II.) ELECTRICAL RESISTIVTY METHODS

These methods involves the measurement of electric potentials developed in the Earth due an applied direct (DC) or low frequency current (generally less that 100 Hz) using electrodes inserted into the ground. Electrical techniques using electrodes are sometimes referred to as galvanic methods.

A.) Fundamentals for Resistivity Methods

1.) Ohm's Law

a.) Ohm’s Law is the constitutive equation giving the relationship between electrical resistivity/conductivity, electric potential and current.

b.) For a resistor in a circuit,

$$\Delta V = I R$$

R = resistance (in Ω), I = current (in A) and ΔV = potential difference (in V)

c.) For an isotropic continuum, we have a generalized form:

$$\mathbf{E} = \rho\, \mathbf{j} \text{ or } \mathbf{j} = \sigma \mathbf{E}$$

$\mathbf{E}$ = electric field strength (in V/m), $\mathbf{j}$ = current density (in A/m^2),

ρ = electrical resistivity of the medium (in Ωm) and

σ = electrical conductivity of the medium (in S/m).

d.) If V is the electric potential in the continuum, then $\mathbf{E} = -\nabla V$ and

$$\nabla V = -\rho\, \mathbf{j}$$

This is the form commonly used in geophysics. This relationship implies that current flow lines are perpendicular to equipotential surfaces in isotropic media.

2.) Continuity Equation (i.e., Conservation of Electrical Charge)

a.) Consider a representative volume element (RVE) in the continuum.

b.) In general, the continuity equation (i.e., the conservation of electrical charge) describe the current flow in the RVE as

$$\nabla \cdot \mathbf{j} = \Im$$

where $\Im$ (in A m^{-3}) corresponds to either a volumetric charge source (i.e., $\Im > 0$) or sink (i.e., $\Im < 0$) term.

(*i.e., the change in the amount of electrical charge in the RVE* ($\nabla \cdot \mathbf{j}$) *is equal to the net charge that enters/leaves the system within the RVE* ($\Im$))

c.) If there are no current sources or sinks (i.e., electrodes) with this RVE, then the conservation of charge implies that all current flow lines entering this volume must also exit it (i.e., there is no accumulation of electrical charge $\Im = 0$).

d.) In this case, the continuity equation becomes

$$\nabla \cdot \mathbf{j} = 0$$

e.) Using the general version of Ohm's Law, this condition is rewritten as

$$\nabla \cdot \left(\sigma \, \nabla V\right) = \nabla \sigma \cdot \nabla V + \sigma \, \nabla^2 V = 0$$

f.) In a homogeneous medium (i.e., where σ does not spatially vary), this expression becomes Laplace's equation:

$$\nabla^2 V = 0$$

B.) Electric Potential and Current Flow in a Homogenous Earth

1.) A single current electrode

a.) Point electrode

1.) Consider a point current electrode C located on the Earth's surface.

2.) It can be shown (see van Nostrand & Cook, 1966) that the potential due to this electrode at a given point in the Earth is

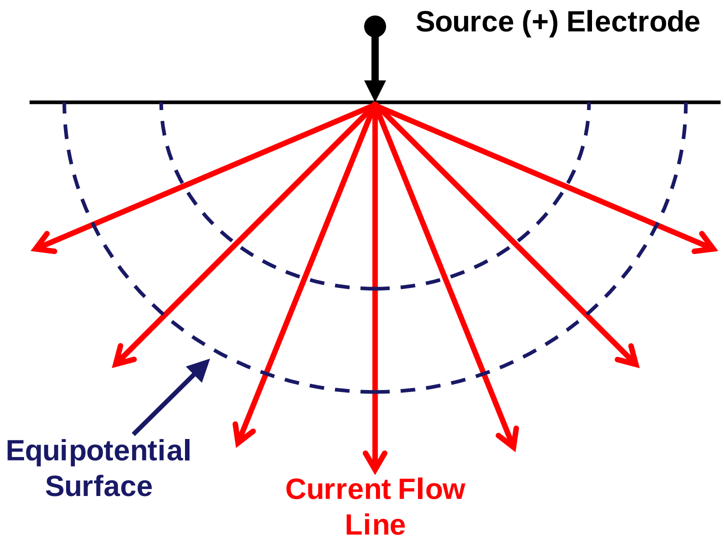
$$V_P = \frac{\rho I}{2\pi r}$$

ρ = resistivity of the Earth

r = distance between point P and electrode C

I = current applied to the electrode

($+I$ for the source/positive electrode and $-I$ for the sink/negative electrode)



