

## II.) ELECTRICAL RESISTIVITY METHODS

### M.) ELECTRICAL PROPERTIES OF ROCKS AND SOILS

*Note: Physical properties are intrinsic properties of the material and are independent of the shape and dimensions of the body*

- 1.) Resistivity ( $\rho$ ) - a measure of a material's ability to impede the flow of an electric current. SI units: Ohm-meter ( $\Omega \text{ m}$ ).
- 2.) Conductivity ( $\sigma$ ) - a measure of a material's ability to permit the flow of an electric current. SI units: Siemens / meter ( $\text{S/m}$ ).

*Note: Conductivity is the reciprocal or inverse of resistivity (i.e.,  $\sigma = 1/\rho$ )*

- 3.) Mechanisms involved in electric current flow in geological materials
  - a.) Electronic conduction
    - i.) Current by electrons in metallic minerals and graphite.
    - ii.) Generally minor in near-surface applications.
  - b.) Electrolytic (or ionic) conduction
    - i.) Current conduction by ions in the pore water.
    - ii.) Since the rock and soil grains are generally insulators, electrolytic conduction is a major, if not controlling, mechanism.
    - iii.) The magnitude of ionic conduction in rocks and soils depends on the pore water conductivity (ionic concentration, ionic species, temperature) and the pore structure (porosity, connectivity/tortuosity).
  - c.) Surface (or interfacial) conduction
    - i.) Electrical structures (i.e., diffuse double layers) occur along solid/pore water interfaces. Current is conducted in these structures.
    - ii.) Due to its large surface area, the presence of clay can lead to significant surface conduction
    - iii.) The magnitude of surface conduction is relatively independent of electrolytic conduction.

4.) Range of resistivity/conductivity for soils and rocks

- a.) Resistivity/conductivity has one of the largest range of values for a physical property of geological materials. Only hydraulic permeability may have a larger range.
- b.) For rocks and soils, this range is primarily due to variations in pore structure, clay content, pore fluid properties and metallic mineral content.
- c.) While there is significant overlapping, there are some general patterns:

- 1.)  $\rho_{unconsolidated\ sediments} < \rho_{sedimentary\ rocks} < \rho_{igneous\ \&\ metamorphic\ rocks}$

