Earth 460 Classnotes (Set 8)

IV.) Traveltime-Offset Distance Relationships for Primary Reflections in a Layered Earth

(Note: Mode conversion at interfaces will be neglected unless explicitly specified for the remainder of this course)

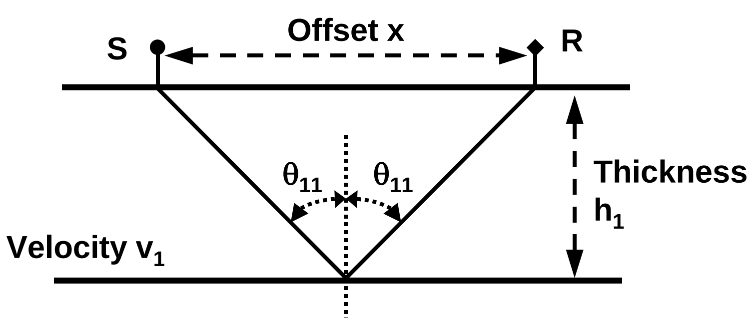
A.) The raypath of a primary reflection undergoes reflection at only its last /deepest interface; it undergoes transmission/refraction at all of the intermediate interfaces.

B.) Single Horizontal Layer

1.) The layer has a thickness h_1 and wave velocity v_1.

2.) The source *S* and receiver *R* are both located on the surface. The source – receiver separation (i.e., *offset*) is x.

3.) The reflection point is located direct beneath the midpoint between *S* and *R*.



4.) The length l_1 of the downgoing and upgoing raypath segments are given by $l_1^2 = x^2/4 + h_1^2$.

5.) The two-way traveltime t for the reflection is related to the offset x by the hyperbolic relationship $t^2(x) = \frac{1}{v_1^2}\left(x^2 + 4h_1^2\right)$.

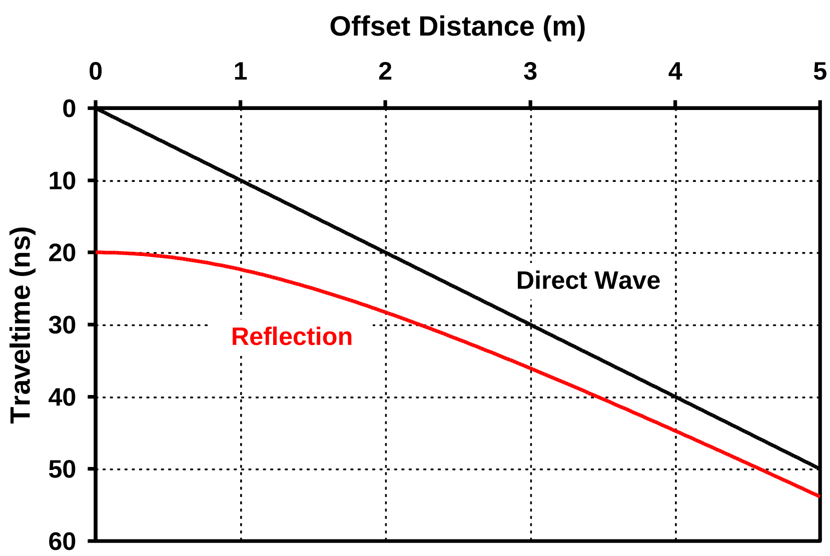
6.) When $x = 0$, then $t(0) = t_0 = 2h_1/v_1$. Hence, we express the traveltime relationship as $t^2(x) = t_0^2 + \frac{x^2}{v_1^2}$.

Example:

Consider the traveltime-offset distance relationships for the single horizontal layer with thickness $h_1 = 1$ m and wave velocity $v_1 = 0.10$ m/ns.

Events: Direct wave and reflection from interface #1

Traveltime-offset relationship for the direct wave: $t_{dir}(x) = x/v_1$.



Note that the reflection relationship asymptotically approaches the direct wave relationship for large offset values.

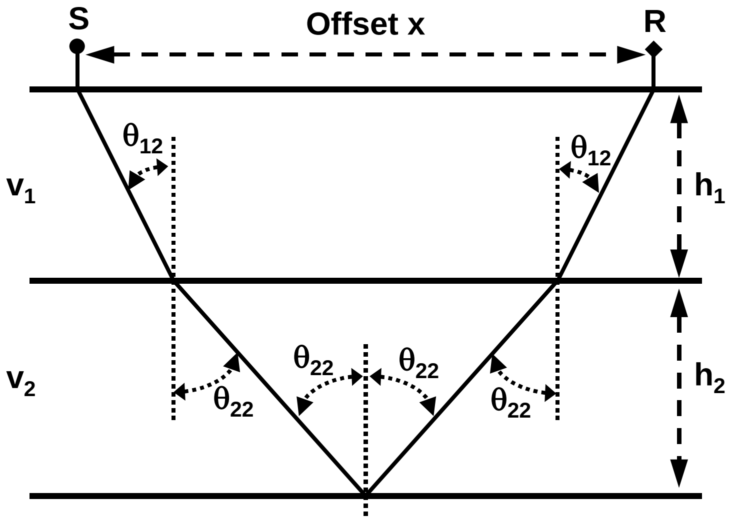
C.) Two Horizontal Layers

1.) The upper layer has a thickness h_1 and wave velocity v_1. The lower layer has a thickness h_2 and wave velocity v_2.

2.) From the inherent symmetry of the problem, it can be shown:

a.) The downgoing and upgoing raypath segments in a given layer have identical lengths.

b.) The reflection point on the lower interface is located direct beneath the midpoint between *S* and *R*.



3.) Derivation of the traveltime-offset distance relationship requires the analysis of each raypath segment.

a.) For the raypath segments in the upper layer:

1.) Segment length $l_1 = h_1/\cos\theta_{12}$

2.) Horizontal segment length $x_1 = l_1 \sin\theta_{12} = h_1 \tan\theta_{12}$

3.) Traveltime $t_1 = l_1/v_1 = h_1/v_1 \cos\theta_{12}$

b.) For the raypath segments in the lower layer:

1.) Segment length $l_2 = h_2/\cos\theta_{22}$

2.) Horizontal segment length $x_2 = l_2 \sin\theta_{22} = h_2 \tan\theta_{22}$

3.) Traveltime $t_2 = l_2/v_2 = h_2/v_2 \cos\theta_{22}$

4.) For the entire raypath:

a.) Offset distance $x = 2\left(x_1 + x_2\right) = 2\left(h_1 \tan\theta_{12} + h_2 \tan\theta_{22}\right)$

b.) Traveltime $t = 2\left(t_1 + t_2\right) = 2\left[\left(h_1/v_1 \cos\theta_{12}\right) + \left(h_2/v_2 \cos\theta_{22}\right)\right]$

5.) From Snell's Law, we have that $p = \sin\theta_{12}/v_1 = \sin\theta_{22}/v_2$ → $\sin\theta_{12} = pv_1$ and $\sin\theta_{22} = pv_2$.

a.) Using $\sin^2\theta + \cos^2\theta = 1$ gives $\cos\theta_{12} = \sqrt{1-(pv_1)^2}$ and $\cos\theta_{22} = \sqrt{1-(pv_2)^2}$.

b.) Using $\tan\theta = \sin\theta/\cos\theta$ gives $\tan\theta_{12} = pv_1\Big/\sqrt{1-(pv_1)^2}$ and $\tan\theta_{22} = pv_2\Big/\sqrt{1-(pv_2)^2}$.

6.) Combining these results, we get the following parametric equations that define the traveltime-offset distance relationship:

$$x(p) = 2p\left[\frac{h_1 v_1}{\sqrt{1-(pv_1)^2}} + \frac{h_2 v_2}{\sqrt{1-(pv_2)^2}}\right]$$

$$t(p) = 2\left[\frac{h_1}{v_1\sqrt{1-(pv_1)^2}} + \frac{h_2}{v_2\sqrt{1-(pv_2)^2}}\right]$$

7.) The raypath of the primary reflection traveling between given source and receiver locations is defined by a unique value of the ray parameter p.

8.) The range of ray parameter values is $0 \le p \le \min(1/v_1, 1/v_2)$

a.) $p = 0$ is the normal incident /zero offset ($x = 0$) raypath

b.) $p = 1/v_2$ is the critical refraction along the interface (limiting case if $v_2 > v_1$).

c.) $p = 1/v_1$ is the direct/horizontal wave from the source (limiting case if $v_1 > v_2$).