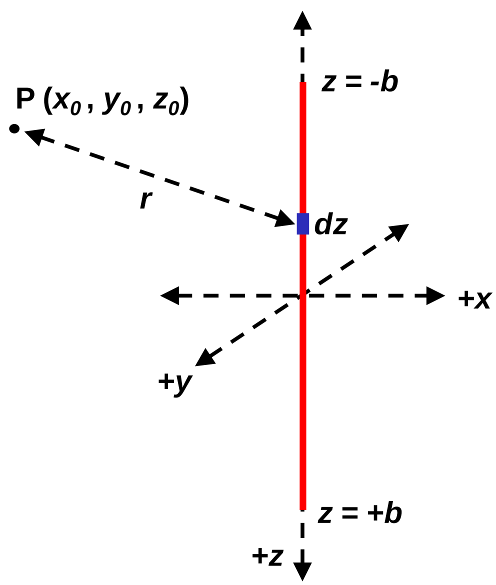
3.) The equipotential surfaces (i.e., loci of points in the Earth with the same V value) are concentric hemispherical shells centered about the electrode. The current flow lines are radial lines.

4.) This result is the basis for determining the response of electrode arrays and the definition of apparent resistivity.

b.) Finite line (i.e., spike) electrode (based on van Nostrand & Cook, 1966)

1.) While point sources are commonly used to represent steel spike electrodes used to acquire resistivity data, a more realistic approximation would be a vertically oriented finite line source.

2.) Consider an infinite homogeneous medium with a finite line source located along the *z* (i.e., vertical) axis between $z = \pm b$.



3.) A total current 2*I* flows from this electrode into the medium. Assume that the same fraction of current emanates from each infinitesimal element *dz* on the line source (i.e., constant current density along the electrode).

4.) Treating each element *dz* as a point source of current $dI = I\,dz/b$, the potential due to this element at a point $\mathrm{P}(x_0, y_0, z_0)$ in the infinite medium is given by

$$dV = \frac{\rho\, dI}{4\pi r} = \frac{\rho I}{4\pi b r} dz$$

where $r = \sqrt{(x_0)^2 + (y_0)^2 + (z - z_0)^2}$ is the distance between the element $dz(0,0,z)$ and the point P.

5.) Integrating along the elements along the line source gives the following result for the potential due to the line source at point P:

$$V_P = \frac{\rho I}{4\pi b} \int_{-b}^{b} \left[(x_0)^2 + (y_0)^2 + (z - z_0)^2 \right]^{-1/2} dz$$

$$= \frac{\rho I}{4\pi b} \ln\left\{ \frac{\left[(x_0)^2 + (y_0)^2 + (b - z_0)^2 \right]^{1/2} + (b - z_0)}{\left[(x_0)^2 + (y_0)^2 + (b + z_0)^2 \right]^{1/2} - (b + z_0)} \right\}$$

6.) The equipotential surfaces from this line source are confocal prolate ellipsoids with foci at $z = \pm b$. The current flow lines follow confocal hyperbolae with the same foci.

7.) It can be argued from symmetry that no current flow lines cross the xy plane (i.e., $z = 0$). Hence, the all of the current *I* flowing from the lower half of the line source remains in the lower half space. Therefore, the electric field and current flow in the lower half space is unaffected by removing the upper half of the line source and considering the upper half space as a medium with infinite resistivity.

