

Solution to Practice 3t

B1(a)

$$\begin{aligned}\langle \vec{u}, \vec{v} \rangle &= u_1 \bar{v}_1 + u_2 \bar{v}_2 = (-2 - 3i)(4 + i) + (2 + i)(4 - i) \\ &= -8 - 2i - 12i - 3i^2 + 8 - 2i + 4i - i^2 \\ &= 4 - 12i\end{aligned}$$

$$\begin{aligned}\langle \vec{v}, \vec{u} \rangle &= v_1 \bar{u}_1 + v_2 \bar{u}_2 = (4 - i)(-2 + 3i) + (4 + i)(2 - i) \\ &= -8 + 12i + 2i - 3i^2 + 8 - 4i + 2i - i^2 \\ &= 4 + 12i\end{aligned}$$

$$\|\vec{u}\| = \sqrt{(-2)^2 + (-3)^2 + 2^2 + 1^2} = \sqrt{18} = 3\sqrt{2}$$

$$\|\vec{v}\| = \sqrt{4^2 + (-1)^2 + 4^2 + 1^2} = \sqrt{34}$$

B1(b)

$$\begin{aligned}\langle \vec{u}, \vec{v} \rangle &= u_1 \bar{v}_1 + u_2 \bar{v}_2 = (3 - i)(1 - i) + (1 + 2i)(2 - i) \\ &= 3 - 3i - i + i^2 + 2 - i_4i - 2i^2 \\ &= 6 - i\end{aligned}$$

$$\begin{aligned}\langle \vec{v}, \vec{u} \rangle &= v_1 \bar{u}_1 + v_2 \bar{u}_2 = (1 + i)(3 + i) + (2 + i)(1 - 2i) \\ &= 3 + i + 3i + i^2 + 2 - 4i + i - 2i^2 \\ &= 6 + i\end{aligned}$$

$$\|\vec{u}\| = \sqrt{3^2 + (-1)^2 + 1^2 + 2^2} = \sqrt{15}$$

$$\|\vec{v}\| = \sqrt{1^2 + 1^2 + 2^2 + 1^2} = \sqrt{7}$$

B1(c)

$$\begin{aligned}\langle \vec{u}, \vec{v} \rangle &= u_1 \bar{v}_1 + u_2 \bar{v}_2 = (4 - 3i)(3) + (2 - i)(2i) \\ &= 12 - 9i + 4i - 2i^2 \\ &= 14 - 5i\end{aligned}$$

$$\begin{aligned}\langle \vec{v}, \vec{u} \rangle &= v_1 \bar{u}_1 + v_2 \bar{u}_2 = (3)(4 + 3i) + (-2i)(2 + i) \\ &= 12 + 9i - 4i - 2i^2 \\ &= 14 + 5i\end{aligned}$$

$$\|\vec{u}\| = \sqrt{4^2 + (-3)^2 + 2^2 + (-1)^2} = \sqrt{30} =$$

$$\|\vec{v}\| = \sqrt{3^2 + (-2)^2} = \sqrt{13}$$

B1(d)

$$\begin{aligned}\langle \vec{u}, \vec{v} \rangle &= u_1 \bar{v}_1 + u_2 \bar{v}_2 = (1 + i)(0) + (-1 + i)(0) \\ &= 0\end{aligned}$$

$$\langle \vec{v}, \vec{u} \rangle = v_1 \bar{u}_1 + v_2 \bar{u}_2 = (0)(1-i) + (0)(-1-i) \\ = 0$$

$$||\vec{u}|| = \sqrt{1^1 + 1^1 + (-1)^2 + 1^2} = \sqrt{4} = 2$$

$$||\vec{v}|| = \sqrt{0^2 + 0^2} = 0$$