- 2.)  $\rho_{fractured\ rock} < \rho_{unfractured\ rock}$

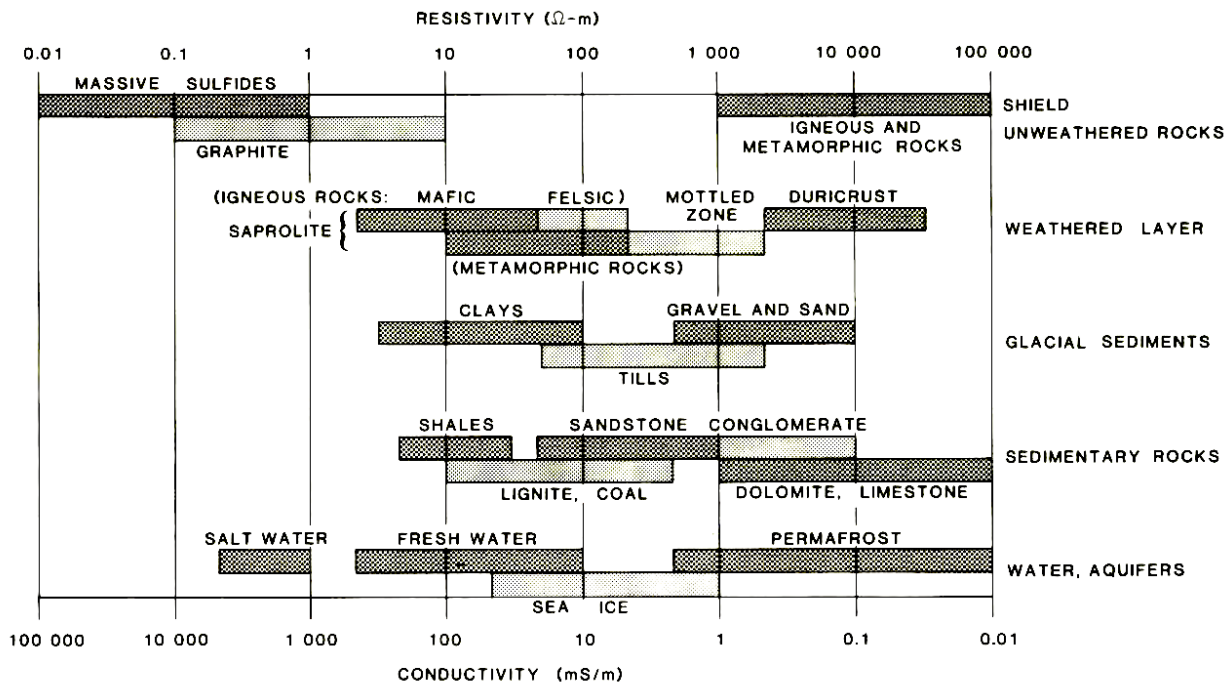


Fig. 1. Typical ranges of resistivities of earth materials.

(Figure from Palacky, 1987)

5.) Electrical properties of pore fluids

- a.) Pure water (i.e., without ions other than  $H^+$  and  $OH^-$ ) is fairly resistive:

$$\rho_w = 2.8 \times 10^5 \Omega m \text{ at } 17.6^\circ C \text{ (Dorsey, 1940).}$$

- b.) In aqueous solutions, electric current is carried by the ions of dissociated salts in fluid. Hence, pore water resistivity is dependent on the ion concentrations (i.e., density of

charge carriers) and their mobilities (i.e., their efficiency as a charge carrier). Mobility varies with ion species and applied conditions (i.e., temperature & pressure).

Equivalent conductances of some common groundwater ions at infinite dilution (Data from Parson, 1960). Units: S/m per meq/ml at 25°C

$\text{Na}^+ = 5.015$	$\text{Cl}^- = 7.635$
$\text{Ca}^{2+} = 5.950$	$\text{SO}_4^{2-} = 8.000$
$\text{K}^+ = 7.350$	$\text{HCO}_3^{2-} = 4.450$
$\text{H}^+ = 34.985$	$\text{OH}^- = 19.670$

c.) The electrical conductivity of an aqueous electrolyte is linear combination of the conductivity due to the individual ionic species. For a solution containing  $N$  ionic species:

$$\sigma_w = \sum_{i=1}^N c_i z_i F v_i$$

where  $c_i$  = concentration of  $i^{th}$  species (moles / unit volume),  
 $z_i$  = valence of  $i^{th}$  species,  $F$  = Faraday's constant (96485.3362 Coulombs / mole) and  
 $v_i$  = mobility of  $i^{th}$  species

d.) Pore water resistivity can be estimated its total dissolved solids (TDS) concentration (i.e., a direct measure of the total ion concentration):

$$\rho_{w,77} = 10000 P / (\text{TDS})$$

where  $\rho_{w,77}$  = pore water resistivity at 77°F (25°C) in  $\Omega$  m.  
TDS is in units of ppm, effectively mg (solids) / l (pore water)

$P$  is an empirical factor that compensates for compositional variations. It ranges from 0.5 for NaCl solutions to 0.9 for alkaline water. A common value used for fresh groundwater is 0.67 (D.G. Jorgensen, 1996)

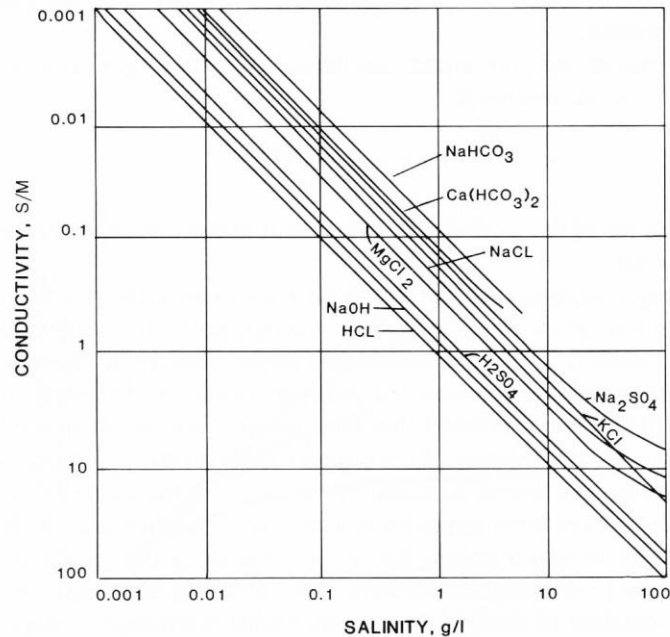


FIG. 16. Conductivity of solutions of various salts as a function of concentration. Temperature of 20° C.