Example #1: (Velocity increasing depth)

Consider the traveltime-offset distance relationships for two horizontal layers:

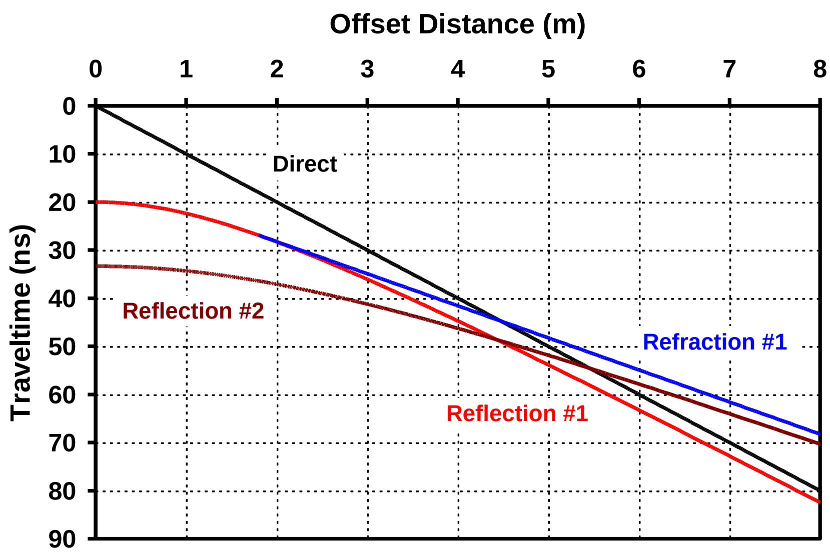
Layer #1: thickness $h_1 = 1$ m and wave velocity $v_1 = 0.10$ m/ns.

Layer #2: thickness $h_2 = 1$ m and wave velocity $v_2 = 0.15$ m/ns.

Events: Direct wave, reflections from interfaces #1 and #2 and critical refraction along interface #1.

Traveltime-offset relationship for the critical refraction along interface #1:

$$t_{refr\,1}(x) = \frac{x}{v_2} + \frac{2h_1\sqrt{(v_2)^2-(v_1)^2}}{v_1 v_2} \quad \text{where} \quad x_{crit\,1} = \frac{2\,h_1\,v_1}{\sqrt{(v_2)^2-(v_1)^2}}.$$



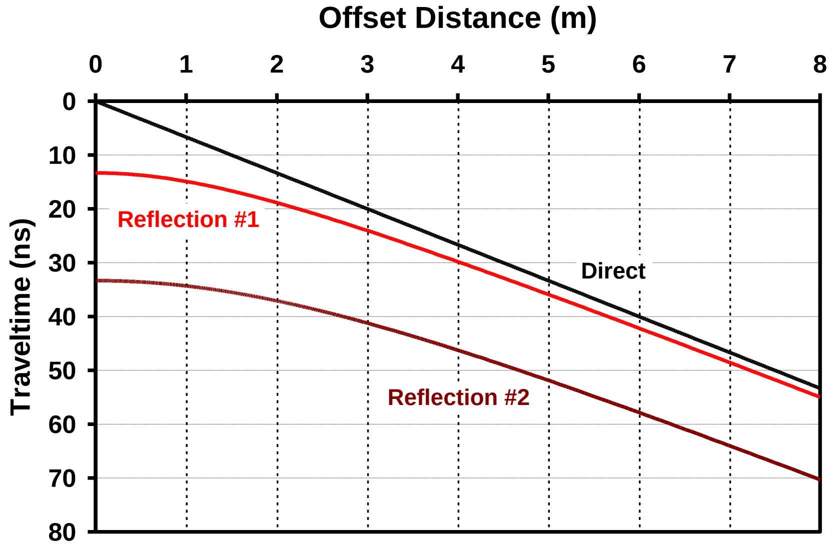
Example #2: (Velocity decreasing depth)

Consider the traveltime-offset distance relationships for two horizontal layers:

Layer #1: thickness $h_1 = 1$ m and wave velocity $v_1 = 0.15$ m/ns.

Layer #2: thickness $h_2 = 1$ m and wave velocity $v_2 = 0.10$ m/ns.

Events: Direct wave and reflections from interfaces #1 and #2.



Note: The relationship between the reflections events depends on the nature of the velocity gradient (i.e., increasing with depth versus decreasing with depth).

D.) Multiple Horizontal Layers (i.e., General Case)

1.) The Earth model

a.) Earth consists of N horizontal layers

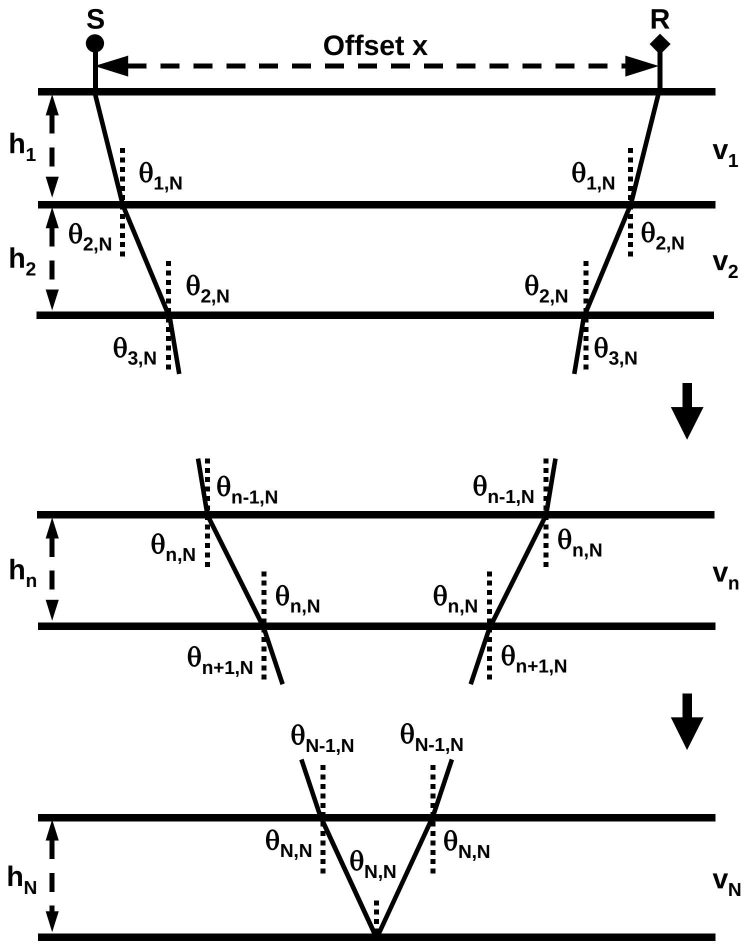
b.) The n^{th} layer ($n = 1, 2, \cdots, N$) has thickness h_n and wave velocity v_n.

c.) The $(n\text{-}1)^{th}$ and n^{th} interfaces are the top and bottom, respectively, of the n^{th} layer.

2.) Symmetry considerations

a.) The reflection point on the N^{th} interface is located directly below the midpoint between the source and receiver locations.

b.) The upward and downward raypath segments in any given layer have identical characteristics.



3.) For the raypath segments in the n^{th} layer

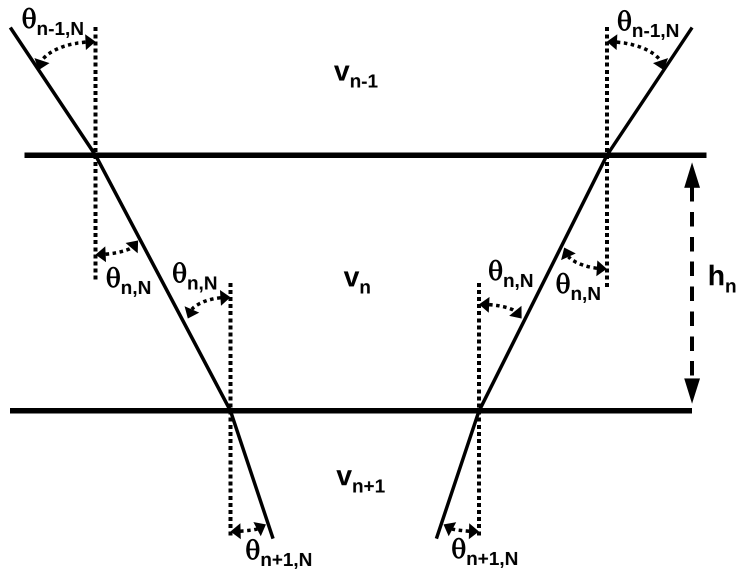
a.) Segment length $l_n = h_n / \cos\theta_{n,N}$

b.) Horizontal segment length $x_n = l_n \sin\theta_{n,N} = h_n \tan\theta_{n,N}$

c.) Traveltime $t_n = l_n / v_n = h_n / v_n \cos\theta_{n,N}$

d.) At the upper/(n-1)th interface, $p = \sin\theta_{n-1,N} / v_{n-1} = \sin\theta_{n,N} / v_n$

e.) At the lower/nth interface, $p = \sin\theta_{n,N} / v_n = \sin\theta_{n+1,N} / v_{n+1}$



