
- flow is aligned with x-direction
- dilute solution (no density effects)

For an ideal, non-reactive tracer (e.g.,  $\text{Cl}^-$ )  $G = 0$ .

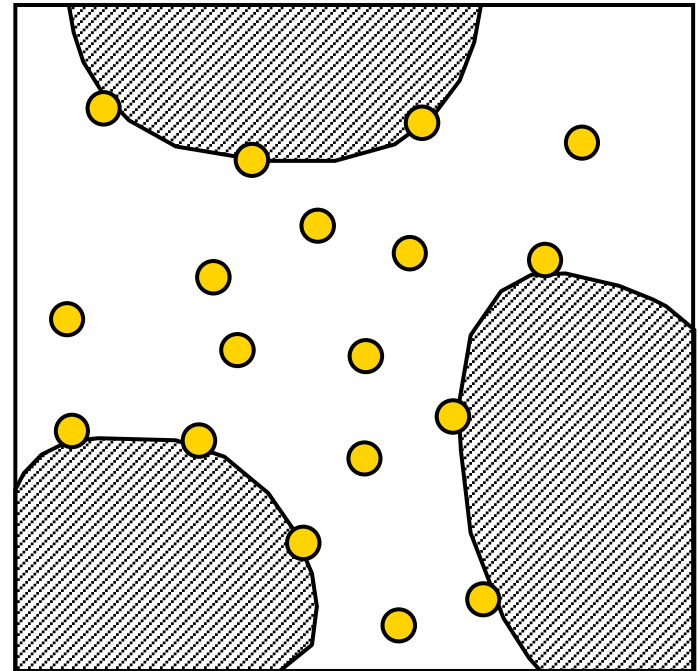
# Transport of Reactive Solutes



## Sorption and Retardation

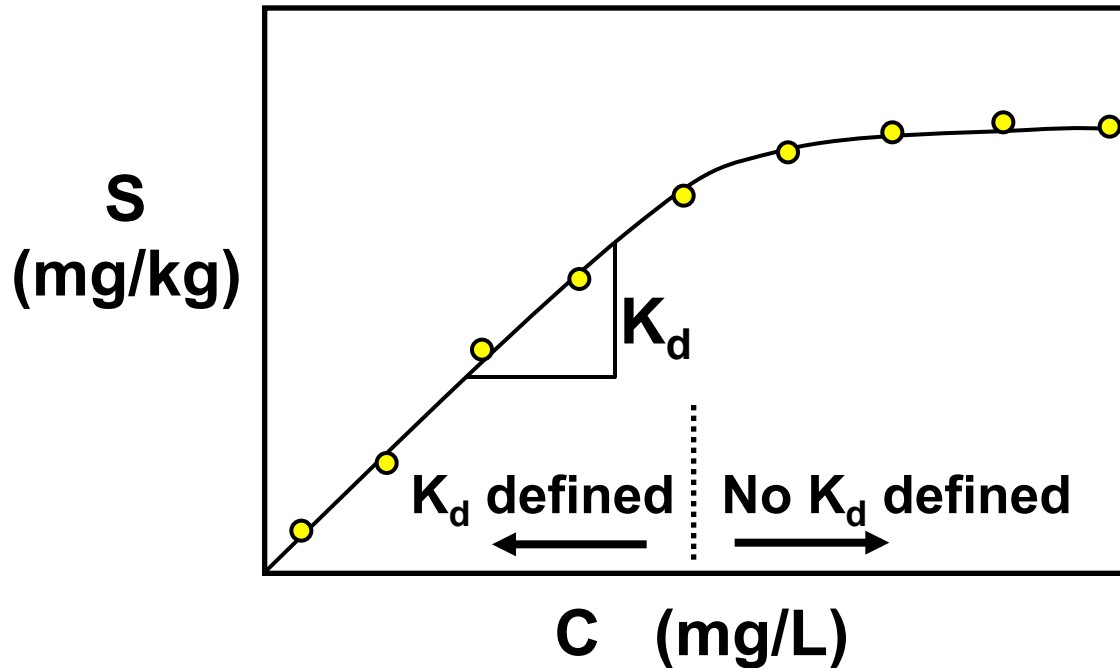
Solutes are either dissolved in water or attached to particle surfaces. Sorption refers to the process where solute molecules attach to the surface of solid particles in a porous medium.

Solutes partition between the solid and aqueous phases. The relationship between the concentration of solute in aqueous solution ( $C$ ) to the mass sorbed on the solid surfaces ( $S$ ) is called an isotherm.