(Figure from Keller, 1987)

e.) Immiscible pore fluids (e.g., air, oil, gasoline, chlorinated solvents) have very high values of resistivity (generally  $\rho_{fluid} > 10^{+6} \Omega m$  for organic liquids) relative to aqueous solutions such as groundwater. Hence, they are treated as insulators in the soil/rock system.

Examples of the electrical resistivity/conductivity of immiscible fluids

Hydrocarbons (Jones, 2010):

Crude Oil:  $1 \times 10^{+10} \Omega m$  ( $1 \times 10^{-10} S/m$ )

Gasoline:  $4 \times 10^{+10} \Omega m$  ( $2.5 \times 10^{-11} S/m$ )

Kerosene (jet fuel):  $1.67 - 20 \times 10^{+9} \Omega \text{ m}$  ( $5 - 60 \times 10^{-11} \text{ S/m}$ )

Diesel Fuel:  $4.42 - 100 \times 10^{+9} \Omega \text{ m}$  ( $1 - 22.6 \times 10^{-11} \text{ S/m}$ )

Lubricating Oil:  $7.09 - 25000 \times 10^{+8} \Omega \text{ m}$  ( $4 - 14100 \times 10^{-13} \text{ S/m}$ )

Chlorinated solvents (Lucius et al., 1992):

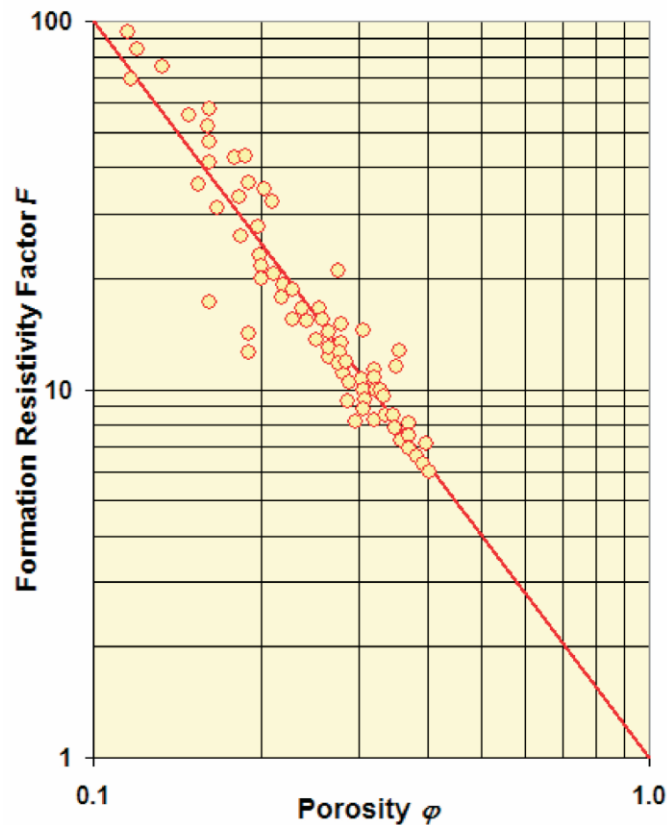
Tetrachloroethene ( $\text{C}_2\text{Cl}_4$ ):  $1.8 \times 10^{+11} \Omega \text{ m}$  ( $5.56 \times 10^{-12} \text{ S/m}$ )

Trichloroethene ( $\text{C}_2\text{HCl}_3$ ):  $1.25 \times 10^{+6} \Omega \text{ m}$  ( $8 \times 10^{-7} \text{ S/m}$ )

## 6.) Petrophysical Relationships for Electrical Resistivity of Rocks & Soils

### a.) Electrical properties of clay-free (clean) materials

- 1.) It assumes that the geological material is "clean" (i.e., clay-free). Hence, electrolytic conduction through the pore space is the only mechanism for current flow.
- 2.) Archie's (1942) equation is an empirical relationship for porous rocks and soils



Data used by Archie (1942) for Nacatoch sandstone

(Figure from Kennedy, 2007)

3.) For fully water-saturated rocks and soils,

$$\rho = a\phi^{-m}\rho_w \text{ or } \sigma = \frac{1}{a}\phi^m\sigma_w$$

$\rho, \sigma$  = effective resistivity, conductivity,  $\rho_w, \sigma_w$  = pore water resistivity, conductivity,

$\phi$  = porosity (in volume fraction),  $m$  = cementation exponent ( $1.3 \leq m \leq 2.5$ ) and

$a$  = empirical constant ( $0.5 \leq a \leq 2.5$ )

Cementation exponent  $m$  implicitly incorporates pore structure information into Archie's equation.