Hence, the above expression represents the potential due to a line source electrode of length *b* with an input current *I* inserted into the surface of the Earth.

2.) Two current electrodes

a.) Consider two point current electrodes located on the Earth's surface. One electrode is a source/positive electrode (A) and the other is a sink/negative electrode (B).

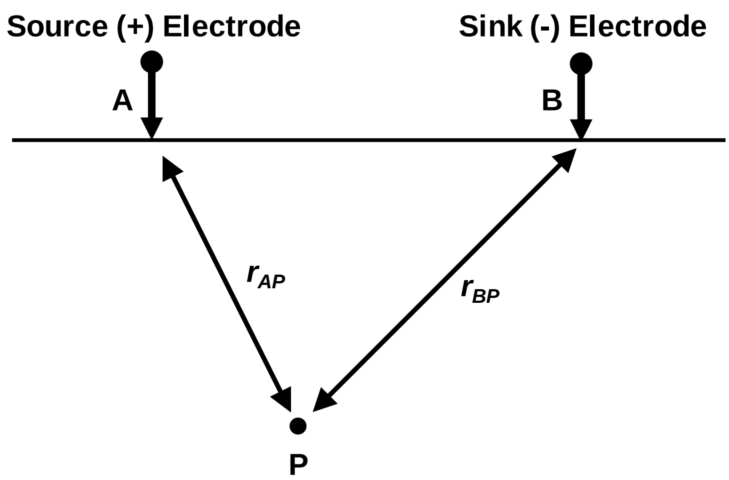
b.) The effects of the two electrodes are additive. The potential at a point P in the Earth (V_P) is given by

$$V_P = V_P^+ + V_P^- = \frac{+\rho I}{2\pi r_{AP}} + \frac{-\rho I}{2\pi r_{BP}}$$

$$= \frac{\rho I}{2\pi} \left(\frac{1}{r_{AP}} - \frac{1}{r_{BP}} \right)$$

r_{AP} = distance between source electrode A and point P

r_{BP} = distance between sink electrode B and point P



(Note, an analogous expression can be obtained for two finite line / spike electrodes.)

c.) This result produces a more complex system of equipotential surfaces and current flow lines.

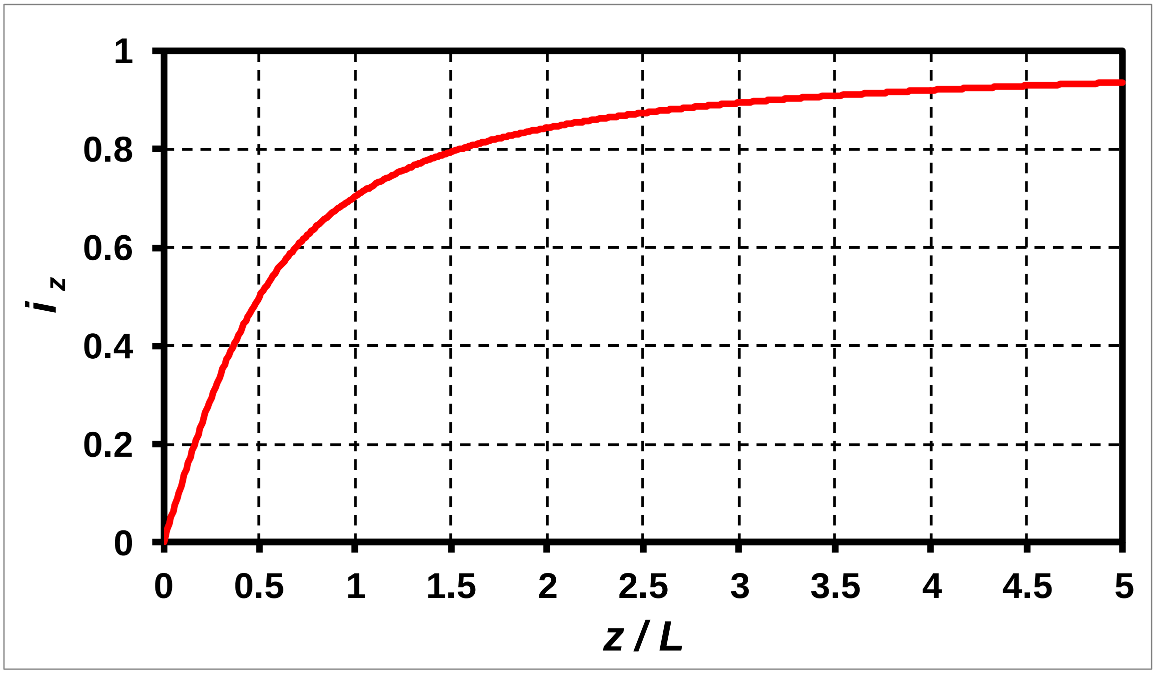
d.) In particular, consider the current flow lines passing through the vertical plane midway between these two electrodes forming the equipotential surface where $V_P = 0$. These are horizontal flow lines parallel to the electrode array orientation. It can be shown (van Nostrand & Cook, 1966) that the fraction of the total current flow that occurs above a given depth z (i.e., i_z):