4.) For the entire raypath

a.) Traveltime $t = 2\sum_{n=1}^{N} t_n = 2\sum_{n=1}^{N}\left(h_n / v_n \cos\theta_{n,N}\right)$

b.) Offset distance $x = 2\sum_{n=1}^{N} x_n = 2\sum_{n=1}^{N} h_n \tan\theta_{n,N}$

5.) For each layer:

a.) Snell's Law gives $\sin\theta_{n,N} = pv_n$

b.) Trigonometric identities $\cos\theta_{n,N} = \sqrt{1-(pv_n)^2}$ and

$\tan\theta_{n,N} = pv_n \Big/ \sqrt{1-(pv_n)^2}$

6.) Combining these results, we get the following parametric equations that define the traveltime-offset distance relationship:

$$x(p) = 2p\sum_{n=1}^{N}\left[\frac{h_n v_n}{\sqrt{1-(pv_n)^2}}\right] \text{ and } t(p) = 2\sum_{n=1}^{N}\left[\frac{h_n}{v_n\sqrt{1-(pv_n)^2}}\right].$$

7.) The range of ray parameter values is $0 \le p \le \min\left(1/v_1, \cdots, 1/v_N\right)$

a.) $p = 0$ is the normal incident/zero offset ($x = 0$) raypath b.) $p = 1/v_n$ is the horizontal ray in the nth layer (e.g., critical refraction along (n-1)th interface).

E.) Hyperbolic Approximations for the Traveltime-Offset Distance Relationship

1.) For offset distances x that are small relative to the interface depth, the traveltime-offset distance relationship can be represented in terms of a hyperbolic approximation of the form $t^2(x) = t_0^2 + x^2/V^2$ where V is an "average velocity" along the effective raypath assumed by the approximation and $t_0 = t(x=0)$ is the two-way traveltime for the vertical/normal incident raypath.

a.) The hyperbolic approximation permits the $x^2 - t^2$ analysis of the traveltime-offset distance relationship.

b.) It defines a simple NMO correction for the effects of offset distance on traveltime, allowing the compositing of CMP gathers.

2.) Straight raypath approximation

a.) This approximation neglects the raypath bending that occurs at interfaces due to Snell's Law.

b.) For the upgoing and downgoing raypath segments in the nth layer:

1.) Length $l_n = h_n/\cos\alpha$

2,) Traveltime: $t_n = h_n/v_n \cos\alpha$

c.) The traveltime for the entire raypath is $t(x) = \dfrac{2}{\cos\alpha}\left[\sum_{n=1}^{N} h_n/v_n\right] = \dfrac{t_0}{\cos\alpha}$

where $t_0(N) = 2\sum_{n=1}^{N} h_n/v_n$

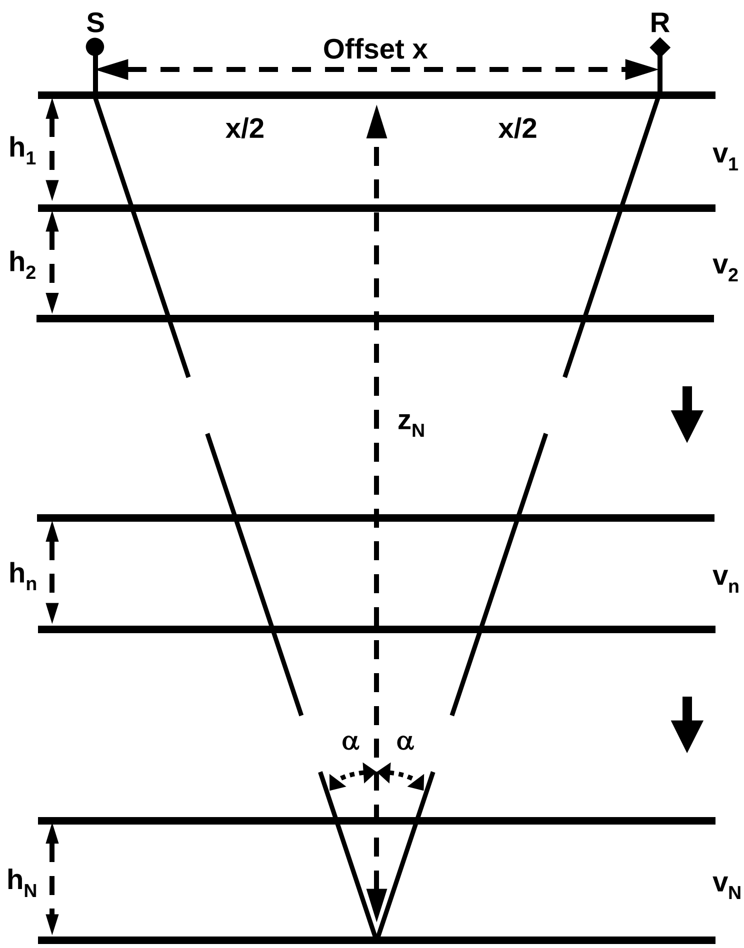
d.) The geometry of the straight raypath gives $\cos\alpha = z_N \Big/ \sqrt{(x/2)^2 + z_N^2}$

where $z_N = \sum_{n=1}^{N} h_n$ is the depth to the Nth interface.

e.) Hence, $t(x) = \frac{t_0(N)}{z_N}\sqrt{(x/2)^2 + z_N^2}$ →

$t^2(x) = [t_0(N)]^2 + \frac{1}{[V_{aver}(N)]^2} x^2$ where

$V_{SR}(N) = \frac{2z_N}{t_0(N)}$ → $V_{SR}(N) = \left[\sum_{n=1}^{N} v_n \tau_n\right] \Big/ \left[\sum_{n=1}^{N} \tau_n\right]$ (a linear weighting of the interval velocities) and $\tau_n = 2h_n/v_n$ (i.e., two-way traveltime for the vertical/normal incidence raypath in the nth layer).



3.) Power series expansion

a.) The traveltime-offset distance relationship for the Nth interface can be expressed as the following power series expanded about $x = 0$:

$$t^2\left(x^2\right) = c_0 + c_1\, x^2 + c_2\, x^4 + c_3\, x^6 + \cdots$$

1.) $c_0 = t^2\left(x^2\right)\Big|_{x^2=0} = t_o^2\left(N\right)$.

2.) $c_1 = \left.\dfrac{d\left(t^2\right)}{d\left(x^2\right)}\right|_{x^2=0} = \dfrac{1}{V_{RMS}^2\left(N\right)}$ where $V_{RMS}^2\left(N\right) = \left[\sum_{n=1}^{N} v_n^2\ \tau_n\right] \Big/ \left[\sum_{n=1}^{N} \tau_n\right]$.

3.) $c_2 = \left.\dfrac{d^2\left(t^2\right)}{d\left(x^2\right)^2}\right|_{x^2=0} = \dfrac{1-\left[V_{RMS}^4\left(N\right)/V_{RMS}^2\left(N\right)\right]^2}{4t_o^2\left(N\right)\left[V_{RMS}^2\left(N\right)\right]^2}$ where

$V_{RMS}^4\left(N\right) = \left[\sum_{n=1}^{N} v_n^4\ \tau_n\right] \Big/ \left[\sum_{n=1}^{N} \tau_n\right]$.

Note: The coefficients of the power series are obtained from the parametric equations and the parametric derivative formulae (e.g., if $x = x\left(p\right)$ and $y = y\left(p\right)$

$$\rightarrow \frac{dy}{dx} = \frac{\left[dy/dp\right]}{\left[dx/dp\right]},\ \frac{d^2y}{dx^2} = \frac{\left[d\left[dy/dx\right]/dp\right]}{\left[dx/dp\right]}\ \ldots)$$

b.) If only the first two terms of the power series are used, we get the hyperbolic approximation $t^2\left(x\right) = \left[t_0\left(N\right)\right]^2 + \dfrac{1}{\left[V_{RMS}\left(N\right)\right]^2}\, x^2$.

c.) Unlike the straight ray approach, the power series expansion does include to some degree the effects of raypath bending.

