Solution to Practice 3g

B7(a) First, we need to find a polar form for $1 + \sqrt{3}i$. First we compute $r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$. Now we want θ such that $\cos\theta = 1/2$ and $\sin\theta = \sqrt{3}/2$. We can use $\theta = \pi/3$, and get that a polar form for $1 + \sqrt{3}i$ is $2e^{i\pi/3}$. And this means that $(1+\sqrt{3}i)^4 = 2^4 e^{i4\pi/3} = 16e^{i4\pi/3}$. Back in standard form, this is $16(\cos(4\pi/3) + i\sin(4\pi/3)) = 16(-1/2 + i(-\sqrt{3}/2)) = -8 - 8\sqrt{3}i$. Note that either answer (polar or standard) is acceptable.

B7(b) First, we need to find a polar form for $-2 - 2i$. First we compute $r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = 2\sqrt{2}$. Now we want θ such that $\cos\theta = -2/2\sqrt{2} = -1/\sqrt{2}$ and $\sin\theta = -2/2\sqrt{2} = -1/\sqrt{2}$. We can use $\theta = 5\pi/4$, and get that a polar form for $-2-2i$ is $2\sqrt{2}e^{i5\pi/4}$. And this means that $(-2-2i)^3 = (2\sqrt{2})^3 e^{(3)(i5\pi)/4} = (2^{3/2})^3 e^{i15\pi/4} = 16\sqrt{2}e^{i7\pi/4}$. Back in standard form, this is $16\sqrt{2}(\cos(7\pi/4) + i\sin(7\pi/4)) = 16\sqrt{2}(1/\sqrt{2}2 + i(-1/\sqrt{2})) = 16 - 16i$. Note that either answer (polar or standard) is acceptable.

B7(c) First, we need to find a polar form for $\sqrt{3} - i$. First we compute $r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = 2$. Now we want θ such that $\cos\theta = \sqrt{3}/2$ and $\sin\theta = -1/2$. We can use $\theta = -\pi/6$, and get that a polar form for $\sqrt{3} - i$ is $2e^{-i\pi/6}$. And this means that $(\sqrt{3} - i)^4 = 2^4 e^{-i4\pi/6} = 16e^{-i2\pi/3}$. Back in standard form, this is $16(\cos(-2\pi/3) + i\sin(-2\pi/3)) = 16(-1/2 + i(-\sqrt{3}/2)) = -8 - 8\sqrt{3}i$. Note that either answer (polar or standard) is acceptable. It is also worth noting that this is the same answer we got in B7(a), so we have found that $(\sqrt{3} - i)^4 = (1 + \sqrt{3}i)^4$.

B7(d) First, we need to find a polar form for $-2 + 2\sqrt{3}i$. First we compute $r = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = 4$. Now we want θ such that $\cos\theta = -2/4 = -1/2$ and $\sin\theta = 2\sqrt{3}/4 = \sqrt{3}/2$. We can use $\theta = 2\pi/3$, and get that a polar form for $-2 + 2\sqrt{3}i$ is $4e^{i2\pi/3}$. And this means that $(-2 + 2\sqrt{3}i)^5 = 4^5 e^{(5)(i2\pi)/3} = 1024e^{i10\pi/3} = 1024e^{i4\pi/3}$. Back in standard form, this is $1024(\cos(4\pi/3) + i\sin(4\pi/3)) = 1024(-1/2 + i(-\sqrt{3}/2)) = -512 - 512\sqrt{3}i$. Note that either answer (polar or standard) is acceptable.