Solution to Practice 2a

A1(a) Since $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = (1)(2)+(2)(-1) = 0$, the set is orthogonal. Since $\left\| \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\| = \sqrt{1^2+2^2} = \sqrt{5}$, and since $\left\| \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\| = \sqrt{2^2+(-1)^2} = \sqrt{5}$, we see that the corresponding orthonormal set is $\left\{ \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix} \right\}$.

A1(b) Since $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = 2+1+1 = 4 \neq 0$, the set is not orthogonal.

A1(c) We see that

$$\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} = 3+0-3 = 0, \text{ and}$$

$$\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -10 \\ 3 \end{bmatrix} = 1-10+9 = 0, \text{ and}$$

$$\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -10 \\ 3 \end{bmatrix} = 3+0-3 = 0,$$

so the set is orthogonal. We now compute

$$\left\| \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\| = \sqrt{1^2+1^2+3^2} = \sqrt{11}, \text{ and}$$

$$\left\| \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \right\| = \sqrt{3^2+0^2+(-1)^2} = \sqrt{10}, \text{ and}$$

$$\left\| \begin{bmatrix} 1 \\ -10 \\ 3 \end{bmatrix} \right\| = \sqrt{1^2+(-10)^2+3^2} = \sqrt{110},$$

so the corresponding orthonormal set is $\left\{ \begin{bmatrix} 1/\sqrt{11} \\ 1/\sqrt{11} \\ 3/\sqrt{11} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{10} \\ 0 \\ -1/\sqrt{10} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{110} \\ -10/\sqrt{110} \\ 3/\sqrt{110} \end{bmatrix} \right\}$.

A1(d) Since $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} = -2 + 0 + 1 + 0 = -1 \neq 0$, the set is not orthogonal.

B1(a) Since $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -1 \end{bmatrix} = (1)(3) + (3)(-1) = 0$, the set is orthogonal. Since $\left\| \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\| = \sqrt{1^2 + 3^2} = \sqrt{10}$, and since $\left\| \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$, we see that the corresponding orthonormal set is $\left\{ \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{10} \end{bmatrix} \right\}$.

B1(b) We see that

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = 2 + 0 - 2 = 0, \text{ and}$$

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = 4 - 5 + 1 = 0, \text{ and}$$

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = 2 + 0 - 2 = 0,$$

so the set is orthogonal. We now compute

$$\left\| \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}, \text{ and}$$

$$\left\| \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \right\| = \sqrt{1^2 + 0^2 + (-2)^2} = \sqrt{5}, \text{ and}$$

$$\left\| \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \right\| = \sqrt{2^2 + 5^2 + 1^2} = \sqrt{30},$$

so the corresponding orthonormal set is $\left\{ \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{5} \\ 0 \\ -2/\sqrt{5} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{30} \\ 5/\sqrt{30} \\ 1/\sqrt{30} \end{bmatrix} \right\}$.

B1(c) Since $\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = -1+0+0 = -1 \neq 0$, the set is not orthogonal.

B1(d) Since $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} = 2+0+0+1 = 3 \neq 0$, the set is not orthogonal.