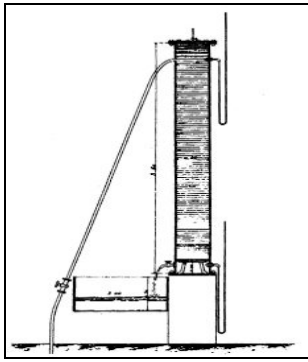
Groundwater Flow and Darcy's Law

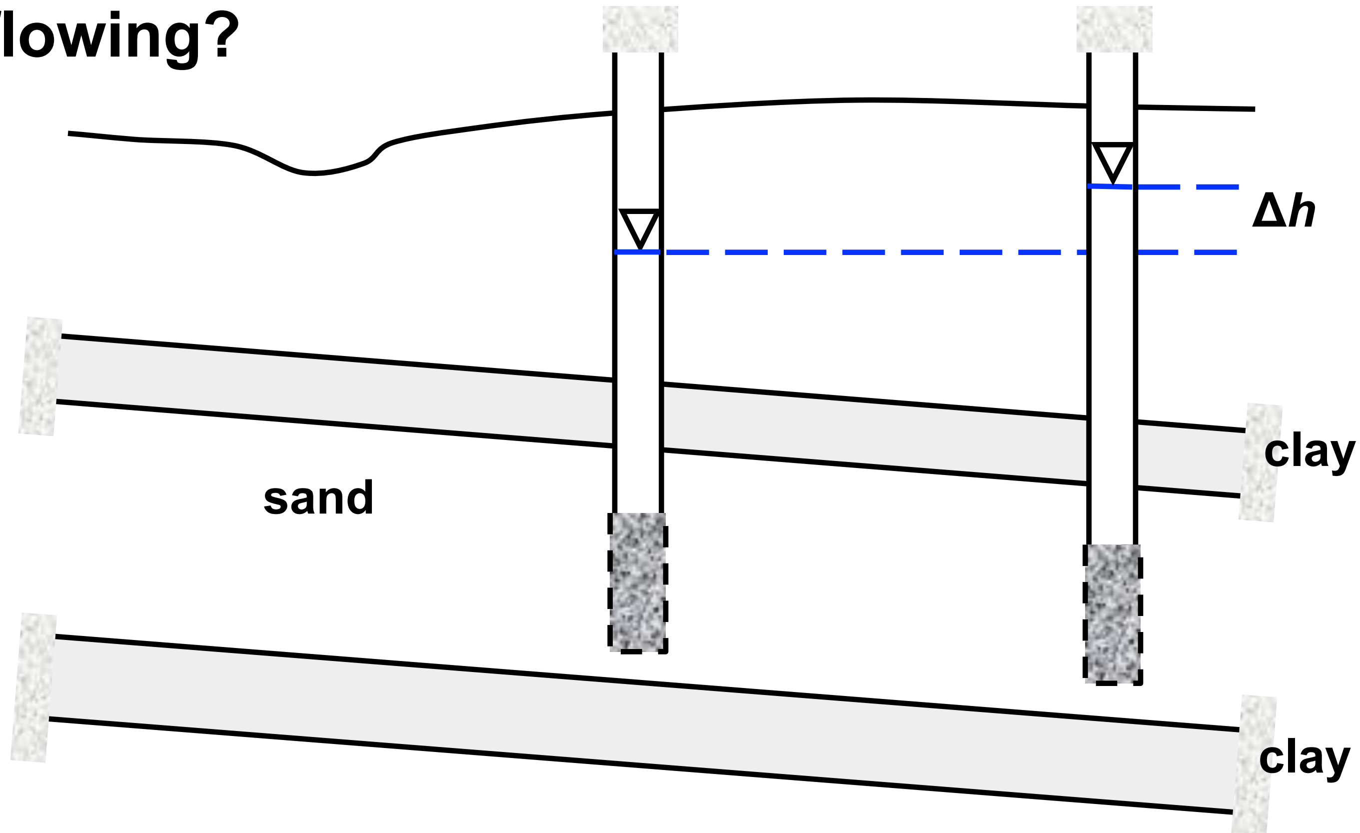
• In all natural systems, fluids will flow in the direction of decreasing energy or potential. This is true for all fluids (e.g., water, air, oil, etc.). In groundwater systems, we use hydraulic head to measure potential and thus water flows from areas of high hydraulic head to low hydraulic head.



• The rate of fluid flow is proportional to the rate of change in hydraulic head. This empirical relationship is known as Darcy's Law and represents one of the fundamental building blocks of groundwater science.

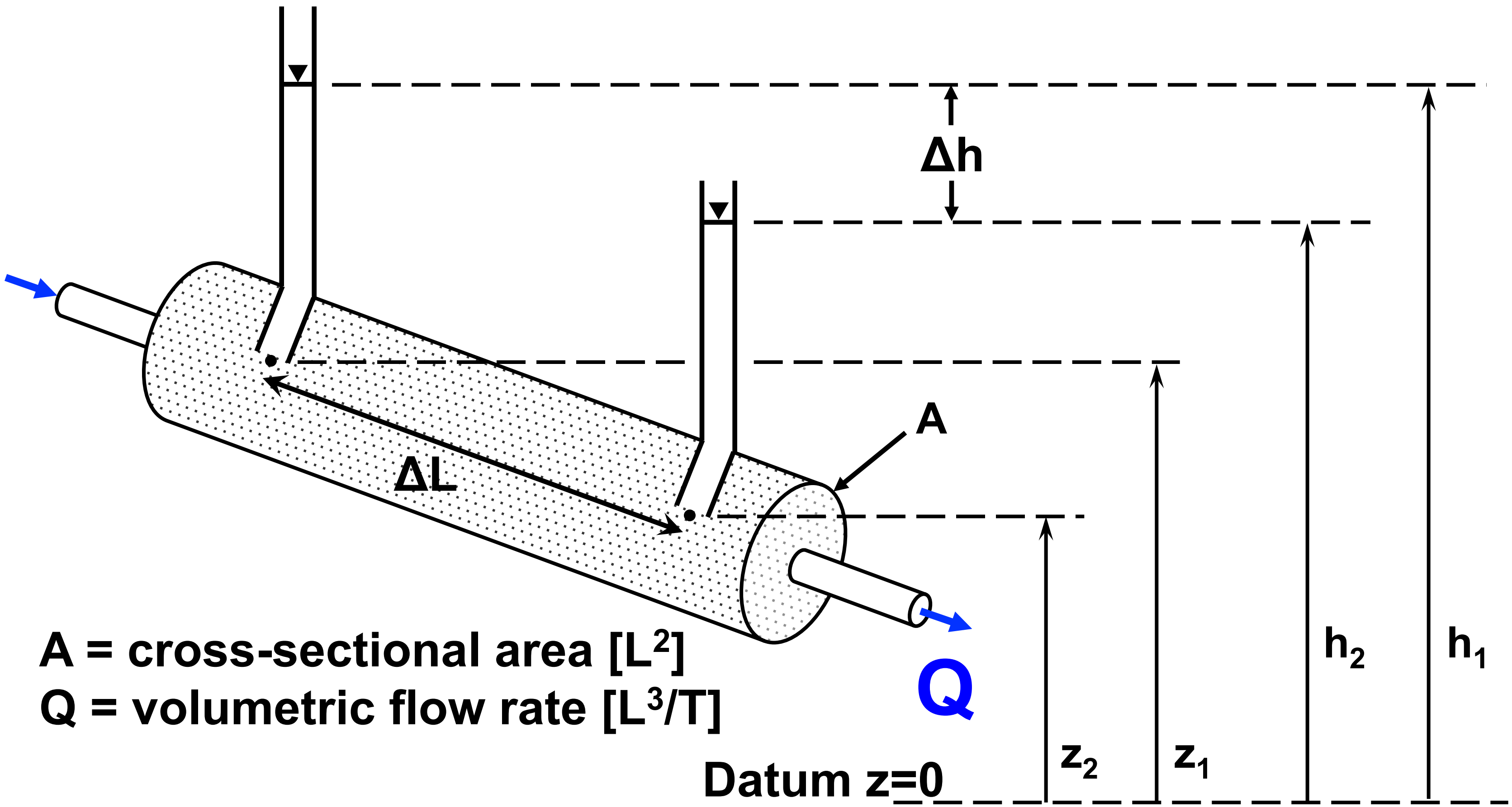
Recall:

Flow rate is proportional to the difference in head, Δh. What direction do you suppose water is flowing?



Darcy’s Experiment (1856)

A French engineer studying water flow through sand columns for the “treatment” of water.



A = cross-sectional area [L^2]
Q = volumetric flow rate [L^3/T]

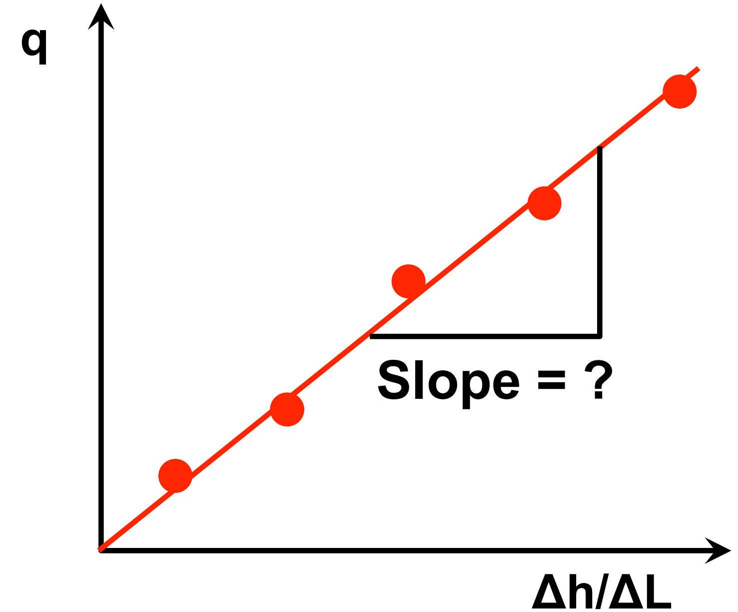
Darcy discovered the following empirical relationship:

$$Q \propto \frac{\Delta h}{\Delta L} A$$

Dividing the volumetric flow, Q, by the cross-sectional area, A, gives us a quantity "q" known as the <u>specific discharge</u> or <u>Darcy flux</u>.

$$q = \frac{Q}{A} \propto \frac{\Delta h}{\Delta L} \qquad \left[\frac{\mathrm{L}^3}{\mathrm{T}} \cdot \frac{1}{\mathrm{L}^2} = \frac{\mathrm{L}}{\mathrm{T}} \right]$$

Note the units are L/T (e.g., m/s), but it is NOT a true velocity.



