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Kant, Bolzano and the Emergence of Logicism

There are many ways to look at logicism. My aim today is to present one of those versions, embedding it in a broader movement which includes the rigorization of the calculus, Frege's and Russell's theories of arithmetic, and Poincaré's and Hilbert's geometric conventionalism. I shall try to convince you that these episodes belong to an as yet unnamed natural kind which, for lack of a better word, I will call 'conceptualism.'[1]

Conceptualism is defined by an enemy, a goal, and a strategy: the enemy was Kant, the goal was the elimination of pure intuition from scientific knowledge, and the strategy was the creation of semantics as an independent discipline. Although this third component is by far the most important, here I shall have time to discuss in some detail only the other two.

I. THE ENEMY

Semantics, properly so called, was born as a response to problems which were implicit in the (tacit) semantics of rationalism and em-

From *The journal of philosophy*, 74 (1982) 679–89. Reprinted by kind permission of the editors of *The journal of philosophy* and the estate of Alberto Coffa.

[1] 'Conceptualism' is just as inadequate a name as the name of Frege's *Begriffsschrift*.

piricism and which first came to light in Kant's philosophy. In order to contrast these semantic doctrines, the first question we must raise is this: How does one find out what Kant and his predecessors thought about meanings?

Concepts, not essences, is what meanings were before they were wedded to the word. If we want to know what our ancestors thought about meanings we should consult not their books about essences but their logic books, where they talked about concepts and judgments, propositions and meanings. What Kant was saying in that famous remark on the permanent value of Aristotelian logic was not that syllogistics was fine—nothing could have been further from his mind[2]—but that "Aristotelian" (i.e., Wolffian, Lambertian, Crusian, etc.) semantics was good enough for him.

The basic notion of that crypto-semantics is that of representation. Knowledge is, of course, expressed in judgments, and for Kant even experience consists of judgments (Ak 23, 24). But judgments consist of the union or separation of representations or of other judgments (e.g., Ak 16, 631/2, Refl. 3046 & 3049; Ak 9, 101).

Since Christian Wolff the word 'representation' had become a technical term designed to play a role analogous to that of the earlier 'idea' in French and English philosophy. Representations or concepts were conceived in terms of what we might call the "chemical" picture, according to which representations are to be thought of on the model of chemical elements, for they are usually complexes (or, in Kant's words, manifolds) and, like them, are subject to analysis. According to this picture, representations have a number of constituents, themselves representations, which might, in turn, be complex. Since Descartes, clarity and distinctness had become the highest virtues of the ethics of representation, and in this tradition distinctness (*Deutlichkeit*) was defined as the outcome of the process through which we identify all the constituents of a representation. The process itself was called *analysis*, and the complete, distinct version of a given concept, its *definition* [e.g., Ak 24, 571 (25-31)]. The ultimate, simple constituents were the indefinables or unanalyzable simples.

[2] Cf Ak 2, 47–61; Ak 29, 32. 'Ak x, y' will mean *Kant's Academy Edition*, volume x, page(s) y.

Whereas most of Kant's German predecessors had identified representation and concept, Kant departed from this doctrine in his *Dissertation* of 1770, where he drew a sharp distinction between two different faculties of representation: the sensible and the intelligible. Soon he came to think there could be no individual concepts. Concepts, he said, are the product of the understanding to which individual objects are never given. Objects are given to human beings only in sensibility, and the representations of sensibility therefore deserve a different name; Kant's choice was 'intuition.' Thus he concluded that representations are of two radically different sorts: concepts and intuitions.

In spite of these innovations Kant remained firmly committed to the traditional chemical picture of representations. Wolff had praised "the great use of magnifying glasses toward gaining distinct notions."[3] Following this lead, Kant explained in his logic classes that, when we look at the Milky Way with our bare eyes, we have a clear but indistinct representation of it, since we see clearly not each star but only a continuous band of light. When we look through a telescope, however, our representation becomes distinct. The same is true of concepts, which usually have constituents (*Bestandstuecke*, *constitutiva*; Ak 8, 229; Ak 24, 753; Ak 11, 34) of which we are not clearly aware. Conceptual analysis is the intellectual telescope with which we come to see, e.g., that the concept of Freedom is contained in the concept of Virtue (Ak 24, 511/2).

As we all know, Kant defined an analytic judgment as one whose predicate concept is "tacitly thought" in the subject concept. One might think that it is a very small step from the chemical picture of the concept to this idea and to its complement, that of a synthetic judgment. Kant did not think so. He was more than willing to grant that many philosophers had recognized the division between the a priori and the a posteriori, but he insisted that no one before had seen the significance of the analytic-synthetic distinction. When Eberhard challenged Kant's originality and said, in effect, that his distinction was old hat, Kant was furious. In an effort toward irony he replied that everything new in science to which nothing can be opposed is eventually "discovered to have been known by the An-

[3] *Logic* (*deutsche Logik*), ch. I, sec. XXII.

cients" (Ak 8, 243).

