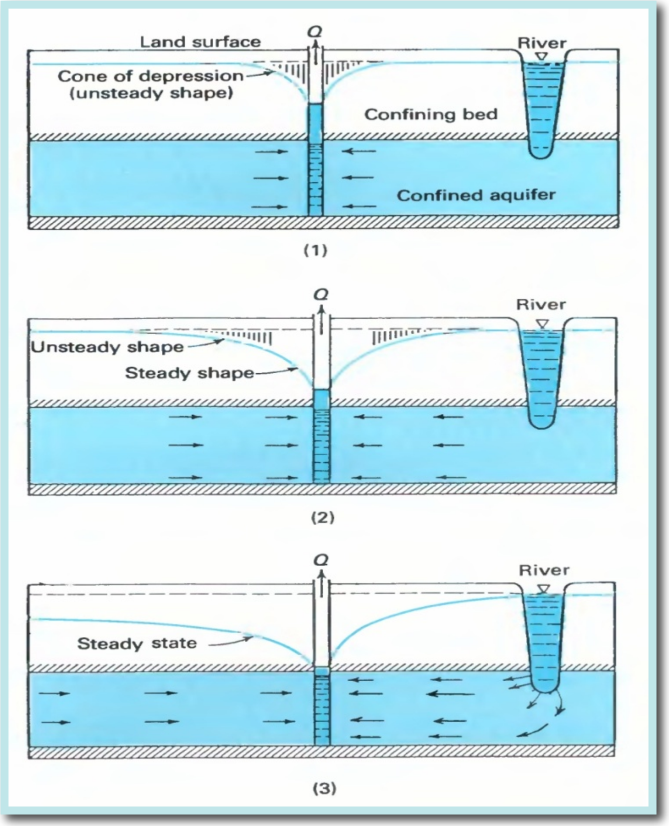
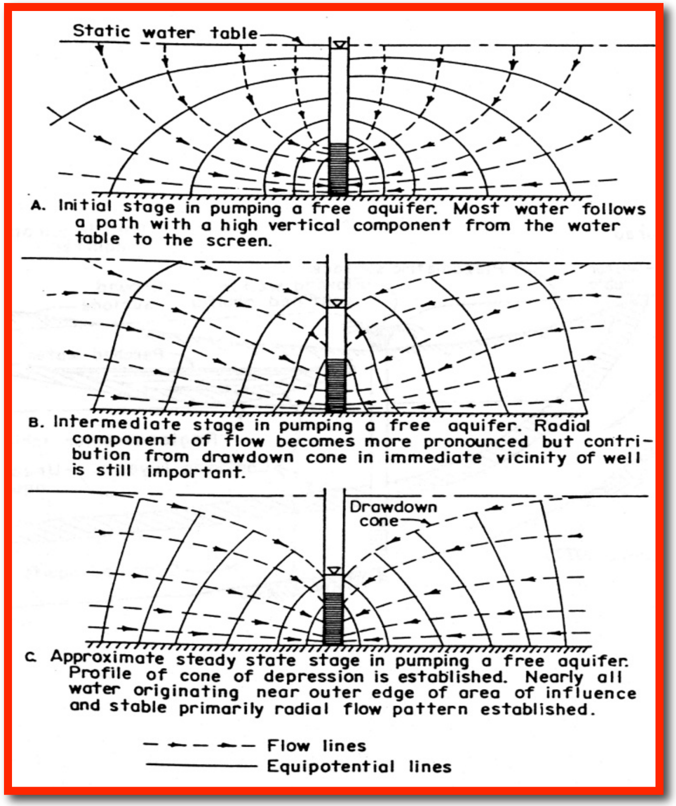
Steady and Transient Groundwater Flow

Recommended Readings: Schwartz and Zhang Chapter 3, 4, and 5



Transient flow in a confined aquifer



Transient flow in an unconfined aquifer

Groundwater Flow Nets

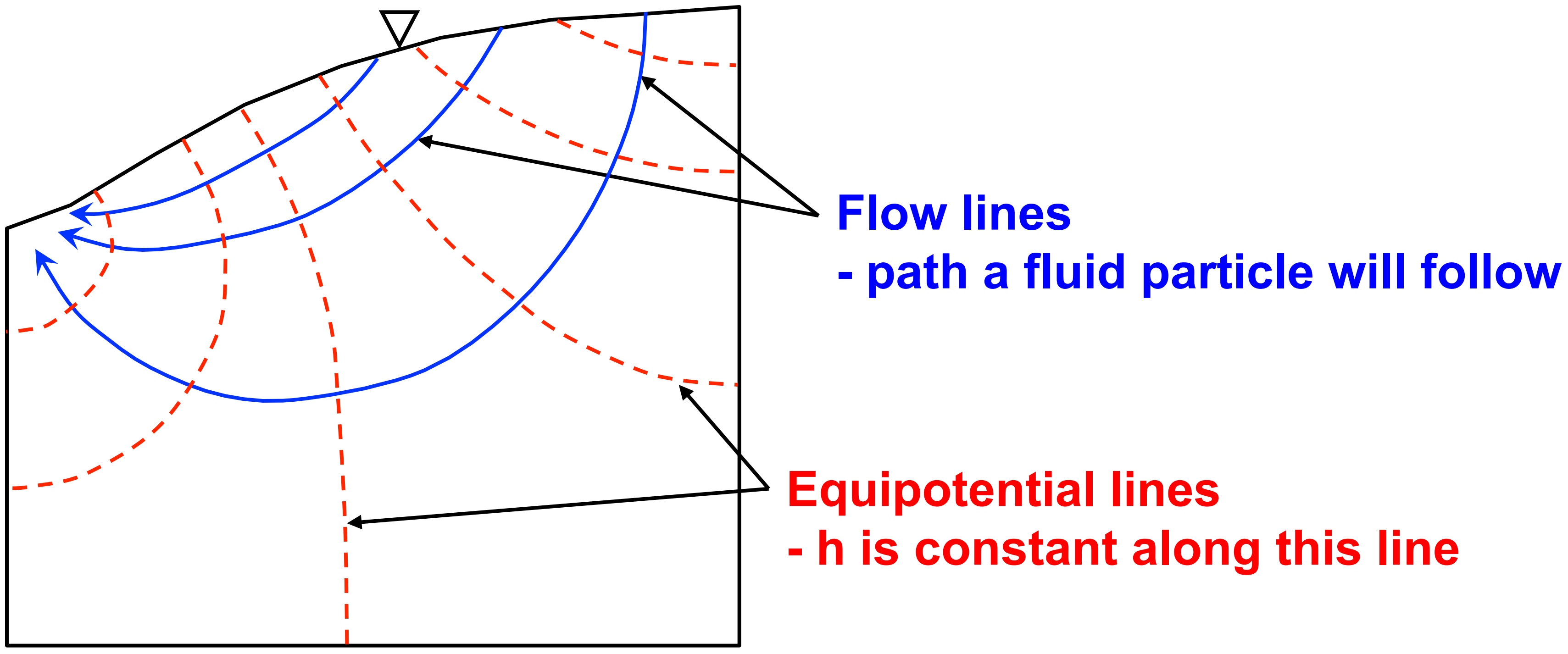
In groundwater flow systems we would like to know the direction of flow or the hydraulic head everywhere in the formation.

For *steady-state* flow conditions, we can graphically represent flow in a 2-dimensional system using a set of intersecting flowlines and equipotential lines. This is termed a flow net.

In reality, a flow net is actually a graphical solution of the steady state groundwater flow equation.

Flow lines (or streamlines): indicate the path a water particle will follow as it moves.

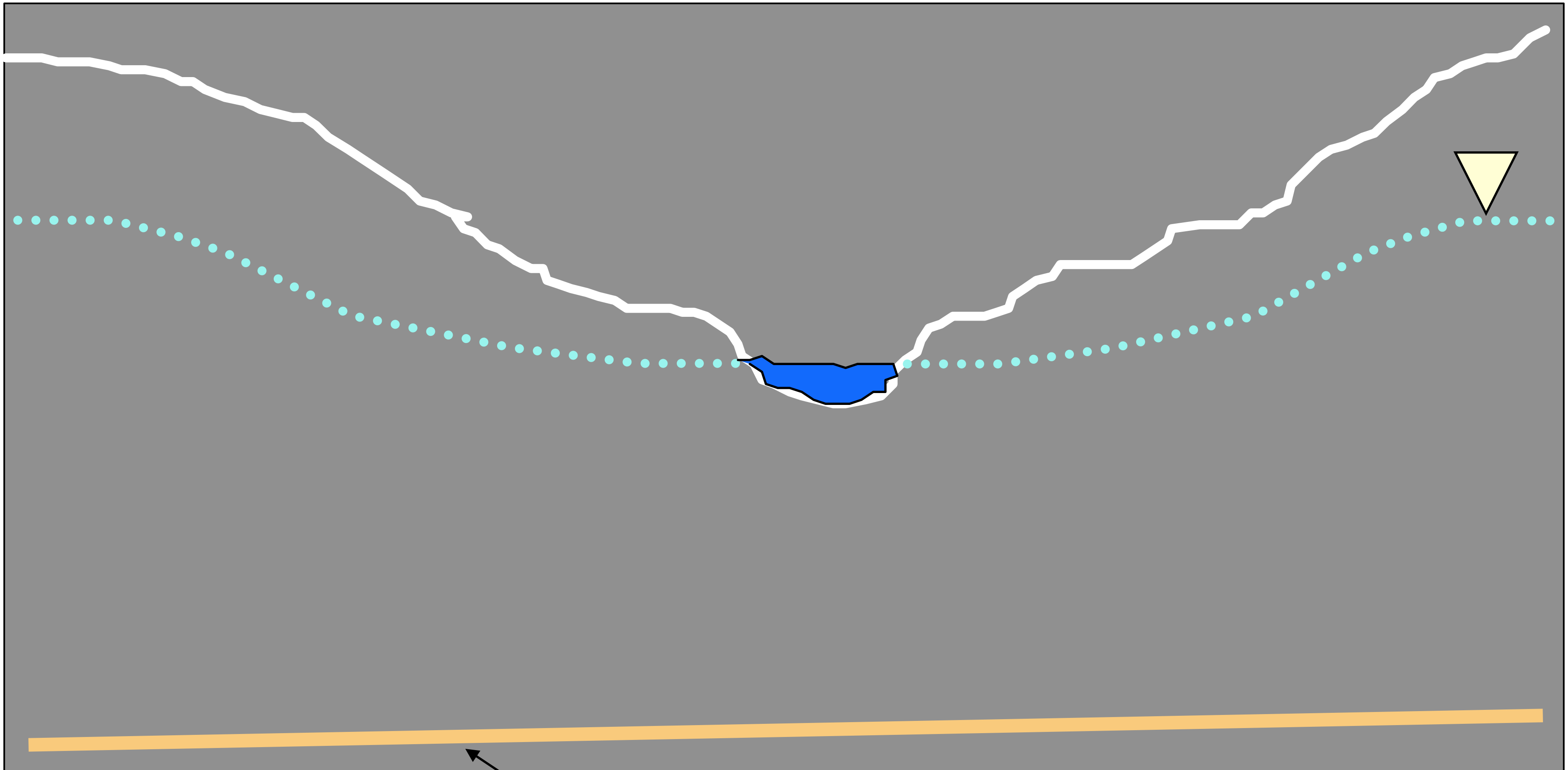
Equipotential lines: contour lines of equal hydraulic head in the subsurface. They intersect streamlines.