Darcy Found:

$$v \propto -\Delta h \quad (\text{if } \Delta h = h_2 - h_1)$$

Eq. 1

$$v \propto \frac{1}{\Delta l}$$

$$gradient = \frac{\Delta h}{\Delta l} \propto v$$

Eq. 2

- **Combining these relationships:**

$$\boxed{v = -K \frac{\Delta h}{\Delta l}}$$

Darcy's Law

Permeability & Hydraulic Conductivity

From Darcy's Law:

$$\boxed{v = -K\left(\frac{\Delta h}{\Delta l}\right)}$$

- Constant of proportionality **K** is a function of:
  - Properties of the porous medium
  - Nature of the fluid

Darcy's Equation is written as,

$$\boxed{q = -K\frac{\Delta h}{\Delta L} \quad \text{or} \quad Q = -KA\frac{\Delta h}{\Delta L}}$$

Darcy's Law

where q = specific discharge (L/T)

Q = volumetric flow rate (L^3/T)

A = cross-sectional area (L^2)

K = hydraulic conductivity (L/T)

Δh = change in hydraulic head (L)

ΔL = distance along the direction of flow (L)

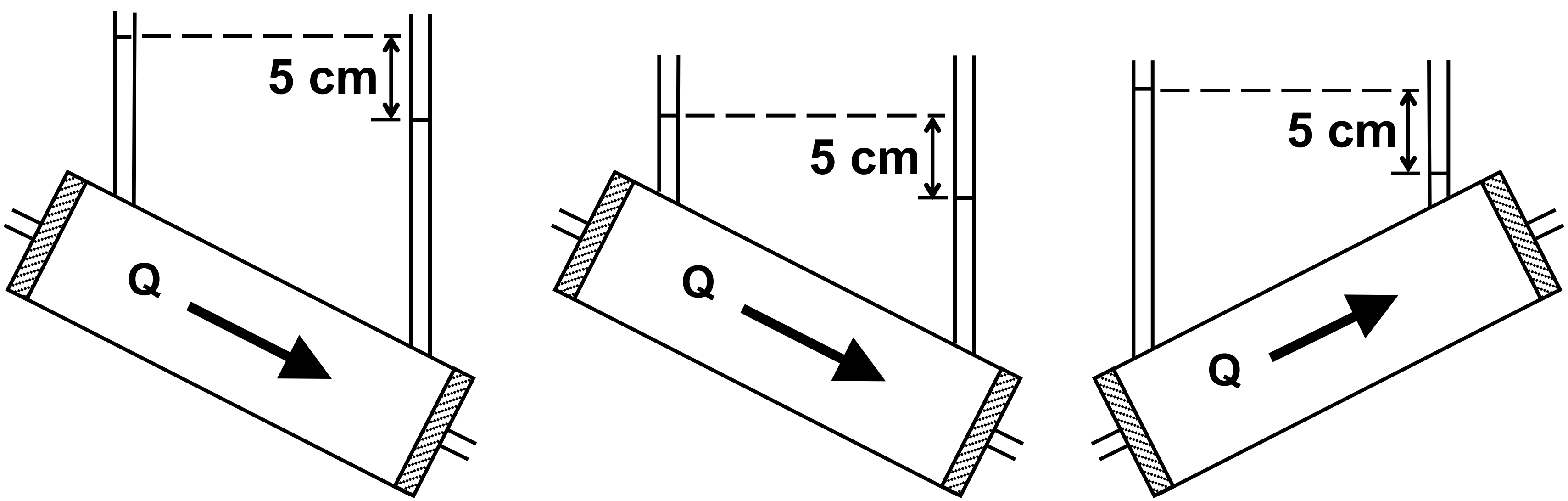
There are two key components to Darcy's Law:

1. ***Hydraulic Gradient, Δh/ΔL or i***
   - **represents the driving force behind flow**
   - **the rate of change in hydraulic head with distance in the direction of flow**
   - **Unitless**

2. ***Hydraulic Conductivity, K***
   - **a proportionality constant for the fluid and the porous medium**
   - **represents the ease with which water will pass through a porous medium**
   - **K is larger for coarse-grained materials and smaller for fine-grained materials**
   - **units of L/T but again not a velocity**

For materials with the same K, the flow rate and direction remain the same regardless of the column angle or the absolute values of *h*.

Note also that *h* consists of *z* and *ψ*. Flow is always from high *h* to low *h*, not necessarily from high pressure to low pressure.



Permeability & Hydraulic Conductivity

Laboratory investigations reveal:

$v \propto d^2$ (grain size)

$v \propto \rho g$ (fluid density)

$v \propto 1/\mu$ (fluid viscosity)

Combining these: $$\boxed{K = \frac{Cd^2 \rho g}{\mu}}$$

Where: C = constant of proportionality (sorting, packing and shape of grain)

Darcy's Law

$$q = -\overbrace{\underbrace{C\,d^2}_{\text{porous medium}}\;\frac{\rho\,g}{\underbrace{\mu}_{\text{fluid}}}}^{\text{K (fluid and porous medium)}}\;\frac{dh}{dx}$$

We define *k* = *Cd*^{2}, where *k* is <u>intrinsic permeability</u> (or simply permeability) and is a property of the porous medium only.

Measuring Hydraulic Conductivity

Because of the wide range in K values for geologic media, it is difficult to determine with great accuracy. Several measurement methods are available:

1. Visual Correlation
- **pick a literature value based on sediment type**

2. Grain Size Methods ***(0.001-0.1 L)***
- **suitable for coarse-grained materials only**
- **e.g., Hazen**
-

3. Permeameter Tests ***(0.01-1 L)***
- **constant head**
- **falling head**

4. Single-Well Tests ***(1-100 L)***
- **slug or bail tests**

5. Pumping Tests ***(1000-10^5 L+)***

Permeability & Hydraulic Conductivity

By convention: $\boxed{\mathbf{k = Cd^2}}$ **(Intrinsic Permeability)**

- **This parameter can be measured in the field or in the lab**

$\boxed{\dfrac{length}{time}}$ **Units:** $\boxed{\dfrac{cm}{s}}$ **or** $\boxed{\dfrac{m}{s}}$ **or** $\boxed{\text{gal./day/ft}^2}$

(has same units as velocity but *is not* a velocity)

Grain Sizes

Photos by B. Conant

Grain Size

Many sediments are classified based on the size of the grains or the distribution of grain sizes.

Grain Size Classification

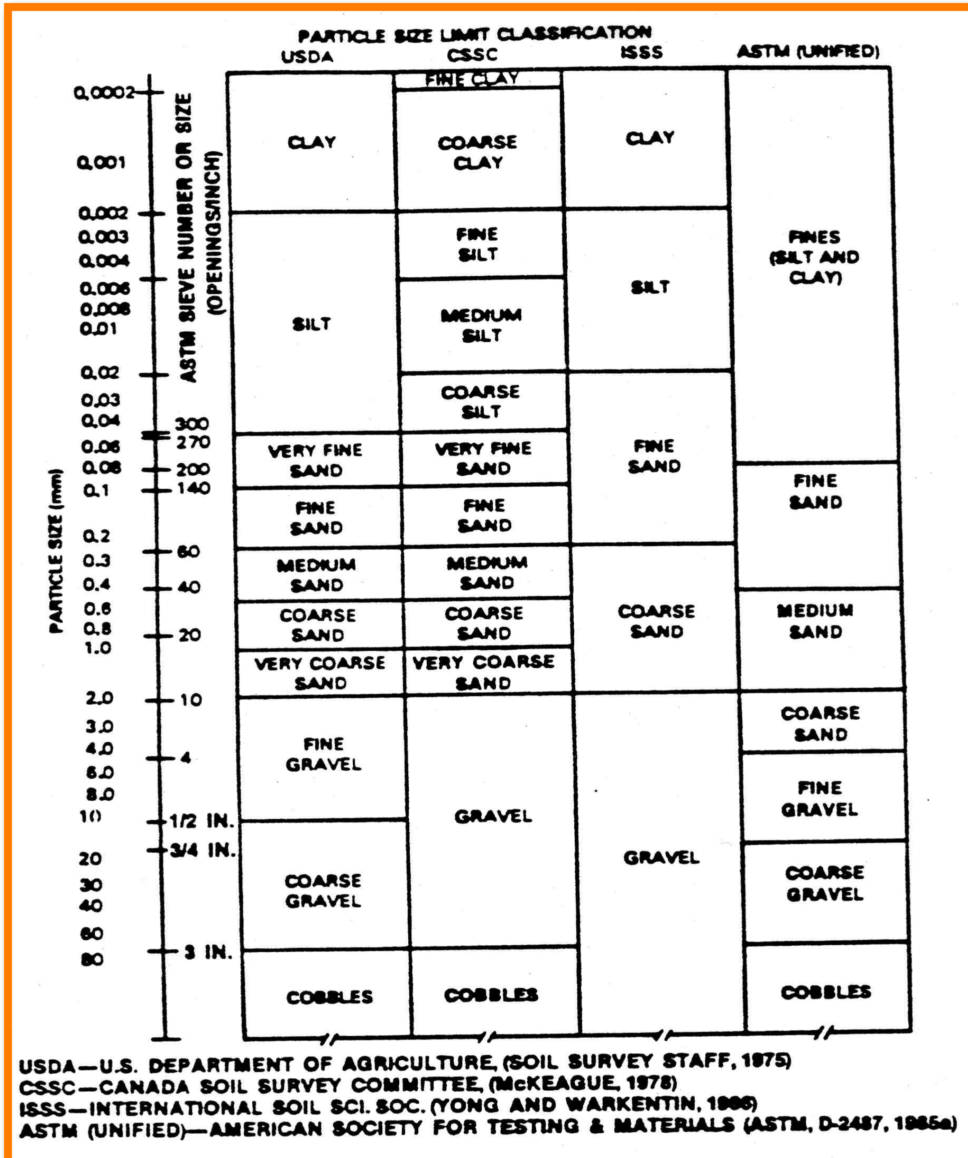
Name	Grain Size Range (mm)
Gravel	> 2.0
Sand	0.062* - 2.0
Silt	0.002 - 0.062*
Clay	<0.002

***varies from 0.050 - 0.075 mm depending on classification system.**

Typically measured using a series of stacked sieves, hydrometer/pipette methods, or laser diffraction.

(Rudolph)

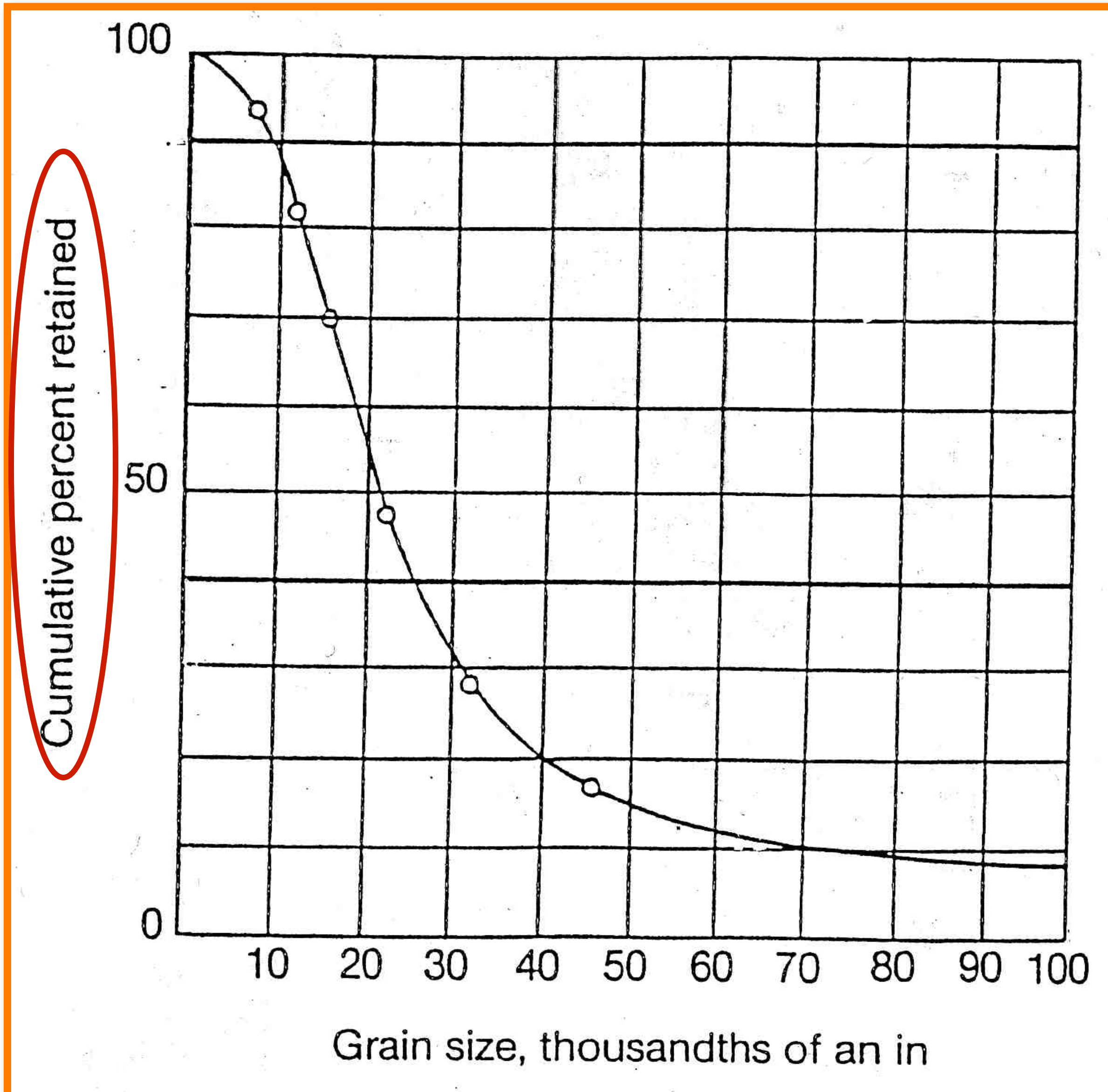
Sediment Classification based on Grain-size