Typical values of the cementation exponent  $m$  for clastic and carbonate hydrocarbon reservoirs: (From Doveton, 1986).

Unconsolidated sands	$m = 1.3$
Very slightly cemented sands	$m = 1.4 - 1.5$
Slightly cemented sands	$m = 1.5 - 1.7$
Moderately cemented sands	$m = 1.8 - 1.9$
Highly cemented sands	$m = 2.0 - 2.2$
Fractured carbonates	$m = 1.4$
Chalky limestones	$m = 1.7 - 1.8$
Crystalline and granular carbonates	$m = 1.8 - 2.0$
Carbonates with vugs	$m = 2.1 - 2.6$

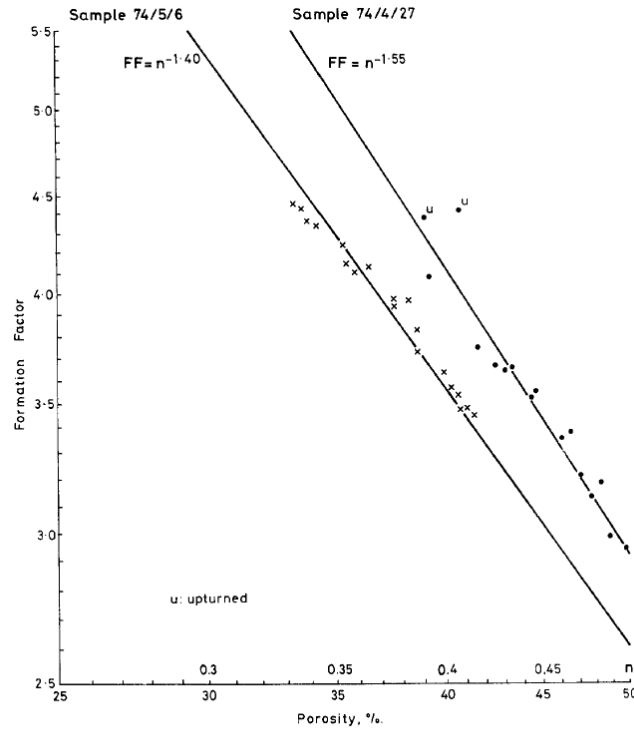


FIG. 4. The formation factor-positivity plots for 2 marine sands (see Figures 3 and 5).

Sample 74/5/6: quartz sand & gravel; Sample 74/4/27: quartz sand & 1% shell  
(Figure from Jackson et al., 1978)

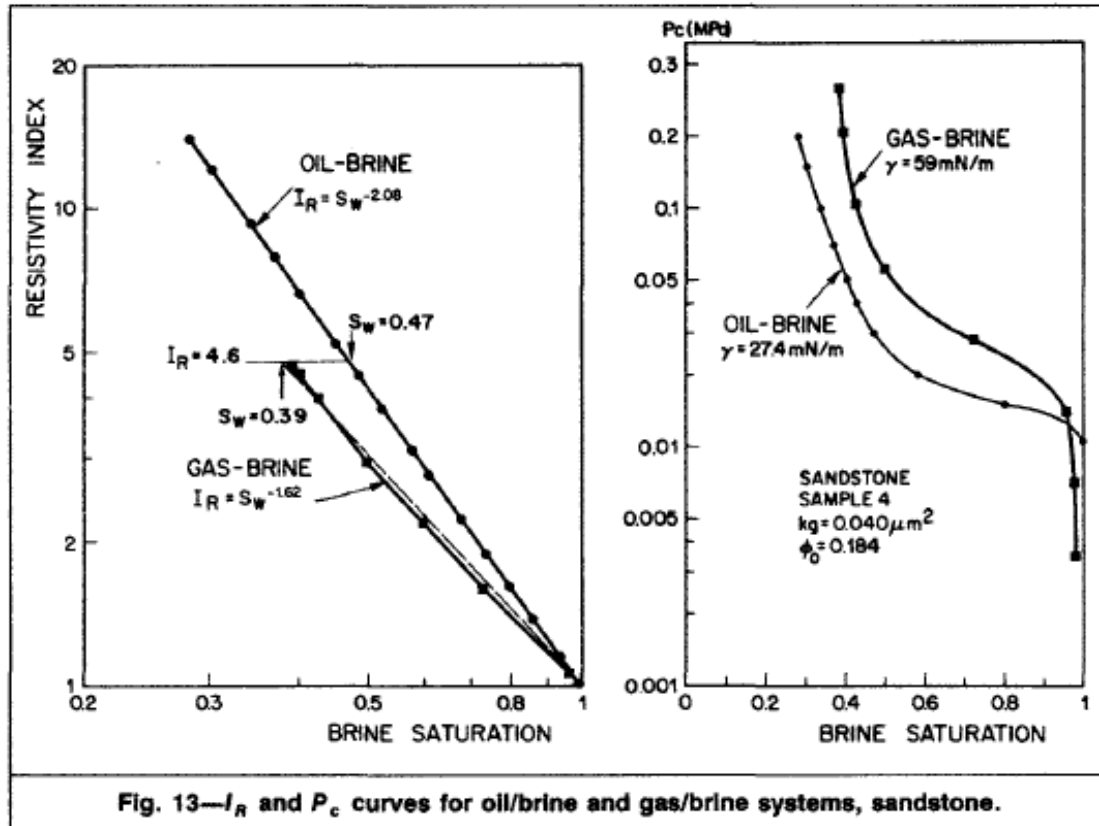
4.) For partially-saturated rocks and soils,