Why didn't Kant think that his distinction was an utterly trivial consequence of the familiar notion of conceptual analysis? The answer I should like to propose is this: when Kant conjoined his notion of synthetic judgment with his casual understanding of semantics—specifically, of concepts—the outcome was an extremely powerful and truly original philosophical thesis, the "principle of synthetic judgments." The most striking proof of the power of this principle lies in the fact that, when conjoined with reasonable assumptions, it implies the existence of a nonempirical kind of intuition. Let us see how.

In the first *Critique* Kant had explained that, just as all analytic judgments are grounded on a single principle, that of identity, or, as he also called it, "the principle of analytic judgments," there is also a single principle involved in the grounding of synthetic judgments; and he sensibly called it "the principle of synthetic judgments." When Eberhard complained that no reader of the *Critique* could tell what the principle in question was, Kant obligingly formulated it once again for him as follows: synthetic judgments "are only possible under the condition that an intuition underlies the concept of their subject" (Ak 8, 241).[4] In fact, he had already explained in the *Critique* that the synthesis of two disjoint concepts must always be mediated by a third representation,[5] an X (A9, B13) not directly present in the judgment (as a constituent). Since he also thought that "from mere concepts only analytic knowledge ... can be derived" (A47, B64; see also A155, B194), this third representation couldn't be a concept; but, since representations are either concepts or intuitions, the ground of the synthesis in a synthetic judgment must lie in intuition. This proves the principle of synthetic judgments. If we now add the assumption that there are synthetic a priori judgments, we can infer that there *is* pure intuition. For, qua synthetic, such judgments must be grounded in intuition, and, qua a priori, they must be grounded in an intuition entirely unlike the empirical sort recognized by empiricists, since it must have the power to confer universality and necessity upon the corresponding judgments.

[4] Kant's formulation of the principle in A158, B197, in that inimitable style of his, may have contributed to the widespread neglect of the import of this principle. Yet his remarks immediately preceding that impenetrable formulation and elsewhere in the *Critique* (e.g., A155, B194; B289) as well as statements in *Prolegomena* and in the reply to Eberhard (e.g., Ak 8, 239/40) should have sufficed to make his unimpressive message clear.

[5] Kant also calls it "das dritte der Anschauung"; see *Textemendationen* to A259 in Ak 23, 49.

II. THE GOAL

I can now be more specific about my proposal. I am inviting you to look at a number of nineteenth-century developments as stages in a process that showed Kant wrong in thinking that his pure intuition plays any role in science. That it plays no role in empirical science was not so hard to see. It was much harder to show that it plays no role either in the calculus or in arithmetic, or—hardest of all—that it plays no role even in geometry. Logicism, as I see it, is one stage in that complex process through which it was finally established that conceptual representations suffice for the construction of every pure a priori science.

One is tempted to compare Kant's discovery of pure intuition as it emerged from the argument above with Leverrier's discovery of Vulcan; but the comparison is unfair to Kant in that there were mathematical "data" that could well be interpreted as justifying Kant's conclusion. During the eighteenth century mathematics and in particular its most productive branch, the calculus, appeared to involve an essential appeal to spatial and temporal intuitions. Newton had presented functions as concerning the motion of points in time, and arithmetic was routinely described as involving processes such as counting. Thus, anyone trying to show Kant wrong in his conclusion that mathematics requires an appeal to pure intuition had two rather considerable tasks to perform, one in philosophy and the other in mathematics. The first was to determine what was wrong with Kant's "deduction" of pure intuition, the second, to show that one can actually construct mathematics in a way that by-passes intuition. It isn't often noticed that a single person initiated the developments that led to the fulfillment of both tasks; his name was Bernard Bolzano.

Let me examine first Bolzano's analysis of Kant's "deduction" of pure intuition. As we saw, the argument depended on two premises:

the assumption that there is synthetic a priori knowledge, and the principle of synthetic judgments. Those determined to reject Kant's pure intuition had to deny one of those premises.