Craig, 1978

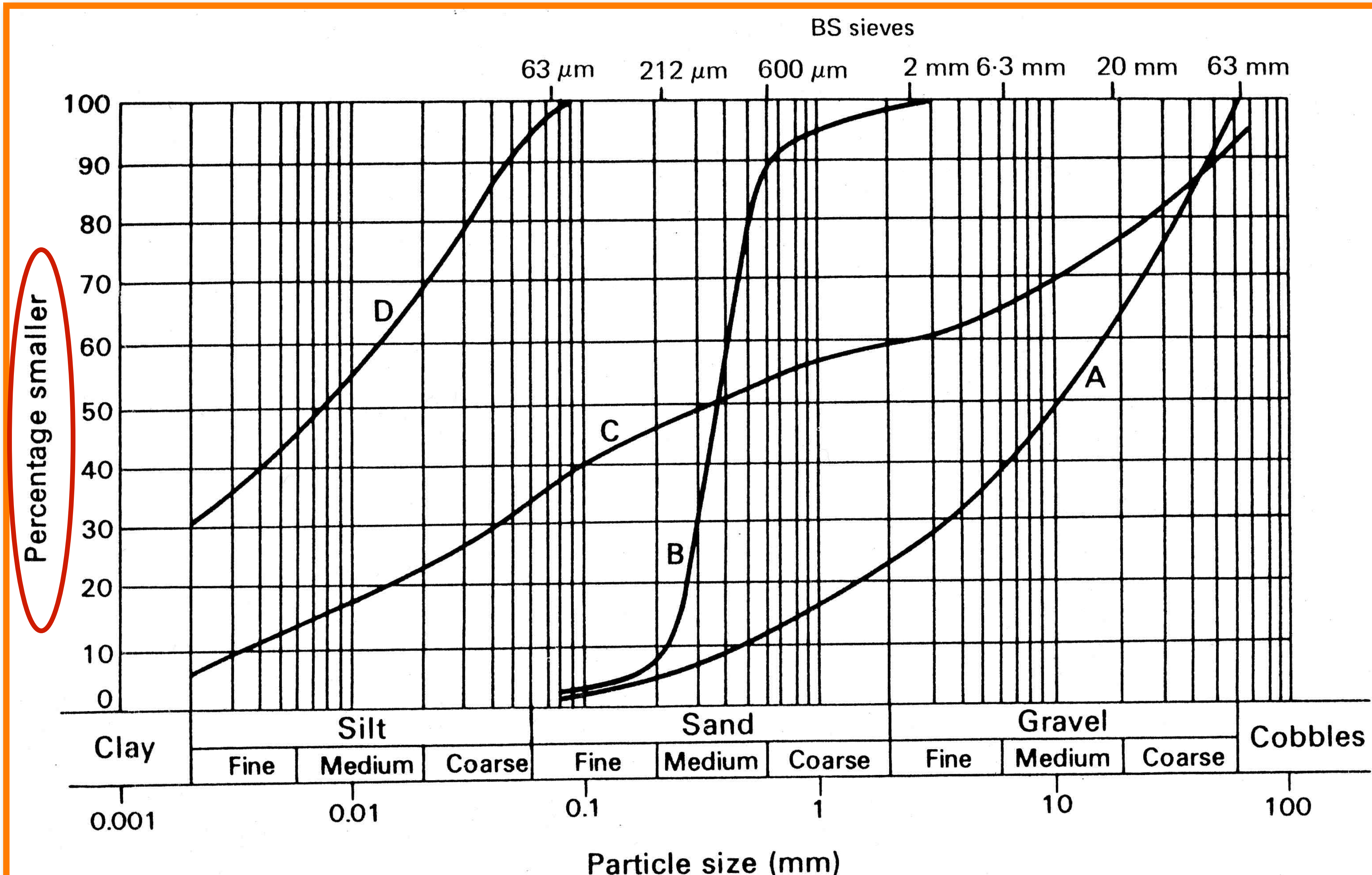
Grain Size Distribution

Plotting method #1



Craig, 1978

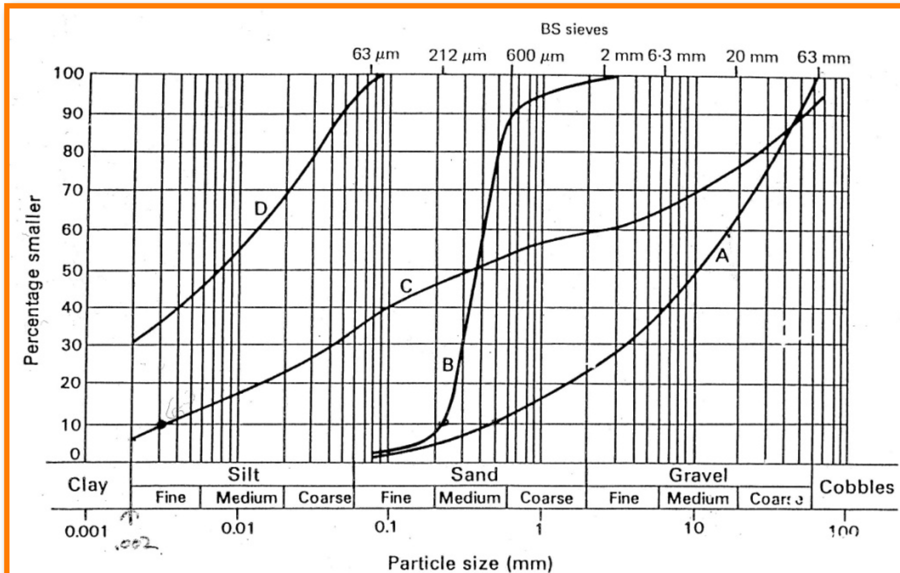
Particle Size Distribution Curves



Craig, 1978

Plotting method #2

Coefficient of Uniformity

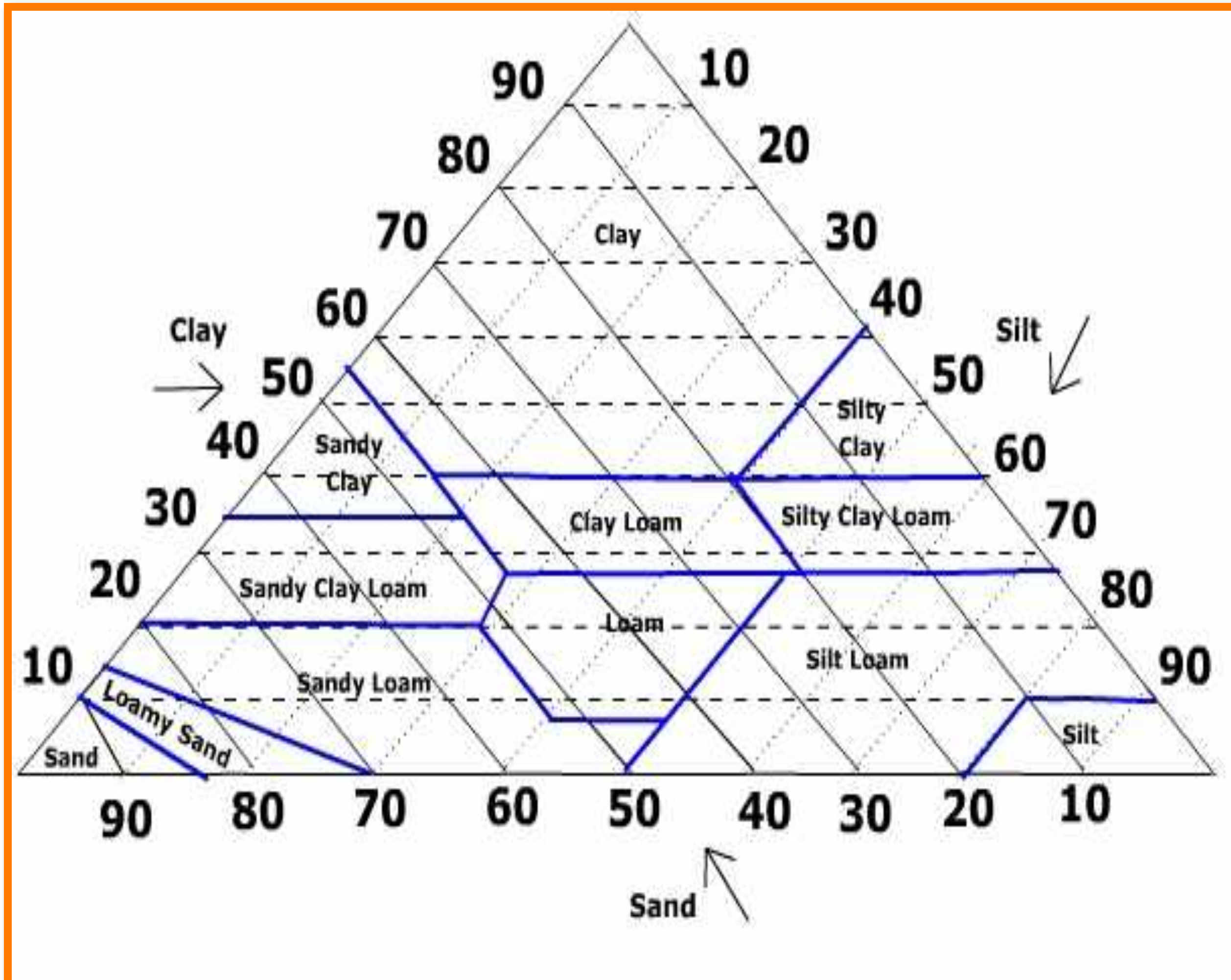


Craig, 1978

$$Uniformity \cdot Coefficient = C_u = \frac{D_{60}}{D_{10}}$$

- **Where: D_{10} indicates that 10% of the sample is smaller than that diameter**

Soil Classification Triangle



Craig, 1978

Parameter Estimation from Grain-Size Analysis

Intrinsic permeability k is estimated from empirical relationships derived primarily on representative grain diameter

General Expression: $\boxed{k = cd^z}$ **(1)**

Where: d is the representative grain diameter and c is a dimensionless constant involving path tortuosity, particle shape, sorting, and porosity

Parameter Estimation from Grain-Size Analysis

General Expression: $\boxed{k = cd^z}$ (1)

- z is commonly taken as 2
- Numerous forms of (1) have been suggested by researchers

Data required for these empirical expressions include:

1. Grain size distribution and uniformity coefficient (C_u)
   - Where $C_u = D_{60}/D_{10}$ or d_{40}/d_{90}
2. Water temperature
3. Empirical data
4. Porosity

Hazen Method (1892)

- Most useful for uniform fine sands or gravels
- Uniformity coefficient less than 5

(Conant)

Hazen Method (1892)

$$K = Ad_{90}^{2}(1.0 + 0.0429T)$$

Alternative #1

Where:

d_{90} = 90% (wt) retained (mm)