$$\rho = a \phi^{-m} S_w^{-n} \rho_w \text{ or } \sigma = \frac{1}{a} \phi^m S_w^n \sigma_w$$

$S_w$  = water saturation (volume fraction of pore space containing water)

$n$  = saturation exponent ( $1.5 \leq n \leq 2.5$ ,  $n \approx 2.0$  commonly assumed)

a.) Saturation exponent  $n$  implicitly incorporates information about the pore fluid distribution in the pore space. It can vary within pore fluid species (i.e., gas vs. liquid), wetting conditions and saturation history (i.e., imbibition vs. drainage).



(Figure from Longeron et al., 1989)

- b.) The commonly assumed value of the saturation exponent ( $n \approx 2.0$ ) is based on experience with oil reservoirs. For air-water systems, such as an aquifer, saturation exponent values tend to be lower:

Ottawa Sand:  $n = 1.56$  (Gorman & Kelly, 1990)

Borden Sand:  $n \approx 1.6$  (Mickle, 2005)

##### 5.) Formation factor and Saturation index

- a.) These are normalized quantities are useful in analyzing resistivity/conductivity measurements.
- b.) For water-saturated rocks and soils,

$$\text{Formation factor } F = \rho / \rho_w = \sigma_w / \sigma = a \phi^{-m}$$



*This normalization removes pore fluid resistivity effects, leaving pore structure information.*

c.) For partially saturated rocks and soils,

$$\text{Saturation (or resistivity) index } I = \rho / \rho_{sat} = \sigma_{sat} / \sigma = S_w^{-n}$$

$$\rho_{sat}, \sigma_{sat} = \text{resistivity / conductivity of the saturated medium}$$

The normalization removes saturated rock effects, leaving pore fluid distribution information.

