Additional Lecture and Reading Notes

Lecture 2f

(from Lecture Presentation)
Definition: Let $\mathbb{S}$ be a k-dimensional subspace of $\mathbb{R}^n$ and let $\mathcal{B} = \{\vec{v}_1, \ldots, \vec{v}_k\}$ be an orthogonal basis of $\mathbb{S}$. If $\vec{x}$ is any vector in $\mathbb{R}^n$, the **projection** of $\vec{x}$ onto $\mathbb{S}$ is defined to be

$$\text{proj}_{\mathbb{S}}\vec{x} = \frac{\vec{x} \cdot \vec{v}_1}{||\vec{v}_1||^2}\vec{v}_1 + \frac{\vec{x} \cdot \vec{v}_2}{||\vec{v}_2||^2}\vec{v}_2 + \cdots + \frac{\vec{x} \cdot \vec{v}_k}{||\vec{v}_k||^2}\vec{v}_k$$

Note that this definition only works if $\mathcal{B}$ is an orthogonal basis–we will not consider projections based on arbitrary bases. Also worth noting is that the textbook only defined projections in relation to orthonormal bases. However, it is easier to first find an orthogonal basis and then normalize the vectors to get the corresponding orthonormal basis than it is to be constantly using normal vectors. As such, I have provided the more general definition here (which even the textbook uses in it construction of an orthonormal basis), but you can easily see that if $\mathcal{B}$ were in fact an orthonormal basis, then we would have $||\vec{v}_i||^2 = 1$ for all i, which leads to the definition given in the text.