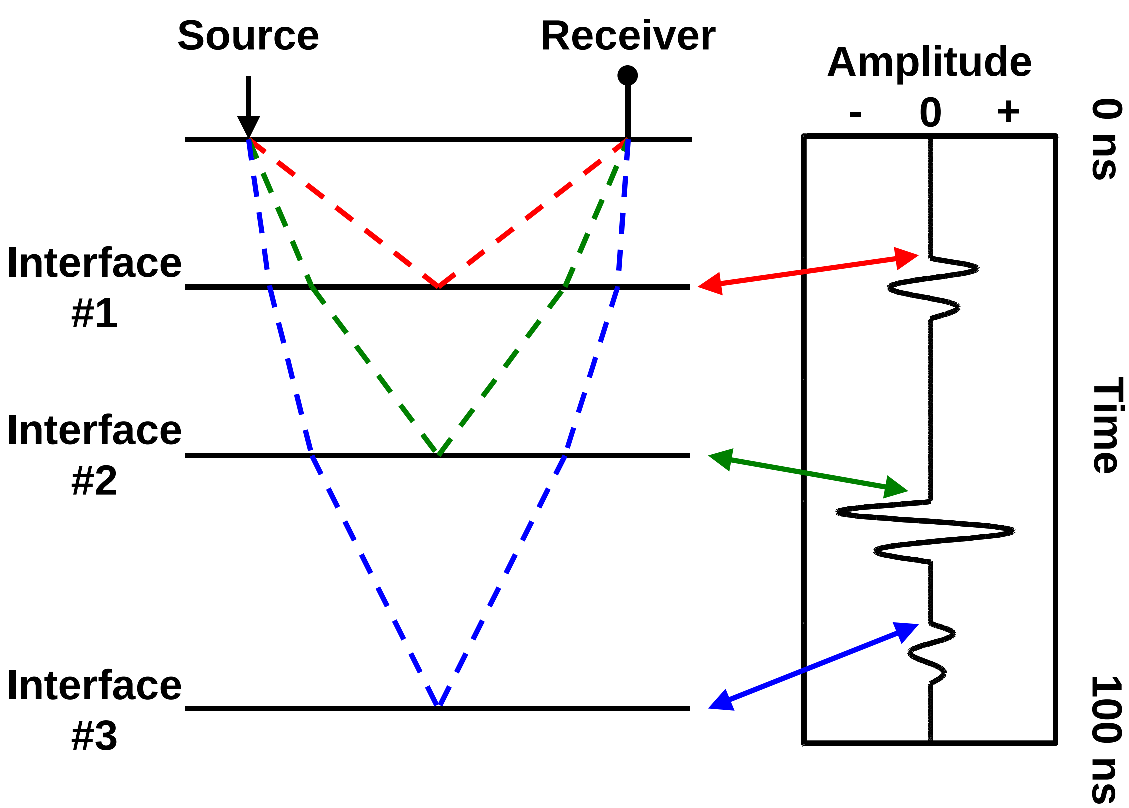
Earth 460 Classnotes (Set 4)

I.) Time Series Analysis – Continuous Signals

E.) Convolution Model of the Earth

1.) Consider GPR or seismic reflection sounding performed on a horizontally layered Earth. A source inputs a signal with a wavelet $w(t)$ into the Earth; the reflection events from the interfaces are recorded at the receiver.



2.) Neglecting spherical divergence, absorption and transmission losses, the reflection signal from the ith interface $s_i(t)$ can be expressed as

$$s_i(t) = R_i\, w(t - T_i)$$

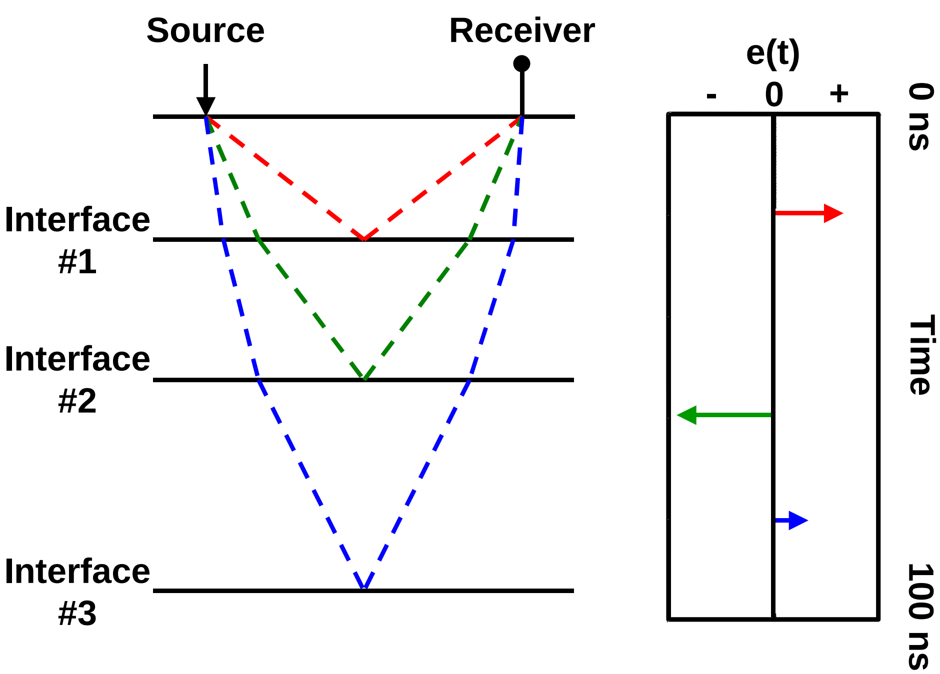
where R_i is the reflection coefficient and T_i is the two-way traveltime, respectively, for the ith interface.

3.) The GPR/seismic trace $s(t)$ for a layered Earth with N interfaces can be expressed as

$$s(t)=\sum_{i=1}^{N}R_i\,w(t-T_i)=\sum_{i=1}^{N}R_i\left[w(t)*\delta(t-T_i)\right]=w(t)*\left[\sum_{i=1}^{N}R_i\,\delta(t-T_i)\right]$$

4.) Hence, the GPR/seismic trace $s(t)$ can be viewed as the convolution of the source wavelet $w(t)$ with a series of scaled delta functions that correspond to the interfaces in the Earth.

This series of scaled delta functions is the impulse response of the Earth $e(t)$ (i.e., $e(t)=\sum_{i=1}^{N}R_i\,\delta(t-T_i)$).



5.) Therefore, the GPR/seismic trace $s(t)$ is the convolution of the source wavelet $w(t)$ with is the impulse response of the Earth $e(t)$:

$$s(t) = w(t) * e(t)$$

This expression is the convolution model of the Earth for a GPR/seismic trace.

6.) Spectral relationships for the convolution model of the Earth

a.) Fourier transform pairs for this model:

$s(t) \Leftrightarrow S(f) = A_S(f)e^{i\theta_S(f)}$, $w(t) \Leftrightarrow W(f) = A_W(f)e^{i\theta_W(f)}$ and $e(t) \Leftrightarrow E(f) = A_E(f)e^{i\theta_E(f)}$