It is widely thought that one of the common features of the anti-Kantian analytic tradition is its rejection of synthetic a priori knowledge. On the contrary, virtually all the members of the semantic tradition we are considering here, from Bolzano, Frege, and Russell to the leading members of the Vienna Circle, recognized the existence and decisive significance of synthetic a priori judgments in Kant's definitional sense. It was not until Quine started The Long March back to J. S. Mill that the analytic tradition seriously questioned the possibility of necessary knowledge that is not analytic in Kant's sense. The challenge emerging from the conceptualist movement against Kantian dogma focused on the *other* premise, the principle of synthetic judgments. Wherever the members of this movement may have stood vis-à-vis the principle of synthetic judgments and whatever they may have said or implied about it, my point is this: that everything these people did concerning foundational issues which was of lasting value, tended to undermine Kant's principle.

Bolzano was the first to recognize the fallacy behind the principle of synthetic judgments. The crucial step in Kant's inference for the need to appeal to intuition in synthetic judgments was the premise that from concepts alone only analytic knowledge can be derived. Astonishingly, there isn't a single argument in the *Critique* for this claim; all Kant says about it is that "it is evident" (A47, B64).[6] What is evident, instead, is that Kant had confused *true in virtue of concepts* with *true in virtue of definitions*, or, in his own language, he had erroneously identified judgments whose predicate is not contained in their subject-concept with judgments that extend our knowledge (*Erweiterungsurteile*). Against this, Bolzano was the first to make a point that even Frege would miss: that Kant's analytic judgments, far from exhausting the grounding power of the conceptual resources of our language, mobilize only a very modest fraction of them, the logical concepts. Bolzano's characterization of analyticity is well known, and it has often been noted that it anticipates not Frege's proof-theoretic treatment but the more modern

[6]For a very modest effort toward an argument, see Ak 20, 400.

semantic approach by means of interpretations. What is less well known is the reasoning that led Bolzano to this proposal. After reviewing a number of attempts to explain the point of Kant's notion of analyticity, Bolzano comments that "none of these explanations singles out what makes these [analytic] propositions important. I believe that this consists in the fact that their truth or falsity does not depend upon their constituent representations but remains unaltered, whatever changes one may make in some of these representations .... This is the ground of my preceding definition."[7] Thus, the *reason* why Bolzano came to his celebrated insight on the semantic characterization of logical truth is that he saw that Kant's analytic judgments, far from being those grounded on the information implicit in the constituent concepts, were grounded on only a few of those concepts, thus concluding that a proper definition of analyticity should emphasize the extent to which all other concepts are to be *ignored*.

Having been led by a confused semantics to think that all conceptual information available in a judgment is to be used up in the grounding of analytic judgments, Kant was naturally led to appeal to intuition to ground the rest. Bolzano, on the other hand, having recognized that analytic judgments are judgments grounded on a handful of concepts, was prepared to explore the possibility that *all* of our pure a priori knowledge—including synthetic a priori knowledge—could be stated and grounded on concepts alone. And he proceeded to put his ideas to the test in the field of mathematics. The development known as the rigorization of the calculus is often presented as the first, longest leg of a reductionist program, and it is certainly that. But there is another side to that development which becomes apparent when we look at matters from our present perspective and which is less easy to accommodate within the now fashionable methodologies.

During the eighteenth century the calculus was a Kuhnian's paradise: lots of wonderful exemplars fuelled a staggeringly successful puzzle-solving tradition. The growing ranks of practitioners shared several basic symbolic generalizations, formulas expressing equations and inequalities, which were fruitfully applied to all sorts of prob-

[7] *Wissenschaftslehre*, Hamburg: Meiner, 1929, Vol. II, § 148, p. 88.

lems. To make matters even better, no one really knew what exactly was going on. Even though there was wide agreement on which symbolic generalizations one should assent to when prompted by appropriate stimuli, most people had their own strange interpretation of what these formulas said, and most or all of these interpretations made only modest sense. Moreover, as Kuhn might have predicted, it was a philosopher, the good Bishop Berkeley, who decided to issue a public complaint (in *The analyst*) about the incoherence of the whole enterprise, and specifically about the fact that no meanings were attached to the symbols most crucial to the calculus. (As a "good" philosopher, he wanted definitions, he was worried about "essences," he was worried about meaning rather than theory, etc. etc. In short, he violated every rule in the book of post-positivistic methodology.) As Kuhn would also have predicted, Berkeley's complaints did not stop anyone from following the proto-Kuhnian advice attributed to d'Alembert: "Allez en avant et la foi vous viendra!"

But around 1800 Kuhnian predictions started to go awry. For, this time, mathematicians themselves started to talk about meaning, to try to figure out what exactly was the meaning of each of the basic expressions of the calculus, the meaning of continuity, differentiability, infinitesimal, function, and so on. The ultimate purpose of this emerging project was not to produce some new theorems or to solve some new puzzles but to figure out what exactly it is that the calculus *says*, what its content is, what information it conveys. The question was clearly a semantical question. It is therefore not entirely surprising that the person who took semantics out of the swamp in which it had been sinking since Descartes was also the one who took the first decisive step in the philosophico-mathematical project known as the rigorization of the calculus. That first step was taken in Bolzano's paper of 1817 on the intermediate-value theorem. It is worth pausing for a moment to consider what Bolzano did in that paper.