A = empirical coefficient

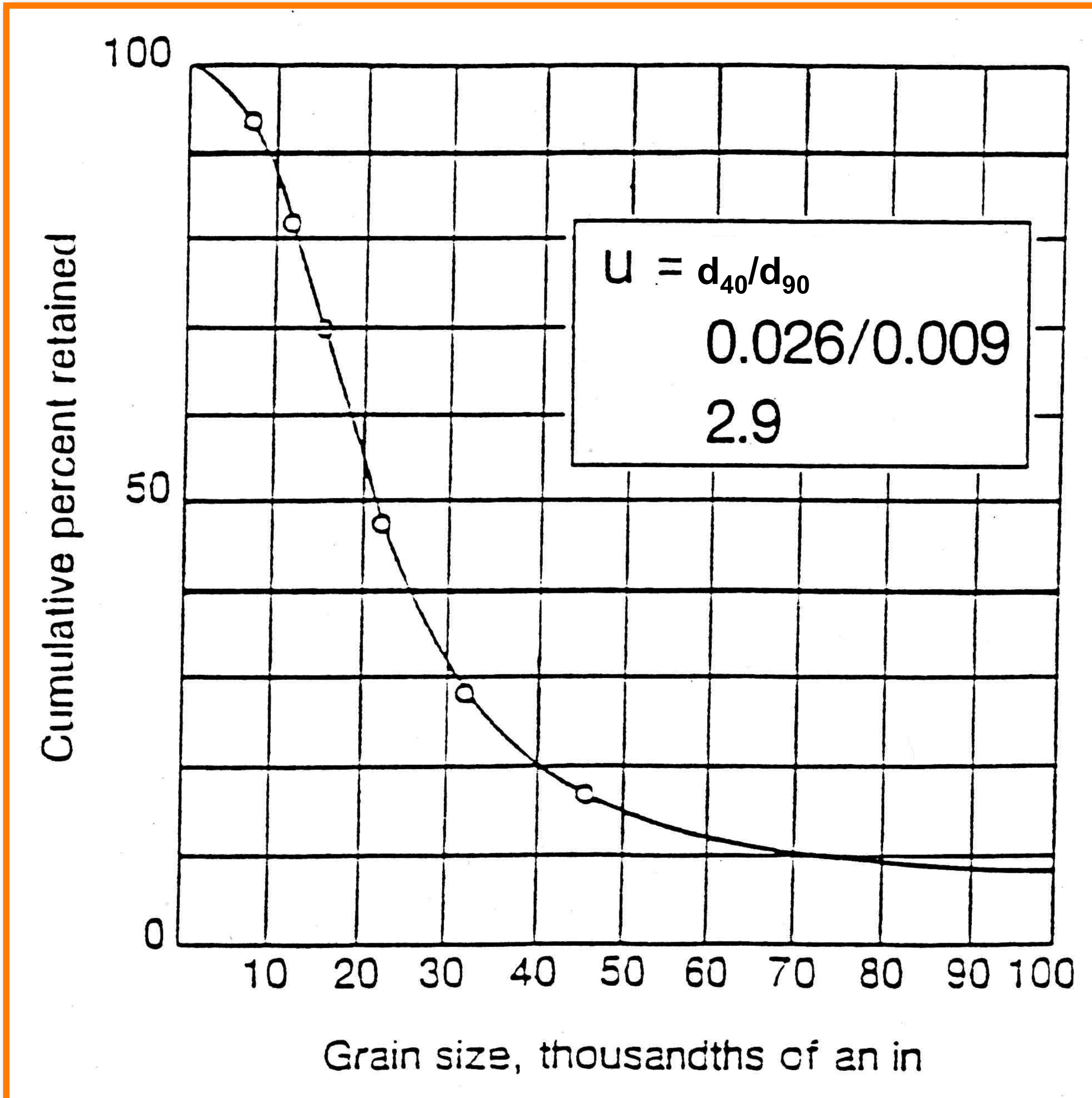
K = cm/s

T = water temperature (°C)

A is related to grain uniformity

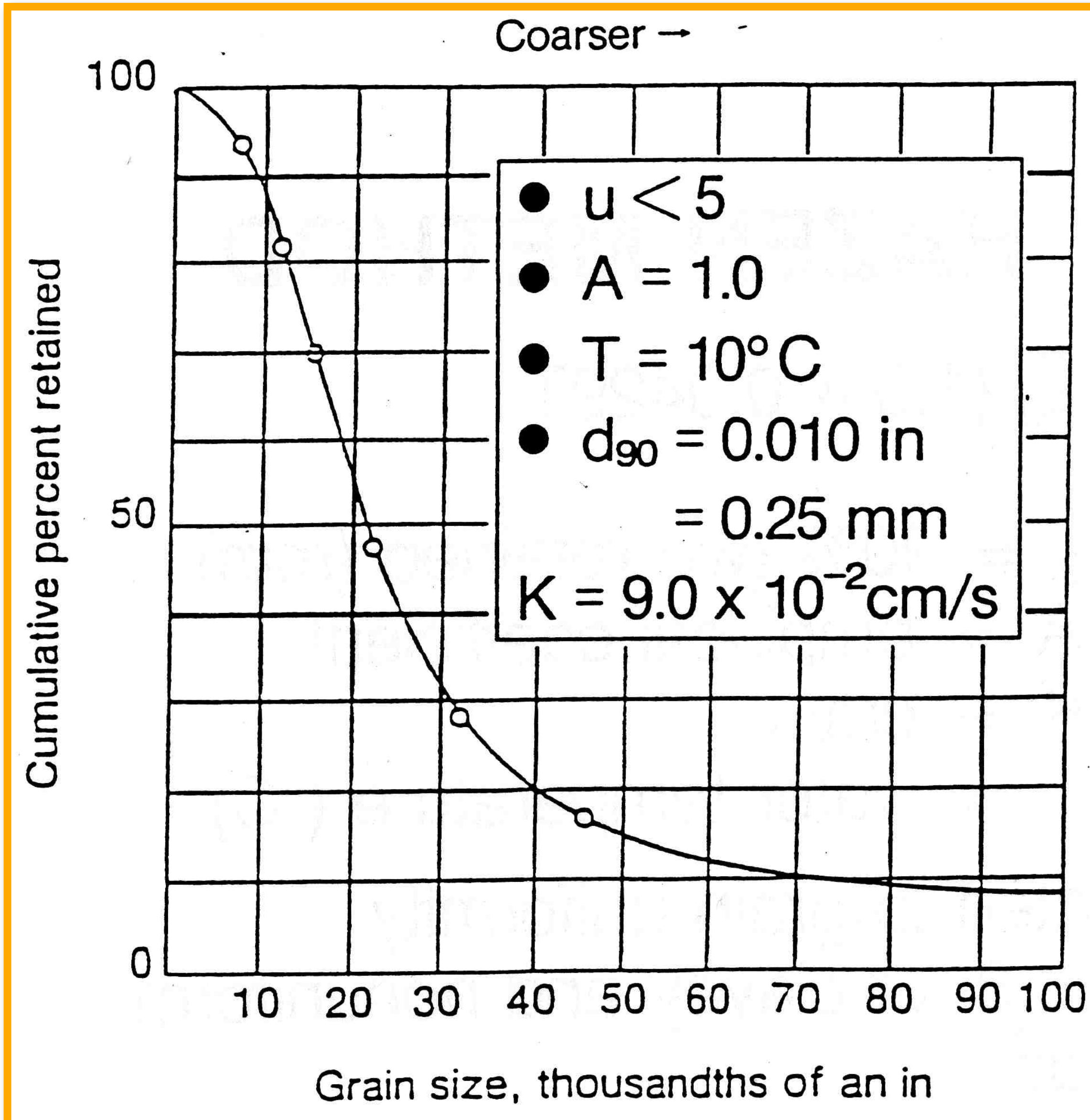
- 0.4 – 0.8 for clayey & non-uniform sands
- 0.8 – 1.2 for clean & uniform sands

Example



U = d_{40}/d_{90}
0.026/0.009
2.9
Cumulative percent retained
100
50
0
10
20
30
40
50
60
70
80
90
100
Grain size, thousandths of an in

What effect does temperature have?



Example

Hazen Method (Hydraulic Conductivty)

$$K = C_{HZ}\left(d_{90}\right)^2$$

Alternative #2
(Schwartz and Zhang, P. 53)

K – Hydraulic conductivity (cm/s)

d_{90} - 90% retained (cm)

where C_{HZ} is a dimensionless constant (see table below).

Description	C_{HZ}
Very fine sand, poorly sorted	40 - 80
Fine sand with appreciable fines	40 - 80
Medium sand, well sorted	80 - 120
Coarse sand, poorly sorted	80 - 120
Coarse sand, well sorted	120 - 150

Schlichter Method (1905)

- Most useful for medium grained sands with $d_{90}(D_{10})$ between 0.01 and 5.0 mm
- Includes porosity

Schlichter Method (1905)

$$K = 0.1022 \times d_{90}^2 \times {}^{P_{corr}}\!/_{T_{corr}}$$

Where:

P_{corr} = 1.0301 x $n^{3.3334}$

T_{corr} = 0.017194 $e^{-0.025926T}$

And:

d_{90} = 90% (wt) retained (mm)

n = porosity (fraction of 1.0)

T = water temperature (^{0}C)

K = cm/s

Permeameter Tests

- Samples taken during drilling operations are loaded into a confining cell and a steady-state flow rate test is conducted (Darcy's experiment) or a transient test is designed

(Rudolph)

Permeameters

Constant Head

- a constant head is maintained on both ends of a sample
- essentially the Darcy apparatus (see pg. 4)
- calculate K directly from Darcy's equation

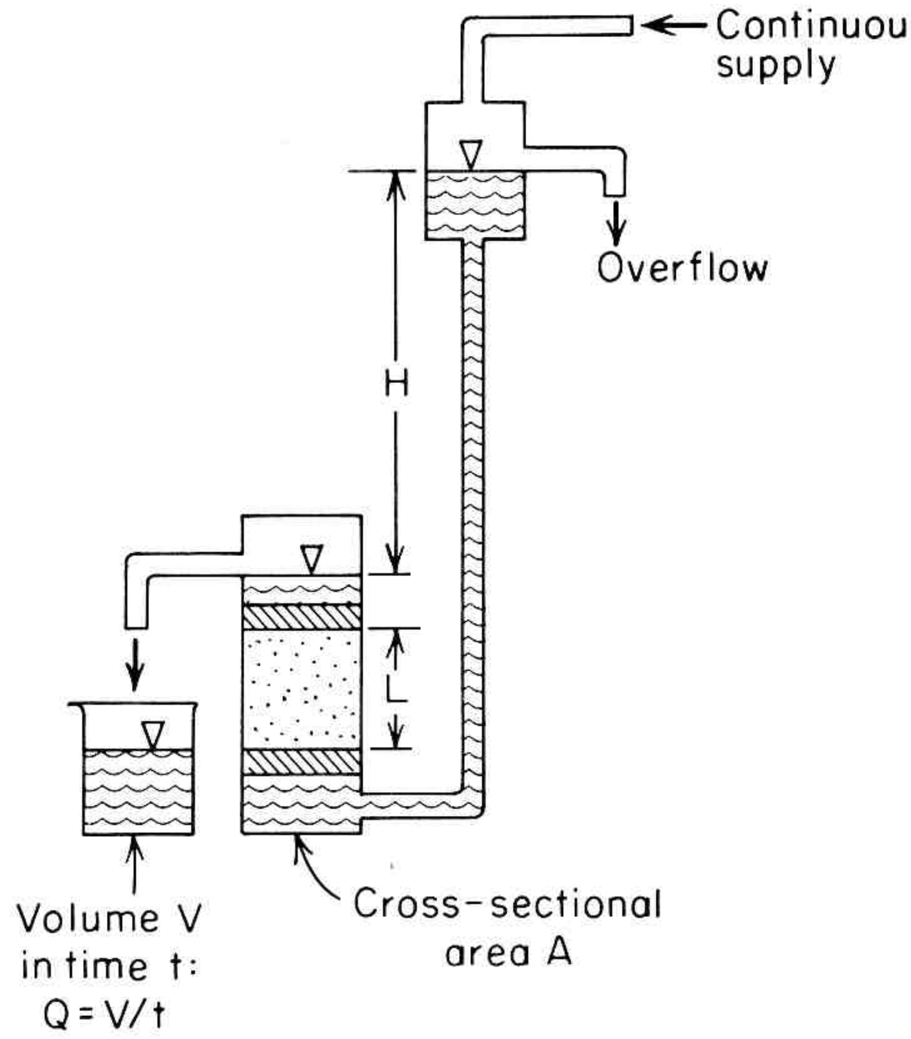
Falling Head

- a sample is connected to a vertical tube in which the water level is allowed to fall with time
- thus, the head difference and discharge through the sample vary with time
- K is calculated from the rate of decline in the water level in the tube

Constant-Head Permeameter

$$K = \frac{QL}{AH}$$

(High K sediments)