If isotherm is linear we can define a **Distribution Coefficient**

$$K_d = dS/dC = \text{Slope of linear portion of isotherm}$$



$$K_d = \frac{dS}{dC} = \frac{\text{Mass Solute Adsorbed/Mass of Solids}}{\text{Mass Solute Dissolved/Volume of Solution}} \quad (\text{L}^3/\text{M})$$

We will define the **Retardation Factor, R**, as:

$$R \equiv \left( 1 + \frac{\rho_b}{n} K_d \right)$$

and the A-D equation becomes:

$$\frac{D_L}{R} \frac{\partial^2 C}{\partial x^2} - \frac{v}{R} \frac{\partial C}{\partial x} = \frac{\partial C}{\partial t}$$

**1-D Advection  
Dispersion Equation  
with Linear Sorption**

**\*Important Note:**

We do not need to change the form of the A-D equation. We can simply replace  $D_L$  with  $D_L/R$  and  $v$  with  $v/R$ . This applies to all forms of the AD equation and its myriad solutions.

## What does R mean physically?

The retardation factor provides a measure of the mean velocity of a reactive contaminant,  $v_c$ , relative to the mean velocity of a non-reactive contaminant (or average groundwater velocity),  $v$ .

Sorption (or retardation) slows the advection of the contaminant and reduces the dispersion of the contaminant.