$$i_z = \frac{2}{\pi}\tan^{-1}\left(\frac{2z}{L}\right)$$ where L = current electrode separation.

1.) Using the expression for i_z, we find that for

$z = 0.5L \;\Rightarrow\; i_z = 0.50$ $\qquad$ $z = 1.0L \;\Rightarrow\; i_z = 0.70$

$z = 2.0L \;\Rightarrow\; i_z = 0.84$ $\qquad$ $z = 3.0L \;\Rightarrow\; i_z = 0.90$



2.) This pattern can be used as a measure of the depth of penetration and sample weighting.

3.) The depth of penetration is increase by increasing the electrode separation *L*.

C.) Electric Potential and Current Flow in an Inhomogeneous Earth

1.) The electric potential and current flow will be affected by spatial variations in resistivity. These effects can be determined solving Laplace's equation with the appropriate boundary conditions. In most cases, this requires considerable mathematical effort or numerical methods. However, the nature of these effects can be seen by examining the case of a planar interface between two media with contrasting values of resistivity.

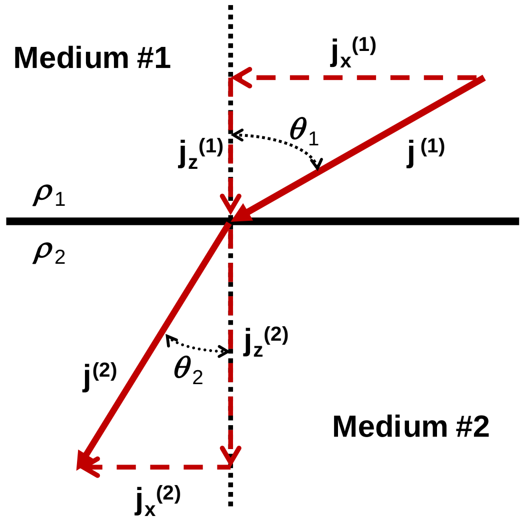
2.) The following two conditions are met at the boundary between two homogeneous material:

a.) Continuity of potential along the boundary: $V_1 = V_2$

b.) Conservation of charge crossing the boundary: $(\mathbf{j}\cdot\mathbf{n})_1 + (\mathbf{j}\cdot\mathbf{n})_2 = 0$ where **n** is the normal to the interface.

3.) Effect of a planar interface on current flow

a.) Consider a horizontal interface between two media with differing ρ.



b.) The current flow across this interface is controlled by the two following conditions.