Hand-Drawn Flow Net Assumptions

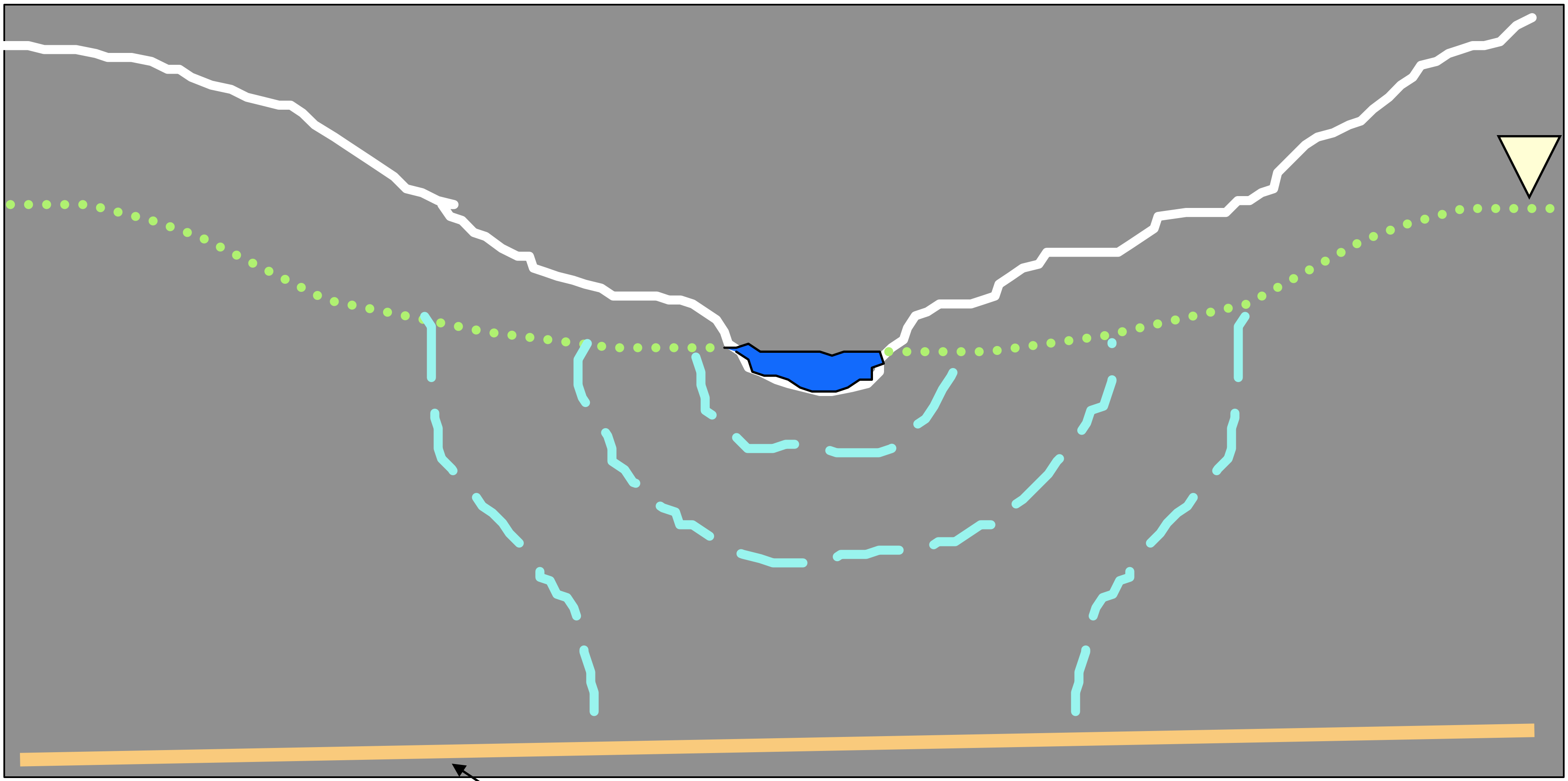
- Homogeneous and isotropic K.
- Fully saturated conditions and steady-state flow (hydraulic heads do not change in time).
- Darcy’s Law is valid.
- *All boundary conditions are known.*

Eg. Sketching groundwater flow to a river.

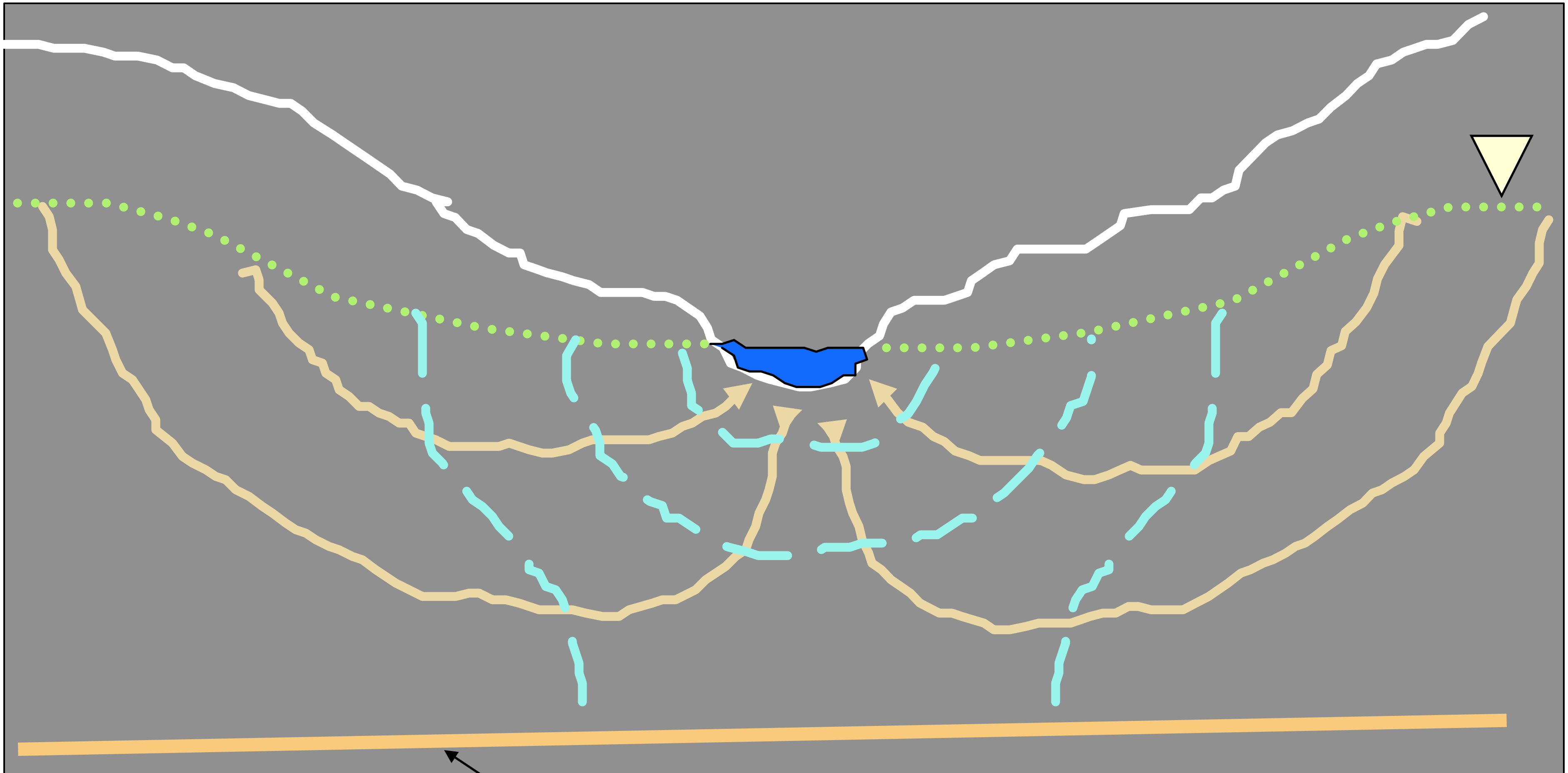


Impermeable boundary

Sketch equipotential lines



Impermeable



Impermeable

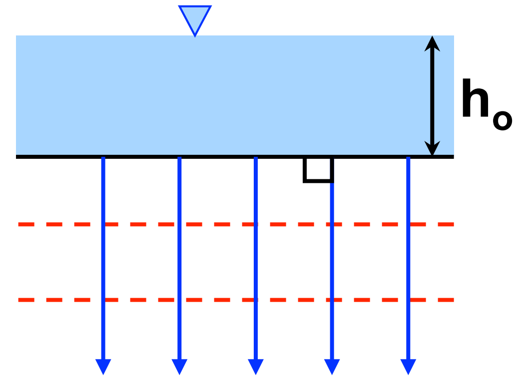
Sketch flow lines.

