Earth Sciences 460

Assignment #1 **Due: Friday January 19, 2018**

<u>**For this assignment, assume that *t* has units of seconds.**</u>

Problem 1.

Derive the mathematical expressions for the exponential Fourier series coefficients (c_n) of the following periodic functions whose definitions over one period ($-5 \le t < 5$) are given by

(1a.) $g(t) = \sin(2\pi t)$

$$c_n = \begin{cases} i/2 & n = -10 \\ -i/2 & n = +10 \\ 0 & n \ne \pm 10 \end{cases}$$

(1b.) $g(t) = \begin{cases} 0 & -5 \le t < 0 \\ \sin(2\pi t) & 0 \le t < 5 \end{cases}$

$$c_n = \begin{cases} i/4 & n = -10 \\ -i/4 & n = +10 \\ \dfrac{10}{2\pi}\left[\dfrac{\left(1-(-1)^n\right)}{100-n^2}\right] & n \ne \pm 10 \end{cases}$$

(1c.) $g(t) = \begin{cases} 0 & -5 \le t < 0 \\ e^{-t} & 0 \le t < 5 \end{cases}$

$$c_n = \frac{\left[1+(-1)^{n+1}e^{-5}\right](5-i\pi n)}{2\left[25+(\pi n)^2\right]}$$

(1d.) $g(t) = \begin{cases} 0 & -5 \le t < 0 \\ e^{-t}\sin(2\pi t) & 0 \le t < 5 \end{cases}$

$$c_n = 5\pi\left[1+(-1)^{n+1}e^{-5}\right]\left\{\frac{\left[\pi^2\left(100-n^2\right)+25\right]-10\pi n i}{\left[\pi^2\left(10-n\right)^2+25\right]\left[\pi^2\left(10+n\right)^2+25\right]}\right\}$$

Problem 2.

a.) Give the mathematical expressions for the real and imaginary parts of the exponential Fourier series coefficients (c_n) for all of the periodic functions from Problem 1.

b.) Use these expressions for the real and imaginary parts to calculate the amplitude spectra for each of the periodic functions. Plot the results for these amplitude spectra for the frequency range $f_n = \pm 10\,\text{Hz}$.

c.) Compare and contrast the evolution of the amplitude spectra of the sinusoidal signal as is transformed into a “causal” signal (i.e., 1a. → 1b.) and modulated by an exponential taper (i.e., 1b. → 1d. using 1c.).

Problem 3.

a.)Use the expressions for the real and imaginary parts from Problem 2 to calculate the phase spectra for the signals given by 1(c.) and 1(d.). Plot these results for the frequency range $f_n = \pm 10\,\text{Hz}$.

b.) Compare and contrast these two phase spectra.

Problem 4.

Suppose the wavelet signal given by 1(d.) is "padded out" with additional zeros so that the periodic function becomes

$$g(t) = \begin{cases} 0 & -15 \le t < 0 \\ e^{-t}\sin(2\pi t) & 0 \le t < 5 \end{cases}$$

a.) Show that the mathematical expression for the exponential Fourier series coefficients (c_n) for this signal is the following

$$c_n = 10\pi\left[1-(-i)^n e^{-5}\right]\left\{\frac{\left[\pi^2\left(400-n^2\right)+100\right]-20\pi n i}{\left[\pi^2(20-n)^2+100\right]\left[\pi^2(20+n)^2+100\right]}\right\}$$

b.) Derive the mathematical expressions for the real and imaginary parts of the exponential Fourier series coefficients (c_n) for this padded signal.

c.) Using these real and imaginary parts, calculate and plot the amplitude and phase spectra of this padded signal for the frequency range $f_n = \pm 10\,\text{Hz}$.

d.) Compare and contrast its amplitude and phase spectra with the "unpadded" version determined above in Problems 2 & 3.