$$v_c = v / R$$

$$D_c = D_L / R$$

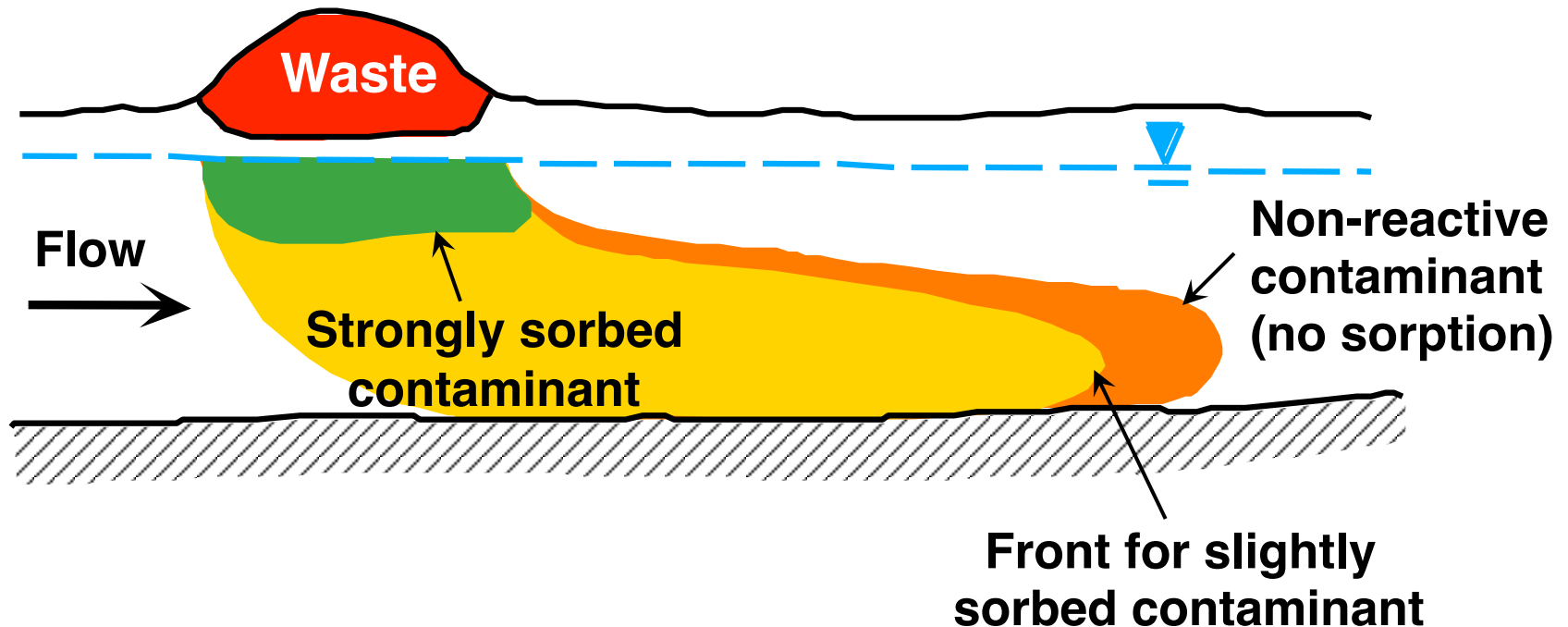


## **Effect of Sorption on Plumes**

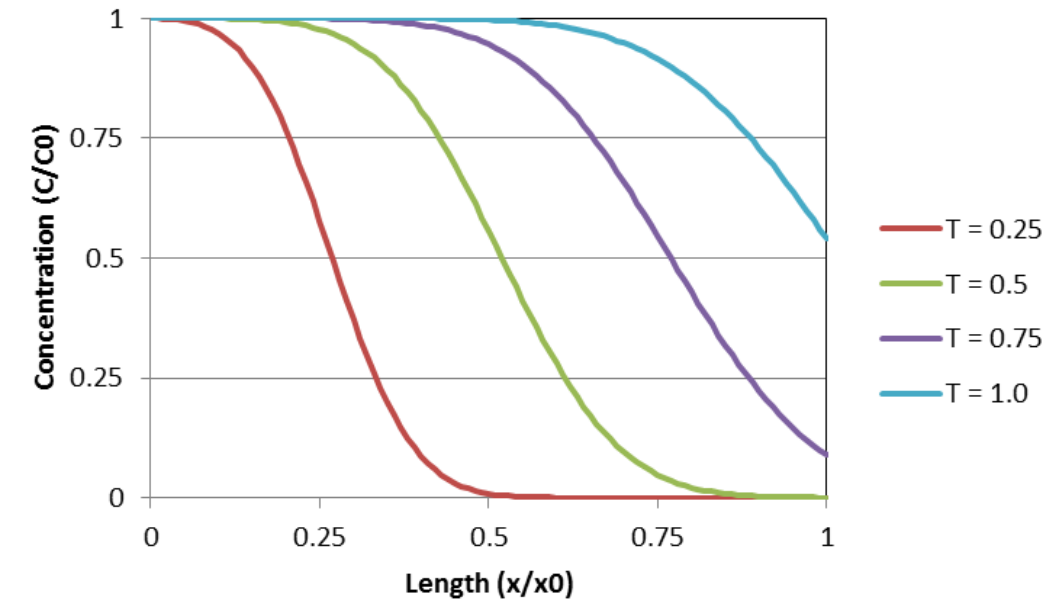
- Sorption in reality can be reversible, partly reversible, or non-reversible
- Contaminants undergoing sorption will travel more slowly than non-reactive contaminants in a plume
  - this is termed **retardation**.
- Strong sorption can cause a contaminant to be immobile over a relevant time scale, such as many decades or centuries

## Effect of Sorption on Plumes

- each contaminant has its own particular sorption affinity (i.e.,  $K_d$  or R value) and therefore will move at its own rate



# Analytical Solution to the 1-D Advection-Diffusion Equation: Ogata-Banks 1-D Transport Solution (Ogata 1970)



Concentration Distribution Profile  
Solved with the Ogata-Banks Solution.