Dirichlet Boundary
-First Type Boundary

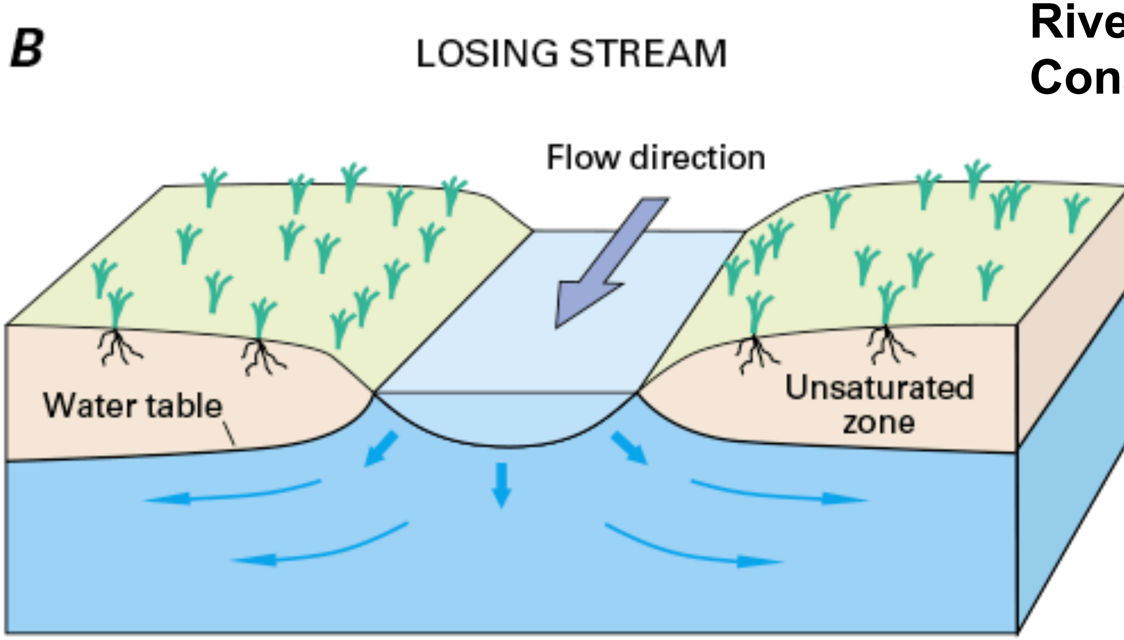
Constant Head Boundaries

• equipotential lines are parallel and flow lines are perpendicular to constant head boundaries

e.g., surface water bodies



River Boundary
Constant Head

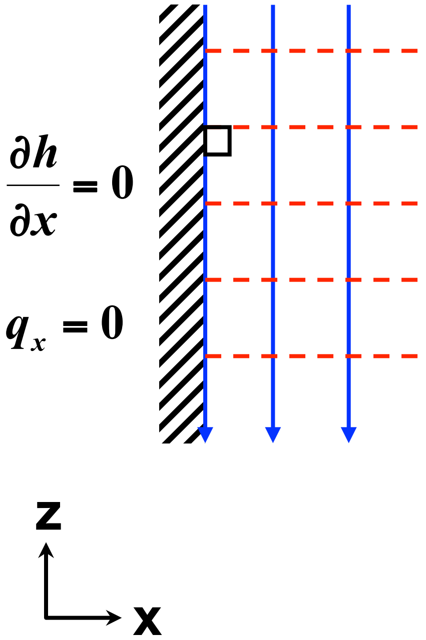


Neumann Boundary
-Second Type Boundary

Impermeable (No Flow) Boundaries

• equipotential lines are perpendicular and flow lines are parallel to impermeable boundaries

e.g., impermeable rock or strata, sheet pile walls



Neumann Boundary
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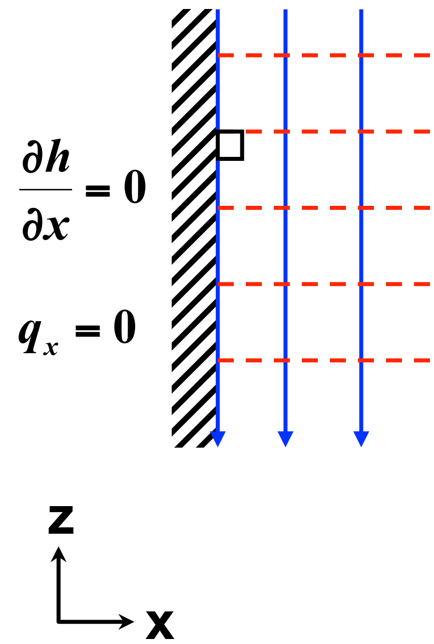
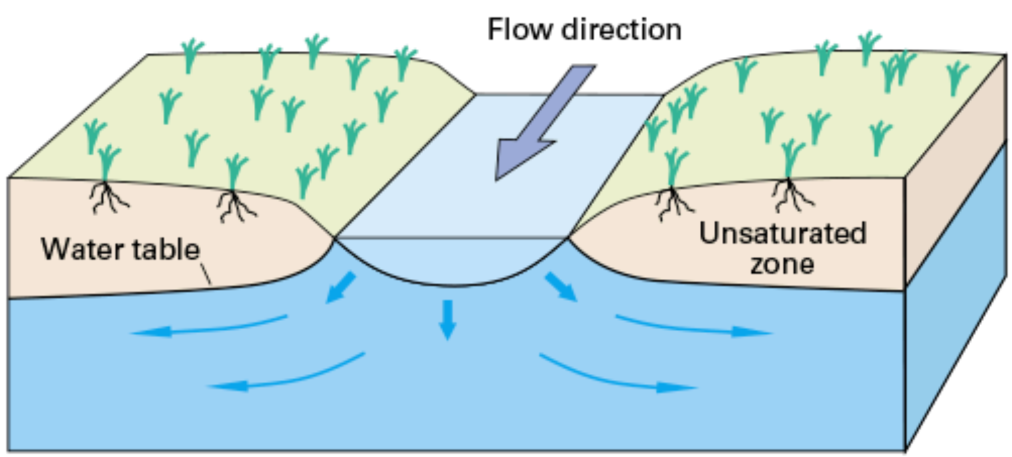
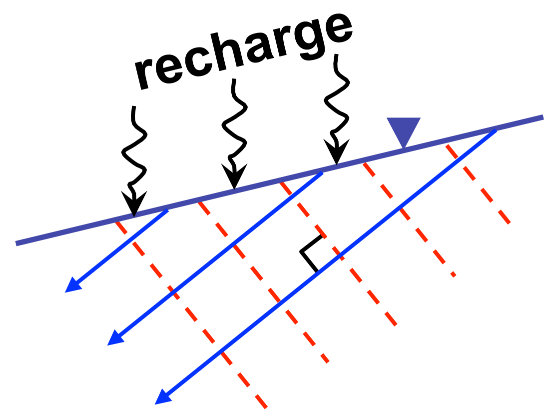




Bedrock No Flow

Cauchy Boundary
-Third Type Boundary

Water Table Boundaries

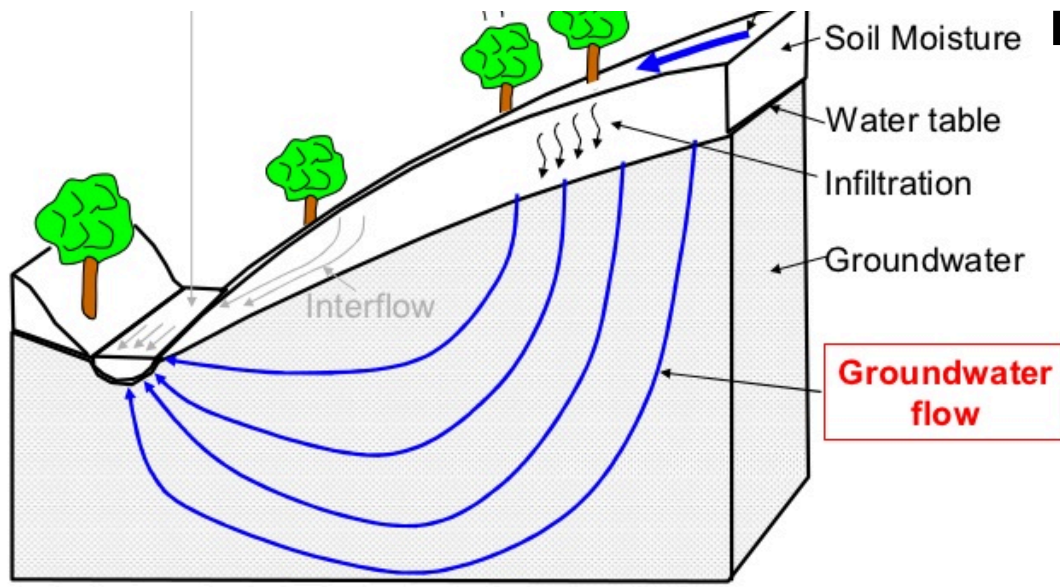


- **h = z since pressure head, ψ = 0**

If there is recharge or discharge, the water table is neither a flow line nor an equipotential line.