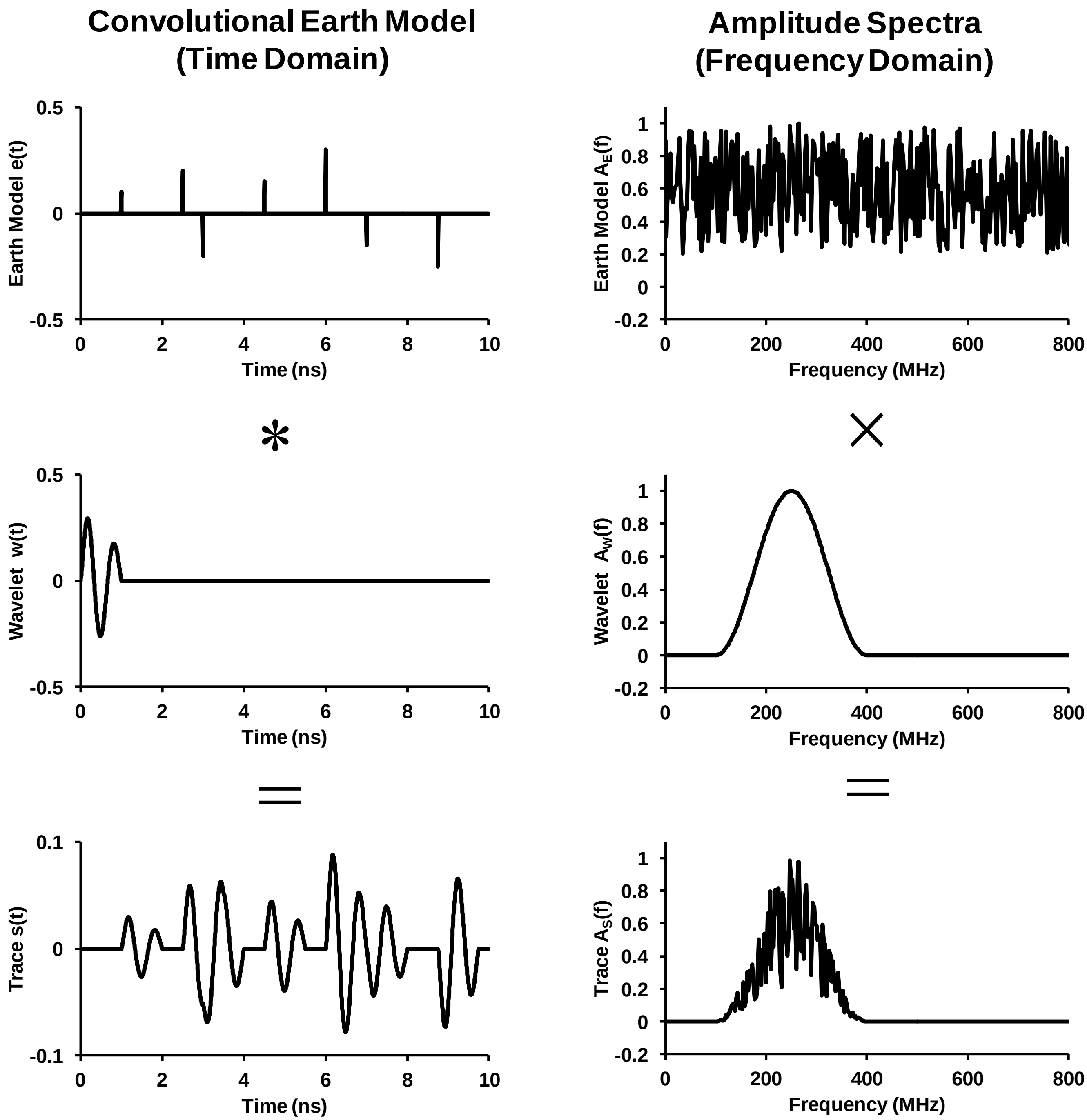
b.) Given the convolutional relationship:

$s(t) = w(t) * e(t) \Leftrightarrow S(f) = W(f)E(f) \rightarrow A_S(f) = A_W(f)A_E(f)$ and $\theta_S(f) = \theta_W(f) + \theta_E(f)$

c.) The information contained in Fourier transform of earth impulse response $E(f)$ generally covers a wide bandwidth of frequencies.

d.) In comparison, the Fourier transform of the source wavelet $W(f)$ has a very band limited spectrum with its energy concentrated about its dominant frequency.

e.) Hence, the amplitude spectrum of the GPR/seismic trace $A_S(f)$ contains band limited information about $E(f)$ that is modulated by $A_W(f)$.



F.) Source Wavelets

1.) A number of idealized wavelets are used to represent source signatures.

2.) Sinc and Ormsby wavelets

a.) The sinc wavelet is a band-limited, zero phase wavelet with a uniform amplitude spectrum (identical in form to band-pass filter given above).

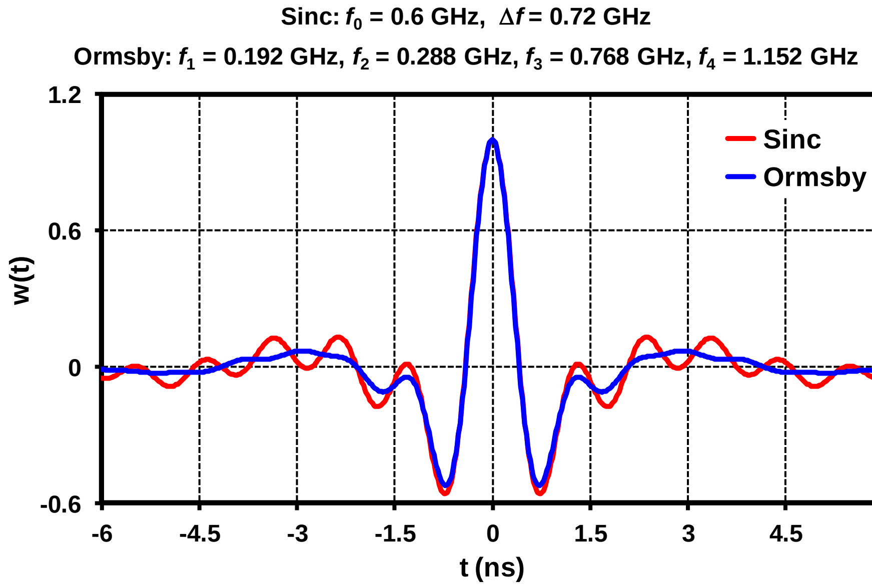
$$w(t) = \cos(2\pi f_0 t)\,\mathrm{sinc}(\Delta f\, t) \Leftrightarrow W(f) = \frac{1}{2\Delta f}\left[\Pi\left(\frac{f-f_0}{\Delta f}\right) + \Pi\left(\frac{f+f_0}{\Delta f}\right)\right]$$

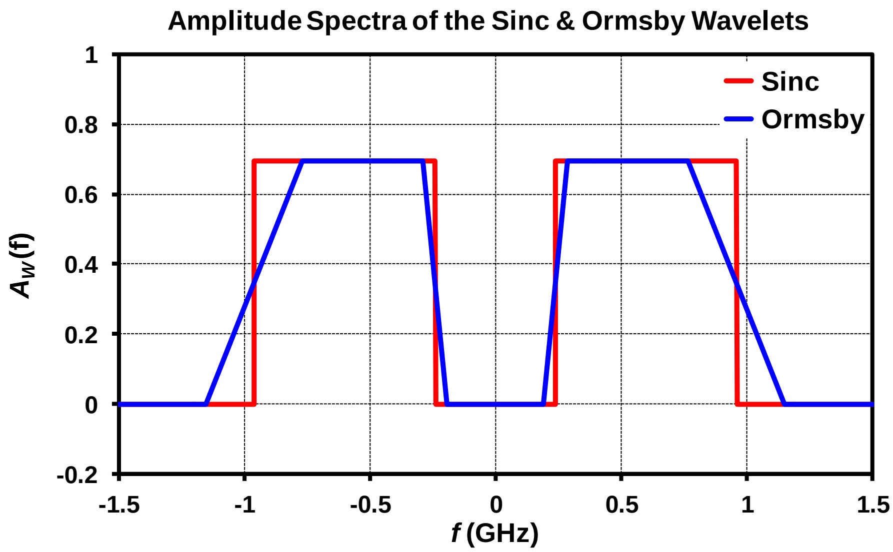
where f_0 is the center/dominant frequency and Δf is the spectral bandwidth.

b.) The related Ormsby wavelet reduced the side lobes by using linear tapers at the limits of the frequency band

$$w(t) = \left[(f_4+f_3)-(f_2+f_1)\right]^{-1}\left\{\left\{(f_4-f_3)^{-1}\left[(f_4)^2\,\mathrm{sinc}^2(f_4 t)-(f_3)^2\,\mathrm{sinc}^2(f_3 t)\right]\right\} - \left\{(f_2-f_1)^{-1}\left[(f_2)^2\,\mathrm{sinc}^2(f_2 t)-(f_1)^2\,\mathrm{sinc}^2(f_1 t)\right]\right\}\right\}$$

where f_1 and f_2 cutoff and pass frequencies, respectively, at the low frequency limit; f_3 and f_4 pass and cutoff frequencies, respectively, at the high frequency limit.