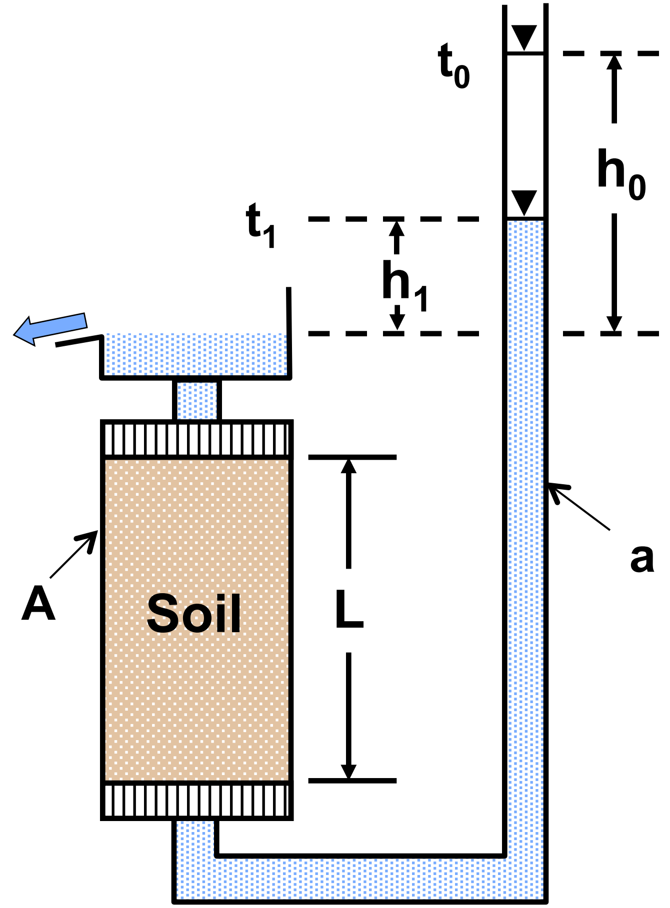


Freeze & Cherry, 1979

Falling Head Permeameter

K is given as:

$$K = \frac{a}{A}\frac{L}{(t_1 - t_0)}ln\left(\frac{h_0}{h_1}\right)$$



a = cross-sectional area of riser tube

A = cross-sectional area of soil sample

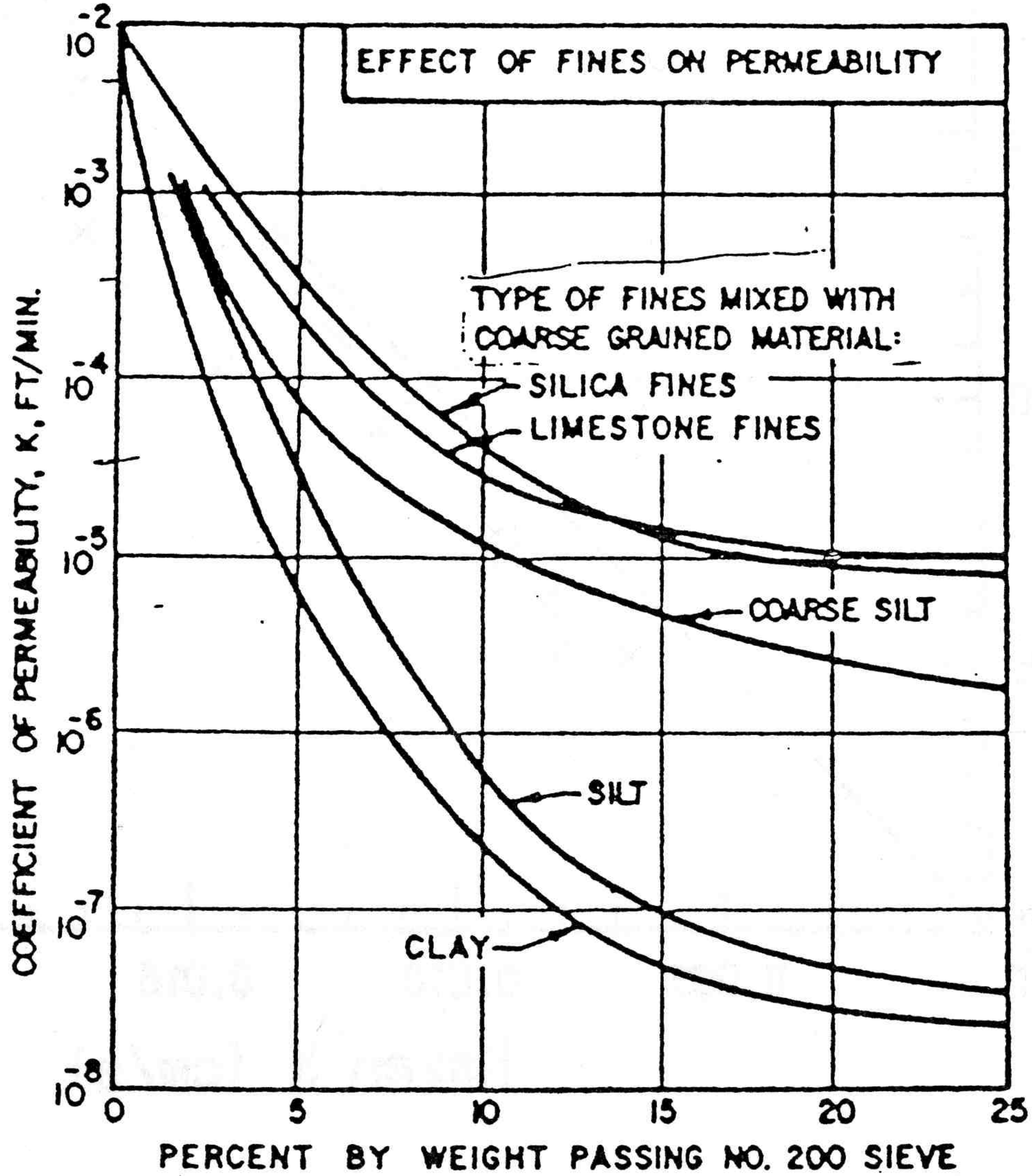
h_0 = head difference across the sample at time t_0

h_1 = head difference across the sample at time t_1

Additional Points

- Coarse, uniform sand or gravel can be repacked to original density and will generally function as undisturbed sample
- Remixed heterogeneous samples can provide significantly different values of K from the undisturbed values
- Samples generally taken vertically so Kv or Kz is measured with this test

Effect of Fines on Permeability



Recall the following relations for specific discharge based on Darcy's Law can be shown:

$$q \propto d^2$$

d = "mean" grain diameter

$$q \propto \rho \cdot g$$

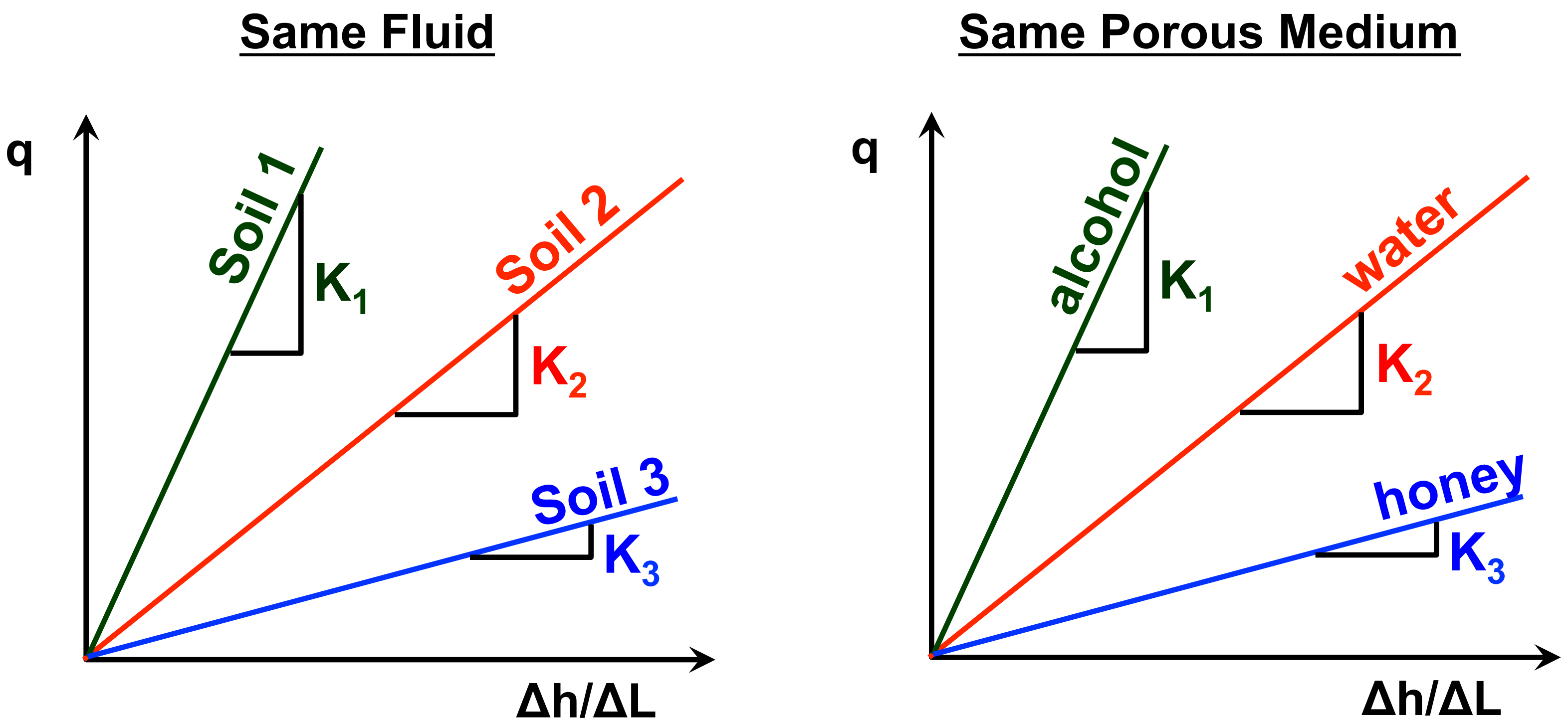
$$q \propto \frac{1}{\mu}$$

Darcy's Law can be written,

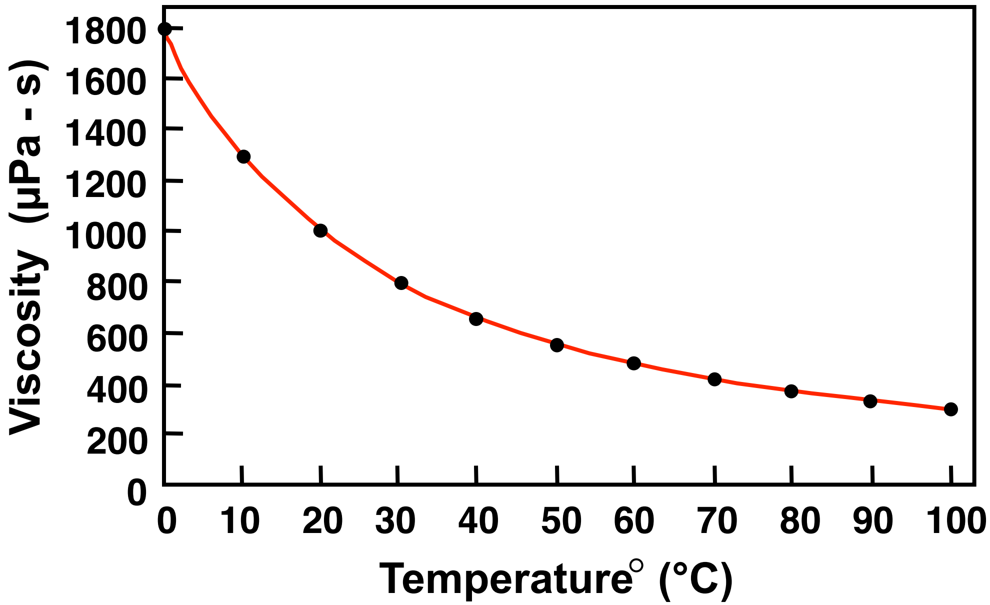
$$\boxed{q = -Cd^2 \frac{1}{\mu} \rho g \frac{dh}{dx}}$$

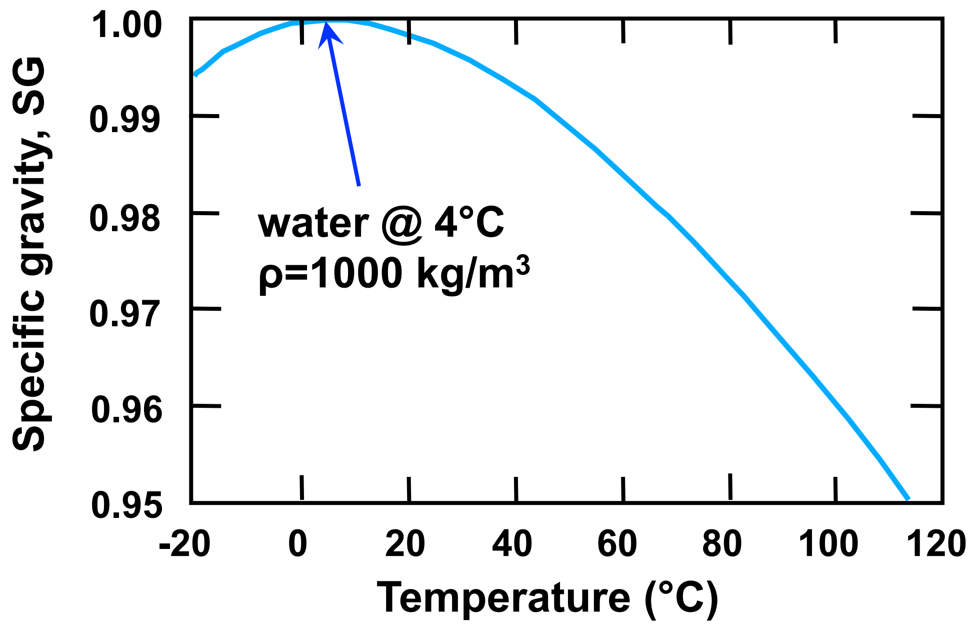
Hydraulic Conductivity and Permeability

Remember that hydraulic conductivity was a property of both the fluid and the porous medium.