An alternate derivation of the RMS form of the average velocity V in the hyperbolic approximation:

Rewriting the hyperbolic approximation as $t\left(x\right) = \sqrt{t_0^2 + x^2/V^2}$, it can be shown that

$$\frac{dt}{dx} = \frac{1}{2}\left[t_0^2 + x^2/V^2\right]^{-1/2}\left[2\left(x/V\right)\right]\left[1/V\right] = \frac{x}{V^2\ t\left(x\right)}$$

Considering the actual raypath in the layered medium, the horizontal segment length in the Nth layer can be expressed as $x_n = l_n \sin\theta_{n,N} = v_n t_n \sin\theta_{n,N}$. Using the fact that the ray parameter p is constant along a given raypath, then

$$p = \sin\theta_{n,N} / v_n = \sin\theta_{1,N} / v_1 \rightarrow \sin\theta_{n,N} = v_n \sin\theta_{1,N} / v_1 \text{ and } x_n = \frac{v_n^2 t_n}{v_1} \sin\theta_{1,N}$$

Hence, the parametric relationship for the offset distance can be written as

$$x = 2\sum_{n=1}^{N} x_n = 2\sum_{n=1}^{N} \frac{v_n^2 t_n}{v_1} \sin\theta_{1,N} = 2\,\frac{\sin\theta_{1,N}}{v_1} \sum_{n=1}^{N} v_n^2 t_n \rightarrow \frac{\sin\theta_{1,N}}{v_1} = x\left[\; 2\sum_{n=1}^{N} v_n^2 t_n \right]^{-1}$$

Since ray parameter p is the reciprocal of the instantaneous apparent horizontal velocity of the wavefront along the surface at $x(p)$ (i.e., $p = \sin\theta_{1,N} / v_1 = [dx/dt]^{-1}$ $= dt/dx$), equating the results from the parametric relationship and the hyperbolic approximation gives

$$x\left[\; 2\sum_{n=1}^{N} v_n^2 t_n \right]^{-1} = \frac{x}{V^2\, t(x)} \rightarrow V^2 = \frac{2\sum_{n=1}^{N} v_n^2 t_n}{t(x)}.$$

This expression is derived for an arbitrary raypath; so consider the vertical/normal incident raypath (i.e., $2t_n \rightarrow \tau_n$ and $t(x) \rightarrow t_0 = \sum_{n=1}^{N} \tau_n$) and we obtain $V^2 = V_{RMS}^2 = \sum_{n=1}^{N} v_n^2 \tau_n \Big/ \sum_{n=1}^{N} \tau_n$.

4.) Comparison of the straight ray and power series hyperbolic approximations.

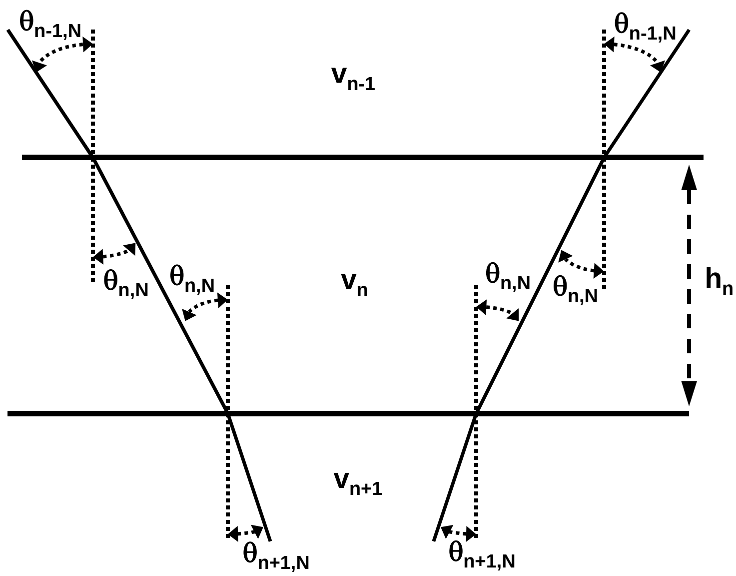
a.) Both approximations agree with the actual traveltime relationship at $x = 0$ (i.e., all give $t_0(N)$).

b.) Because of the method of derivation, the power series approximation is tangent to the actual traveltime relationship at $x = 0$. The straight ray approximation does not possess this trait.

c.) Because it includes bending effects, the power series approximation gives a more precise fit to the actual traveltime-offset relationship.

F.) Intercept-Slowness ($\tau - p$) Formulation for the Traveltime Relationship

1.) Consider the raypath segments in the nth layer.



It can be shown that for a given ray segment $v_n\, t_n = \left(x_n^2 + h_n^2\right)^{1/2}$ where x_n is the horizontal length of the raypath segment and t_n is the one-way traveltime in the nth layer.

2.) Using the expressions $\sin\theta_{n,N} = x_n \big/ \left(x_n^2 + h_n^2\right)^{1/2} = x_n / v_n t_n$ and

$\cos\theta_{n,N} = h_n \big/ \left(x_n^2 + h_n^2\right)^{1/2} = h_n / v_n t_n$, the above ray segment relationship can be rewritten as

$$v_n\, t_n = \left(x_n^2 + h_n^2\right) \big/ \left(x_n^2 + h_n^2\right)^{1/2} = x_n \sin\theta_{n,N} + h_n \cos\theta_{n,N} \rightarrow$$

$$v_n\, t_n = \left(x_n \sin\theta_{n,N} / v_n\right) + \left(h_n \cos\theta_{n,N} / v_n\right) = p_n x_n + \eta_n h_n$$

where $p_n = \sin\theta_{n,N} / v_n = \sin\theta_{n,N} u_n$ is the horizontal slowness,

$\eta_n = \cos\theta_{n,N} / v_n = \cos\theta_{n,N} u_n$ is the vertical slowness and

$u_n = 1/v_n = \left(p_n^2 + \eta_n^2\right)^{1/2}$ is the n[th] layer slowness.

3.) The total traveltime along a given raypath (i.e., $p_n = p$, a constant value) is obtained by summing over the downgoing and upgoing raypath segments:

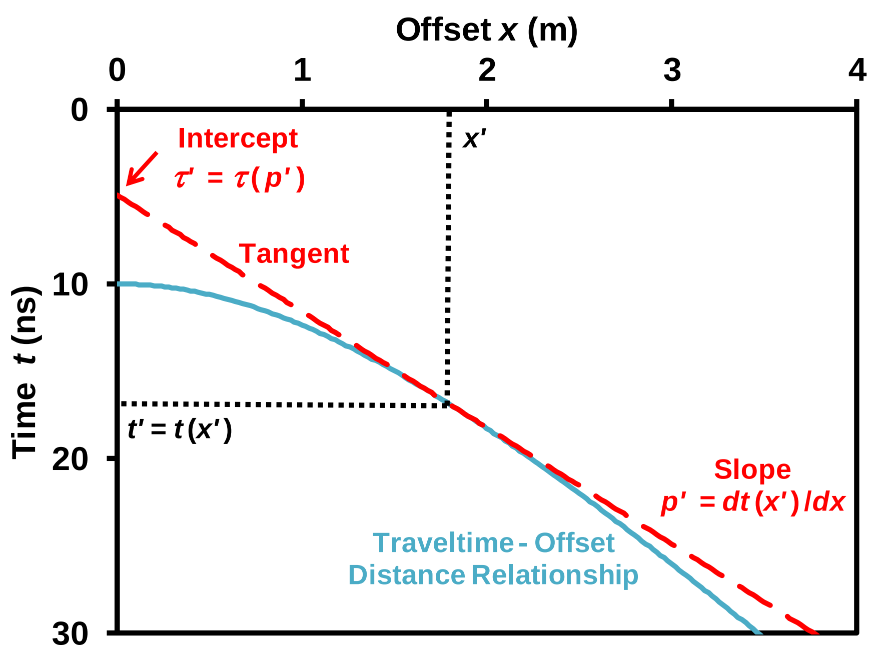
$$t(x) = 2\sum_{n=1}^{N} t_n = 2\sum_{n=1}^{N} p_n x_n + 2\sum_{n=1}^{N} \eta_n h_n = px + 2\sum_{n=1}^{N} \eta_n h_n$$

where x is total horizontal travel distance (i.e., the source-receiver offset distances)

4.) This equation can be expressed as $t(x) = px + \tau(p)$ where

$$\tau(p) = 2\sum_{n=1}^{N} \eta_n h_n = \sum_{n=1}^{N}\left[\left(v_n^{-2} - p^2\right)^{1/2} h_n\right] = \sum_{n=1}^{N}\left[\left(u_n^2 - p^2\right)^{1/2} h_n\right].$$

a.) Consider a given point $x', t' = t(x)$ on the traveltime-offset distance relationship.



b.) The tangent line to the traveltime-offset curve at this point has a slope $p' = dt(x')/dx$ and a time axis intercept $\tau' = \tau(p')$.

c.) The values of p and τ vary along the traveltime-offset relationship of a reflection event. Hence, the reflection event can be described in terms of either (x,t) or (p,τ). The (p,τ) description is referred to as the *intercept-slowness or $\tau - p$ formulation*.

d.) Using the ray parameter p as the independent variable instead of offset distance x, then $\tau(p) = t(p) - p\,x(p)$ (*Note, expressions for $t(p)$ and $x(p)$ were obtained above for a horizontally layered earth*).