If no recharge, then water table is a flow line.

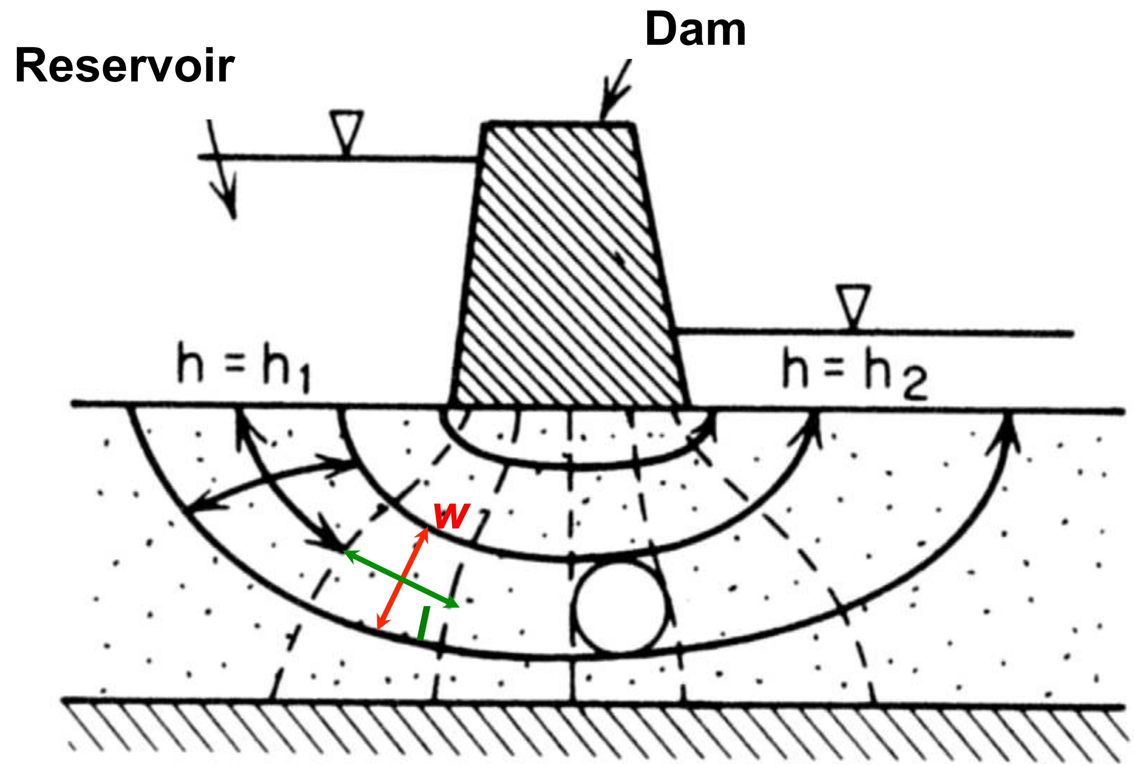
Groundwater Recharge



If the medium is homogeneous and isotropic, flow lines and equipotential lines are <u>everywhere</u> perpendicular.

This results in an orthogonal grid of curvilinear squares.

For a proper flow net:
$l \approx w$



Flow net of seepage beneath a dam
(Freeze and Cherry, Groundwater, 1979)

Additional “Rules” for Flow Nets

1. Hydraulic head drops between adjacent equipotential lines are the same.
2. The discharge or flow rate between adjacent flow lines (known as a streamtube) is constant throughout the flow net.
3. The same quantity of water flows between each set of adjacent flow lines (i.e., streamtubes).
4. Flow lines end at extraction wells, drains and gaining streams. They start at injection wells and losing streams.
5. Lines of symmetry dividing a flow system are streamlines.
6. The plane that the flow net is drawn on must be roughly parallel to groundwater flow.

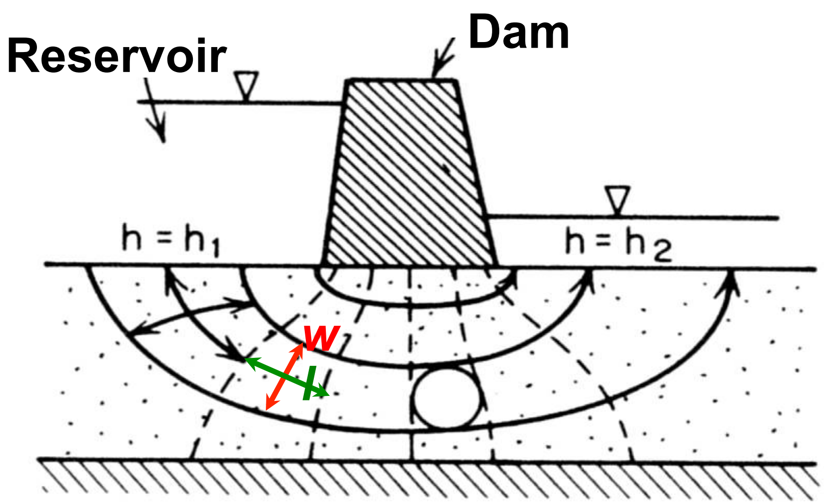
Helpful Hints

1. Identify boundary conditions first.
2. Use boundary conditions (and common sense) to evaluate GW flow directions. Flow from regions of high h to low h.
3. Look for symmetry to reduce the size of the problem.
4. Decide on number of flow tubes. Typically 3-5 is a good start and can always be refined later.
5. Where flow lines converge, higher gradients are needed (i.e., equipotentials are closer). Conversely, where flow lines diverge, gradients decrease accordingly to maintain curvilinear squares.

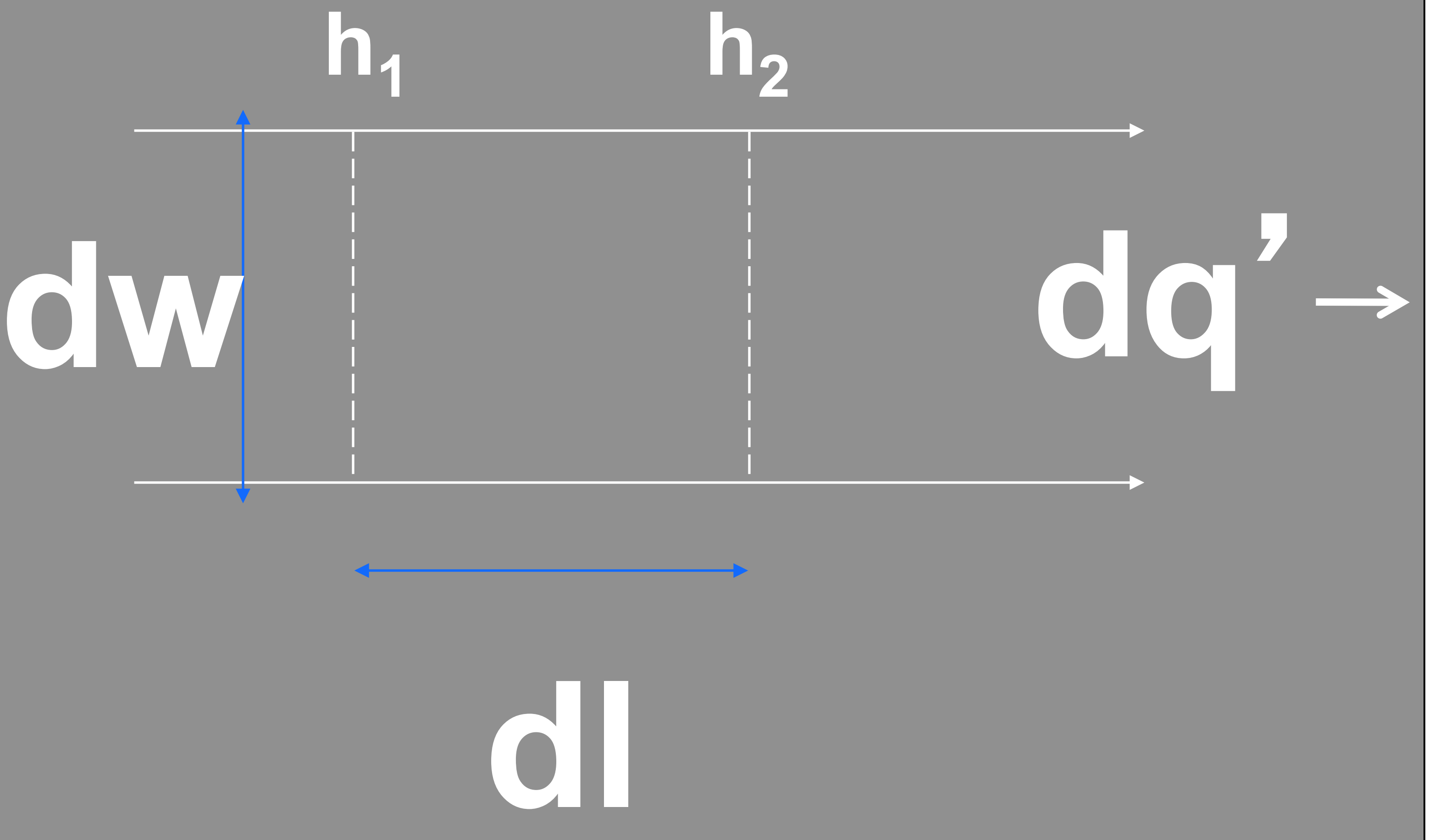
A trial-and-error process. Use a pencil!