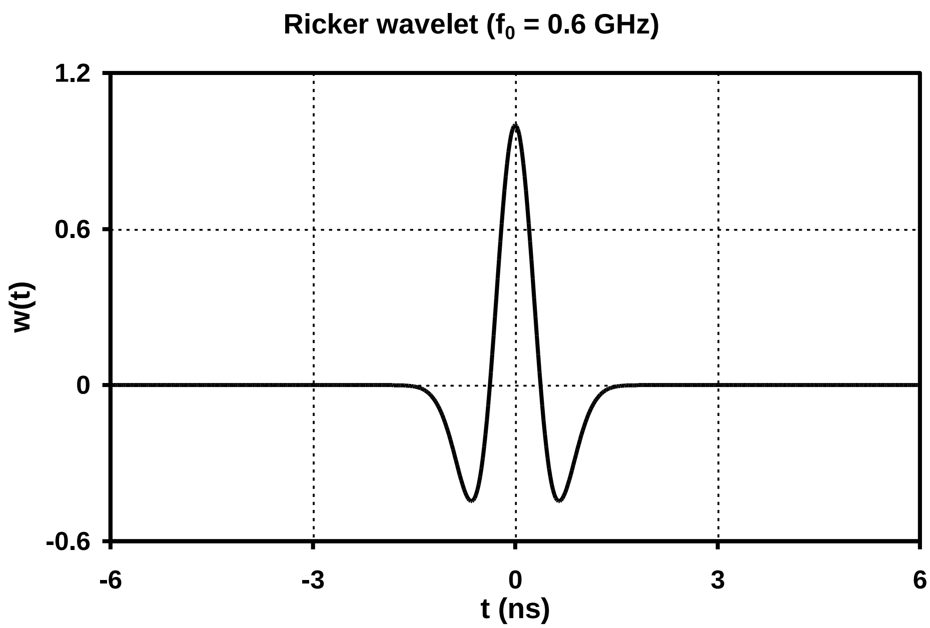

Sinc: f_0 = 0.6 GHz, Δf = 0.72 GHz
Ormsby: f_1 = 0.192 GHz, f_2 = 0.288 GHz, f_3 = 0.768 GHz, f_4 = 1.152 GHz
Sinc
Ormsby
w(t)
1.2
0.6
0
-0.6
-6
-4.5
-3
-1.5
0
1.5
3
4.5
6
t (ns)

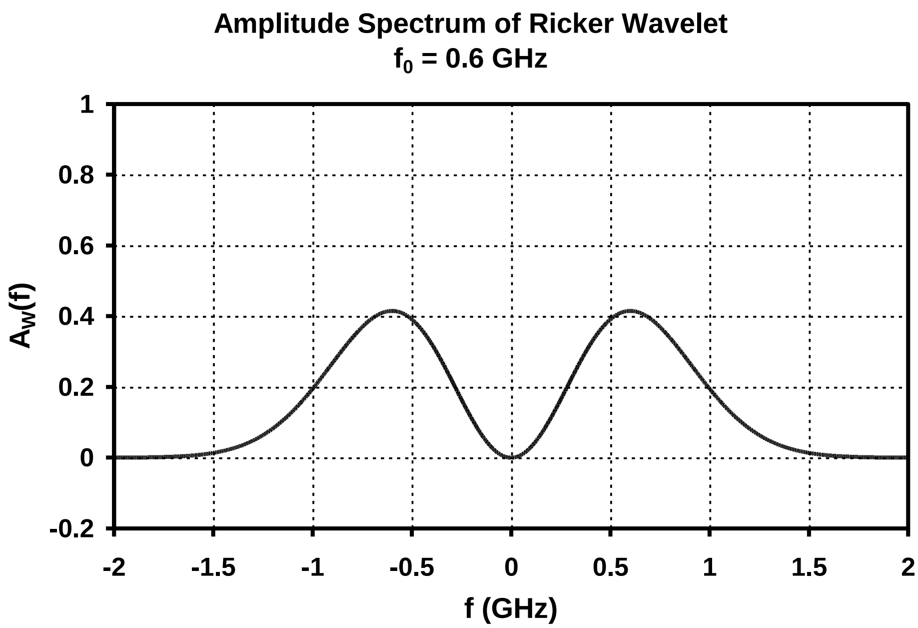

Amplitude Spectra of the Sinc & Ormsby Wavelets
Sinc
Ormsby
$A_W(f)$
1
0.8
0.6
0.4
0.2
0
-0.2
-1.5
-1
-0.5
0
0.5
1
1.5
f (GHz)

3.) Ricker wavelet

A zero phase wavelet with its amplitude spectrum concentrated about its center/dominant frequency f_0 .

$$w(t) = \left[1 - 2(\pi f_0 t)^2\right] e^{-(\pi f_0 t)^2} \Leftrightarrow W(f) = \left(\frac{2}{\sqrt{\pi}}\right)\left(\frac{f}{f_0}\right)^2 e^{-(f/f_0)^2}$$





4.) Berlage wavelet

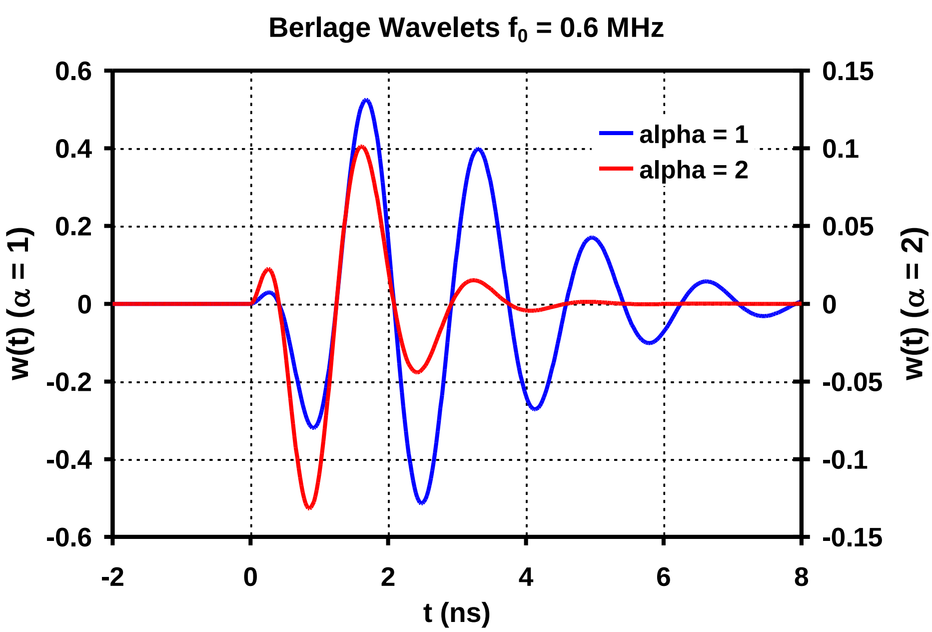
A causal wavelet with its amplitude spectrum concentrated about its center/dominant frequency f_0. The time amplitude envelope is defined by the time exponent n and the exponential decay factor α. The parameter ϕ_0 is the initial phase angle of the wavelet.

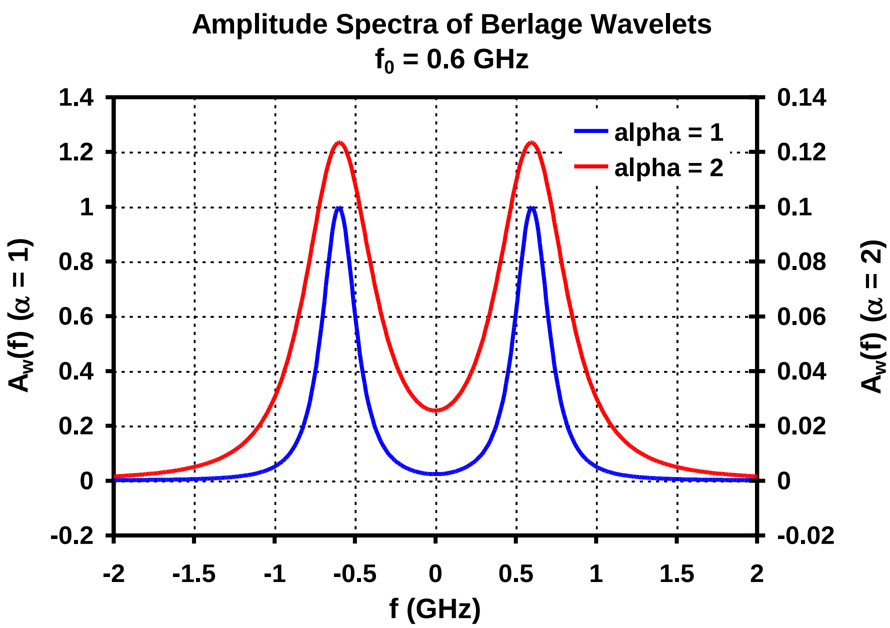
$$w(t) = At^n e^{-\alpha t}\cos\left(2\pi f_0 t + \phi_0\right)H(t) \Leftrightarrow W(f) = \frac{A\Gamma(n+1)}{2(i2\pi)^{n+1}}\left[\frac{e^{+i\phi_0}}{(f-F_1)^{n+1}} - \frac{e^{-i\phi_0}}{(f-F_2)^{n+1}}\right]$$

where $F_1 = +f_0 + i(\alpha/2\pi)$ and $F_2 = -f_0 + i(\alpha/2\pi)$

The effective bandwidth of the amplitude spectrum is directly related to α

Consider two Berlage wavelets, both with $n = 2$ and $\phi_0 = 0$





G.) Deconvolution and Inverse Filters

1.) Deconvolution is a procedure that compensates for the effects of wavelet $w(t)$ on the GPR/seismic trace $s(t)$, in effect reversing the convolution between $w(t)$ and $e(t)$.