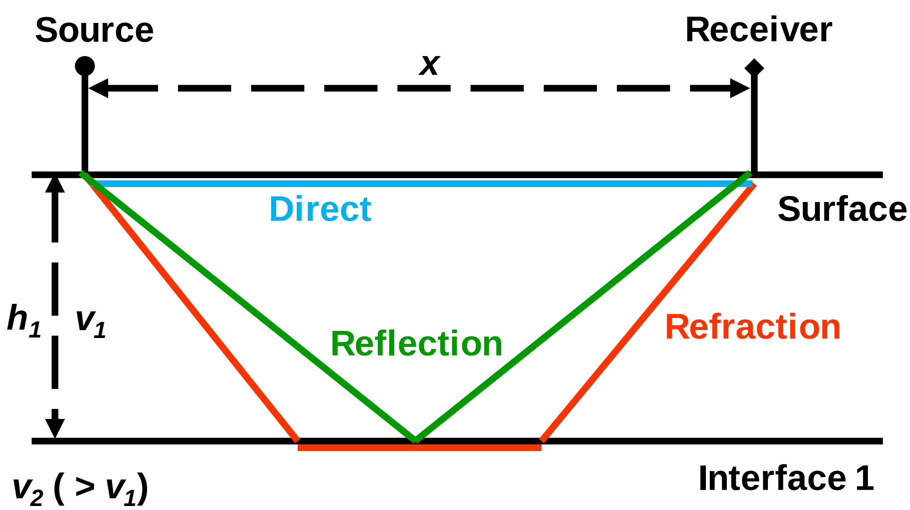
Differentiating this expression with respect to p:

$$\frac{d\tau}{dp} = \frac{dt}{dp} - p\frac{dx}{dp} - x(p) = \frac{dt}{dx}\frac{dx}{dp} - p\frac{dx}{dp} - x(p) = p\,\frac{dx}{dp} - p\frac{dx}{dp} - x(p) = -x(p).$$

Hence, the slope of the tangent lines to the $\tau(p)$ relationship are $-x(p)$.

5.) Nature of the $\tau(p)$ relationship for a horizontally layered earth

a.) A single layer over a half space with $v_2 > v_1$.



1.) In this case, we will consider the direct, primary reflection and critical refraction events.

2.) Using the expression for $\tau(p)$ from above, the relationship for the primary reflection is given by

$$\tau(p) = 2\left(u_1^2 - p^2\right)^{1/2} h_1 = \tau(p) = 2\left(v_1^{-2} - p^2\right)^{1/2} h_1$$

3.) This expression can be rewritten as

$$\frac{v_1^2\left[\tau(p)\right]^2}{4h_1^2}+v_1^2p^2=1$$

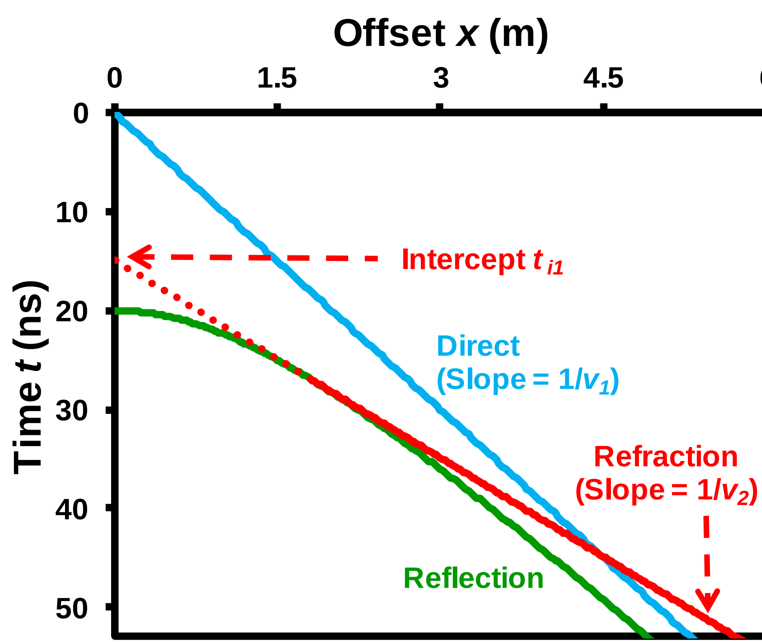
a.) This form is an ellipse with axes that coincide with the p and τ axes.

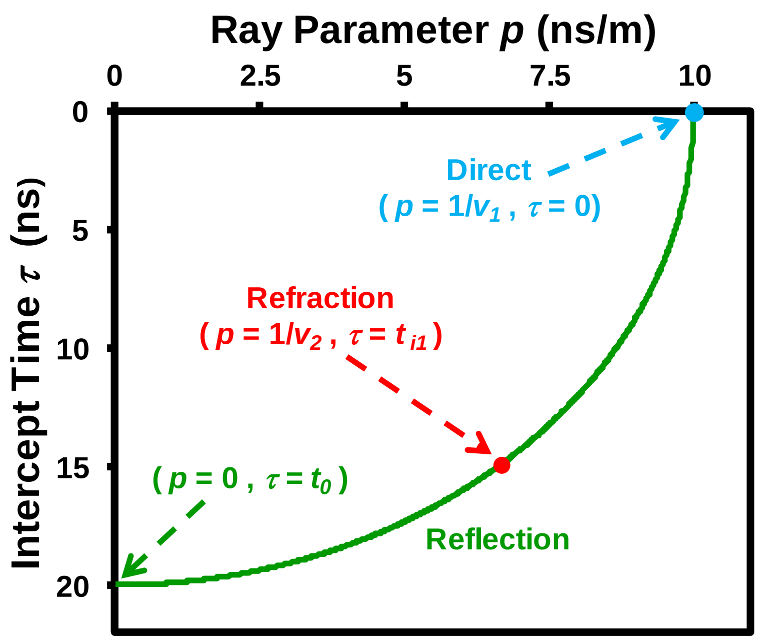
b.) The τ axis (i.e., $p=0$) intercept is $\tau=2h_1/v_1=t_0$ (i.e., the vertical two-way traveltime).

c.) The p axis (i.e., $\tau=0$) intercept is $p=1/v_1$.

4.) The direct wave event is a linear $x-t$ relationship with a slope $m=1/v_1$ and a time axis intercept $t=0$. Hence, it is represented in the $p-\tau$ graph as the single point $(1/v_1,0)$. This point coincides with the p axis (i.e., $\tau=0$) intercept of the reflection event ellipse.

5.) The critical refraction event is also a linear $x-t$ relationship with a slope $m=1/v_2$ and a projected time axis intercept $t_{i1}=2h_1\left(v_1^{-2}-v_2^{-2}\right)^{1/2}$. Hence, it is represented in the $p-\tau$ graph as the single point $\left(1/v_2,t_{i1}\right)$ on the reflection event ellipse.





b.) Multi-layered Earth

1.) Reflection events are represented on the $p-\tau$ graph by elliptical segments.