Estimation of Total Groundwater Flow with Flow Net

- **Use Darcy's Law.**
- **Quantity of flow per unit width (Q') is the same in each stream tube.**



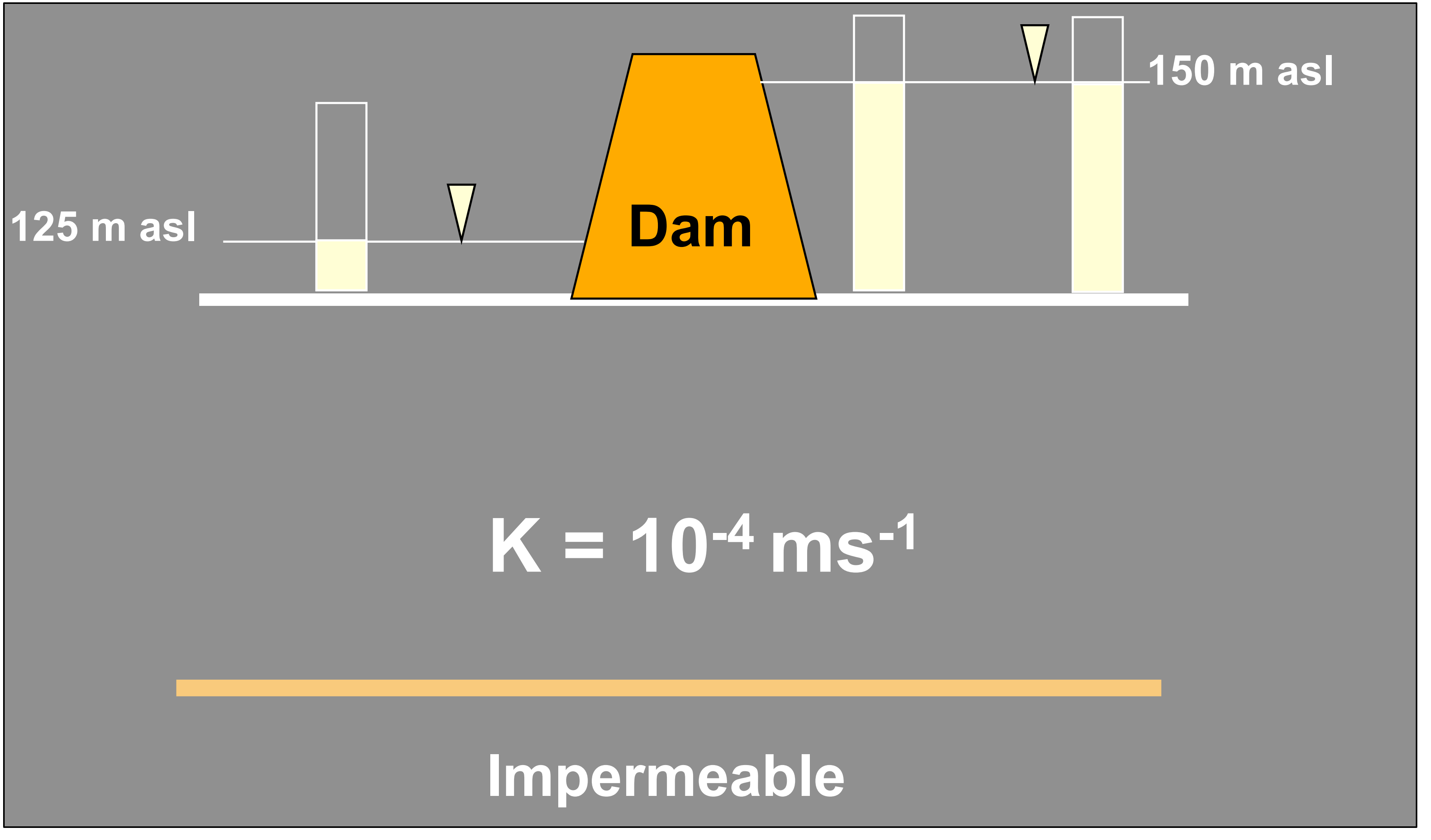
Flow Along a single stream tube: $dq' = K \frac{dh}{dl} dw$

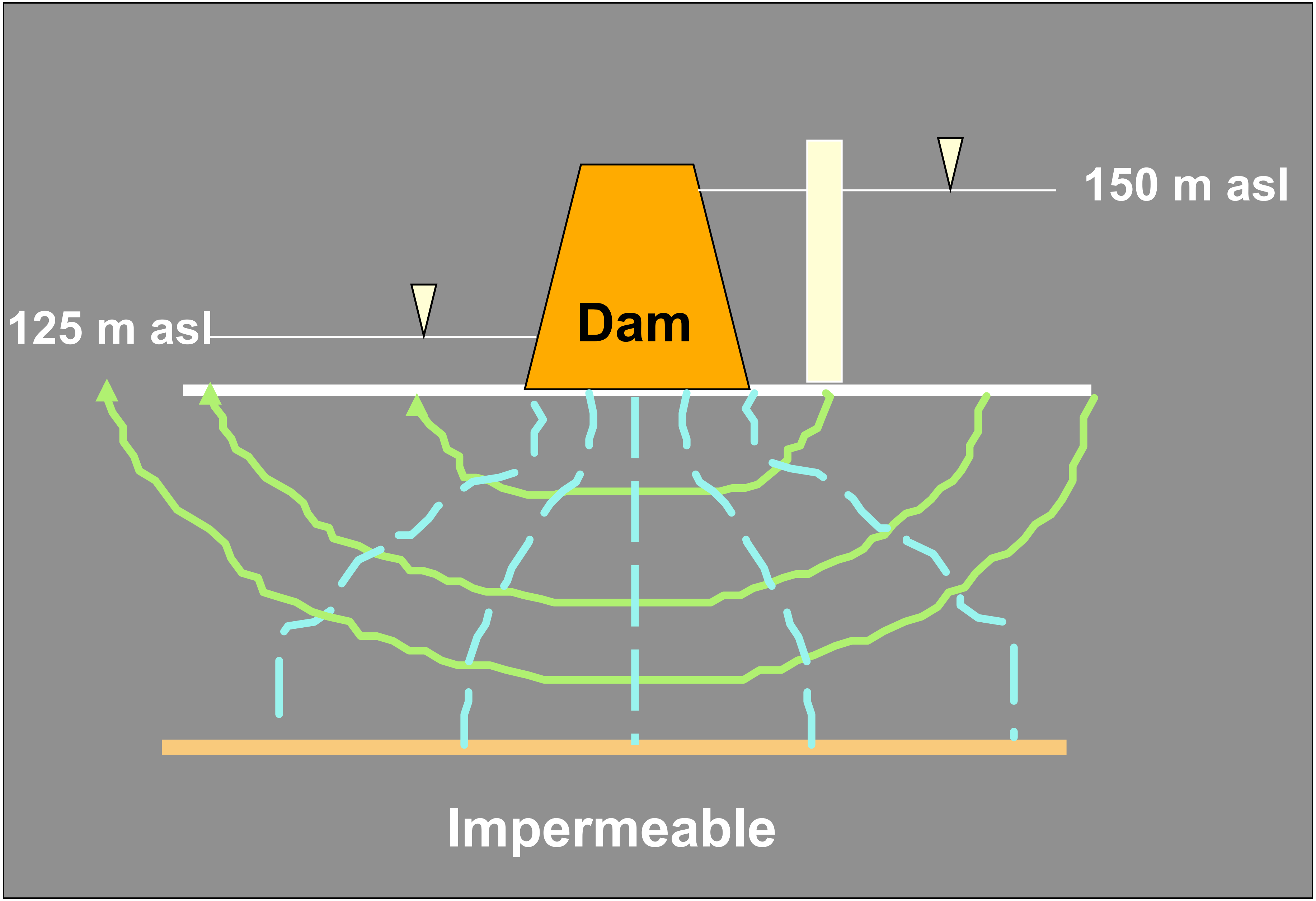


If flow net has perfect squares:

- dq' = K (dh/dl) dw
- dw = dl
- dq' = K dh for one streamtube.
- For m streamtubes, Q' = mKdh
- dh = H/f
- If total head loss is H with f divisions of head, Q' = mKH/f

Calculate Flow Beneath a Dam



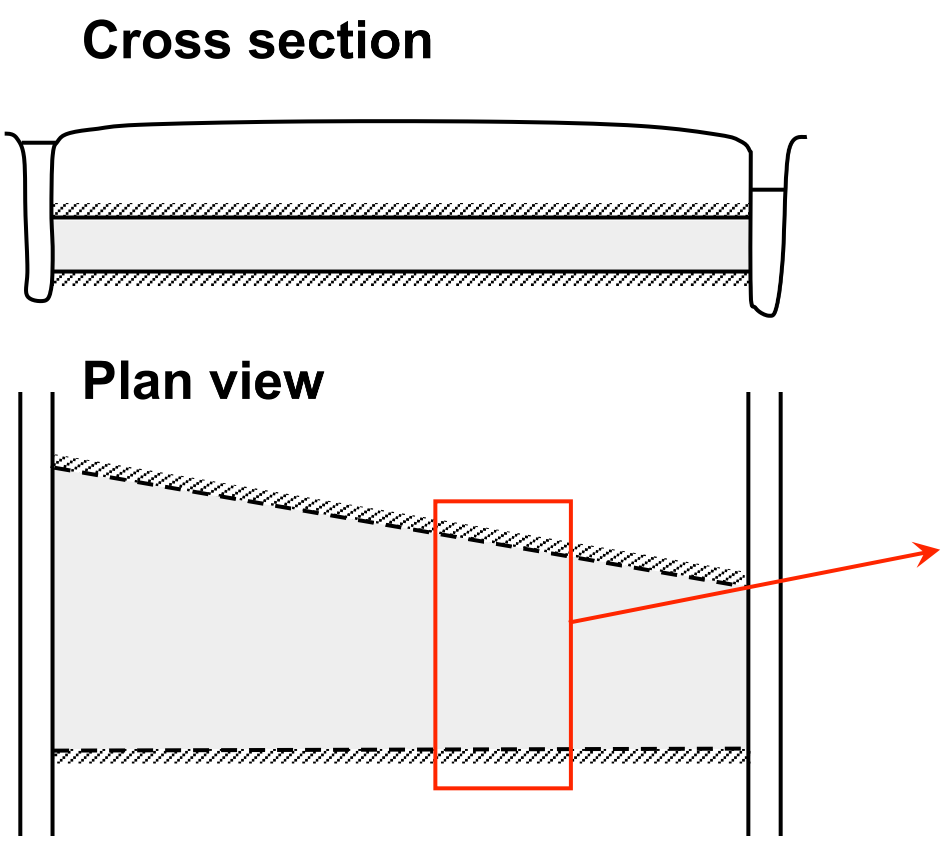

150 m asl
Dam
125 m asl
Impermeable

Approximate Calculation of q'

