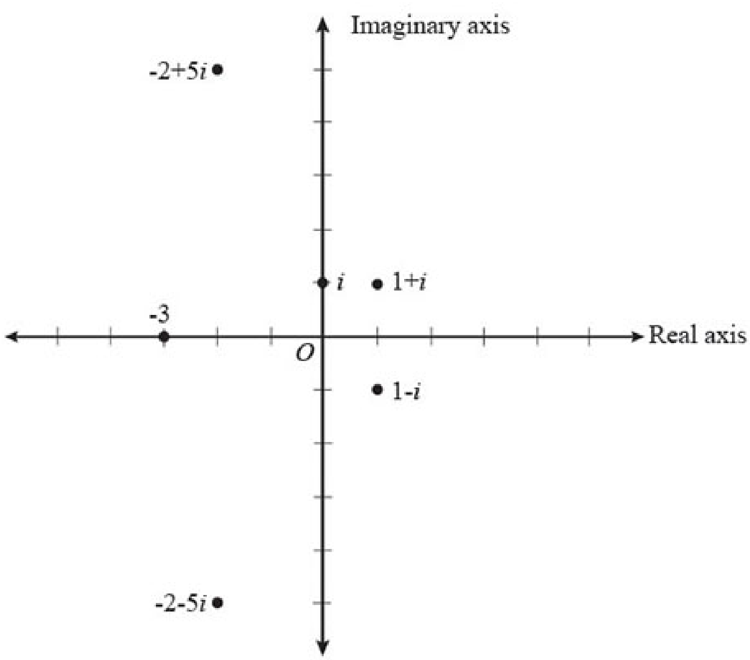
Lecture 3e

The Complex Plane

(page 399)

There are some ways in which the complex numbers seem like $\mathbb{R}^2$. When we add them, it is done in a component-wise fashion, as we add the real parts together and the imaginary parts together, with neither calculation affecting the others. And if we multiply a complex number $x+yi$ by a real number s we get $sx+syi$, which is reminiscent of scalar multiplication. And so it is that when we want to graph complex numbers, we graph them in terms of their two "components", the real part and the imaginary part, as though they were elements of $\mathbb{R}^2$. We label the x_1 axis the "real axis", and the x_2 axis the "imaginary axis", and equate a complex number $x+yi$ with the point (x,y).

Example: We plot the points $1+i$, $\overline{1+i}$, $-2-5i$, $\overline{-2-5i}$, -3 and i as:



Since complex addition is done "componentwise", we still have a parallelogram rule for addition. And multiplication by a real number also works the same as in $\mathbb{R}^2$. But multiplication by a complex number does not have a counterpart in $\mathbb{R}^2$. Starting in the next lecture, we will use this visualization of the complex numbers to develop a new way of thinking about complex numbers, that will provide us with a geometrical interpretation of complex multiplication, and has many other uses as well.

Example: Plot the complex numbers $2+i$, $1+3i$, $(2+i)+(1+3i) = 3+4i$, $3(2+i) = 6+3i$, and $(2+i)(1+3i) = -1+7i$