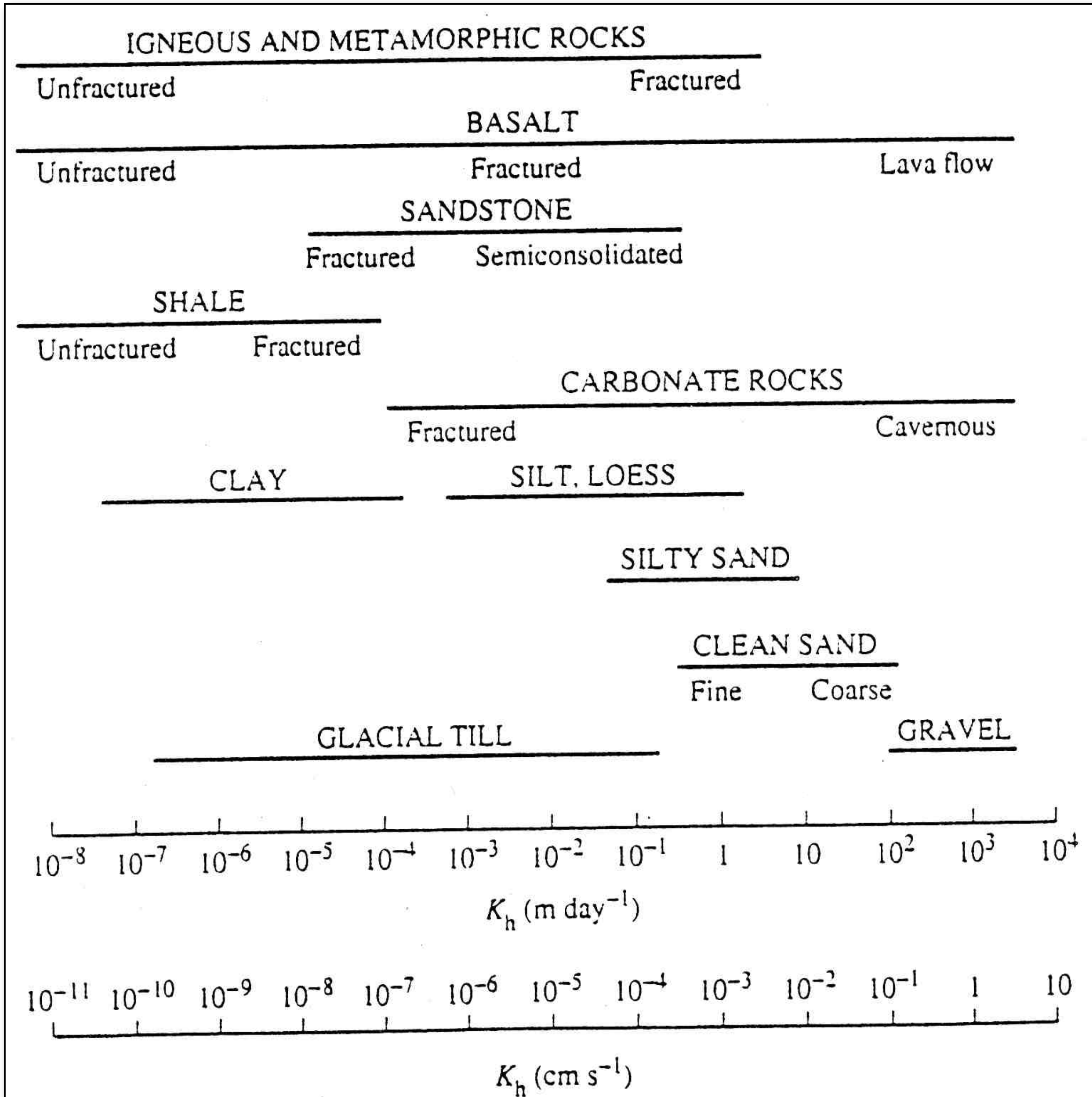


Temperature Effects on K





Source: CRC Handbook - H_2O properties



Hydraulic Conductivity of Selected Geologic Materials

Dingman (2002)

Ranges of Hydraulic Conductivity *and* Permeability

Table 2.2 Range of Values of Hydraulic Conductivity and Permeability

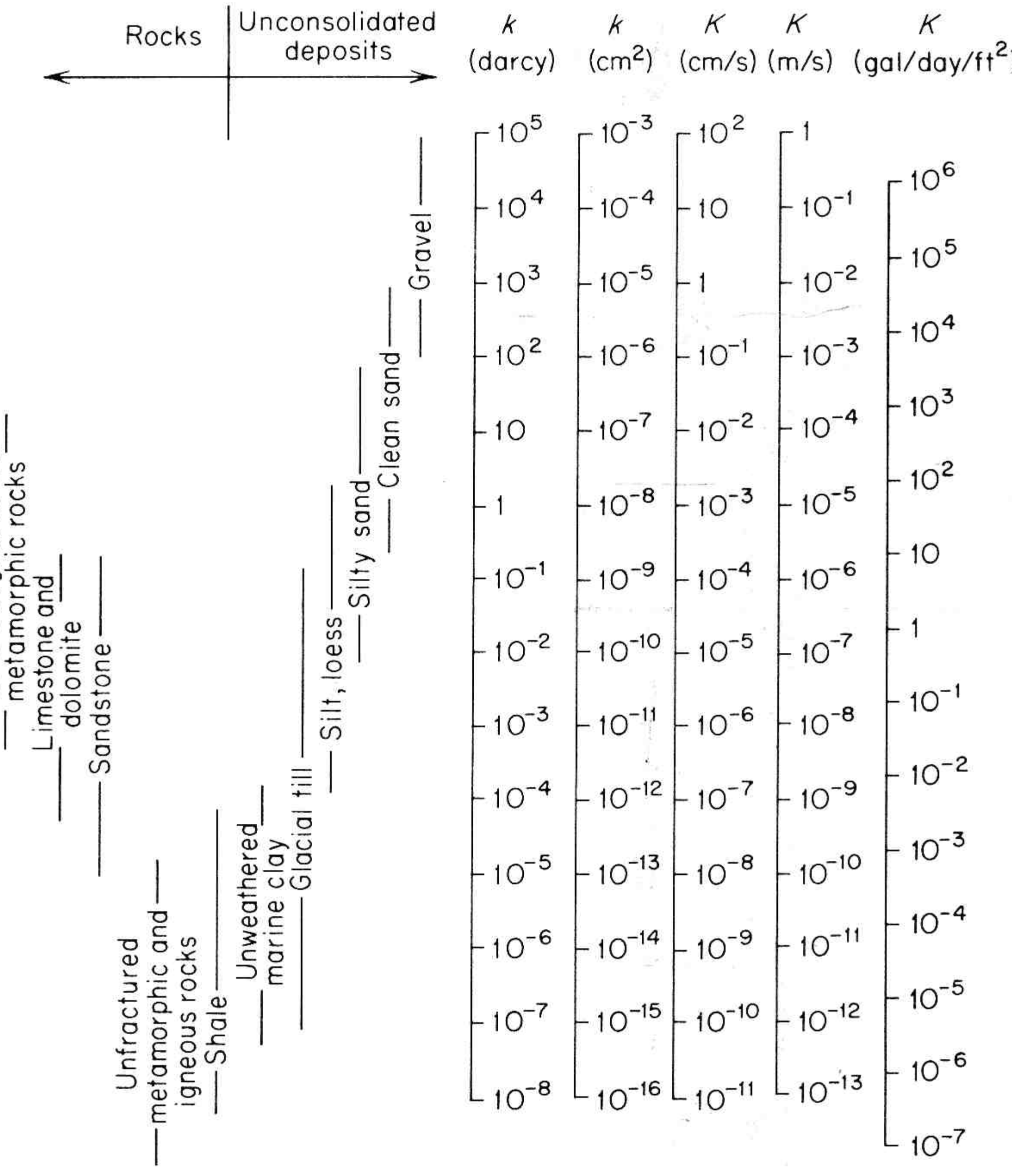
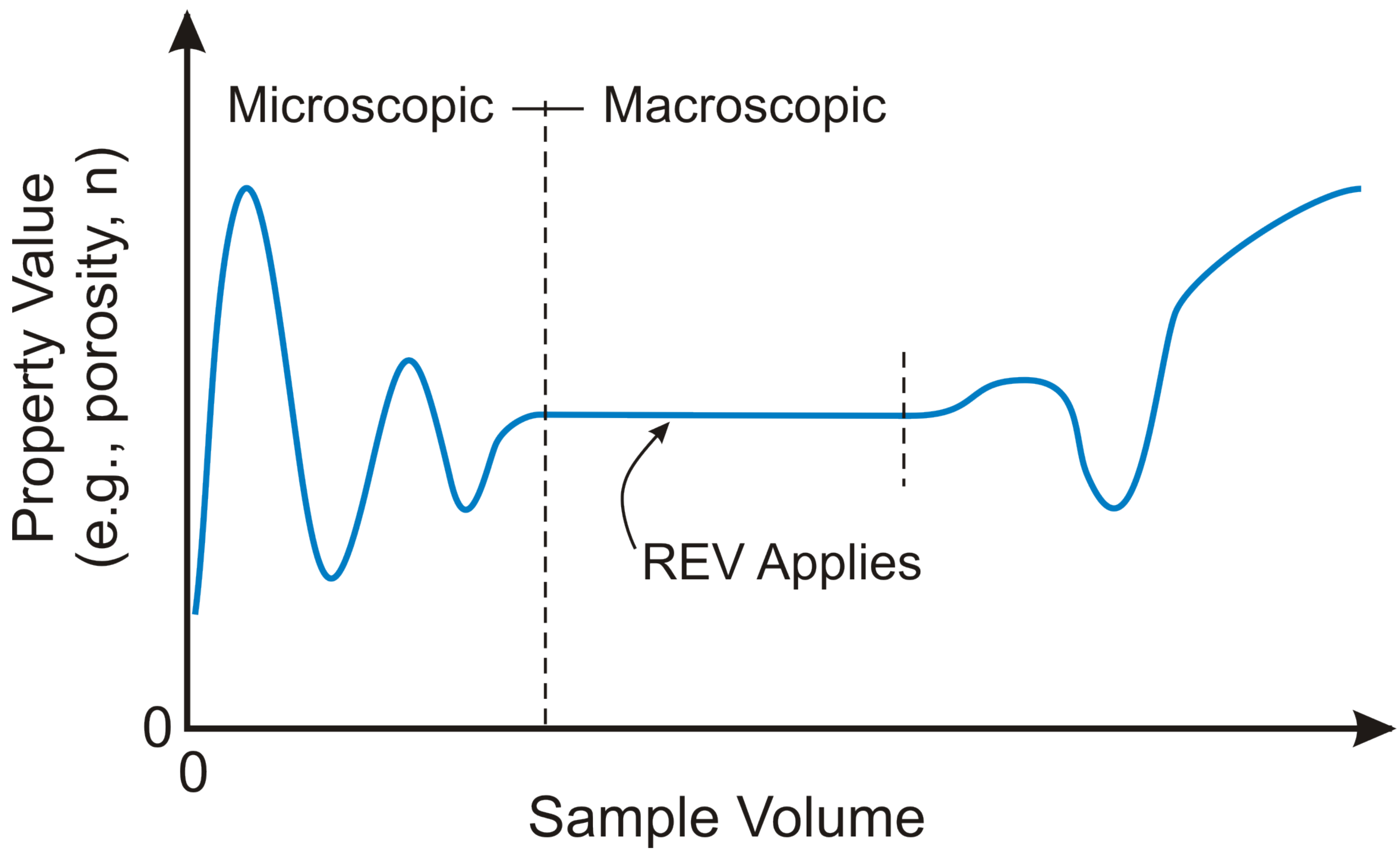