Velocity = 1.0

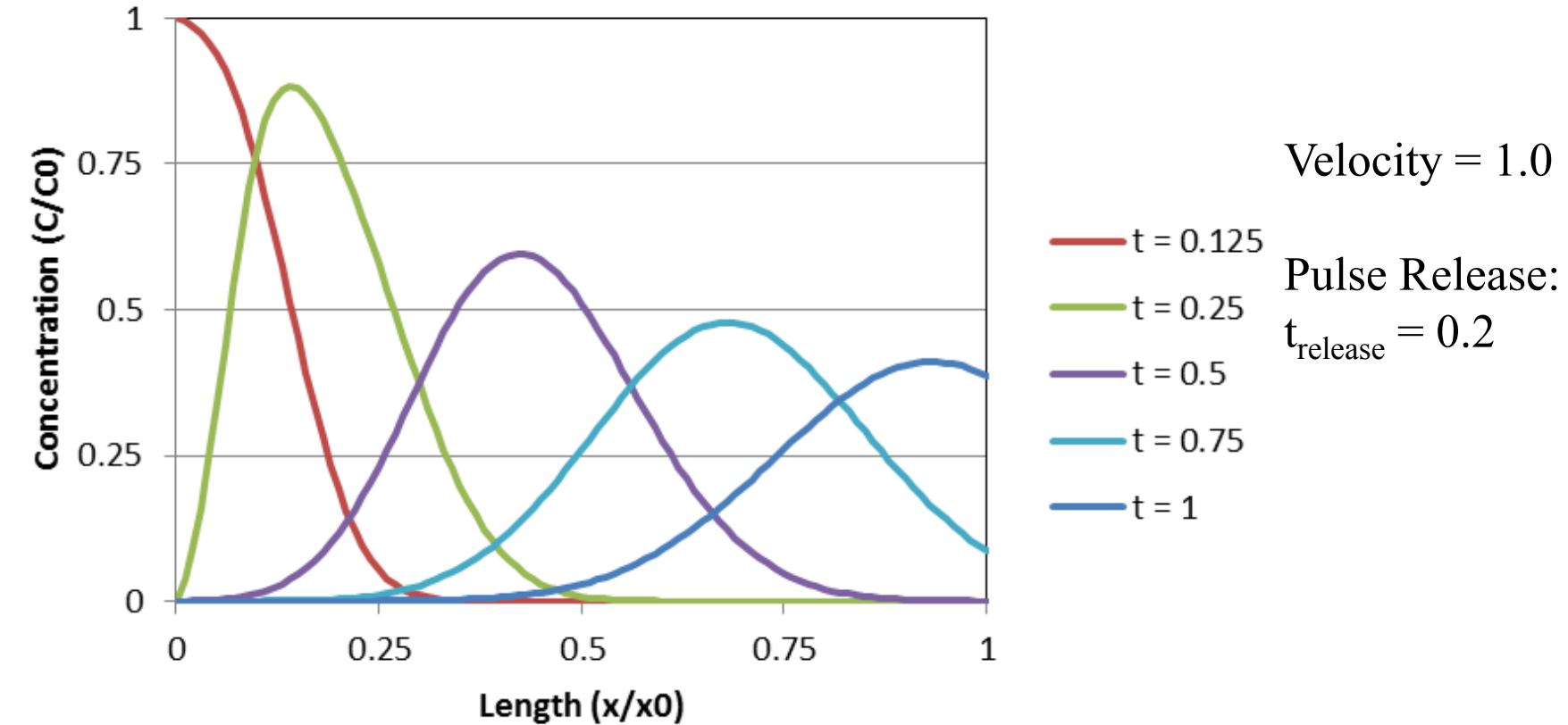
Left hand boundary is constant  
concentration

Ogata-Banks Equation

$$C = \frac{C_0}{2} \left[ \operatorname{erfc} \left( \frac{L - v_x t}{2\sqrt{D_L t}} \right) + \exp \left( \frac{v_x L}{D_l} \right) \operatorname{erfc} \left( \frac{L + v_x t}{2\sqrt{D_L t}} \right) \right]$$

# Ogata-Banks 1-D Transport for Pulse Release

## Concentration Distribution Profile



$$C = \frac{C_0}{2} \left[ \operatorname{erfc} \left( \frac{L - v_x t}{2\sqrt{D_L t}} \right) + \exp \left( \frac{v_x L}{D_l} \right) \operatorname{erfc} \left( \frac{L + v_x t}{2\sqrt{D_L t}} \right) \right] - \frac{C_0}{2} \left[ \operatorname{erfc} \left( \frac{L - v_x (t_{\text{release}} - t)}{2\sqrt{D_L t}} \right) + \exp \left( \frac{v_x L}{D_l} \right) \operatorname{erfc} \left( \frac{L + v_x (t_{\text{release}} - t)}{2\sqrt{D_L t}} \right) \right]$$

**Error Function** (also called the Gauss error function) is encountered in integrating the normal distribution.

Error Function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Complementary error function:

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$

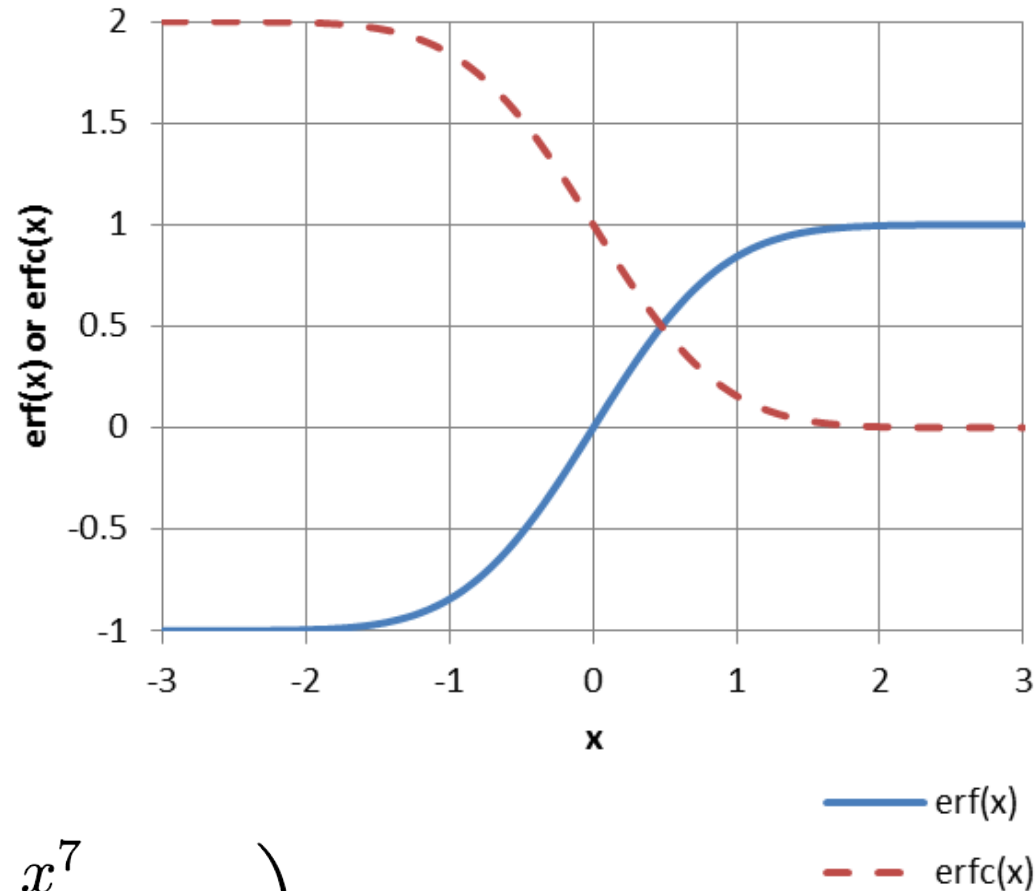
$$\operatorname{erfc}(-x) = 2 - \operatorname{erfc}(x)$$

Error function approximated as:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right)$$

Complementary error function approximated as:

$$\operatorname{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right)$$



# Error Function and Complimentary Error Function

x	erf(x)	erfc(x)		x	erf(x)	erfc(x)		x	erf(x)	erfc(x)		x	erf(x)	erfc(x)
-3	-1	1.99998		-1.5	-0.9661	1.96611		0	0	1		1.5	0.96611	0.03389
-2.9	-1	1.99996		-1.4	-0.9523	1.95229		0.1	0.11246	0.88754		1.6	0.97635	0.02365
-2.8	-0.9999	1.99992		-1.3	-0.934	1.93401		0.2	0.2227	0.7773		1.7	0.98379	0.01621
-2.7	-0.9999	1.99987		-1.2	-0.9103	1.91031		0.3	0.32863	0.67137		1.8	0.98909	0.01091
-2.6	-0.9998	1.99976		-1.1	-0.8802	1.88021		0.4	0.42839	0.57161		1.9	0.99279	0.00721
-2.5	-0.9996	1.99959		-1	-0.8427	1.8427		0.5	0.5205	0.4795		2	0.99532	0.00468
-2.4	-0.9993	1.99931		-0.9	-0.7969	1.79691		0.6	0.60386	0.39614		2.1	0.99702	0.00298
-2.3	-0.9989	1.99886		-0.8	-0.7421	1.7421		0.7	0.6778	0.3222		2.2	0.99814	0.00186
-2.2	-0.9981	1.99814		-0.7	-0.6778	1.6778		0.8	0.7421	0.2579		2.3	0.99886	0.00114
-2.1	-0.997	1.99702		-0.6	-0.6039	1.60386		0.9	0.79691	0.20309		2.4	0.99931	0.00069
-2	-0.9953	1.99532		-0.5	-0.5205	1.5205		1	0.8427	0.1573		2.5	0.99959	0.00041
-1.9	-0.9928	1.99279		-0.4	-0.4284	1.42839		1.1	0.88021	0.11979		2.6	0.99976	0.00024
-1.8	-0.9891	1.98909		-0.3	-0.3286	1.32863		1.2	0.91031	0.08969		2.7	0.99987	0.00013
-1.7	-0.9838	1.98379		-0.2	-0.2227	1.2227		1.3	0.93401	0.06599		2.8	0.99992	7.5E-05
-1.6	-0.9763	1.97635		-0.1	-0.1125	1.11246		1.4	0.95229	0.04771		2.9	0.99996	4.1E-05
												3	0.99998	2.2E-05

The error function can be analytically approximated within 0.7% by the following equation:

$$\text{erf}(x) = \sqrt{1 - \exp \frac{-4x^2}{\pi}}$$

## **Applications of the ADE**

We can use our understanding of transport processes and the advection-dispersion equation to gain insights into many important environmental and geological problems.

