Earth Sciences 460

Assignment #5 **Due: Monday February 27, 2017**

Problem 1.

Let $\mathbf{x} = (x_0, x_1, x_2, x_3) = (1, 2, 3, -1)$ be a discretely sampled time signal.

Compute the DFT $\mathbf{X} = (X_0, X_1, X_2, X_3)$ of this signal directly from the definition $X_k = \sum_{n=1}^{N-1} x_n \, e^{-2\pi i n k / N}$.

If the uniform time sampling interval is $\Delta t = 0.2 \sec$, what are the values of f_k for each X_k?

Determine the discrete phase and amplitude spectra and plot each as a function of frequency (assuming $\Delta t = 0.2 \sec$).

Problem 2.

Compute $\mathbf{x} * \mathbf{y}$ for the following discrete time signals using the "fold and shift" method and the z-tranform.

a.) $\mathbf{x} = (x_{-2}, x_{-1}, x_0, x_1, x_2) = (1, -1, 2, -2, 1)$ and $\mathbf{y} = (y_1, y_2, y_3) = (1, 0, -1)$

b.) $\mathbf{x} = (x_0, x_1, x_2, x_3) = (1, 1, 1, 1)$ and $\mathbf{y} = (y_0, y_1, y_2, y_3) = (1, 1, 1, 1)$

c.) $\mathbf{x} = (x_{-2}, x_{-1}, x_0, x_1, x_2) = (1, 2, 3, 2, 1)$ and $\mathbf{y} = (y_{-1}, y_0, y_1) = (1, 1, 1)$

Plot $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{x} * \mathbf{y}$ for each.

Problem 3

Using the z transform and polynomial division, find the first 5 **nonzero** terms of two different inverse filters for the following discrete impulse responses

a.) $\mathbf{x} = (x_0, x_1, x_2) = (1, -1, 2)$ b.) $\mathbf{x} = (x_{-2}, x_{-1}, x_0) = (3, 2, 1)$

c.) $\mathbf{x} = (x_0, x_1, x_2) = (1, 0, 1)$

Plot the two versions of the inverse filters for each case.

Problem 4

Using the z transform and polynomial division, determine the causal version of the inverse filter truncated to the following number of **nonzero** terms: 2, 3, 4, 5, 6, and 7 for the discrete impulse response $\mathbf{x} = (x_0, x_1, x_2) = (1, 0, -1/2)$.

Determine the error energy for each case. Briefly comment on the effects of the inverse filter length.

Problem 5

Determine the optimal least-squares 3 term inverse filter for the discrete impulse response $\mathbf{x} = (x_0, x_1, x_2) = (1, 0, -1/2)$.

Compare this inverse filter in terms of error energy with the corresponding (i.e., same length) inverse filter from Problem 4.

Problem 6

Compute the cross-correlation $\mathbf{x} \otimes \mathbf{y}$ and the auto-correlations $\mathbf{x} \otimes \mathbf{x}$ and $\mathbf{y} \otimes \mathbf{y}$ for the following discrete time signals using the "shift" method.

$\mathbf{x} = (x_{-2}, x_{-1}, x_0, x_1, x_2) = (1, -1, 2, -2, 1)$ and $y = (y_1, y_2, y_3) = (1, 0, -1)$

Plot the discrete signals and the correlations.