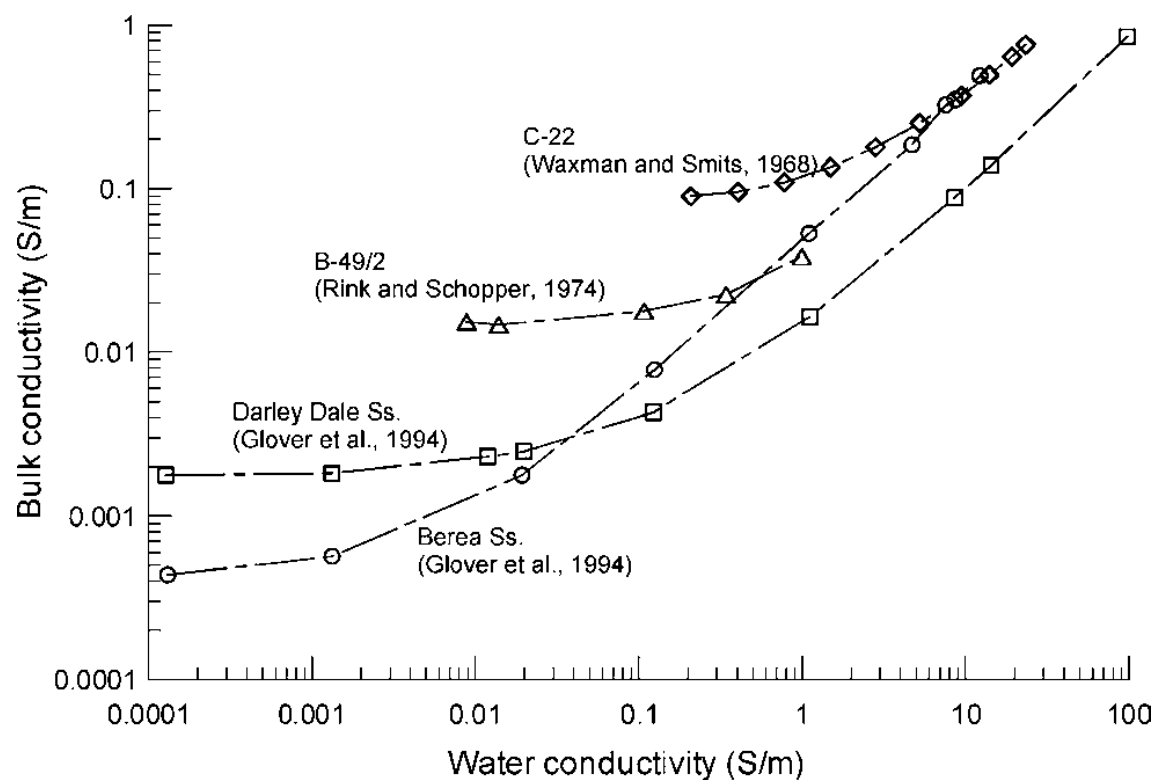
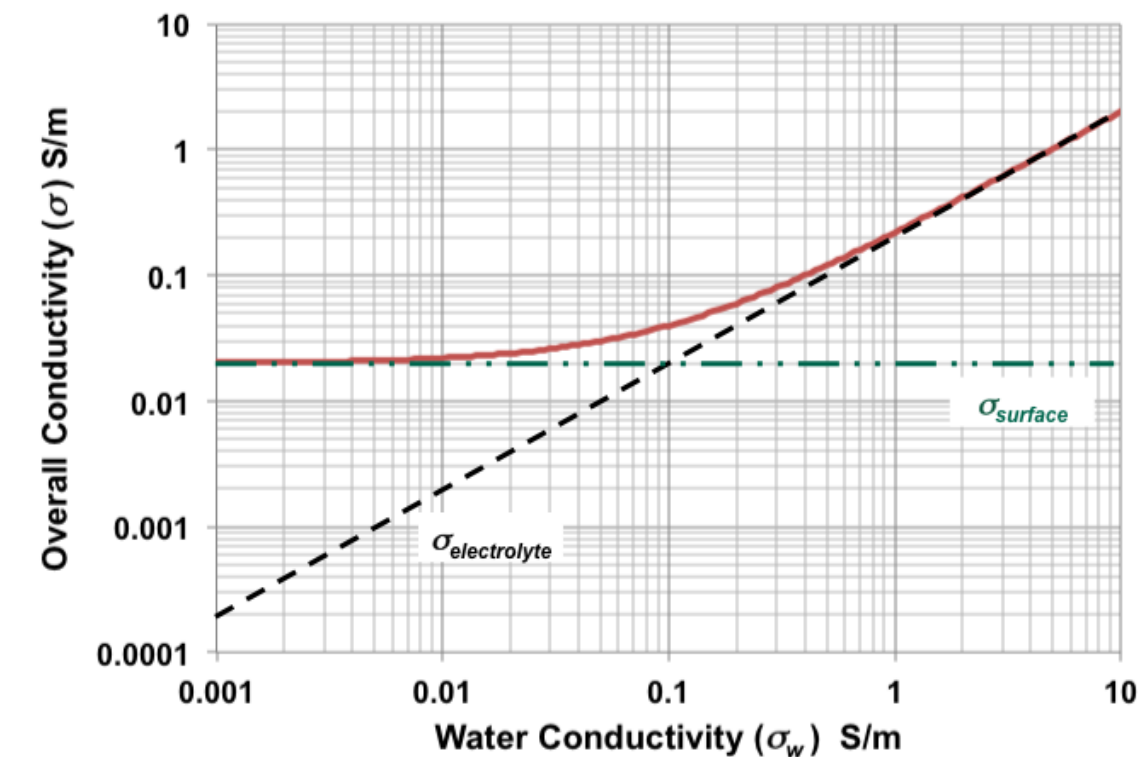
b.) Electrical Properties of clayey / shaly sands materials

- 1.) Surface conduction  $\sigma_{surface}$  along the solid/water interfaces is assumed to act as part of a parallel conductor system in a porous rock/soil. The electrolytic conduction  $\sigma_{electrolyte}$  through the pore space forms the other part.

$$\sigma = \sigma_{electrolyte} + \sigma_{surface}$$

where  $\sigma_{electrolyte}$  obeys Archie's equation.