2.) The relationship between consecutive reflection events id dependent on the nature of their velocity contrast (i.e., is velocity increasing or decreasing with depth)

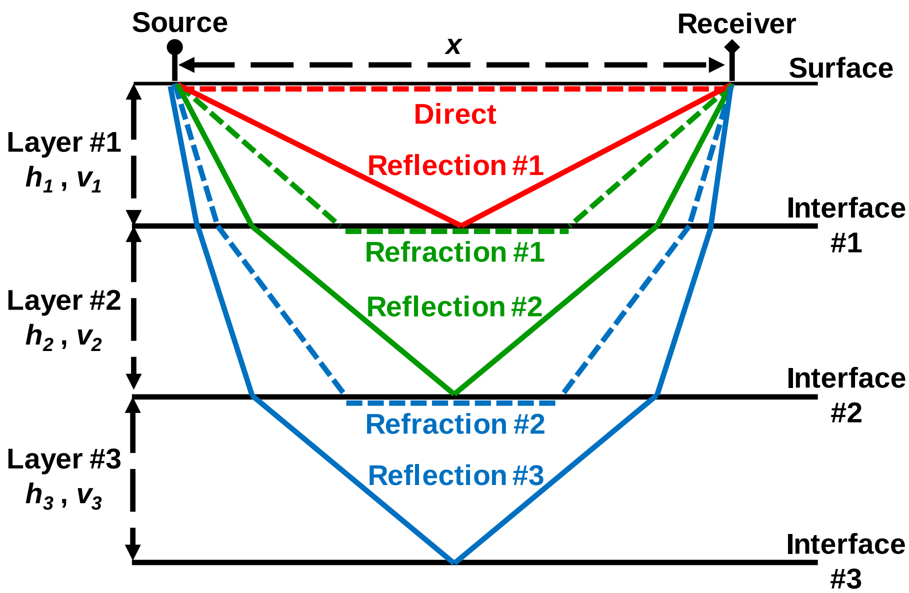
Example #1

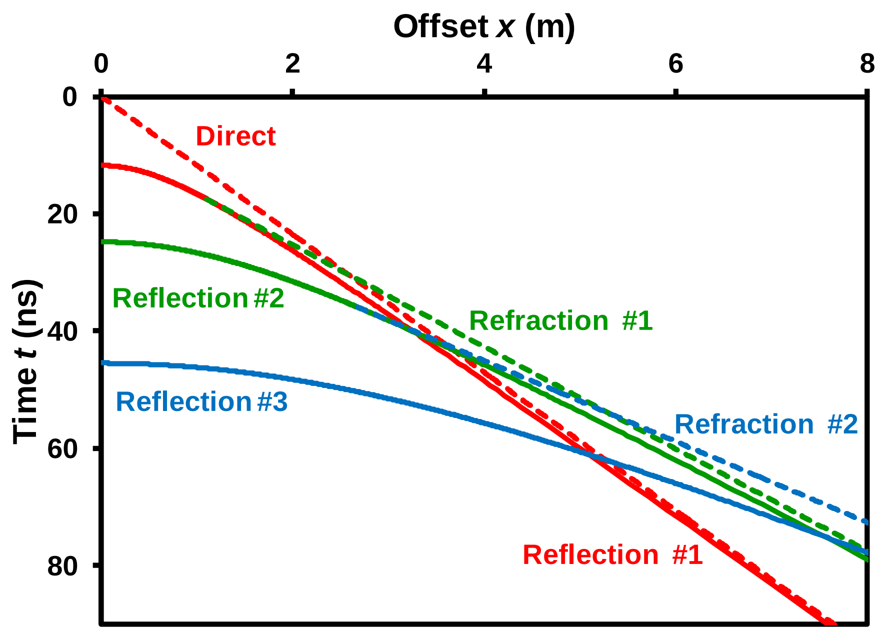
Three layers with velocities $v_1 < v_2 < v_3$ (i.e., there are critical refraction events along Interfaces #1 & #2.)

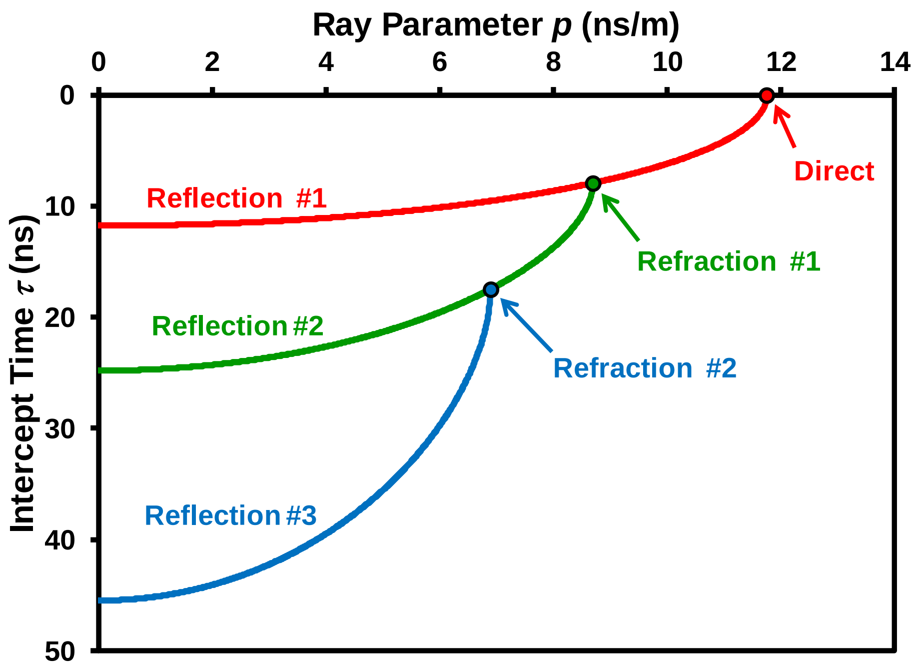
Layer #1: $v_1 = 0.085\ \mathrm{m/ns}$, $h_1 = 0.50\ \mathrm{m}$

Layer #2: $v_2 = 0.115\ \mathrm{m/ns}$, $h_2 = 0.75\ \mathrm{m}$

Layer #2: $v_3 = 0.145\ \mathrm{m/ns}$, $h_3 = 1.50\ \mathrm{m}$


Source
Receiver
x
Surface
Direct
Reflection #1
Layer #1
h1 , v1
Interface #1
Refraction #1
Reflection #2
Layer #2
h2 , v2
Interface #2
Refraction #2
Reflection #3
Layer #3
h3 , v3
Interface #3


Offset x (m)
0
2
4
6
8
Time t (ns)
0
20
40
60
80
Direct
Reflection #2
Refraction #1
Reflection #3
Refraction #2
Reflection #1



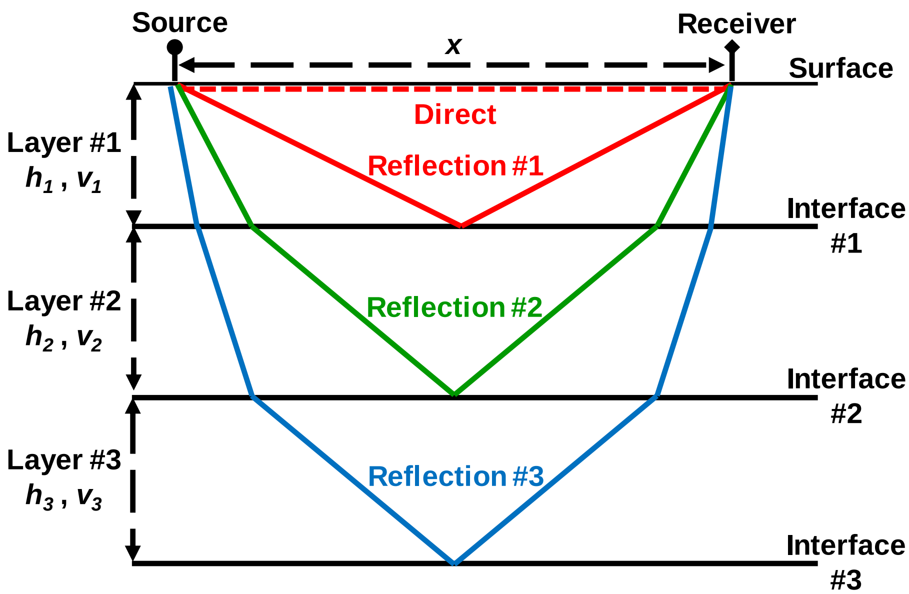
Example #2

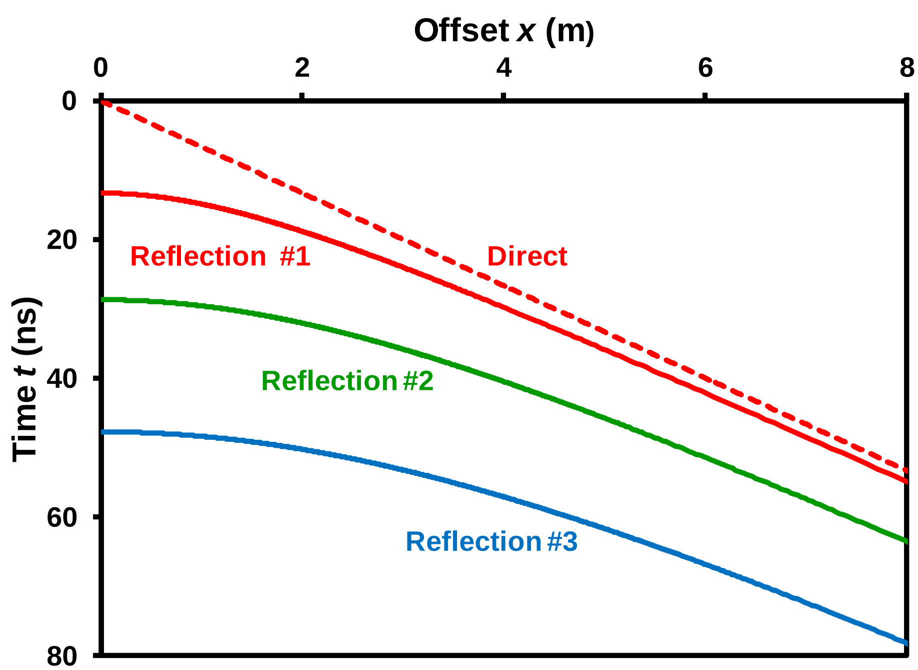
Three layers with velocities $v_1 > v_2 > v_3$ (i.e., there are no critical refraction events along Interfaces #1 and #2.)