Let's examine some potential scenarios and consider what factors we would consider and what equations might be applicable.

# (1) Landfill leachate

When does is  $C/C_0 = 0.1$  at the well?

$L = 100\text{m}$

$K = 10\text{E-}5 \text{ m/s}$

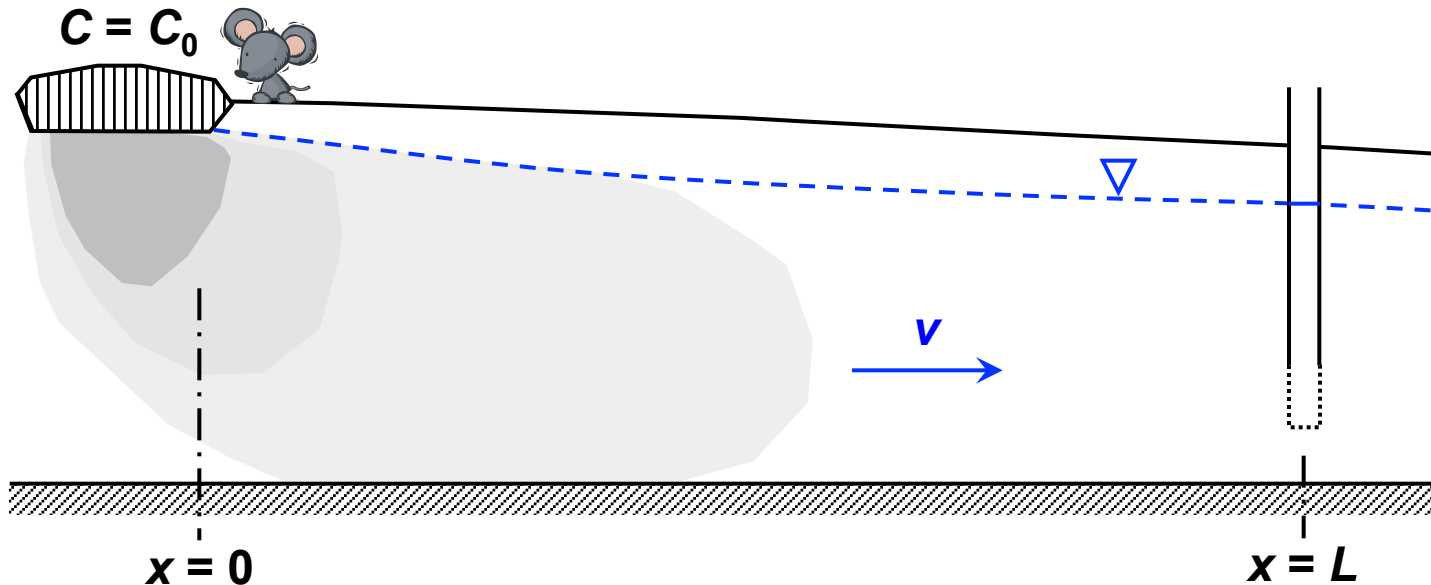
$dh/dx = -0.01$

$\alpha_L = 0.25 \text{ m}$

$D^* = 1\text{E-}9 \text{ m}^2/\text{s}$

$n = 0.25$

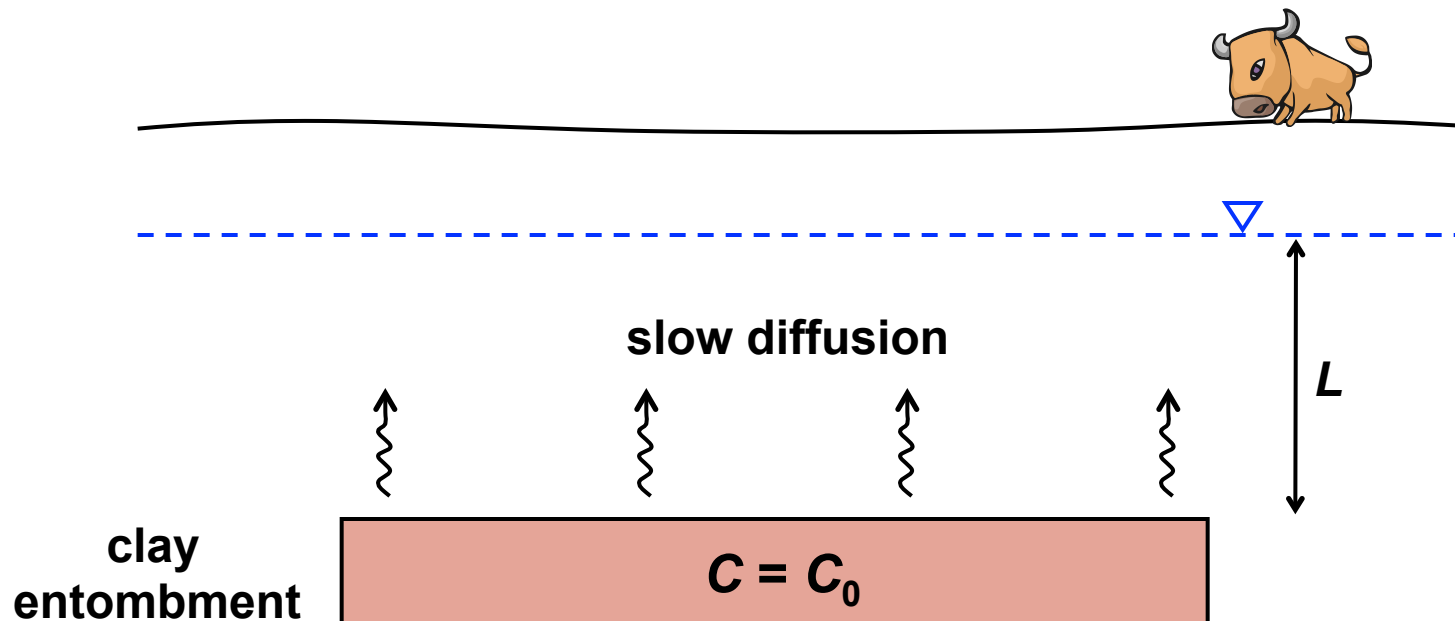
$$D_L = \alpha_L v + D^*$$





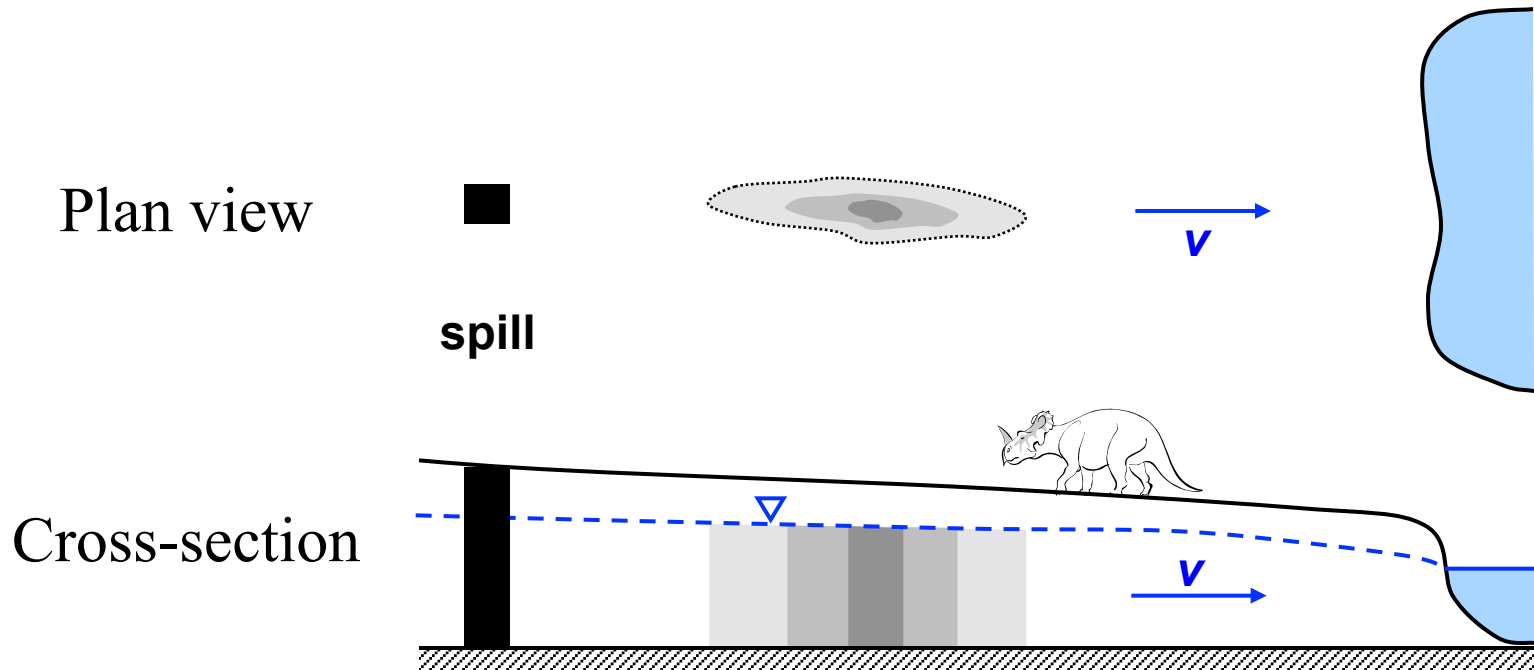
