

Earth Sciences 460

Assignment #3

Due: Friday February 3, 2017

Problem 1.

Consider the impulse response a moving time average filter $\Pi(t)$.

- a.) Using the convolution integral, derive the following mathematical expressions of the impulse responses for the repeated applications of a moving time average filter:

$$\Pi(t) * \Pi(t) = \begin{cases} t+1; & -1 \leq t \leq 0 \\ 1-t; & 0 \leq t \leq +1 \\ 0; & \text{otherwise} \end{cases}$$

$$[\Pi(t) * \Pi(t)] * \Pi(t) = \begin{cases} (t^2/2) + (3t/2) + (9/8); & -3/2 \leq t \leq -1/2 \\ -(t^2) + (3/4); & -1/2 \leq t \leq +1/2 \\ (t^2/2) - (3t/2) + (9/8); & +1/2 \leq t \leq +3/2 \\ 0; & \text{otherwise} \end{cases}$$

- b.) Plot the impulse responses of moving time average filter $\Pi(t)$ and its repeated applications $\Pi(t) * \Pi(t)$ and $[\Pi(t) * \Pi(t)] * \Pi(t)$.
- c.) For these plots, discuss the nature of the averaging process as the moving time average filter is repeated applied.

Problem 2.

- a.) Determine the mathematical expressions of the transfer functions for the moving time average filter $\Pi(t)$ and its repeated applications $\Pi(t) * \Pi(t)$ and $[\Pi(t) * \Pi(t)] * \Pi(t)$.
- b.) Plot the amplitude spectra of moving time average filter $\Pi(t)$ and its repeated applications $\Pi(t) * \Pi(t)$ and $[\Pi(t) * \Pi(t)] * \Pi(t)$.
- c.) For these plots, discuss the effects on a signal's amplitude spectrum as the moving time average filter is repeated applied.

Problem 3.

Using the following two Fourier transform pairs

$$\Delta f \operatorname{sinc}(\Delta f t) \Leftrightarrow \Pi(f/\Delta f) \quad \text{and} \quad \cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

and the convolution relationships for the Fourier transform, derive the following Fourier transform pairs:

$$\cos(2\pi f_0 t) \operatorname{sinc}(\Delta f t) \Leftrightarrow \frac{1}{2\Delta f} \left[\Pi\left(\frac{f - f_0}{\Delta f}\right) + \Pi\left(\frac{f + f_0}{\Delta f}\right) \right]$$

Problem 4.

The time signal in Problem 3 is an excellent example of a band-limited zero phase wavelet.

a.) Plot the wavelets corresponding to the following three series of parameter values for mean frequency f_0 and bandwidth Δf :

Series #1

$f_0 = 10\text{Hz}$ and $\Delta f = 5\text{Hz}$ $f_0 = 10\text{Hz}$ and $\Delta f = 10\text{Hz}$ $f_0 = 10\text{Hz}$ and $\Delta f = 15\text{Hz}$

Series #2

$f_0 = 10\text{Hz}$ and $\Delta f = 15\text{Hz}$ $f_0 = 15\text{Hz}$ and $\Delta f = 15\text{Hz}$ $f_0 = 20\text{Hz}$ and $\Delta f = 15\text{Hz}$

Series #3

$f_0 = 10\text{Hz}$ and $\Delta f = 12\text{Hz}$ $f_0 = 15\text{Hz}$ and $\Delta f = 18\text{Hz}$ $f_0 = 20\text{Hz}$ and $\Delta f = 24\text{Hz}$

b.) Discuss the role of mean frequency f_0 and bandwidth Δf in defining the form of the time domain wavelet.