1.) Conservation of charge carriers across the boundary: $j_{z1} = j_{z2}$

2.) Continuity of potential across the boundary (equivalent to continuity of the horizontal electric field component: $E_{x1} = \rho_1 j_{x1} = \rho_2 j_{x2} = E_{x2}$

c.) Combining these conditions gives $\rho_1 \left(j_{x1}/j_{z1} \right) = \rho_2 \left(j_{x2}/j_{z2} \right) \quad \Rightarrow$

$$\rho_1 \tan\theta_1 = \rho_2 \tan\theta_2$$

This relationship is called the Electrical Law of Refraction.

d.) There is a bending of current flow lines at the interface between two geological units. The sense of the bending depends on the relative magnitude of ρ_1 and ρ_2:

1.) If $\rho_1 > \rho_2$, then $\theta_1 < \theta_2 \quad \Rightarrow$ flow lines bend away from normal.

2.) If $\rho_1 < \rho_2$, then $\theta_1 > \theta_2 \quad \Rightarrow$ flow lines bend towards normal.

e.) This bending determines the relative current densities in each medium.

1.) If $\rho_1 > \rho_2$, then current density is greater in Medium #2 than in Medium #1.

2.) If $\rho_1 < \rho_2$, then current density is less in Medium #2 than in Medium #1.

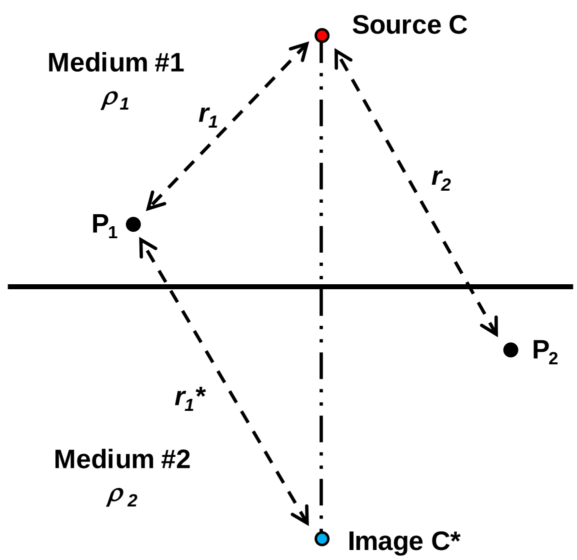
4.) Effect of a planar interface on electric potential

a.) Consider a horizontal interface between two media with differing ρ.

b.) A point source C located in Medium #1.

c.) A simple approach for determining the electric potential at any point in Medium #1 and #2 is called the "*method of images*" which is an analogy to geometrical optics. This analogy is based that both current density and light ray intensity decrease as the inverse square of the distance from the source.

d.) In this case, the interface is treated as a semi-transparent mirror.



1.) For a point P_1 in Medium #1, the potential is given by

$$V_{P1} = \frac{\rho_1 I}{4\pi}\left(\frac{1}{r_1} + \frac{k}{r_1 *}\right)$$

where r_1 and $r_1{}^*$ are the distances between the point P_1 and the real and image sources C and C*, respectively;

and $k = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is the reflection coefficient.

2.) For a point P_2 in Medium #2, the potential is given by

$$V_{P2} = \frac{\rho_2 I}{4\pi}\left(\frac{1-k}{r_2}\right)$$

where r_2 is the distance between the point P_2 and the real source C.

(While its applications are limited, the method of images has been used to examine a variety of problems that arise in the use of resistivity methods, including the response of a horizontally layered Earth. These can be found in van Nostrand & Cook (1966), as well as numerous textbooks.)

D.) Anisotropy

1.) If the measured value of a physical property is dependent on the direction, the medium is anisotropic. (So far, we have assumed isotropic media, meaning that the value of resistivity/conductivity is the same regardless of the measurement direction).