## (2) Nuclear waste disposal

How long will it take to have  $C/C_0 = 0.001$  near the water table? How does it compare to the half life of  $^{226}\text{Ra}$  ( $t_{1/2} = 1600$  yr)? Note that  $v = 0$  in this problem.



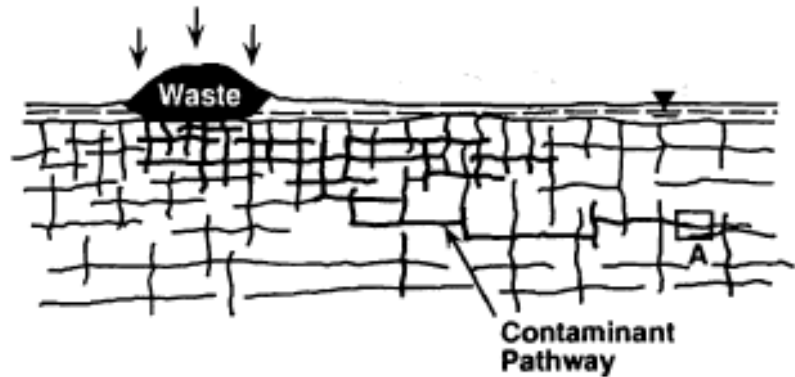
### (3) Chemical spill

A spill of pesticide generated a plume that is being transported in an aquifer toward a lake. When the plume reaches the lake, what is the concentration of pesticide?

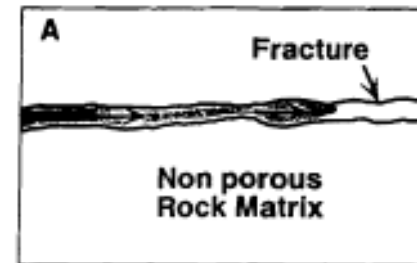


## PLUME IN FRACTURED NON - POROUS ROCK

- Advection occurs in the fracture network