2.) Ideal conditions

a.) We could design an inverse filter $w'(t)$ such that $w'(t)*w(t)=\delta(t)$

b.) If this inverse filter is applied to the trace $s(t)$, we would get

$$s'(t)=w'(t)*s(t)=w'(t)*\left[w(t)*e(t)\right]=\left[w'(t)*w(t)\right]*e(t)=\delta(t)*e(t)=e(t)$$

c.) Frequency domain relationships between wavelet $w(t)$ and inverse filter $w'(t)$:

$$w'(t)*w(t)=\delta(t)\Leftrightarrow W'(f)W(f)=1 \quad \rightarrow \quad W'(f)=1/W(f)$$

3.) Actual conditions

a.) The wavelet $w(t)$ has a band-limited frequency spectrum (i.e., $W(f)=0$ beyond the wavelet cutoff frequencies).

b.) The best that can be achieved is the designed of an inverse filter $w'(t)$ such that $w'(t)*w(t)=\delta'(t)$ where $\delta'(t)$ is a band-limited approximation of $\delta(t)$ (e.g., sinc wavelet)

c,) Frequency domain relationships between wavelet $w(t)$ and inverse filter $w'(t)$:

$$W'(f)=\begin{cases} 1/W(f) \text{ for } f_{Low} \le |f| \le f_{High} \\ 0 \text{ otherwise} \end{cases}$$

d.) Application of the inverse filter gives

$$s'(t)=w'(t)*s(t)=w'(t)*\left[w(t)*e(t)\right]=\left[w'(t)*w(t)\right]*e(t)=\delta'(t)*e(t)$$

4.) The effect of deconvolution in the time domain is the compression of the time duration of the effective source wavelet. This process reduces the overlapping/interference between closely space reflection events (i.e., improves resolution of the reflections).

5.) Prewhitening

a.) Commonly, the source wavelet spectrum $W(f)$ has very small amplitude values over some frequency bands.

b.) Conversely, the inverse filter spectrum $W'(f)$ has very large amplitude values over these frequency bands.

c.) This situation can result in the significant magnification of noise contained within these frequency bands.

d.) To avoid this problem, a small, but significant constant ε is added to $W(f)$ when determining $W'(f)$ (i.e., $W'(f) = 1/\left[W(f)+\varepsilon\right]$.

e.) This approach does not substantially change the inverse filter over frequencies where $W(f)$ has significantly large amplitude as $1/\left[W(f)+\varepsilon\right] \approx 1/\left[W(f)\right]$.

f.) Noise amplification is limited

g.) The addition of the constant ε is equivalent adding a small amount of white noise to $w(t)$; hence the term prewhitening.

6.) Types of deconvolution processes

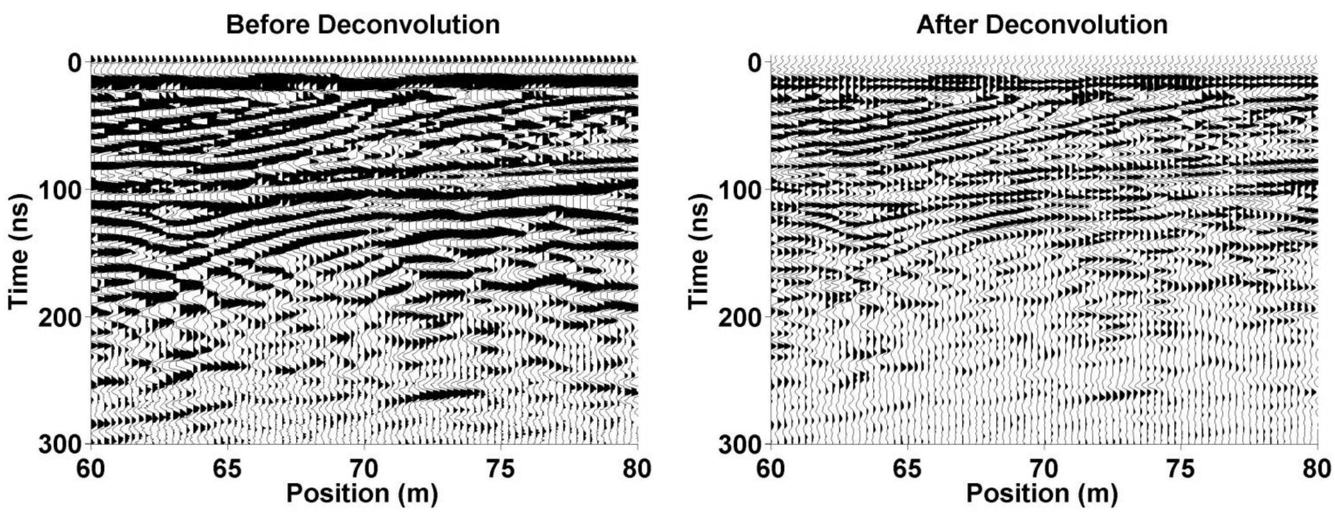
a.) The deconvolution process required information about the nature of source wavelet $w(t)$.

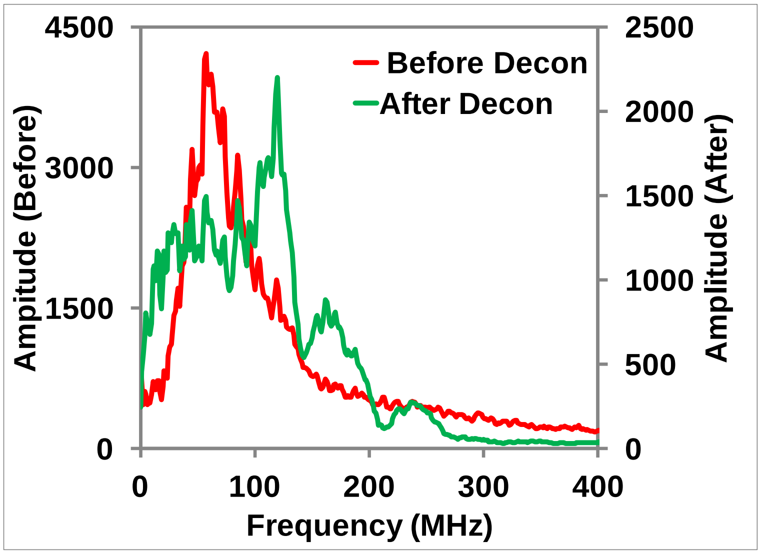
b.) If we can directly measure $w(t)$, we can design a "deterministic inverse filter".

c.) Frequently, we cannot directly measure $w(t)$. In this case, it is assumed that $w(t)$ and $e(t)$ have specific properties/characteristics. Depending on these assumptions, different types of deconvolution processes have been developed (e.g., least-squares (Wiener), shaping and predictive deconvolution)

Example: GPR 100 MHz reflection profiling data from stratigraphic imaging project (Line 4, Elmira Moraine Site)

Deconvolution parameters: Frequency = 70 MHz; Filter Width = 42 ns; Delay = 21 ns; Spike Width = 4.3 ns; Whiten = 0.1





7.) Other applications of inverse filtering

a.) Acquisition equipment, such as geophones and seismographs, act as linear systems and distort the incoming seismic signal:

$$\tilde{s}(t) = r(t) * g(t) * s(t)$$

where $\tilde{s}(t)$ is the recorded seismogram, $r(t)$ is the impulse response of the seismograph, $g(t)$ is the impulse response of the geophone and $s(t)$ is the incoming seismic signal.

b.) Seismic equipment can be directly tested to determine their impulse responses. From this information, deterministic inverse filters can be designed to the distortion due to the equipment.