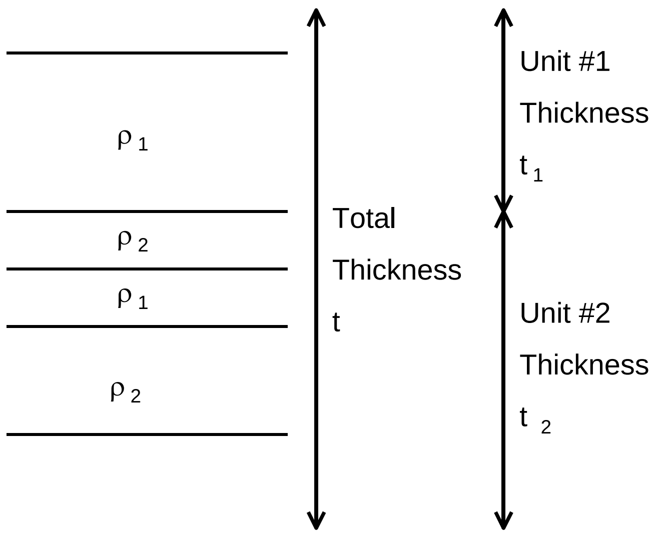
2.) For an anisotropic material, the continuum form of Ohm's Law becomes

$$\mathbf{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{xy} & \rho_{yy} & \rho_{yz} \\ \rho_{xz} & \rho_{yz} & \rho_{zz} \end{bmatrix} \begin{bmatrix} j_x \\ j_y \\ j_z \end{bmatrix} = \boldsymbol{\rho}\ \mathbf{j}$$

where $\boldsymbol{\rho}$ is the resistivity tensor (instead of the scalar previously used).

3.) There are a variety of mechanisms that can produce anisotropy on either the microscopic or macroscopic scale.

4.) A common mechanism for electrical resistivity anisotropy at the macroscopic scale is thinly laminated horizontal bedding.



a.) Consider a horizontally layered material composed of thin layers of two different materials. Each of these components are isotropic with resistivity values ρ_1 and ρ_2.

1.) The resistivity measured in the vertical direction is $\rho_v = \phi_1 \rho_1 + \phi_2 \rho_2$ where $\phi_1 = t_1/t$ and $\phi_2 = t_2/t$ (i.e., the volume fraction of each component with $\phi_1 + \phi_2 = 1$).

2.) The resistivity measured in the horizontal direction is $\rho_h = \left[(\phi_1/\rho_1) + (\phi_2/\rho_2)\right]^{-1}$.

b.) This system can be treated as a transversely isotropic medium with:

1.) Transverse resistivity $\rho_\perp = \rho_v$

2.) Longitudinal resistivity $\rho_\parallel = \rho_h$

3.) Coefficient of anisotropy $\lambda = \sqrt{\rho_\perp/\rho_\parallel}$

4.) Ohm's Law becomes $\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \rho_\parallel & 0 & 0 \\ 0 & \rho_\parallel & 0 \\ 0 & 0 & \rho_\perp \end{bmatrix} \begin{bmatrix} j_x \\ j_y \\ j_z \end{bmatrix}$

5.) The effects of anisotropy on current flow and electrical potential.

a.) Consider a point current electrode located on the surface of a transversely isotropic half space at the origin of the coordinate system.

b.) It can be shown (e.g., Keller & Frischknecht, 1966) that the potential at point P (x_P, y_P, z_P) within the earth is given

$$V_P = \frac{\lambda\left(\rho_{\|}\right)^{3/2} I}{2\pi\left(\rho_{\|}\right)^{1/2}\left[x_P^2 + y_P^2 + \lambda^2 z_P^2\right]^{1/2}}$$

c.) Equipotential surfaces are ellipses revolved about the z (vertical) axis.

d.) From Ohm's Law, it can be seen that the current density $\mathbf{j}$ is not necessarily oriented in the same alignment as the potential gradient ∇V → current flow lines may not be perpendicular to equipotential surfaces.

Note: Other potential sources of macroscopic anisotropy includes fracture and jointing networks with preferred orientation(s) where the spacing is small relative to the electrode spacing.

References:

G. V. Keller & F. C. Frischknecht, 1966. *Electrical Methods in Geophysical Prospecting*, Pergamon Press, Oxford.

R. G. van Nostrand & K. L. Cook, 1966. *Interpretation of Resistivity Data*, U. S. Geological Survey Professional Paper 449.