- 2.) The value  $\sigma_{surface}$  is relatively independent of the pore water conductivity  $\sigma_w$ . Hence,  $\sigma_{surface}$  dominates the overall conductivity  $\sigma$  at lower  $\sigma_w$  values. At higher  $\sigma_w$  values, electrolytic conduction dominates and Archie's equation works.



(Figure from de Lima et al., 2005)

- 3.) Using this basic model, Waxman & Smits (1968) came up with the following empirical formula for the electrical conductivity of water-saturated clayey/shaly materials:

$$\sigma = \frac{1}{F}(\sigma_w + BQ_v)$$

$B$  = charge mobility in double layer (empirical parameter)

$Q_v$  = total double layer charge per unit pore volume (which is related to the cation exchange capacity (CEC)).

- 4.) More recently, theoretical analyses of the interfacial conduction phenomenon suggest the following (Ruffet et al., 1995):

$$\sigma_{surface} \cong \frac{\Sigma_s \Lambda}{\tau_s}$$

$\Sigma_s$  = surface conductivity (in S)

$\Lambda$  = specific surface area of the pore space (i.e., pore surface area / pore volume) (in 1/m)

$\tau_s$  = pore surface tortuosity

- 5.) For a partially saturated medium, the Waxman & Smit equation becomes

$$\sigma = \frac{S_w^n}{F} \left( \sigma_w + \frac{BQ_v}{S_w} \right)$$

$Q_v$  is replaced by  $Q_v/S_w$  because same number of double layer charge carriers are concentrated into a less pore water volume.

7.) Temperature Dependence of Electrical Resistivity

- a.) Temperature variations can significantly change the electrical resistivity/conductivity of rocks and soils.
  - 1.) To extract information about other parameters (e.g., clay content, water content/saturation) from resistivity measurements, it is necessary to account for the effects of temperature variations.
  - 2.) This can be accomplished by applying a temperature correction that converts the resistivity/conductivity values obtained at an arbitrary temperature  $T$  to an equivalent resistivity that would be measured at a reference temperature  $T_{ref}$ .
- b.) Temperature effects on the resistivity of aqueous solutions
  - 1.) Temperature affects the mobility of ionic charge carriers in aqueous solutions.
  - 2.) As temperature increases, water viscosity decreases; and the ions become more mobile. This condition leads to a decline in  $\rho_w$  with elevated temperature.
  - 3.) For NaCl solutions, the following relationship can be used to compensate for temperature effects below 100°C (from Arps, 1953):

$$\rho_w(T_2) = \rho_w(T_1) \frac{T_1 + 21.5}{T_2 + 21.5}$$

where  $T_1$  = reference temperature (in °C)

$\rho_w(T_1)$  = water resistivity at temperature  $T_1$  (in  $\Omega \text{ m}$ )

$T_2$  = actual water temperature (in °C)

$\rho_w(T_2)$  = water resistivity at temperature  $T_2$  (in  $\Omega \text{ m}$ )

- c.) Temperature effects on the electrical resistivity of soils.