- Diffusion into rock matrix is insignificant



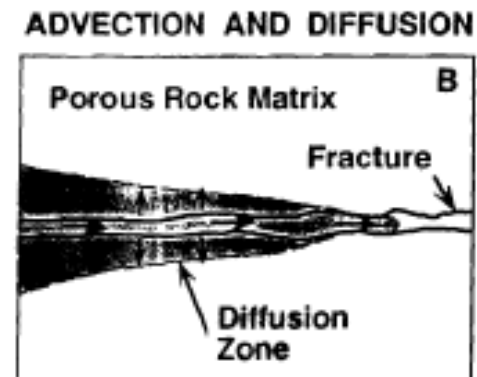
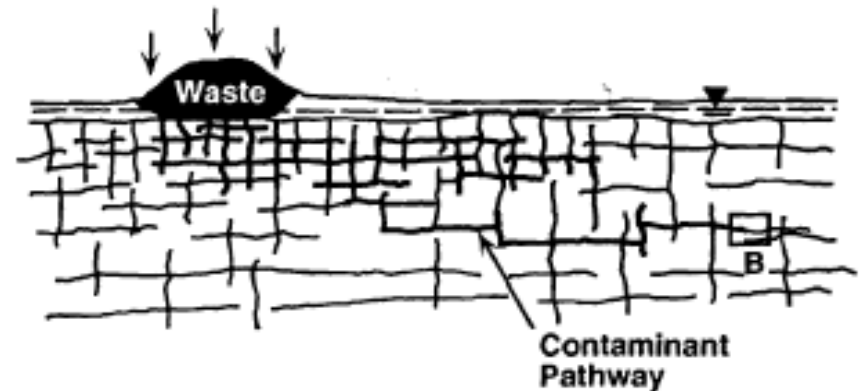
ADVECTION IN FRACTURES



(Cherry)

# PLUME IN FRACTURED POROUS ROCK

- Contaminant migration governed by advection in fractures and diffusion in matrix.
- Mass transfer from the fracture into the matrix due to diffusion causes retardation of the contaminant front relative to groundwater velocity
- Transport processes in fractured clay are similar



(Cherry)