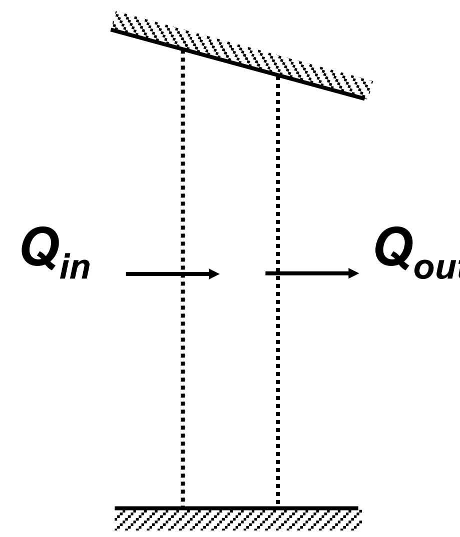
- 4 streamtubes, m.
- 6 head divisions, f.
- total head difference of 25 m, H.
- Q' = mKH/f
- Q' = 144 m^3/day per metre of dam

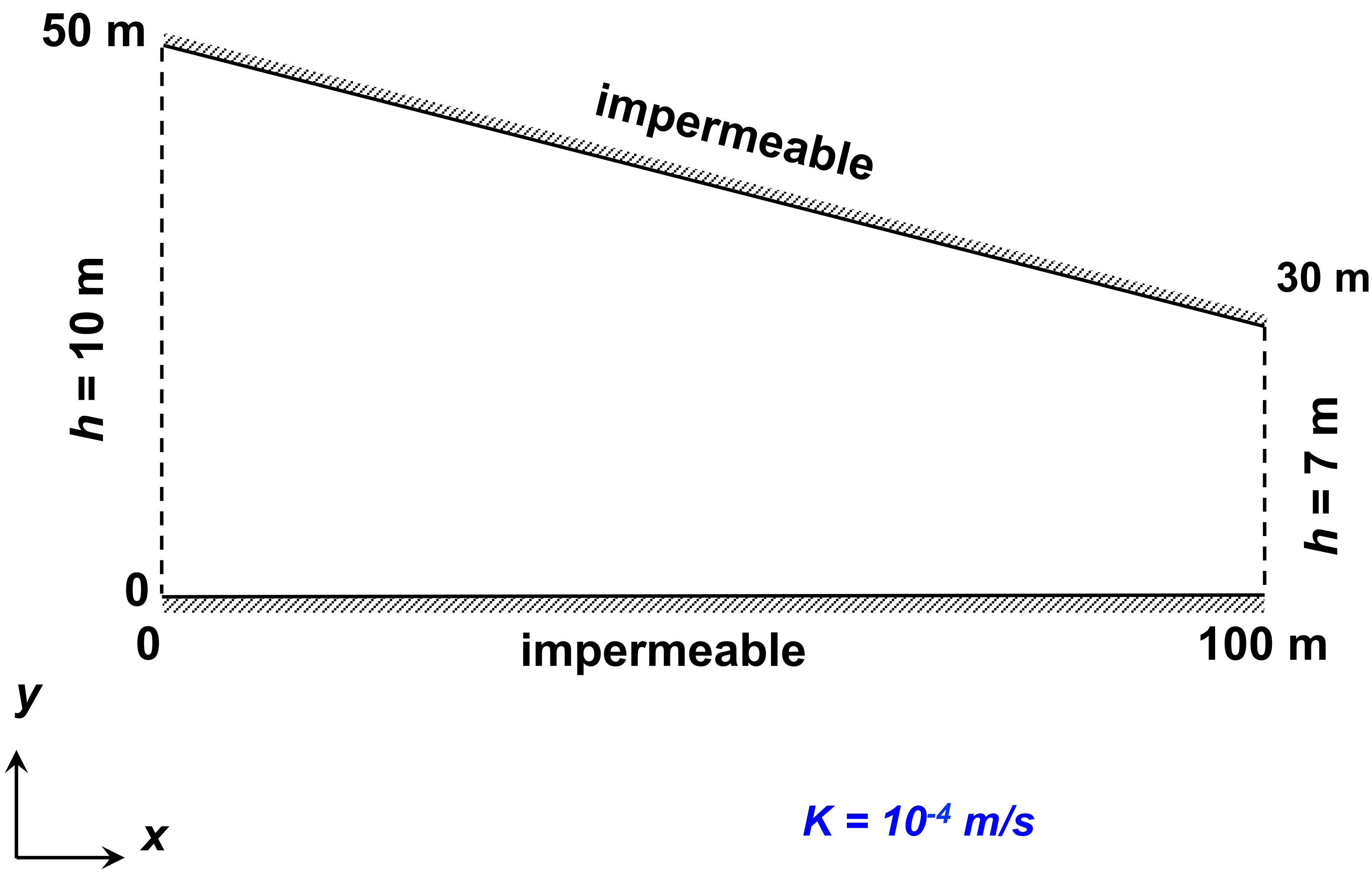
Streamtubes and Discharge Calculations

Consider a confined aquifer that has a somewhat irregular shape in plan view.