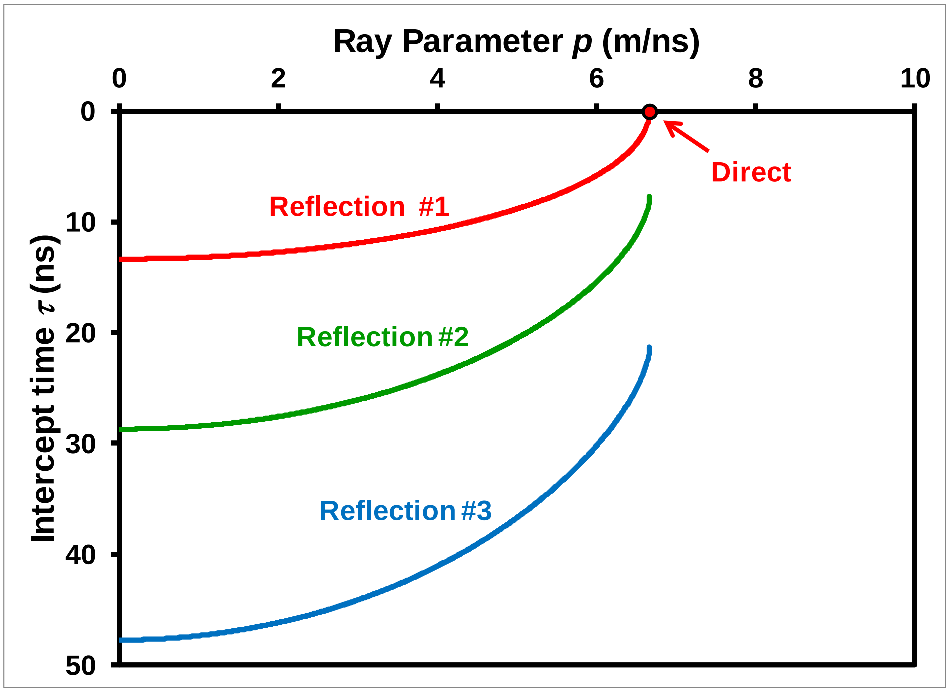
Layer #1: $v_1 = 0.150\ \mathrm{m/ns}$, $h_1 = 1.00\ \mathrm{m}$

Layer #2: $v_2 = 0.130\ \mathrm{m/ns}$, $h_2 = 1.00\ \mathrm{m}$

Layer #2: $v_3 = 0.105\ \mathrm{m/ns}$, $h_3 = 1.00\ \mathrm{m}$


Source
Receiver
x
Surface
Direct
Reflection #1
Layer #1
h1 , v1
Interface
#1
Layer #2
h2 , v2
Reflection #2
Interface
#2
Layer #3
h3 , v3
Reflection #3
Interface
#3


Offset x (m)
0
2
4
6
8
0
20
40
60
80
Time t (ns)
Reflection #1
Direct
Reflection #2
Reflection #3



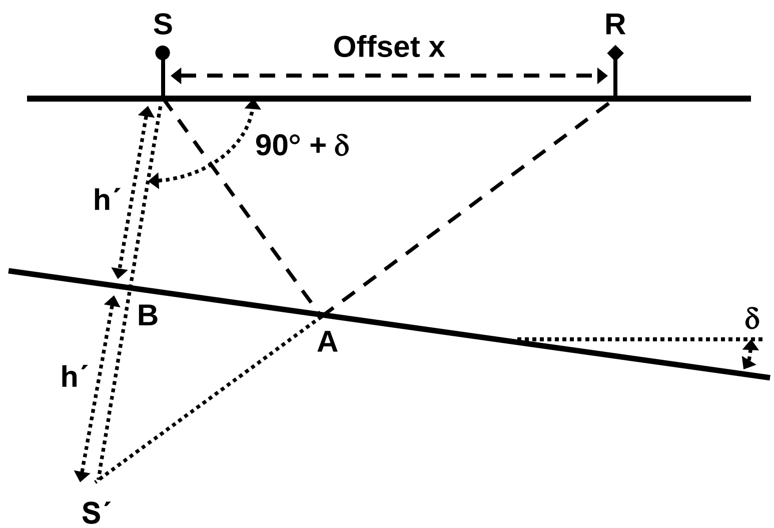
G.) Single Dipping Interface

1.) The interface has a dip δ. The seismic profile line is aligned in the dip direction.

2.) Fixed Source Point Case (i.e., Common Shotpoint or CSP Gather)

a.) First, we consider the case were the source position is fixed. The interface has a normal thickness h' below the source point.

b.) Consider a receiver located an offset x from the source point. The $+x$ direction is the downdip direction; conversely, $-x$ direction is the updip direction.



c.) Given the right angle at B, the raypath segment SA can be projected through the interface to form the triangle S´SR where the side S´R is the length of the total raypath.

d.) Using the Law of Cosines, it can be shown that the total raypath length $l = S'R$ is given by

$$l^2 = x^2 + (2h')^2 - 2(2h')x\cos\left(\frac{\pi}{2} + \delta\right) = x^2 + 4h'^2 + 4h'x\sin\delta$$

e.) The traveltime for this raypath is $v^2 t^2 = x^2 + 4h'^2 + 4h'x\sin\delta$

f.) Completing the square gives $\dfrac{v^2 t^2}{(2h'\cos\delta)^2} - \dfrac{(x + 2h'\sin\delta)^2}{(2h'\cos\delta)^2} = 1$ for the traveltime-offset distance relationship.

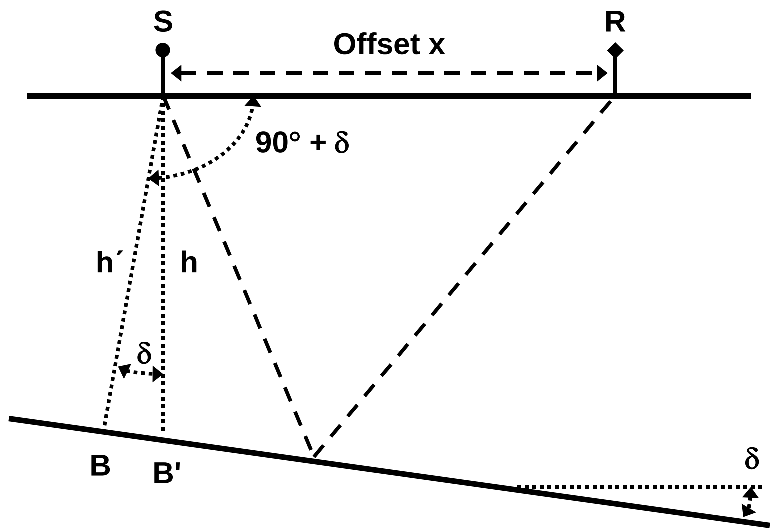
1.) This relationship defines a hyperbola

2.) Unlike the symmetric hyperbola for a horizontal interface, this one is centered at $x = -2h'\sin\delta$ (neg. sign indicates the updip direction).

3.) Effect increase with increasing dip.

g.) It should be noted that as the dip angle in this model increases, the normal projection of source location onto the interface B moves in an updip direction. Further, the vertical projection of source location moves deeper. In general, the entire interface is translating.

h.) An alternate analysis of the dip effects on traveltime is obtained by fixing the location of a point on the interface, such as the vertical projection of source location B' at a vertical depth/layer thickness h.



1.) For this model, $h' = h\cos\delta$.