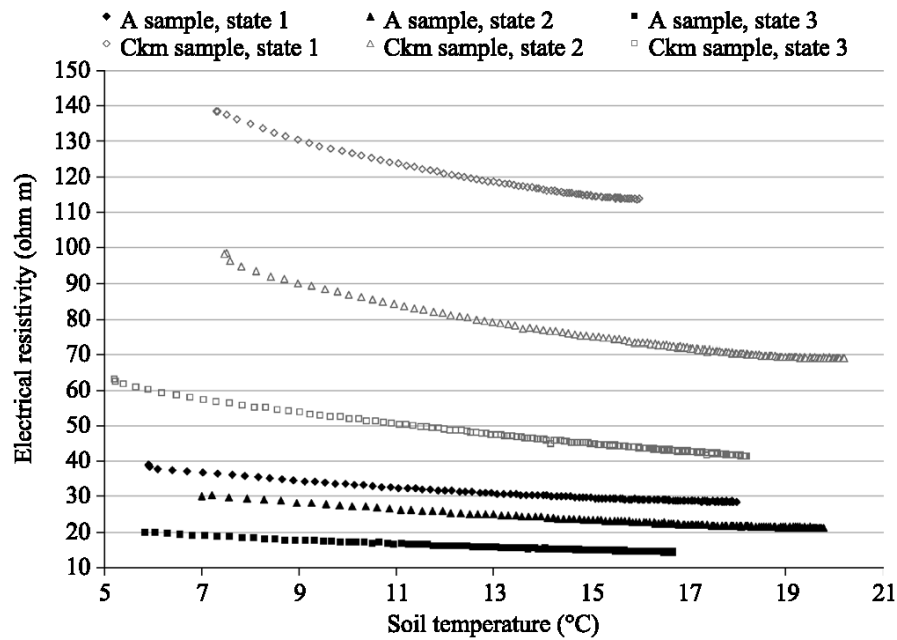


Fig. 2. Evolution of the electrical resistivity versus the temperature. Experimental data.

(Figure from Besson et al. 2008)

- 1.) A number of mathematical relationships have been developed from laboratory data (Ma et al., 2011)
- 2.) In many cases, the following simple linear relationship has been used

$$\sigma(T) = \sigma(T_{ref}) [1 + \delta(T - T_{ref})]$$

where  $\sigma(T)$  = measured conductivity value (in S/m)

at temperature  $T$  (in °C)

and  $\sigma(T_{ref})$  = equivalent conductivity value (in S/m) at the

reference temperature  $T_{ref}$  (in °C)

- 3.) Based on experimental data, values of  $\delta$  for  $T_{ref} = 25^\circ\text{C}$  used in published studies for soils range between 0.017 – 0.023.

## 8.) Relationship between Electrical and Hydraulic Properties

- a.) Both electrical conductivity/resistivity and permeability/hydraulic conductivity are dependent on the characteristics of the pore space (i.e., porosity, tortuosity) in rocks and soils.
- b.) Laboratory results have found good to strong correlation between electrical and hydraulic properties clean (i.e., clay-free) media where electrolytic conduction is the predominate mechanism

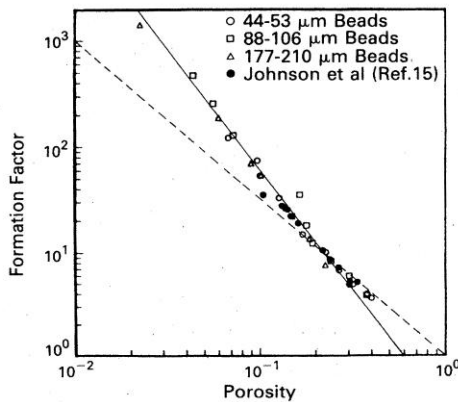


FIG. 3. Formation factor for different fused-glass-beads samples as obtained by resistivity measurements. The dashed line is the prediction of the self-similar model for spherical grains ( $a = 1$ ,  $m = \frac{3}{2}$ ). Data below 20% porosity show substantial deviation from the prediction. They can be approximated by the solid line, which corresponds to  $a = 3.3$  and  $m = 2.3$ .

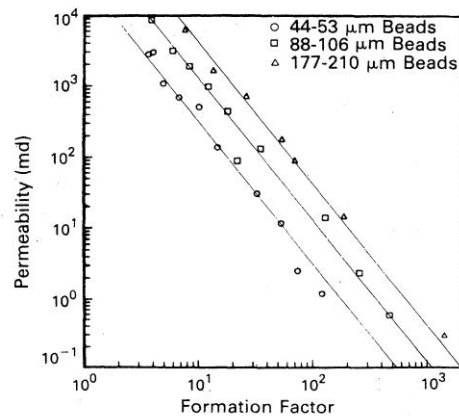


FIG. 5. Log-log plot of permeability vs formation factors for fused-glass-beads samples with different grain sizes. For each grain size, the relationship  $k_r \propto F^{-2}$  is approximately obeyed.

(Figures from Wong et al., 1984)

- c.) In addition, there is a similarity between the two empirical relationships (i.e., Archie's and the Carmen-Kozeny equations) used to describe these two properties.
- d.) Theoretical models based on tube networks prediction a direct mathematical relationship between these two properties.
- e.) The application of this interrelationship between electrical and hydraulic properties to geophysical field measurements is complicated by the other factors that affect electrical properties (e.g., surface conduction, changes in water conductivity).

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