Earth Sciences 460

Assignment #2 **Due: Friday January 27, 2016**

Problem 1.

Using the definition of either the Fourier transform or the inverse Fourier transform, derive the following Fourier transform pairs:

a.) $g(t) = e^{-\alpha t}\cos(2\pi f_0 t)\mathrm{H}(t) = \begin{cases} e^{-\alpha t}\cos(2\pi f_0 t), & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$\Leftrightarrow G(f) = \frac{1}{2}\left[\frac{1}{\alpha + i\,2\pi(f - f_0)} + \frac{1}{\alpha + i\,2\pi(f + f_0)}\right]$$

(for $\alpha > 0$ and $f_0 > 0$)

b.) $g(t) = \mathrm{sinc}(t) \Leftrightarrow G(f) = \prod(f) = \begin{cases} 1, & -1/2 \leq f \leq +1/2 \\ 0, & |f| > 1/2 \end{cases}$

c.) $g(t) = e^{-\alpha|t|}\cos(2\pi f_0 t) = \begin{cases} e^{-\alpha t}\cos(2\pi f_0 t), & t \geq 0 \\ e^{\alpha t}\cos(2\pi f_0 t), & t < 0 \end{cases}$

$$\Leftrightarrow G(f) = \alpha\left[\frac{1}{\alpha^2 + 4\pi^2(f - f_0)^2} + \frac{1}{\alpha^2 + 4\pi^2(f + f_0)^2}\right]$$

(for $\alpha > 0$ and $f_0 > 0$)

Problem 2.

a.) Derive the mathematical expressions for real and imaginary parts for the Fourier transforms in (1a) and (1c).

b.) Compute and plot the amplitude and phase spectra of (1a) and (1c). Assume that $\alpha = 1\,\mathrm{s}^{-1}$ and $f_0 = 1\mathrm{Hz}$. Plot both spectra over the frequency range ±10 Hz.

Problem 3.

Using the Fourier transform pairs from Problem 1 and the operational properties of the Fourier transform, derive the following Fourier transform pairs:

a.) $g(t) = te^{-\alpha t}\cos(2\pi f_0 t)\mathrm{H}(t) = \begin{cases} te^{-\alpha t}\cos(2\pi f_0 t), & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$\Leftrightarrow G(f) = \frac{1}{2}\left\{\frac{1}{\left[\alpha + i\,2\pi(f - f_0)\right]^2} + \frac{1}{\left[\alpha + i\,2\pi(f + f_0)\right]^2}\right\}$$

b.) $g(t) = t^2 e^{-\alpha t}\cos(2\pi f_0 t)\mathrm{H}(t) = \begin{cases} t^2 e^{-\alpha t}\cos(2\pi f_0 t), & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$\Leftrightarrow G(f) = \frac{1}{\left[\alpha + i\,2\pi(f - f_0)\right]^3} + \frac{1}{\left[\alpha + i\,2\pi(f + f_0)\right]^3}$$

c.) $g(t) = \frac{2}{f_2 - f_1}\left[f_2\,\mathrm{sinc}(2f_2 t) - f_1\,\mathrm{sinc}(2f_1 t)\right]$

$$\Leftrightarrow G(f) = \frac{1}{f_2 - f_1}\left[\Pi(f/2f_2) - \Pi(f/2f_1)\right]$$

(for $f_2 > f_1 > 0$)

Problem 4.

a.) Derive the mathematical expressions for real and imaginary parts for the Fourier transforms in (3a) and (3b).

b.) Compute and plot the amplitude and phase spectra of (3a) and (3b). Assume that $\alpha = 1\,\mathrm{s}^{-1}$ and $f_0 = 1\mathrm{Hz}$. Plot both spectra over the frequency range ±10 Hz.

Problem 5.

Compare and contrast the amplitude spectra for (1a), (3a), and (3b).