2.) Rewriting the traveltime-offest distance relationship gives

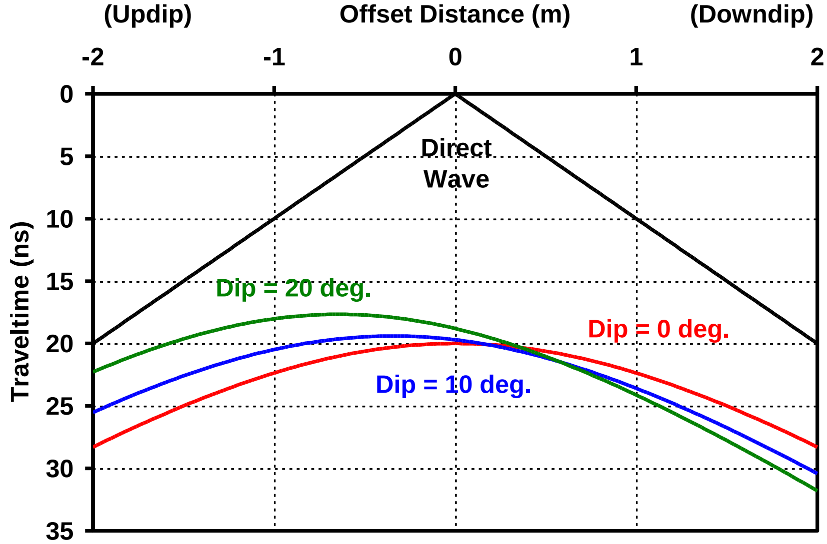
$$\frac{v^2 t^2}{\left(2h\cos^2\delta\right)^2} - \frac{\left(x + 2h\cos\delta\sin\delta\right)^2}{\left(2h\cos^2\delta\right)^2} = 1 \rightarrow$$

$$t^2(x) = \frac{\left(2h\cos^2\delta\right)^2}{v^2} + \frac{\left(x + h\sin 2\delta\right)^2}{v^2}$$

3.) This hyperbola is centered at $x = -h\sin 2\delta$

Example:

Consider the traveltime-offset distance relationships for the single dipping layer with the vertical thickness below the source point of $h = 1$ m and wave velocity $v = 0.10$ m/ns. The following values of dip angle δ were modeled: 0°, 10° and 20°.



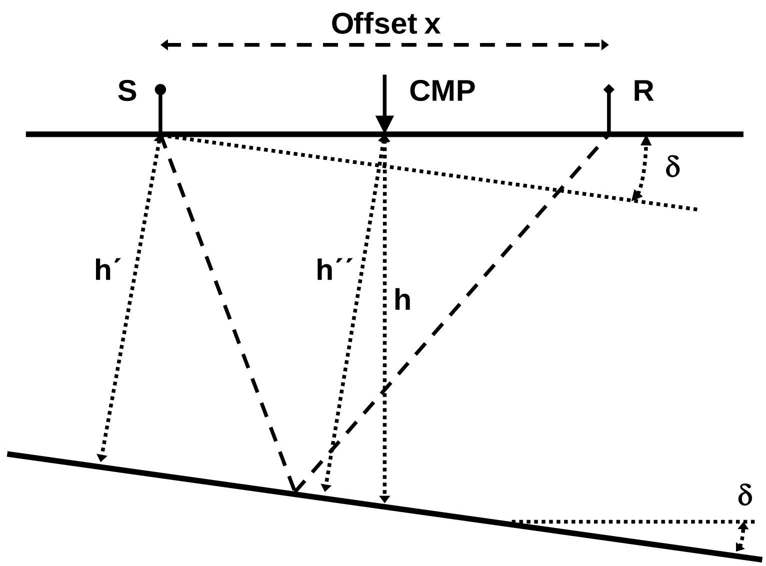
It can be seen that the apex of the hyperbolae are displaced in an increasing updip direction as the dip angle grows.

3.) Fixed Common Midpoint Case

a.) Now we consider the case were the midpoint between the source and receiver positions is fixed as the offset increases. The interface has a normal thickness h'' below the common midpoint.

b.) Using $h'' = h' + (x/2)\sin\delta$ in the expression for traveltime along the raypath (i.e., $v^2 t^2 = x^2 + 4h'^2 - 4h'x\sin\delta$), we obtain the results

$$t^2(x) = \frac{(2h'')^2}{v^2} + \frac{x^2}{(v/\cos\delta)^2} = t_0^2 + \frac{1}{v_{app}^2}x^2$$

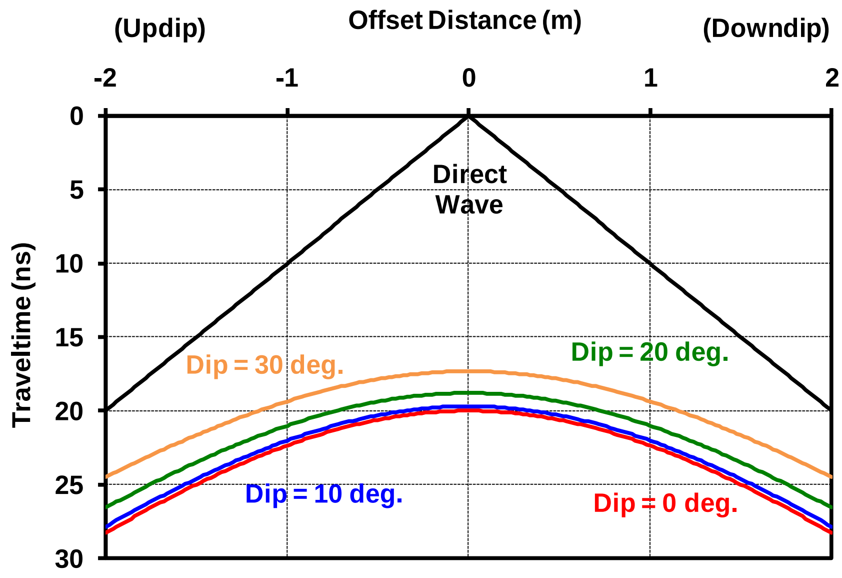


This expression can be rewritten in term of the vertical depth/layer thickness h below the fixed common midpoint location using $h' = h\cos\delta$. Hence, $t_0 = 2h\cos\delta/v$

1.) This a hyperbolic traveltime-offset distance relationship that is symmetric $t^2(x)$ about $x = 0$, similar to the horizontal interface case.
2.) $v_{app} = v/\cos\delta$ is the apparent velocity for the hyperbolic relationship. The apparent velocity v_{app} increases with increasing dip angle δ.
3.) $2h\cos\delta/v = 2h/v_{app}$ is the actual zero offset traveltime. Hence, t_0 decreases with increasing dip angle δ.
4.) The traveltime-offset distance relationship for a dipping layer is the same as that for an equivalent higher velocity horizontal layer.

Example:

Consider the traveltime-offset distance relationships for the single dipping layer with the vertical thickness below the common midpoint (CMP) of $h = 1$ m and wave velocity $v = 0.10$ m/ns. The following values of dip angle δ were modeled: 0°, 10°, 20° and 30°.



H.) Multiple Dipping Layers

1.) Consider an Earth composed of N layers with each interface having an arbitrary dip.

2.) The common midpoint between the source and receiver is held in a fixed location while the source-receiver offset is increased.

3.) It can be shown that the traveltime-offset distance relationship for the Nth interface in this CMP gather is

$$t^2(x) = \left[t_0(N)\right]^2 + \frac{1}{\left[\tilde{V}_{rms}(N)\right]^2}x^2 + \text{H.O.T.} \quad \text{where}$$

$$\tilde{V}^2_{rms}(N) = \frac{1}{t_0(N)\cos^2\beta_0}\sum_{n=1}^{N}\left\{v_n^2\,\Delta t_0(n)\left[\prod_{k=1}^{n-1}\left(\frac{\cos^2\alpha_k}{\cos^2\beta_k}\right)\right]\right\},$$

α_n and β_n are incident and refraction angles along the zero offset raypath,

$t_0(N)$ is the total zero offset traveltime and

$\Delta t_0(n)$ is the two-way traveltime in the nth layer for the zero offset raypath.

4.) For gentle dips and relatively short offset distances, the traveltime-offset distance relationship is approximated by the hyperbolic approximation defined by this expansion.

Note: The reflection point on the N^{th} interface changes with varying offset distance; the amount of change is a function of the dips.

Example: Dipping Interface on CSP and CMP Gathers

Data: GPR 900 MHz reflection profiling, CMP & WARR data from soil moisture monitoring project (Site 5G, Smith Farm, Waterloo Moraine, June 4, 2008)

Processing: Dewow; SEC Gain (Atten = 0.5, Max = 200); Bandpass Filter (0.100 - 0.200 - 1.500 - 2.000 GHz)