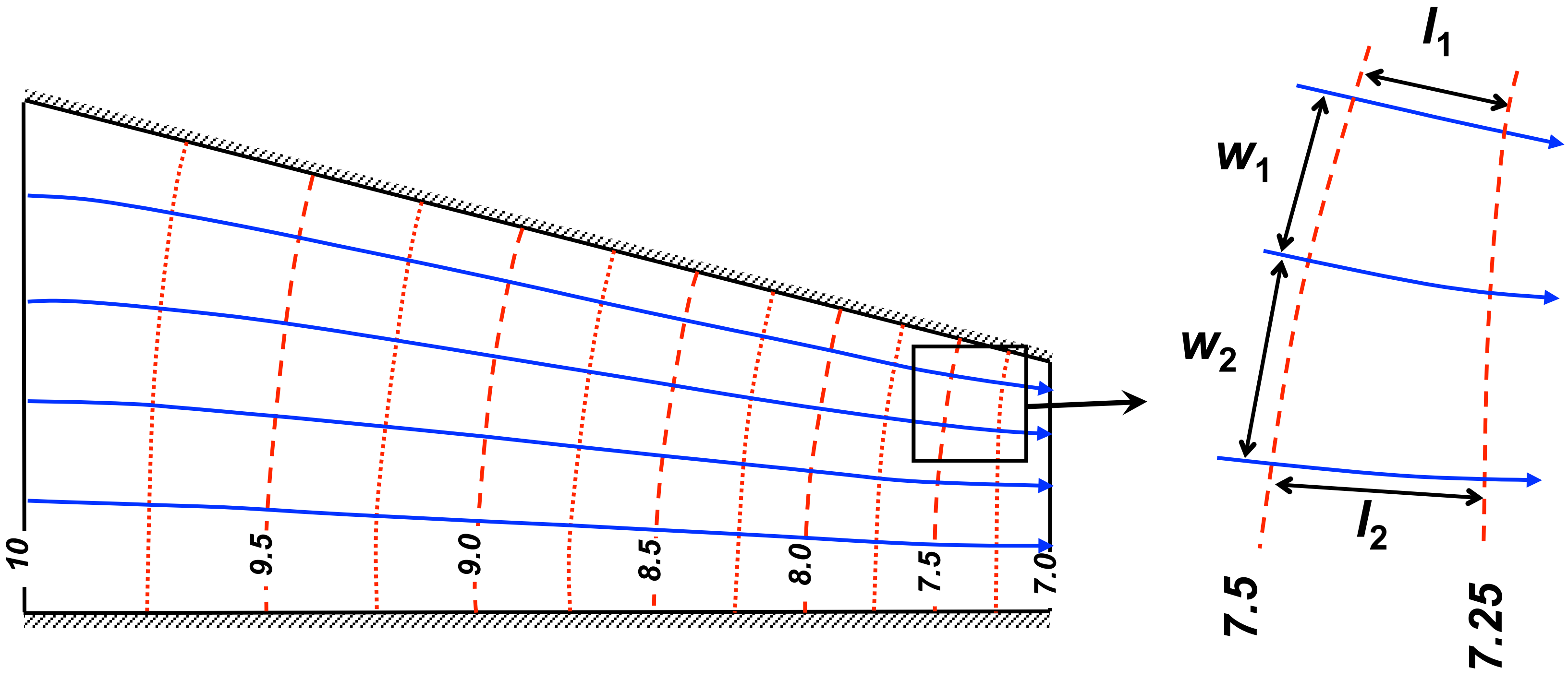


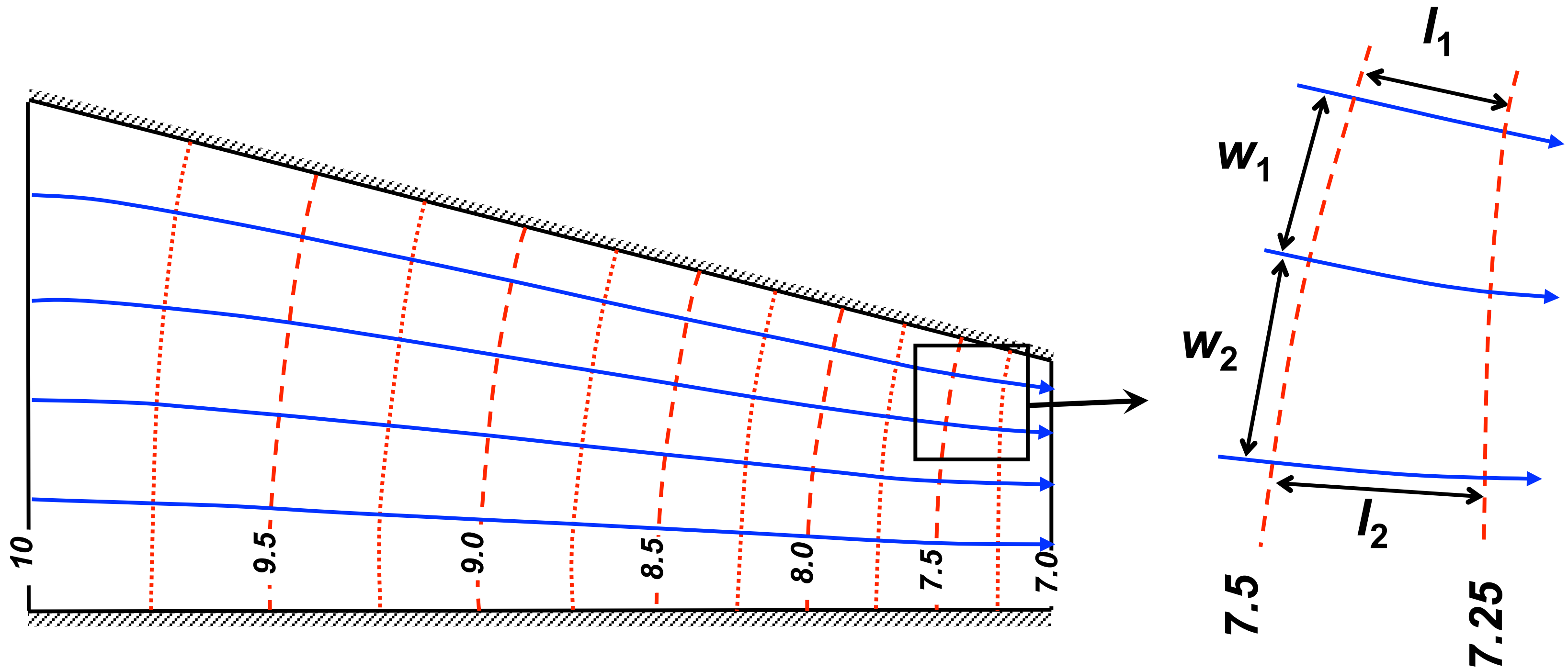
Remember that flow is steady state, so Q_{in} = Q_{out}





50 m
impermeable
30 m
h = 10 m
h = 7 m
0
0
impermeable
100 m
y
x
K = 10-4 m/s


10
9.5
9.0
8.5
8.0
7.5
7.0
l_1
w_1
w_2
l_2
7.5
7.25



Consider flow along streamtube '*i*' for an aquifer of unit thickness (normal to the page). Discharge is:

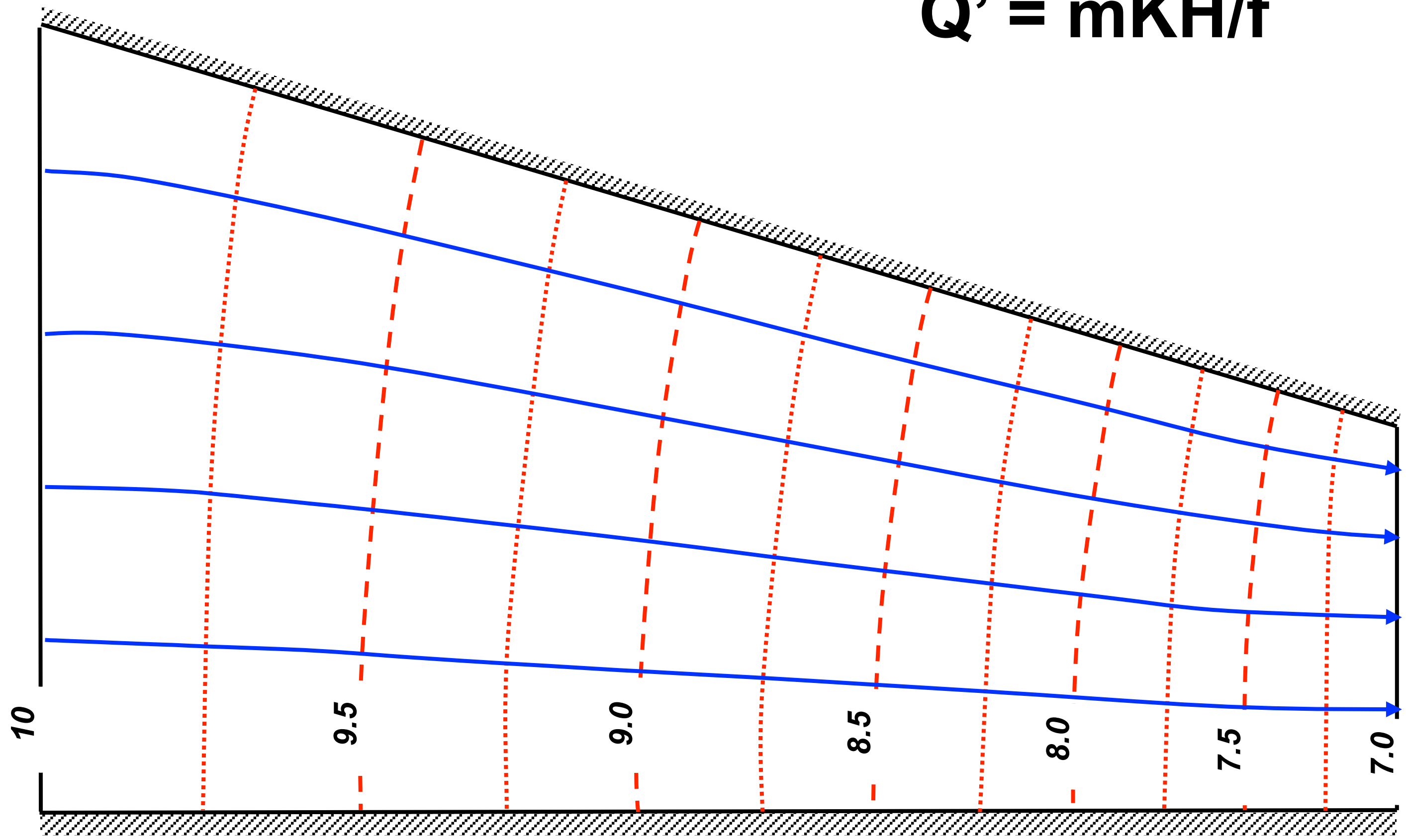
$$Q_i = -KA\frac{dh}{dx} = Kw\frac{\Delta h}{l}$$

Since *w* = *l*: $Q_i = K\Delta h$ **for a single streamtube (per unit width)**

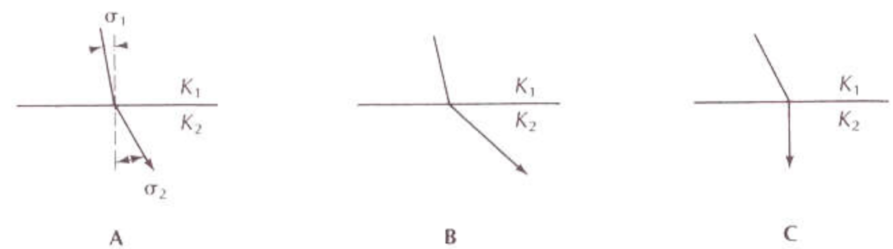
$$Q_{Total} = \sum_{i=1}^{m} Q_i = mK\Delta h$$ **Discharge per unit width**

Calculate Q for the example flow net on pg. 21 assuming that K = 10^{-4} m/s. and the aquifer thickness, b = 4 m.

Q' = mKH/f

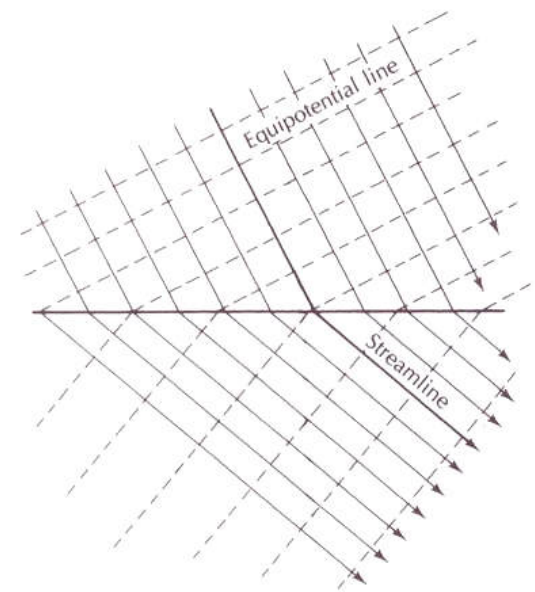


Flow lines crossing hydraulic conductivity boundary



▲ FIGURE 4.14
A. Refraction of a flowline crossing a conductivity boundary. **B.** Refracted flowline going from a region of low to high conductivity. **C.** Refracted flowline going from a region of high to low conductivity.

► FIGURE 4.15
A flow net with flow crossing a conductivity boundary showing refraction of flowlines and equipotential lines. The hydraulic conductivity above the boundary is less than that below the boundary.



