

Solution to Practice 2d

$$\mathbf{A5(a)} \quad \vec{g}_3 \cdot \vec{g}_1 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} = -2 + 4 - 2 = 0$$

$$\vec{g}_2 = \vec{g}_3 \times \vec{g}_1 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \times \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 + 2 \\ 1 - 4 \\ 4 + 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 6 \end{bmatrix}$$

A5(b)

$$\vec{f}_1 = \frac{\vec{g}_1}{\|\vec{g}_1\|} = \frac{1}{\sqrt{(-1)^2 + 2^2 + 2^2}} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$$\vec{f}_2 = \frac{\vec{g}_2}{\|\vec{g}_2\|} = \frac{1}{\sqrt{(6)^2 + (-3)^2 + 6^2}} \begin{bmatrix} 6 \\ -3 \\ 6 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 6 \\ -3 \\ 6 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$\vec{f}_3 = \frac{\vec{g}_3}{\|\vec{g}_3\|} = \frac{1}{\sqrt{2^2 + 2^2 + (-1)^2}} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

$$\text{So } P = \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$\mathbf{A5(c)} \quad [L]_{\mathcal{B}} = \begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) & 0 \\ \sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

A5(d)

$$\begin{aligned}
[L]_{\mathcal{S}} &= P[L]_{\mathcal{B}}P^{-1} \\
&= \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \\
&= \frac{1}{9\sqrt{2}} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} -3 & 3 & 0 \\ 1 & 1 & 4 \\ 2\sqrt{2} & 2\sqrt{2} & -\sqrt{2} \end{bmatrix} \\
&= \frac{1}{9\sqrt{2}} \begin{bmatrix} 5+4\sqrt{2} & -1+4\sqrt{2} & 8-2\sqrt{2} \\ -7+4\sqrt{2} & 5+4\sqrt{2} & -4-2\sqrt{2} \\ -4-2\sqrt{2} & 8-2\sqrt{2} & 8+\sqrt{2} \end{bmatrix}
\end{aligned}$$