

# Additional Lecture and Reading Notes

## Lecture 1f

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(from lecture presentation)

**Definition:** Suppose that  $\mathbb{V}$  is a vector space. Then  $\mathbb{U}$  is a subspace of  $\mathbb{V}$  if it satisfies the following three properties:

**S0.**  $\mathbb{U}$  is a non-empty subset of  $\mathbb{V}$

**S1.**  $\mathbf{x} + \mathbf{y} \in \mathbb{U}$  for all  $\mathbf{x}, \mathbf{y} \in \mathbb{U}$  ( $\mathbb{U}$  is closed under addition)

**S2.**  $t\mathbf{x} \in \mathbb{U}$  for all  $\mathbf{x} \in \mathbb{U}$  and  $t \in \mathbb{R}$  ( $\mathbb{U}$  is closed under scalar multiplication)

Note that I have modified the statement of this definition slightly from the one in the text to emphasize the need to verify that  $\mathbb{U}$  is not the empty set, as well as emphasizing the need to show that  $\mathbb{U}$  is actually a subset of  $\mathbb{V}$ .