H.) Correlation Processes

1.) Cross-Correlation Function

a.) The cross-correlation function $\phi_{xy}(t)$ is a measure of the similar between two different real-valued signals $x(t)$ and $y(t)$ as they are shifted relative to one another. This process is defined as:

$$\phi_{xy}(t) = x(t) \otimes y(t) = \int_{-\infty}^{+\infty} x(s)\, y(s+t)\, ds$$

b.) If $|\phi_{xy}(t)|$ is relative larger at $t = t_0$, then $x(t)$ and $y(t)$ are similar in form when a shift of t_0 is applied to $y(t)$. The sign of $\phi_{xy}(t)$ indicates the relative polarity of signals.

c.) The order in which the signals are correlation is can not be reversed:

$$\phi_{xy}(t) = x(t) \otimes y(t) = \int_{-\infty}^{+\infty} x(s)\, y(s+t)\, ds \neq \phi_{yx}(t) = y(t) \otimes x(t) = \int_{-\infty}^{+\infty} y(s)\, x(s+t)\, ds$$

d.) Mathematical relationships for cross-correlation function

1.) $\phi_{xy}(t) = \phi_{yx}(-t)$

2.) $\phi_{xy}(t) = x(t) * y(-t)$

3.) $\phi_{yx}(t) = x(-t) * y(t)$

Example:

$$\phi_{xy}(t) = \sin(2\pi f_0 t) \otimes \Pi(t/a) = \int_{-\infty}^{+\infty} \sin(2\pi f_0 s)\, \Pi[(s+t)/a]\, ds$$

The form of $\Pi[(s+t)/a]$ determines the limits of integration:

$$\Pi[(s+t)/a] = \begin{cases} 1, & -t-(a/2) \le s \le -t+(a/2) \\ 0, & \text{otherwise} \end{cases}.$$

$$\sin(2\pi f_0 t) \otimes \Pi(t/a) = \int_{-t-(a/2)}^{-t+(a/2)} \sin(2\pi f_0 s)\, ds = -(2\pi f_0)^{-1} \cos(2\pi f_0 s)\Big|_{-t-(a/2)}^{-t+(a/2)}$$

$$= (2\pi f_0)^{-1}\left[\cos(-2\pi f_0 t - \pi f_0 a) - \cos(-2\pi f_0 t + \pi f_0 a)\right]$$

$$= (2\pi f_0)^{-1}\left[\cos(2\pi f_0 t + \pi f_0 a) - \cos(2\pi f_0 t - \pi f_0 a)\right]$$

Using $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$,

$$\sin(2\pi f_0 t) \otimes \Pi(t/a) = -(\pi f_0)^{-1} \sin(\pi f_0 a) \sin(2\pi f_0 t)$$

2.) Autocorrelation Function

a.) The autocorrelation function $\phi_{xx}(t)$ is the cross-correlation of $x(t)$ with itself as defined by

$$\phi_{xx}(t) = x(t) \otimes x(t) = \int_{-\infty}^{+\infty} x(s)\, x(s+t)\, ds$$

b.) Autocorrelation measures the repetitiveness of the signal $x(t)$.

1.) $\phi_{xx}(0)$ is the maximum value of an autocorrelation since $x(t)$ matches itself perfectly when the shift is zero.

2.) If $|\phi_{xx}(t)|$ is relative larger at $t = t_0$, then $x(t)$ potentially has a significant harmonic component with a period $T = t_0$. The sign of $\phi_{xx}(t)$ is related to the polarity of harmonic component.

3.) $\phi_{xx}(t) = \phi_{xx}(-t)$ (i.e., $\phi_{xx}(t)$ is an even function)

Example:

$$\phi_{xx}(t) = e^{-at}\sin(2\pi f_0 t)\,\mathrm{H}(t) \otimes e^{-at}\sin(2\pi f_0 t)\,\mathrm{H}(t) \quad \text{(with } a > 0\text{)}$$

$$= \int_{-\infty}^{+\infty} e^{-as}\sin(2\pi f_0 s)\,\mathrm{H}(s)\,e^{-a(s+t)}\sin\left[2\pi f_0(s+t)\right]\mathrm{H}(s+t)\,ds$$

(Since $\phi_{xx}(t)$ is an even function, we consider $t \geq 0$ and use the form of $\mathrm{H}(s)$ and $\mathrm{H}(s+t)$ to define the limits of integration)

$$= e^{-at}\int_{0}^{+\infty} e^{-2as}\sin(2\pi f_0 s)\sin\left[2\pi f_0(s+t)\right]ds$$

(Using

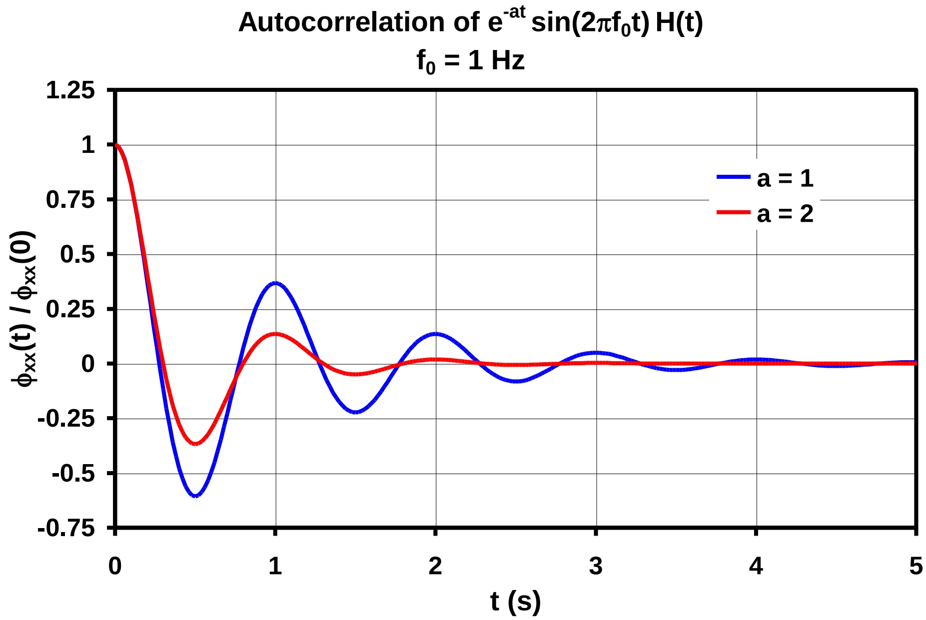
$$\int e^{\alpha s}\sin(\beta s)\sin(\beta s+\gamma)\,ds = \frac{e^{\alpha s}}{2}\left[\frac{\cos(\gamma)}{\alpha} - \frac{\alpha\cos(2\beta s+\gamma)+2\beta\sin(2\beta s+\gamma)}{\alpha^2+4\beta^2}\right])$$

$$= e^{-at}\,\frac{e^{-2as}}{2}\left[\frac{\cos(2\pi f_0 t)}{-2a} - \frac{-2a\cos(4\pi f_0 s+2\pi f_0 t)+4\pi f_0\sin(4\pi f_0 s+2\pi f_0 t)}{4a^2+16\pi^2 f_0^2}\right]\Bigg|_0^{+\infty}$$

(Since $e^{-2as} \to 0$ as $s \to +\infty$)

$$= \frac{-e^{-at}}{2}\left[\frac{\cos(2\pi f_0 t)}{-2a} - \frac{-2a\cos(2\pi f_0 t)+4\pi f_0\sin(2\pi f_0 t)}{4a^2+16\pi^2 f_0^2}\right]$$

$$= \frac{e^{-at}}{4}\left[\frac{4\pi^2 f_0^2\cos(2\pi f_0 t)+2\pi f_0 a\sin(2\pi f_0 t)}{a\left(a^2+4\pi^2 f_0^2\right)}\right]$$



3.) Fourier Transforms for Correlation Functions

a.) For the autocorrelation function

1.) With $x(t) \Leftrightarrow X(f)$,

then $\phi_{xx}(t) = x(t) \otimes x(t) \Leftrightarrow \Phi_{xx}(f) = \overline{X(f)}\,X(f) = \left[A_X(f)\right]^2$

2.) $\Phi_{xx}(f) = \left[A_X(f)\right]^2$ is referred to as the power spectrum, energy density or spectral density of $x(t)$.