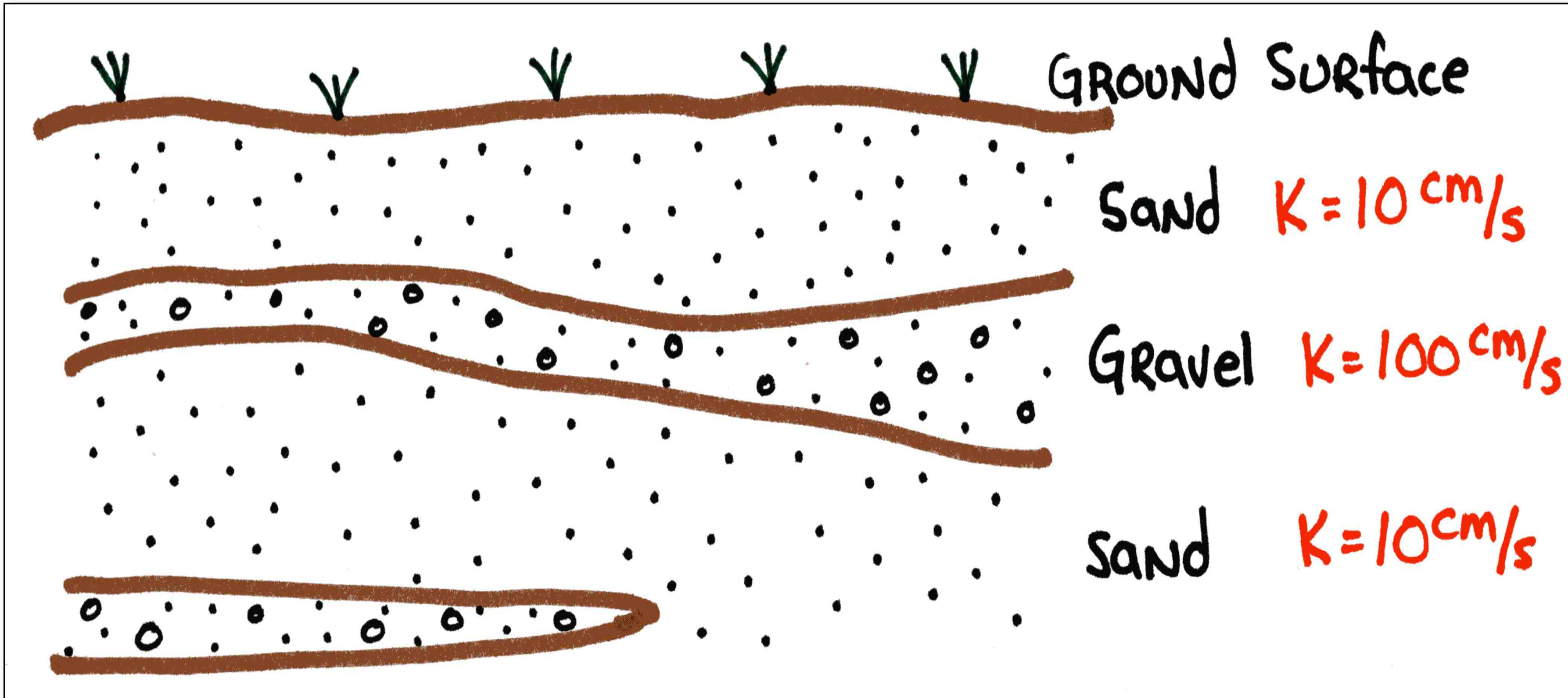
Table 2.3 Conversion Factors for Permeability and Hydraulic Conductivity Units

Freeze and Cherry (1979)

Property values change depending on the scale of measurement. This is a critical concept that allows us to describe and quantify field-scale observations.

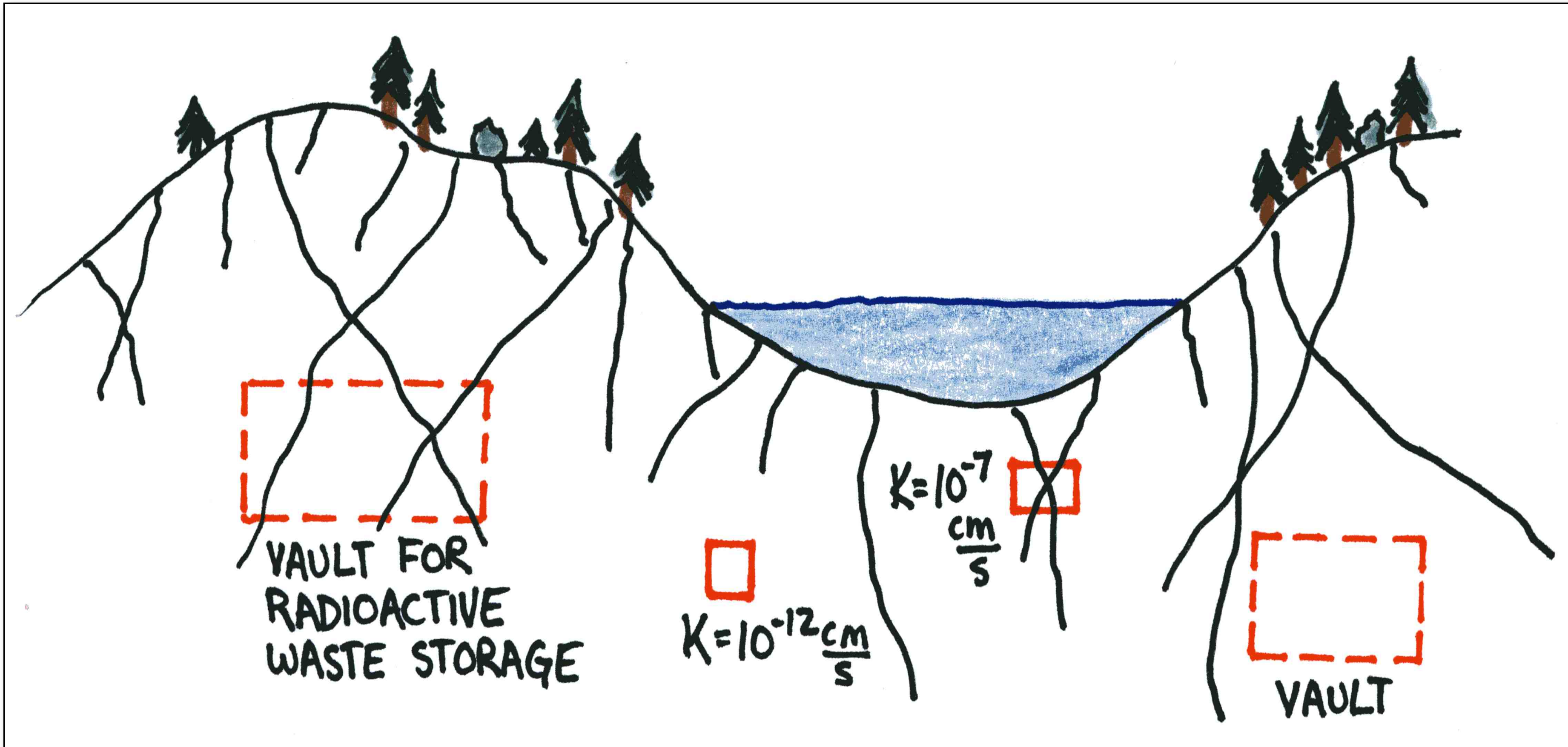


Hydraulic Conductivity at the Field Scale



Sand and Gravel Aquifer System

Hydraulic Conductivity at the Field Scale



Fractured Granite in the Canadian Shield

<u>Specific Discharge and Flow Velocity</u>

Specific discharge, q, has units of m/s, but it does <u>not</u> represent the true average velocity of groundwater flow.

q = specific discharge (or Darcy flux; Darcy "velocity")

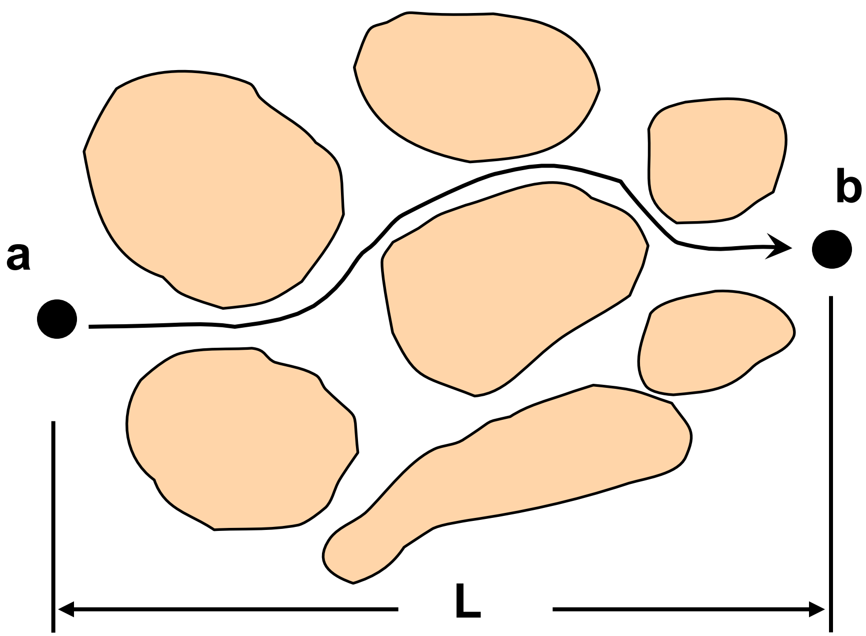
It is a flux and arises from $\dfrac{\text{volume}}{\text{area} \times \text{time}}$ **not** $\dfrac{\text{distance}}{\text{time}}$

Everybody knows that velocity is just distance traveled per unit time. Right?

Let us try to define:

<u>Average groundwater velocity:</u> $\boxed{\bar{v}}$

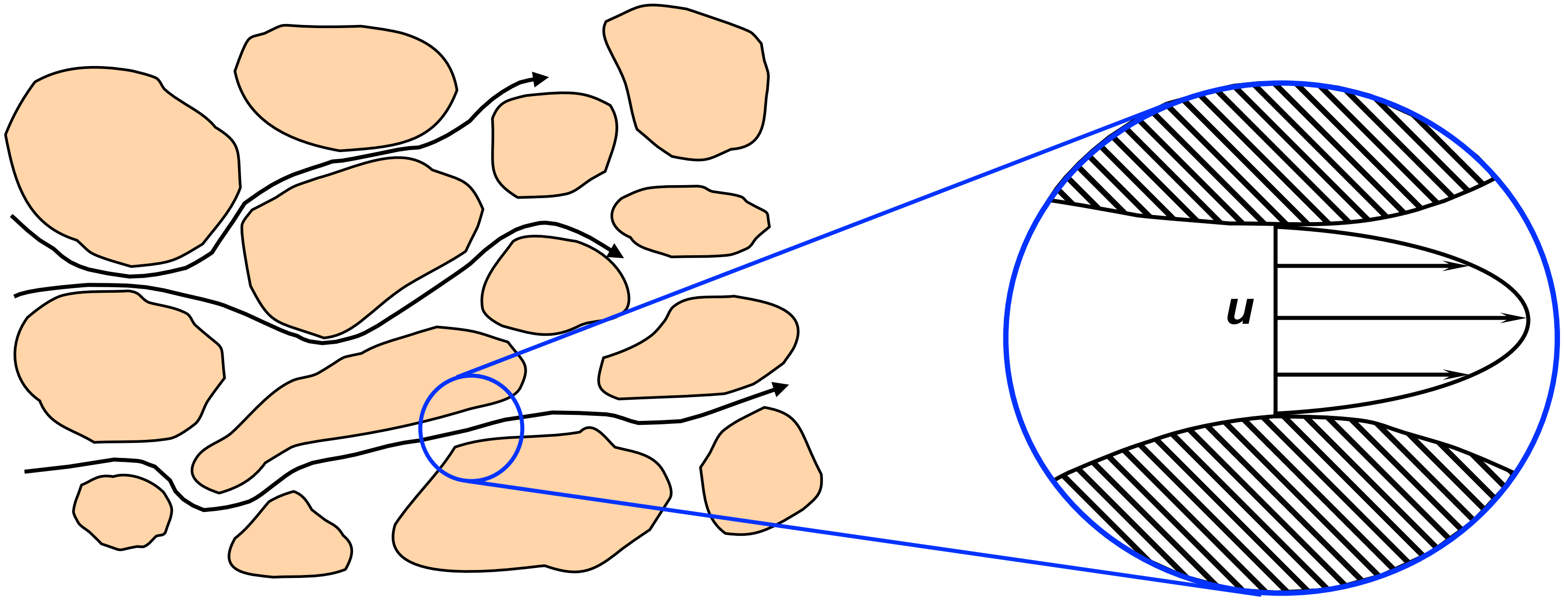
What velocity do we really mean? Consider the microscopic velocity of individual water molecules.