Bolzano's problem was to prove that a continuous real function that takes values above and below zero, must also take a zero value somewhere in between. A Kantian would probably regard this as trivial: if the point whose path we are considering is moving from the negative to the positive quadrant and the path is continuous it must

surely intersect the x axis at some point.[8] Bolzano's problem looks like a problem only to someone who has already understood that intuition is not an indispensable aid to mathematical knowledge, but rather a cancer that has to be extirpated in order to make mathematical progress possible.

Bolzano began by defining continuity in the now standard manner, in terms of epsilons and deltas (not his notation); he then defined convergence for series and gave the criterion now wrongly attributed to Cauchy; then he stated and proved the Bolzano-Weierstrass theorem (modulo a theory of real numbers) and, on that basis, he finally proved the intermediate-value theorem. It was the first time anyone had introduced these concepts and proofs. If Kant had known about Bolzano's paper there can be little doubt that he would have regarded it as a philosophically incoherent effort to prove the obvious. The paper was, instead, one of the landmarks of nineteenth-century mathematics.

Bolzano made many more brilliant contributions to this project of conceptualizing analysis—most of them ignored; but other mathematicians, including Cauchy, Weierstrass, Dedekind, and Cantor, eventually carried the project to completion, establishing that there need be in mathematics no more intuition than there is in arithmetic.

But, how much intuition *is* there in arithmetic? This is the question Frege raised about 1880.

Like Bolzano's, Frege's project started with the decision to take meanings, content seriously. Like Bolzano's, the semantic project of *Begriffsschrift* was put to the test in a theory of mathematics, which is sketched in *Grundlagen*. The most fundamental result of *Grundlagen*, as Frege explained it years later,[9] was his treatment of what he had called "number-statements" (*Zahlangaben*), statements in which we say that there is a certain number of objects of a certain sort. Frege wondered what these statements were about: About counting? About putting things together in the synthetic unity of apperception? About multitudes? The answer was no, in each case. Frege's own verdict, as you recall, is that they are all about

[8] For a Kantian "proof" of a theorem in the calculus, see Ak 2, 400.
[9] In *Grundgesetze*, Hildesheim: Olms, 1962, p. ix.

logical concepts, that is, in virtue of the meanings of logical words, and that, just as the meanings of logical words can ground claims, so can the meanings of all other words.

By the end of this process the death of pure intuition had left only concepts in charge of the job of grounding pure a priori knowledge, either through explicit definition, as Frege wanted, or by the more complex methods displayed by Poincaré and Hilbert. When concepts were finally wedded to the word, a priori knowledge turned from true in virtue of concepts to true in virtue of meanings—as Carnap put it—or true *ex vi terminorum*—as Wilfrid Sellars puts it. Semantics, finally freed from its psychologistic fetters, had produced a coherent, appealing theory that explained a priori and necessary knowledge in a way no longer subject to the idealist temptation.

And then came Quine.

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2

Frege: The Last Logicist

When I was young I was taught a number of fundamental propositions: Frege was the father of logicism—he showed that arithmetic was really only logic (ingeniously disguised), and consequently that it was really analytic, which was really why it was a priori, all of which showed where Kant had gone wrong about arithmetic, and probably about the rest of the alleged synthetic a priori as well.

I was told too that Frege had invented the logic that arithmetic was really only—or at the very least that he was the father of modern logic. Had I stopped to think, it might have occurred to me to question this all too happy coincidence of discovery and invention. At the very least, a decent interval should have been allowed to

From *Midwest studies in philosophy VI*, Peter French et al., eds., Minneapolis: University of Minnesota Press, 1981, 17–35. Reprinted by kind permission of the author.

The first version of this paper was prepared for and delivered to the Chapel Hill Colloquium at the University of North Carolina in October 1976. It also served as the basis for two seminars offered by the author at the University of Minnesota at Morris in February 1980. I am particularly indebted to Steve Wagner, Fabrizio Mondadori, Glenn Kessler, Ian Hacking, David Kaplan, Jim Van Aken, and Hide Ishiguro for their helpful comments on earlier drafts. The final draft was completed while the author was a Fellow at the Center for Advanced Study in the Behavioral Sciences. The support of the Center, the Sloan Foundation, the National Endowment for the Humanities, and Princeton University are gratefully acknowledged.