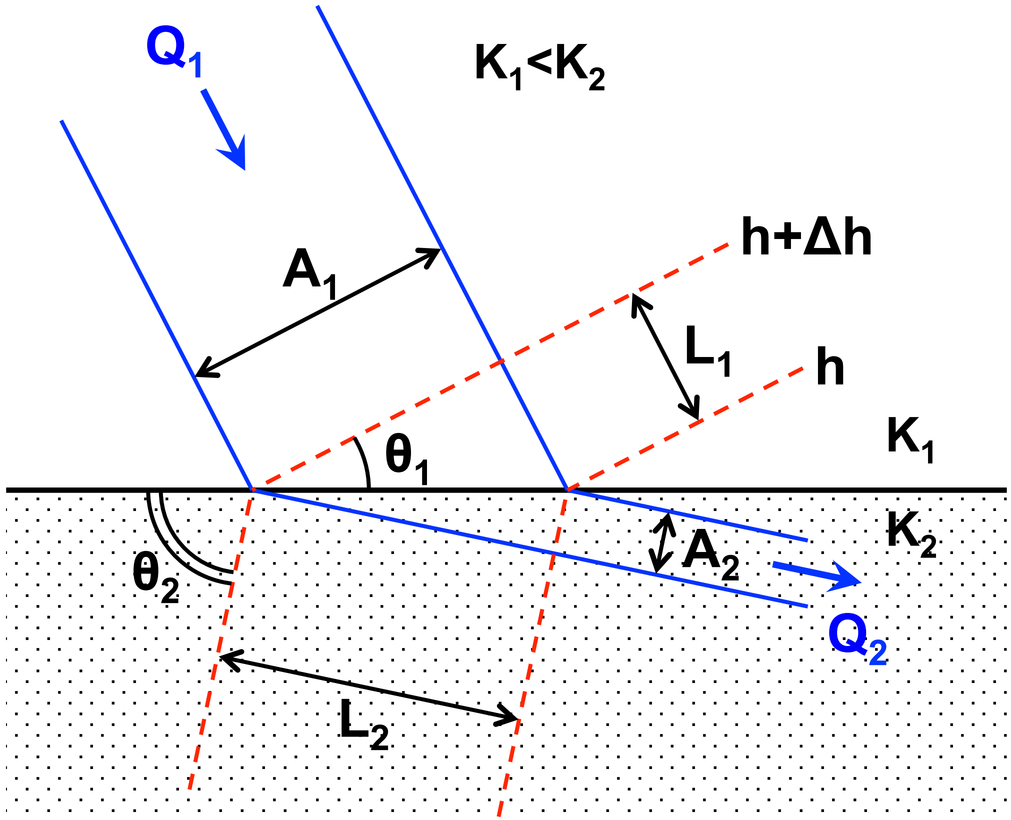
We must have $Q_1 = Q_2$ at the boundary. Since $Q = qA$, we can write,

$$q_1 A_1 = q_2 A_2$$

$$K_1 \frac{\Delta h}{L_1} A_1 = K_2 \frac{\Delta h}{L_2} A_2$$

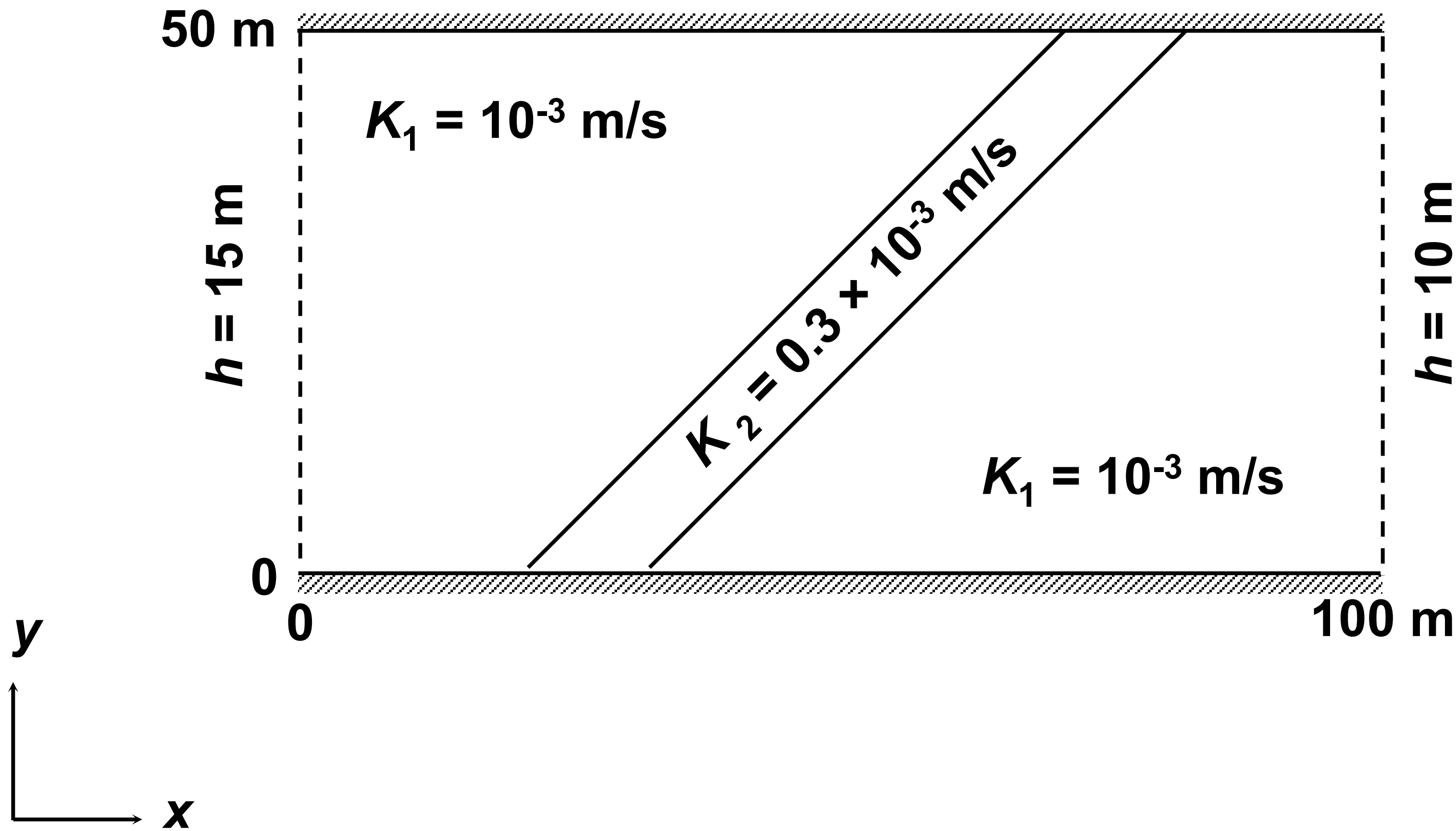
Rearranging yields:

$$\boxed{\frac{K_1}{K_2} = \frac{L_1 / A_1}{L_2 / A_2} = \frac{\tan\theta_1}{\tan\theta_2}}$$

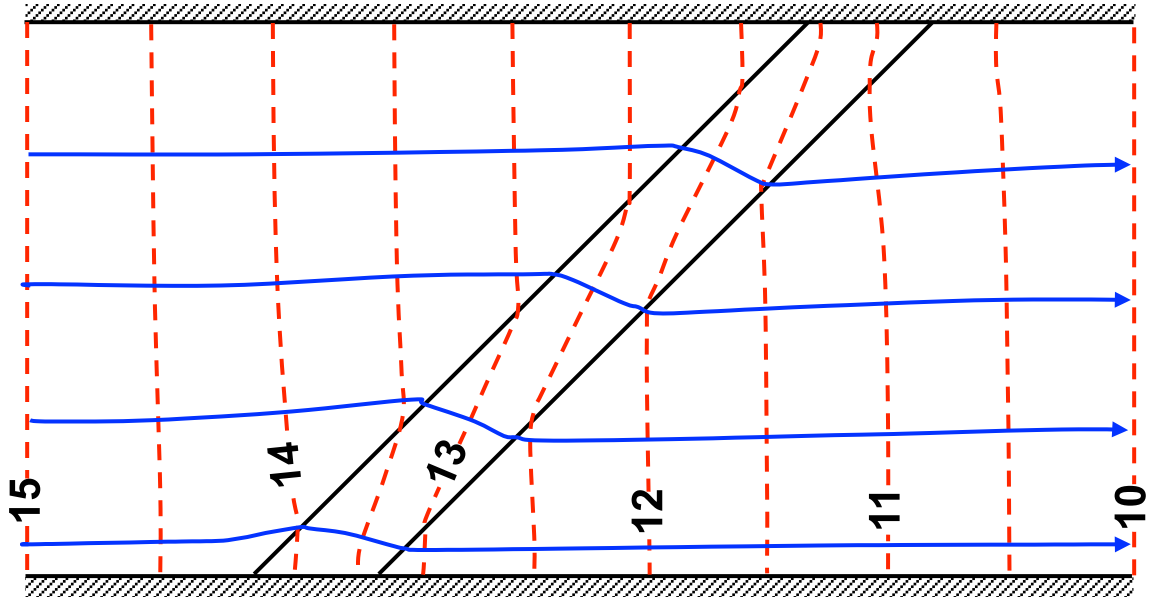


<u>**Heterogeneity and Flow Refraction**</u>

What happens to the flow net in heterogeneous systems, such as the case shown below?



Flowlines (and equipotentials) will refract when they cross permeability boundaries. Note they are still perpendicular.



Groundwater takes the "path of least resistance" by travelling a shorter path through the low-*K* material (i.e., flowing nearly straight across the low K zone).