We can see that water follows tortuous paths around soil grains.

Is this the v we are looking for?

Pore-scale Velocity Variations

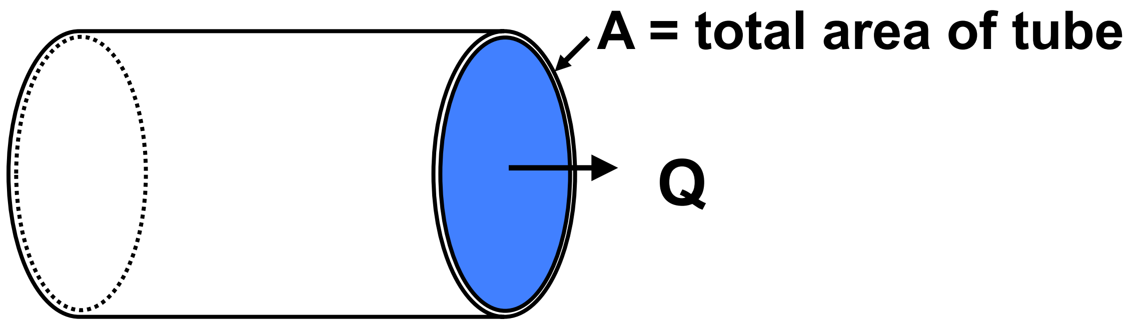


Velocities at the microscopic scale are extremely complex, but are important for transport of contaminants (as we will see later).

We need to determine a groundwater velocity that is averaged over some REV. That is, a macroscopic quantity that we can measure.

If we had a tube with no solid phase, the average velocity is simply Q/A, or q as we defined it.

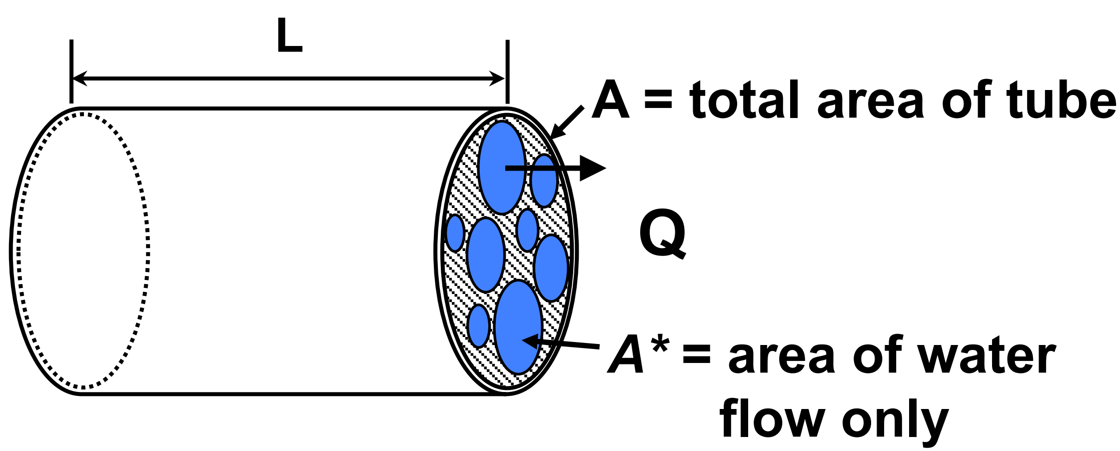
$$q = \frac{Q}{A} = v$$



By adding sediment we effectively reduce the cross-sectional area through which flow occurs.

$$q = \frac{Q}{A}$$

$$v = \frac{Q}{A^*}$$



The relationship between A^* and A is given by the porosity,

$$n = \frac{V_{Voids}}{V_{Total}} = \frac{A_{Voids}}{A_{Total}} = \frac{A^*}{A}$$

This gives the following relationship between the <u>average groundwater velocity, *v*</u>, and specific discharge, q:

$$\boxed{\bar{v} = \frac{q}{n} = -\frac{K}{n}\frac{\Delta h}{\Delta l}(m/s)}$$

Where: $\bar{v}$ represents average *linear* flow path. (i.e. average of all flow path velocities over a linear distance).

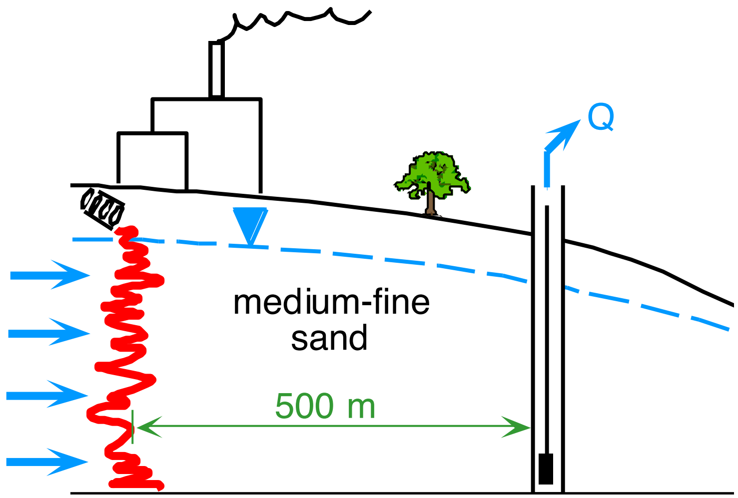
What porosity value do we pick?

We need to pick the porosity for the voids that the water actually flows through. What is this called?

Primary and secondary porosities can differ greatly in a single material.

For example, a clay may have a primary (intergranular) porosity of 0.60, but a fracture porosity of only 0.001. If water flow is predominantly through the fractures it will considerably alter the calculated value for groundwater velocity.

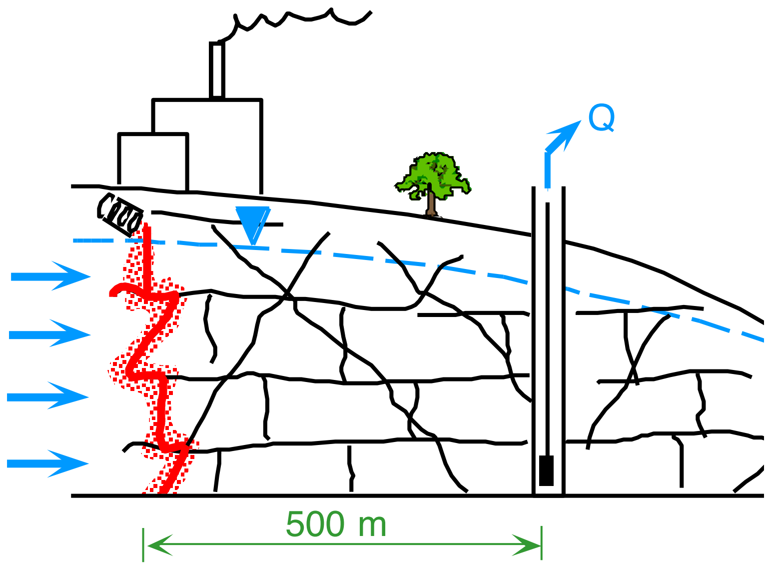
Example: Travel Time Calculation - Porous Medium



As a result of a spill, contaminants entered the groundwater at a location 500 m up-gradient from a water supply well. The average hydraulic gradient between the well and the site of the spill was 0.01, the K of the medium-fine grained sand is 10^{-5} m/s, and the porosity is 0.3.

How long will it take for contaminants to reach the well?

Example: Travel Time Calculation - Fractured Rock



Consider the same situation with the same parameter values, except now flow is occurring in a fractured rock environment. The effective fracture porosity is 10^{-3}.

Now how long will it take for contaminants to reach the well?

Limitations of Darcy's Law

Flow is in the direction of decreasing energy (or head). Is this potential energy lost? What happens to it?

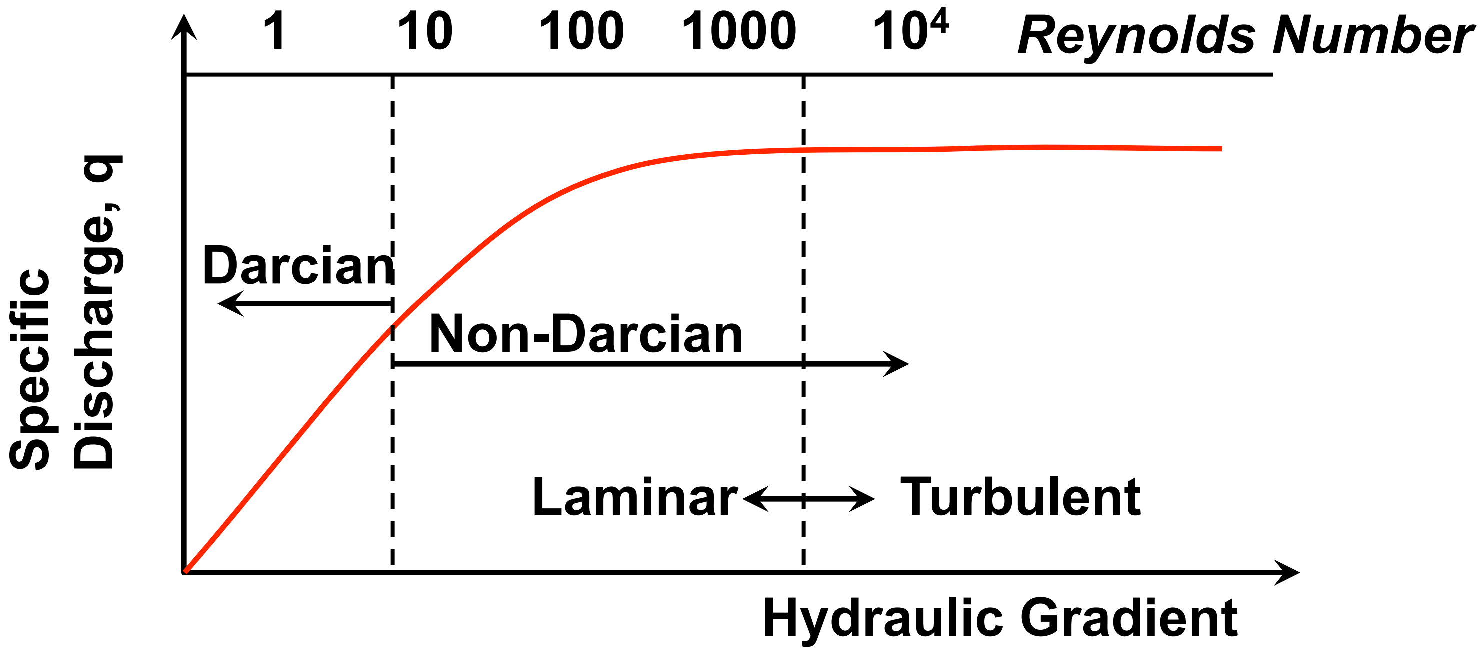
Darcy's law fails when kinetic energy becomes significant in comparison to frictional heat loss. Reynolds number (*Re*) is used as a criterion for judging the applicability of Darcy's law. Note that we ignored inertial (kinetic) forces earlier.

$$Re = \frac{\rho v d}{\mu} \propto \frac{\text{inertial forces}}{\text{viscous forces}}$$

where *v* = linear velocity
d = characteristic length (typically grain size, d_{10})

Darcy's law is known to fail when *Re* approaches 10.

Note that the flow is still perfectly laminar in this range of *Re*. The laminar-to-turbulent transition occurs at *Re* = 2000 or so.



What is the highest groundwater velocity for which Darcy's Law is still valid in sand of d = 0.5 mm?

$$v_x = \frac{10 \times 0.011404}{0.99909 \times 0.0005}$$

$$= \underline{229\ m/day}$$

Extremely fast, only found in very course sediments or fractured/karstic rocks.