Earth Sciences 460

Assignment #3 **Due: Friday February 2, 2018**

Problem 1.

Consider the impulse response a moving time average filter $\prod(t)$.

a.) Using the convolution integral, derive the following mathematical expressions of the impulse responses for the repeated applications of a moving time average filter:

$$\prod(t) * \prod(t) = \begin{cases} t+1; & -1 \le t \le 0 \\ 1-t; & 0 \le t \le +1 \\ 0; & \text{otherwise} \end{cases}$$

$$\left[\prod(t) * \prod(t)\right] * \prod(t) = \begin{cases} \left(t^2/2\right) + \left(3t/2\right) + \left(9/8\right); & -3/2 \le t \le -1/2 \\ -\left(t^2\right) + \left(3/4\right); & -1/2 \le t \le +1/2 \\ \left(t^2/2\right) - \left(3t/2\right) + \left(9/8\right); & +1/2 \le t \le +3/2 \\ 0; & \text{otherwise} \end{cases}$$

b.) Plot the impulse responses of moving time average filter $\prod(t)$ and its repeated applications $\prod(t) * \prod(t)$ and $\left[\prod(t) * \prod(t)\right] * \prod(t)$.

c.) For these plots, discuss the nature of the averaging process as the moving time average filter is repeated applied.

Problem 2.

a.) Determine the mathematical expressions of the transfer functions for the moving time average filter $\Pi(t)$ and its repeated applications $\Pi(t) * \Pi(t)$ and $\left[\Pi(t) * \Pi(t)\right] * \Pi(t)$.

b.) Plot the amplitude spectra of moving time average filter $\Pi(t)$ and its repeated applications $\Pi(t) * \Pi(t)$ and $\left[\Pi(t) * \Pi(t)\right] * \Pi(t)$.

c.) For these plots, discuss the effects on a signal's amplitude spectrum as the moving time average filter is repeated applied.

Problem 3.

Using the following two Fourier transform pairs

$$\Delta f \ \mathrm{sinc}(\Delta f t) \Leftrightarrow \Pi(f/\Delta f) \quad \text{and} \quad \cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$$

and the convolution relationships for the Fourier transform, derive the following Fourier transform pairs:

$$\cos(2\pi f_0 t)\mathrm{sinc}(\Delta f t) \Leftrightarrow \frac{1}{2\Delta f}\left[\Pi\left(\frac{f - f_0}{\Delta f}\right) + \Pi\left(\frac{f + f_0}{\Delta f}\right)\right]$$

Problem 4.

The time signal in Problem 3 is an excellent example of a band-limited zero phase wavelet.

a.) Plot the wavelets corresponding to the following three series of parameter values for mean frequency f_0 and bandwidth Δf :

Series #1

$f_0 = 10\text{Hz}$ and $\Delta f = 5\text{Hz}$ $f_0 = 10\text{Hz}$ and $\Delta f = 10\text{Hz}$ $f_0 = 10\text{Hz}$ and $\Delta f = 15\text{Hz}$

Series #2

$f_0 = 10\text{Hz}$ and $\Delta f = 15\text{Hz}$ $f_0 = 15\text{Hz}$ and $\Delta f = 15\text{Hz}$ $f_0 = 20\text{Hz}$ and $\Delta f = 15\text{Hz}$

Series #3

$f_0 = 10\text{Hz}$ and $\Delta f = 12\text{Hz}$ $f_0 = 15\text{Hz}$ and $\Delta f = 18\text{Hz}$ $f_0 = 20\text{Hz}$ and $\Delta f = 24\text{Hz}$

b.) Discuss the role of mean frequency f_0 and bandwidth Δf in defining the form of the time domain wavelet.