3.) $\Phi_{xx}(f)$ gives the energy in the signal contained between the frequencies $f - df/2$ and $f + df/2$,

4.) Since $\phi_{xx}(t)$ is a even function, $\Phi_{xx}(f)$ is zero phase spectrum. Therefore, $\Phi_{xx}(f)$ possesses only amplitude spectrum information about $x(t)$. Hence, two different signals can have the same power spectrum.

b.) For the cross-correlation function

1.) With $x(t) \Leftrightarrow X(f)$ and $y(t) \Leftrightarrow Y(f)$,

then $\phi_{xy}(t) = x(t) \otimes y(t) \Leftrightarrow \Phi_{xy}(f) = \overline{X(f)}\,Y(f)$

2.) $\Phi_{xy}(f)$ is called the cross-energy spectrum

4.) Vibroseis Correlation

a.) Vibratory energy sources (e.g., Vibroseis) permit control of the source signal into the Earth; hence, source wavelet $w(t)$ is relatively well characterized.

b.) This type of source wavelet generally has a very long duration in comparison to the traveltime differences between reflection events. Hence, there is significant overlapping of reflection events on the seismic trace $s(t) = w(t) * e(t)$.

c.) A compression of the effective source wavelet can be achieved by cross-correlating the $w(t)$ with $s(t)$:

$$\hat{s}(t) = w(t) \otimes s(t) = w(t) \otimes \left[w(t) * e(t)\right] = \left[w(t) \otimes w(t)\right] * e(t) = \phi_{ww}(t) * e(t)$$

d.) The autocorrelation of the source wavelet $\phi_{ww}(t)$ is the effective source wavelet for correlated trace $\hat{s}(t)$. Being an autocorrelation, this effective source wavelet is zero phase.

e.) If $w(t)$ is a linear sweep (e.g., $w(t) = A\cos\left(2\pi\left(f_1 + \frac{f_2 - f_1}{T}t\right)t\right)$ for $0 \le t \le T$ where f_1 and f_2 are the start and end sweep frequencies, respectively), then $\phi_{ww}(t)$ is a Klauder wavelet.

Earth 460 Classnotes (Set 4)

I.) Multi-Dimensional Signal Analysis

1,) The concepts of Fourier transform and its operational properties, convolution, filtering, deconvolution and correlation can be easily extended to two or more dimensions.

2.) Examples of multi-dimensional geophysical data sets/signals:

a.) A collection of seismic /GPR traces (e.g., common shot gather or CMP gather).

b.) A spatial array of geophysical measurements (e.g., grid of magnetic or gravity data)

3.) Two-dimensional Fourier transform

a.) Definition of the Fourier transform pair $g(x,y) \Leftrightarrow G(u,v)$

$$G(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\, e^{-2\pi i(ux+vy)}\,dx\,dy \quad \text{(Fourier transform)}$$

$$g(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(u,v)\, e^{+2\pi i(ux+vy)}\,du\,dv \quad \text{(inverse Fourier transform)}$$

where u is the frequency in the x direction and v is the frequency in the y direction.

Example:

Consider $g(x,y) = {}^2\Pi(x,y) = \Pi(x)\Pi(y) = \begin{cases} 1, |x| \le 1/2 \,\&\, |y| \le 1/2 \\ 0, \text{otherwise} \end{cases}$

$$G(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} {}^2\Pi(x,y)\, e^{-2\pi i(ux+vy)}\,dx\,dy = \int_{-1/2}^{+1/2}\int_{-1/2}^{+1/2} e^{-2\pi i(ux+vy)}\,dx\,dy$$

$$= \int_{-1/2}^{+1/2} e^{-2\pi ivy}\left[\int_{-1/2}^{+1/2} e^{-2\pi iux}\,dx\right]dy = \text{sinc}(u)\int_{-1/2}^{+1/2} e^{-2\pi ivy}\,dy = \text{sinc}(u)\,\text{sinc}(v) = {}^2\text{sinc}(u,v)$$

1.) If x (or y) is a time quantity, then u (or v) is a temporal frequency

2.) If x (or y) is a spatial quantity, then u (or v) is a spatial frequency. Spatial frequency is also called wavenumber

b.) Spatial array of measurements: $g(x,y) \Leftrightarrow G(k_x,k_y)$

$x = \text{distance}$ (meters) $\leftrightarrow k_x =$ wavenumber in x direction (cycles/meter)

$y = \text{distance}$ (meters) $\leftrightarrow k_y =$ wavenumber in y direction (cycles/meter)

(Note: wavenumber k_z is related to wavelength λ_z by $k_z = 1/\lambda_z$ where λ_z units are meters/cycle)

c.) Collection of seismic/GPR traces: $g(t,x) \Leftrightarrow G(f,k)$

$t = \text{time}$ (seconds) $\leftrightarrow f = \text{frequency}$ (cycles/second)

$x = \text{distance}$ (meters) $\leftrightarrow k = \text{wavenumber}$ (cycles/meter)

1.) Events having linear travel-time – offset relationships on the trace gather (i.e., *x-t* space) transform to linear events in *f-k* space.

2.) These linear events in *x-t* space has the form $g(t,x) = \delta(t - t')$ where $t' = \frac{1}{v_a}x + t_0$. This event has an apparent velocity v_a and a time intercept t_0.

3.) The Fourier transform of this linear event is

$$G(f,k) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta(t-t')\, e^{-2\pi i(kx+ft)}\,dt\,dx = \int_{-\infty}^{+\infty} e^{-2\pi ikx}\left[\int_{-\infty}^{+\infty} \delta(t-t')\, e^{-2\pi ift}\,dt\right]dx$$

$$= \int_{-\infty}^{+\infty} e^{-2\pi ikx} e^{-2\pi ift'}\,dx = \int_{-\infty}^{+\infty} e^{-2\pi ikx} e^{-2\pi if\left(\frac{x}{v_a} - t_0\right)}\,dx = e^{2\pi ift_0}\int_{-\infty}^{+\infty} e^{-2\pi i\left(\frac{f}{v_a}+k\right)x}\,dx$$

$$= e^{2\pi i f t_0}\ \delta\left(\frac{f}{v_a} + k\right).$$

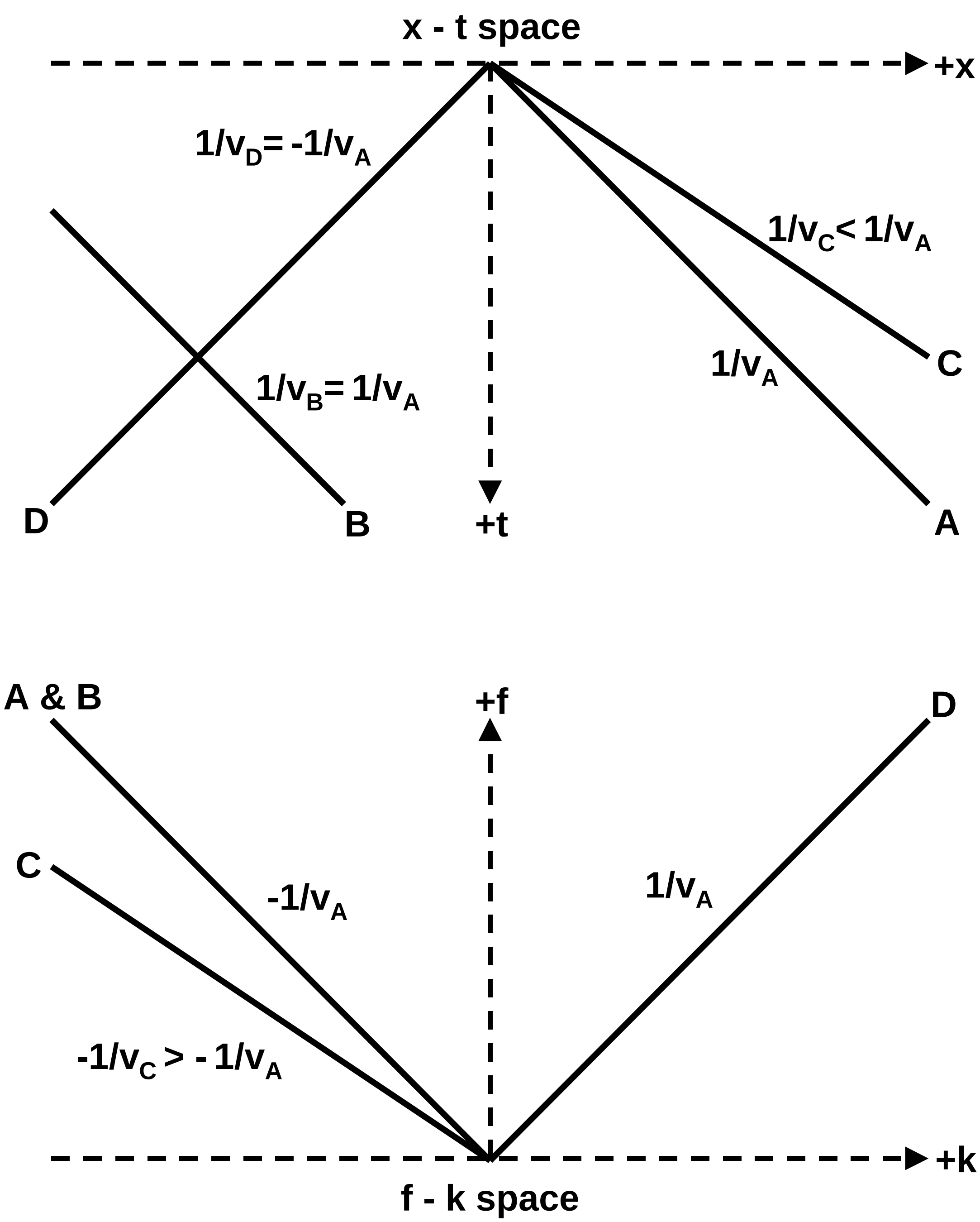
4.) This Fourier transform corresponds to a linear signal on the line $\frac{f}{v_a} + k = 0$ which passes through the origin. Note that all linear events in the in *x-t* space with apparent velocity v_a transform to the same line in *f-k* space; their Fourier transforms differ by the phase term $e^{2\pi i f t_0}$.

