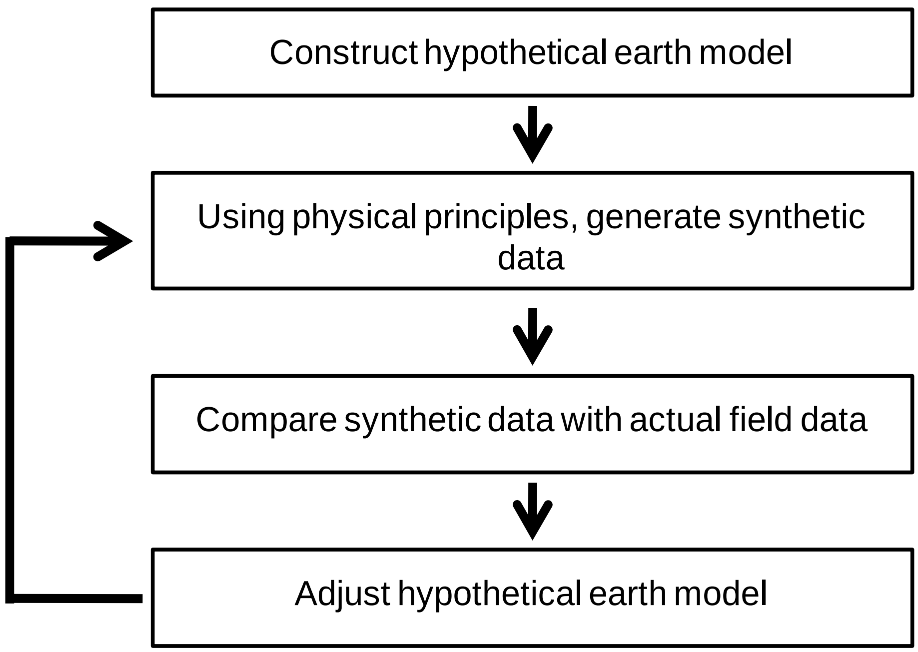
I.) BRIEF INTRODUCTION TO INVERSE PROBLEMS & METHODS

Inverse techniques are being widely used in geophysical data analysis. While inversion techniques are mathematically complicated and vary with geophysical method, there are basic elements underlying all inversion process.

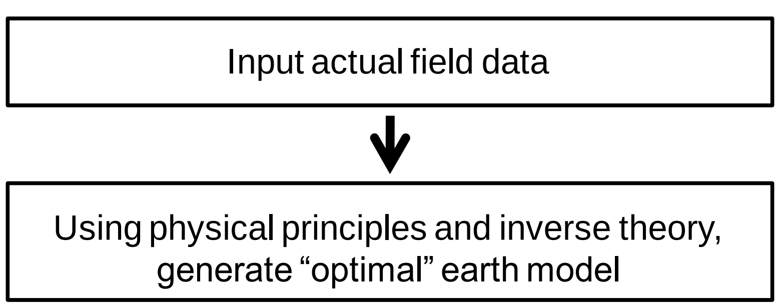
A.) FORWARD MODELLING VERSUS INVERSION

1.) Basic Forms

a.) Forward Modelling:



b.) Inversion:



2.) Analogy Using Linear System of Equations

$$\mathbf{A}\cdot\mathbf{x}=\mathbf{d} \quad \Rightarrow \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

where matrix $\mathbf{A}$ contains coefficients describing the “physical principles” of the problem, vector $\mathbf{x}$ is the model parameters and vector $\mathbf{d}$ is the data.

a.) Forward modeling: Try an initial $\mathbf{x}'$, compute $\mathbf{d}'=\mathbf{A}\cdot\mathbf{x}'$ and compare with $\mathbf{d}$, adjust $\mathbf{x}'$ and repeat.

b.) Inversion: Determine the inverse matrix $\mathbf{A}^{-1}$ and compute $\mathbf{x}$ using $\mathbf{x}=\mathbf{A}^{-1}\cdot\mathbf{d}$.

B.) Conditioning and Fitting of Actual Data

1.) Conditioning is a measure of the susceptibility of the inversion results to the effects of errors and/or noise in actual field data.

2.) Geophysical inverse problems are very commonly “ill-conditioned” (i.e., the inversion results can be greatly affected by small data imperfections).

3.) These effects are most evident when the derived earth model attempts to fix the field data to an inappropriate accuracy level. This produces fictitious features (i.e., artifacts) in the earth model.

4.) This problem can be controlled by requiring that field data be replicated to an appropriate level. There are a various measures available to determine this level of fit (e.g., χ^2 criterion).

C.) Uniqueness

1.) The earth model obtained from an inversion algorithm is *non-unique*. That is, there are an infinite number of possible earth models that reproduce the data as accurately as the inversion solution.

2.) Non-uniqueness is results from one of two causes:

a.) The physical principles for the problem are inherently non-unique (i.e., potential field methods, such as gravity and magnetic).

b.) Imperfect data (discrete sampling, truncated values, noise/errors).

D.) Optimization Criterion

The optimization criterion defines the method the inversion process constructs an earth model. Different criteria emphasize/suppress specific characteristics of the earth model (e.g., do the physical properties vary smoothly or abruptly).

E.) Constraints

1.) The range of physical property values used to construct an earth model can frequently be specified. This range should reflect the values likely to be encountered.

2.) Use of constraints can reduce the degree of non-uniqueness in the inversion results.

3.) Types of constraints:

a.) Hard bounds: model cannot exceed set limits (i.e., $\rho_{lower} \le \rho_{ij} \le \rho_{upper}$).

b.) Soft bounds: model are biased to expected values (e.g., $\min\left|\rho_{ij} - \rho_0\right|$ or $\min\left(\rho_{ij} - \rho_0\right)^2$).

F.) Initial (A Priori) Earth Model

1.) The earth model obtained from an inversion process is dependent on the initial model used to start the process.

2.) This initial model should attempt to incorporate known information about the geology and physical conditions of the site

G.) Discretization and Parameterization

1.) While physical properties can vary continuous throughout the subsurface and boundaries between units usually have irregular geometries, most inversion methods represent the earth in terms on specifically shaped elements (e.g., horizontal layers, cubes) with uniform physical properties.

2.) It necessary to make sure that these elements and their dimensions are appropriate for the situation.

H.) Inherent Assumptions/Simplifications

1.) To simplify the computations, inversion processes can make significant assumptions about:

a.) Representation of the physics (e.g., straight ray vs bending ray)

b.) Dimensionality (i.e, 1-D, 2-D and 3-D)

c.) Physical properties (e.g., isotropic vs anisotropic)

2.) These assumptions need to be reasonable for the application.