Earth Sciences 460

Assignment #4 **Due: Monday Feb. 13, 2017**

Problem 1.
Consider a continuous cosine signal (i.e., $g(t) = \cos(2\pi f_0 t)$) with frequency $f_0 = 250\,\text{Hz}$.

a.) What will be the recovered cosine signal if the sampling interval Δt is 0.001 sec?

b.) What will be the recovered cosine signal if the sampling interval Δt is 0.003 sec?

For both a & b, justify your answers by drawing a labeled diagram of the Fourier spectra of the discretely sampled cosine (showing the original and the first order replications) and the recovered cosine.

Problem 2.

Consider a continuous time signal $g(t) = 2 f_0 \operatorname{sinc}(2 f_0 t)$ with $f_0 = 250\,\text{Hz}$. A discrete time signal $d(t)$ is obtained using a uniform sampling interval Δt.

Consider the following value of the sampling interval Δt:

(a.) 0.001 sec, (b.) 0.0025 sec, (c.) 0.003 sec and (d.) 0.0035 sec.

For each value of Δt, do the following five steps:

1.) Draw a labeled diagram showing $D(f)$, showing the original and the first order replications.
2.) Draw a labeled diagram showing Fourier transform of the recovered signal.
3.) Give the mathematical expression for the Fourier transform of the recovered signal.
4.) Give the mathematical expression for the recovered time signal.
5.) Plot the recovered time signal with the original signal $g(t)$.

Using the results for these four sampling interval values, comment on the effects of increasing the value of the sampling interval Δt on the recovered time signal.

Problem 3.

Another consideration in time series analysis is windowing of the time data. Practically, we cannot handle an infinitely long time signal. Also, we may want to do a Fourier analysis on a small portion of the signal.

Time windowing is equivalent to multiplying a signal by an appropriate boxcar.

Let us consider the original time signal $g(t) = \cos(2\pi f_0 t)$. We apply the windowing function $win(t) = \frac{f_0}{n}\Pi\left(\frac{f_0}{n}t\right)$ that covers n cycles of the cosine signal, giving the windowed time data

$$g_w(t) = g(t) \times win(t) = \cos(2\pi f_0 t) \times \left[\frac{f_0}{n}\Pi\left(\frac{f_0}{n}t\right)\right]$$

a.) Using the operational and convolution properties of the Fourier transform, derive the following Fourier transform pair for the windowed time data:

$$g_w(t) = \cos(2\pi f_0 t) \times \left[\frac{f_0}{n}\Pi\left(\frac{f_0}{n}t\right)\right] \Leftrightarrow$$

$$G_w(f) = \frac{1}{2}\left\{\text{sinc}\left[\frac{n}{f_0}(f - f_0)\right] + \text{sinc}\left[\frac{n}{f_0}(f + f_0)\right]\right\}$$

Please start this derivation with the following two Fourier transform pairs:

$$\Pi(t) \Leftrightarrow \text{sinc}(f) \;\;\&\;\; \cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$$

b.) Assuming that $f_0 = 20\,\text{Hz}$, determine and plot $G_w(f)$ for $n = 1, 2, 5$ and 10 over the frequency range ±100 Hz.

c.) From these results, discuss the effects of windowing on the Fourier spectrum of a signal. In particular,

(1) How does windowing affect the information at a particular frequency value in the original signal.

(2) What is the nature of the information at a particular frequency value in the windowed signal.

d.) Briefly discuss the potential impact of windowing on determining the appropriate sampling interval Δt to obtain a discrete time series.