5.) The directionality of linear events in *x-t* space are preserved by the Fourier transform in *f-k* space.

Forward traveling event: $g_F(t,x) = \delta(t - t'); \quad t' = \frac{1}{v_a} x + t_0 \Leftrightarrow e^{2\pi i f t_0}\ \delta\left(\frac{f}{v_a} + k\right)$

Reverse traveling event: $g_R(t,x) = \delta(t - t'); \quad t' = \frac{-1}{v_a} x + t_0 \Leftrightarrow e^{2\pi i f t_0}\ \delta\left(\frac{f}{v_a} - k\right)$

(The Fourier transform of the reverse events is a linear signal on the line $\frac{f}{v_a} - k = 0$.)

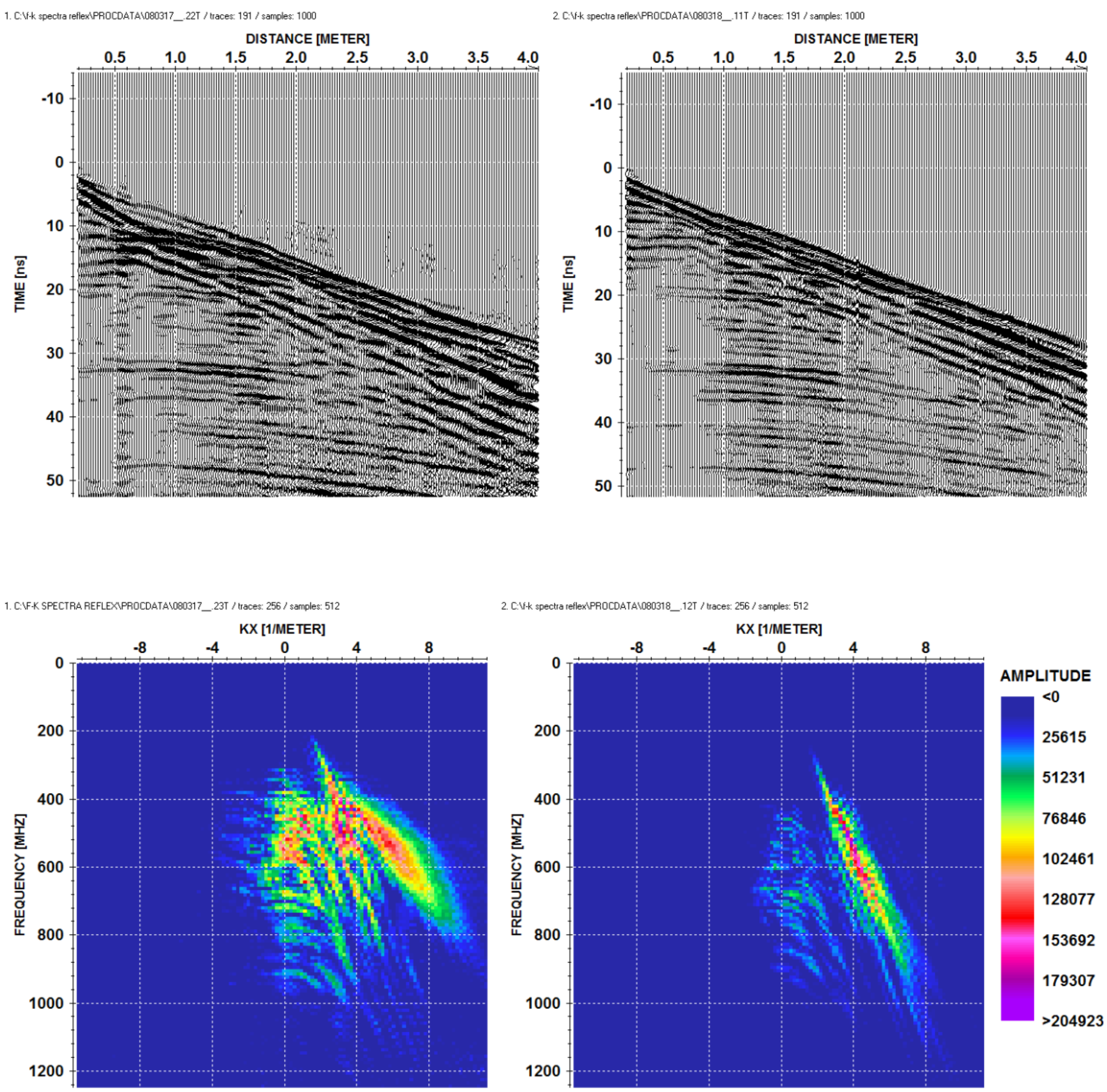

x - t space
+x
$1/v_D = -1/v_A$
$1/v_C < 1/v_A$
$1/v_A$
C
$1/v_B = 1/v_A$
D
B
+t
A
A & B
+f
D
C
$-1/v_A$
$1/v_A$
$-1/v_C > -1/v_A$
+k
f - k space

Example:

f-k analysis of dispersive wave guide phenomenon in GPR common midpoint (CMP) sounding data.

Data: GPR 900 MHz CMP data from soil moisture monitoring project (Site A, Smith Farm, Waterloo Moraine, March 17-18, 2008)

March 17 2008 March 18 2008



4.) Two-dimensional filtering

a.) Spatial dimensions of a potential field anomaly (i.e., gravity and magnetic) are related to size of the causative body. To enhance/emphasize anomalies of a given size range, spatial filters can be applied to the data array

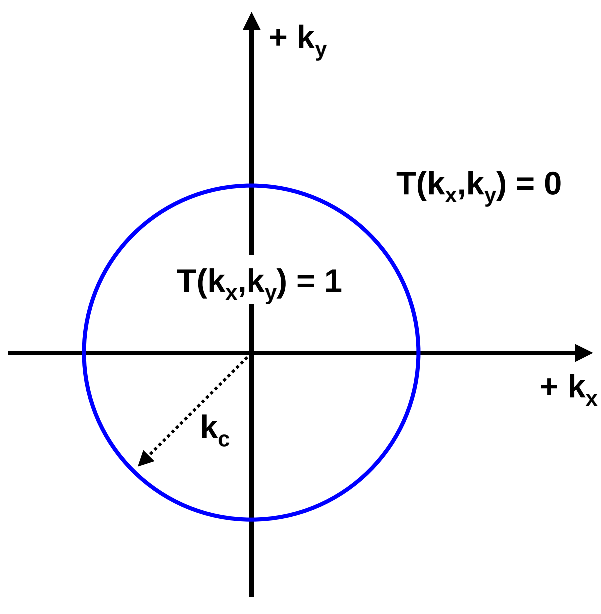
b.) Transfer functions for ideal two-dimensional filters

1.) Low-pass spatial filter

$$T\left(k_x,k_y\right)=\begin{cases}1, & k \le k_c = 1/\lambda_c \\ 0, & k > k_c = 1/\lambda_c\end{cases} \quad \text{where} \quad \begin{aligned} k_c &= \text{cutoff wavenumber} \\ \lambda_c &= \text{cutoff wavelength}\end{aligned} \quad \text{and} \quad k=\sqrt{k_x^2+k_y^2}$$

The pass region for this filter is a circular region in the k_x, k_y plane.

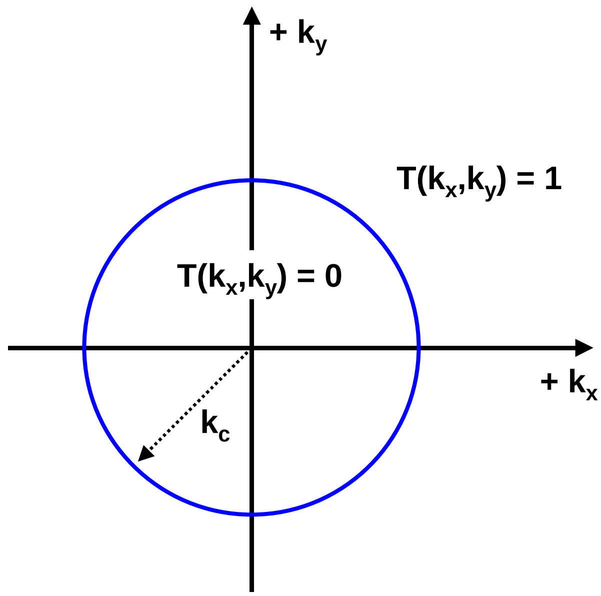
The definition of effective wavenumber k accommodates features that are oblique to the x and y directions.



(Note: Since k is inversely related to λ, lower k implies longer λ. Conversely, higher k implies shorter λ.)

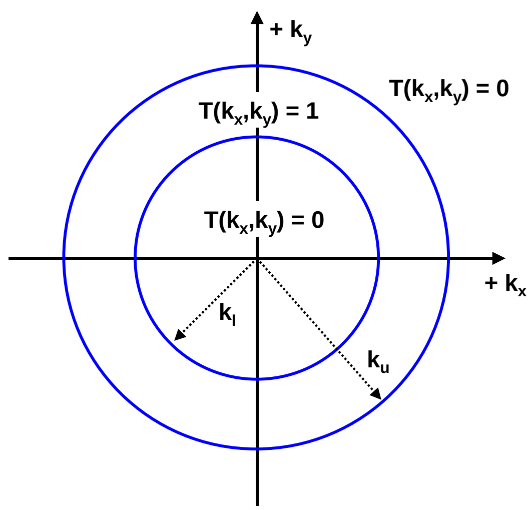
2.) High-pass spatial filter

$$T\left(k_x,k_y\right)=\begin{cases}1, & k \geq k_c = 1/\lambda_c \\ 0, & k < k_c = 1/\lambda_c\end{cases}$$



3.) Band-pass spatial filter

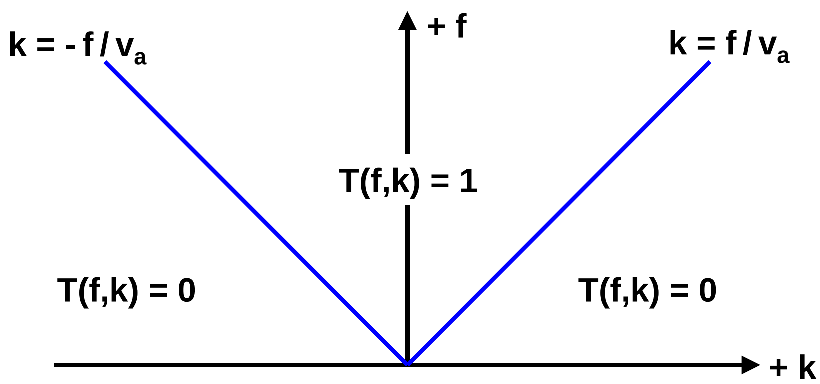
$$T\left(k_x,k_y\right)=\begin{cases}1, & k_l \leq k \leq k_u \\ 0, & \text{otherwise}\end{cases} \quad \text{where} \quad \begin{aligned} k_l &= \text{lower cutoff wavenumber} \\ k_u &= \text{upper cutoff wavenumber}\end{aligned}$$



c.) Two-dimensional filters can be designed to pass/reject signals on the basis of directionality. For seismic & GPR data, this feature can be used to design filters pass/reject events on the basis of their apparent velocity (i.e., dip, fan, move-out or pie-slice filtering).

Example:

$T(f,k) = \begin{cases} 1, & |k| \le f/v_a \\ 0, & |k| > f/v_a \end{cases}$ passes linear events with apparent velocities equal or greater than v_a while rejecting slower events.



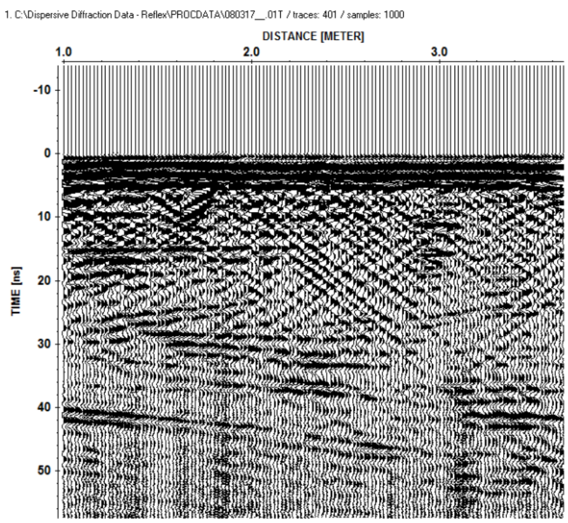
Example #1:

Suppression of shallow scattering/diffractions in GPR reflection profiling data using a spatial filter (i.e., a moving horizontal trace averaging).

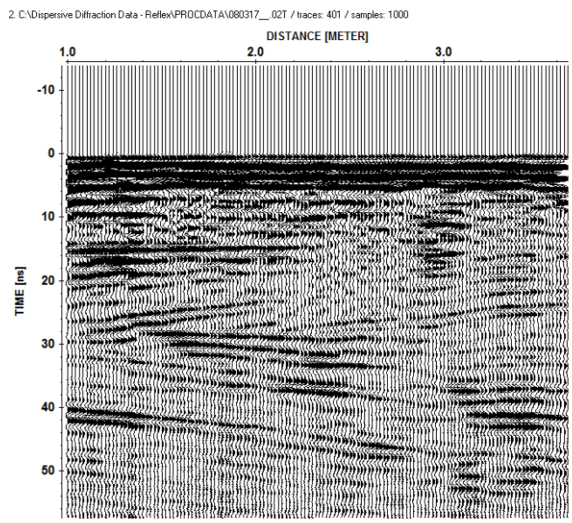
Data: GPR 900 MHz reflection profiling data from soil moisture monitoring project (Site A, Smith Farm, Waterloo Moraine, March 17, 2008)

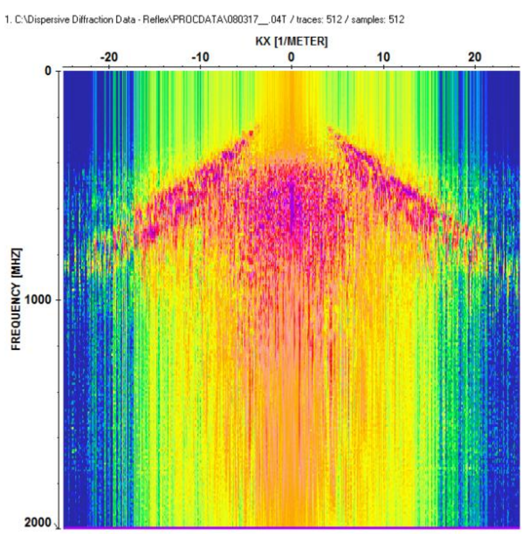
Moving Averaging Window: 0.10 meters (5 traces)

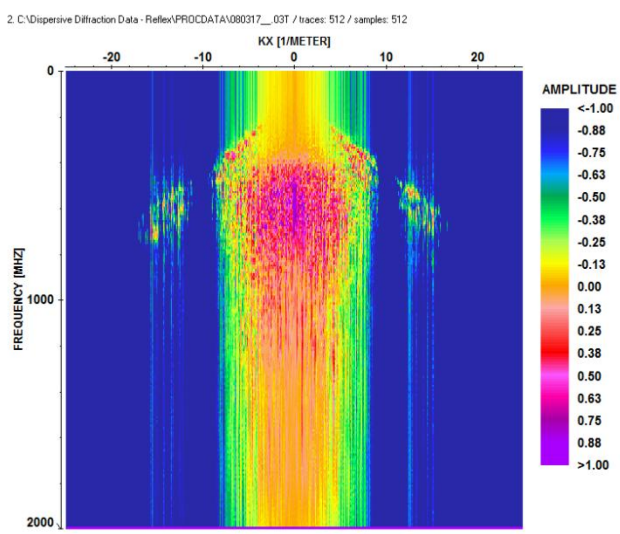
Unfiltered



Filtered







Example #2:

Suppression of shallow scattering/diffractions in GPR reflection profiling data using an f-k velocity filter.

Data: GPR 900 MHz reflection profiling data from soil moisture monitoring project (Site A, Smith Farm, Waterloo Moraine, March 17, 2008)

F-k velocity filter: 0.00 – 0.09 m/ns reject window.

Unfiltered

Filtered