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Revolutionary Fictionalism: A Call to Arms[†]

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This paper responds to John Burgess's 'Mathematics and *Bleak House*'. While Burgess's rejection of hermeneutic fictionalism is accepted, it is argued that his two main attacks on revolutionary fictionalism fail to meet their target. Firstly, 'philosophical modesty' should not prevent philosophers from questioning the truth of claims made within successful practices, provided that the utility of those practices as they stand can be explained. Secondly, Carnapian scepticism concerning the meaningfulness of *metaphysical* existence claims has no force against a *naturalized* version of fictionalism, according to which our ordinary standards of scientific evidence may show that we have no reason to believe the mathematical existence claims made within the context of our mathematical and scientific theories.

In 'Mathematics and *Bleak House*', John P. Burgess [2004] argues that any defence of fictionalism regarding mathematics ought to be of the revolutionary kind rather than the hermeneutic kind. According to revolutionary fictionalism, while mathematicians may well believe their mathematical assertions, they ought not, or need not, do so. In contrast, hermeneutic fictionalism holds that mathematicians do, in general, only mean their mathematical assertions in a fictional/non-literal sense. Burgess rightly rejects hermeneutic fictionalism on the grounds that it is not supported by the evidence of mathematical practice. In the absence of explicit denials, we ought to interpret at least some mathematicians as meaning their mathematical assertions to be understood literally.

Burgess goes on to attack revolutionary fictionalism on two grounds. First of all, he claims that it is laughably immodest to hold any philosophical view according to which the majority of successful practitioners of a discipline are involved in making systematic and repeated mistakes. However, such arguments from 'philosophical modesty' have no force against revolutionary fictionalism, as I will show in part one of this response. We have no reason to hold back from attributing even systematic error to practitioners of a discipline so long as we can account for the usefulness of

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such error. If we can do so, we may even defend the continued practice of the error-prone discourse. ‘Revolutionary’ fictionalism need not advocate a revolution in practice, only in our understanding of that practice.

Burgess’s second reason for rejecting revolutionary fictionalism is more nuanced, proceeding via a Carnapian argument against the empirical significance of the revolutionary fictionalist’s hypothesis. The revolutionary fictionalist holds that the best understanding of mathematical assertions is as merely fictional. Against this, Burgess argues that the question of whether the objects asserted to exist within a given linguistic framework ‘capital-R Really’ exist is, if not as Carnap thought strictly meaningless, at least lacking in empirical meaning. There are strong reasons for being suspicious that the metaphysical ‘Reality’ or ‘un-Reality’ of numbers would make no difference to our experience of the world. Burgess contrasts this situation with the situation we have in the case of more straightforward debates over whether a given story is fact or fiction. In these cases, in contrast with the mathematical case, the debates are given empirical meaning since we do have some sense of what the world would have to be like in order for the stories in question to be really true.

It is curious, though, that Burgess uses this argument *against* revolutionary fictionalism, since it is precisely this observation that provides some of the force to recent fictionalist rejections of the indispensability argument *for* the existence of mathematical objects. Against Quine’s claim that we are committed to the existence of mathematical objects because of their role in our empirically supported scientific theories, recent fictionalists have noted that it is not at all clear how empirical evidence could bear on the question of the existence of mathematical objects. In part two of this paper I will consider briefly how this point can be used to reject mathematical realism, and therefore to defend fictionalism.

Given such a defence of fictionalism, revolutionary fictionalism is thus seen not as a metaphysical position concerning the rejection of capital-R Realism (realism of a metaphysical bent that is somehow meant to be defended or rejected based on non-empirical considerations). Rather, the fictionalist’s target is a more common-or-garden realism of the Quinean sort concerning mathematical entities, which proceeds from consideration of the commitments of our scientific theories. As a result, revolutionary fictionalism, properly understood as a view about the correct understanding of our scientific theories, remains a plausible response to the Quinean defence of realism with a small ‘r’.

1. Revolutionary Fictionalism and Philosophical Modesty

According to hermeneutic fictionalism, when mathematicians and scientists make assertions that apparently commit them to belief in the existence of mathematical objects, they should in fact be understood as meaning only to make such assertions within the context of some fiction. Thus, although they might say ‘Three is a prime number’, their commitment to that assertion is not as a literal truth, but rather as a fictionally correct (but literally false) claim, on a par with ‘Holmes is a detective’.[1] Burgess is quite rightly suspicious of the hermeneutic fictionalists’ claim that, in casting mathematical assertions as on a par with fictional assertions, they have uncovered what mathematicians meant all along. While some mathematicians, when pushed, *will* explicitly defend a fictionalist interpretation of their mathematical assertions, many mathematicians are not immediately drawn to such an interpretation.

Indeed, as Burgess notes, many mathematicians would not be sure what to make of the question of what it is they ‘really mean’ when they make their mathematical assertions. Others will stand firm to a *literal* interpretation of their mathematical existence claims. If one hopes to defend a fictionalist approach to mathematics on the grounds that mathematicians, when pushed to elaborate on what they mean by their mathematical assertions, are themselves exposed as fictionalists, one is fighting a losing battle. There is at least as much diversity within the mathematical community as within the philosophical community regarding the correct *metaphysical* interpretation of the assertions of accepted mathematical theories.

Given the diversity of views among mathematicians regarding the interpretation of their mathematical assertions, Burgess is surely right to question the plausibility of the hermeneutic fictionalists’ claim that mathematicians are really fictionalists at heart. It is odd, then, that Burgess himself goes on to impose yet another interpretation on the mathematicians’ claims. According to Burgess, our default interpretation of assertions within a discourse must be that they are intended as assertions of literal truth, unless it is explicitly stated otherwise. As Burgess puts it, ‘There is a *presumption* that people mean and believe what they say’ (Burgess [2004], p. 26). As a result, despite the diversity of views among mathematicians regarding how to interpret their mathematical assertions, Burgess proposes to interpret these assertions literally.

It is not clear, however, that the most sympathetic interpretation of the assertions of mathematicians when doing mathematics is to take them as believing what they say. The widespread confusion regarding the interpretation of the claims of accepted mathematical theories, along with the refusal of many mathematicians to entertain questions regarding the interpretation of their assertions, suggests, on the contrary, that in many cases mathematicians *do not know* what they mean by their mathematical

[1] To take an example from Sarah Hoffman’s paper in the same issue of *Philosophia Mathematica* (Hoffman [2004]).

assertions.[2] And even in cases where mathematicians are clear about what they mean by their assertions, many such mathematicians still don't know whether to believe what they say. (One is reminded of Bertrand Russell's assertion that 'Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true' (Russell [1901], p. 84)[3].

While Burgess is right, then, to reject hermeneutic fictionalism, on the grounds that it rests on an implausible claim regarding what mathematicians 'really' mean, he makes an analogous mistake in insisting on a literal interpretation of mathematical discourse. Indeed, we might label Burgess's alternative view 'hermeneutic literalism', and wonder whether this view fares any better than hermeneutic fictionalism as an account of what mathematicians really mean.

Having agreed with Burgess in rejecting hermeneutic *fictionalism* though, the alternative 'fictionalist' position we are left with is the view Burgess labels 'revolutionary fictionalism'. Burgess characterizes revolutionary fictionalism against a hermeneutic literalist background. While the default interpretation of mathematicians is, for Burgess, a literal one, the revolutionary fictionalist argues that mathematicians ought to abandon this literalism and instead make their mathematical assertions only within the context of some fiction. Clearly, given our worries about Burgess's insistence on a literal interpretation of mathematicians, this definition of 'revolutionary' fictionalism is somewhat questionable. However, the spirit of revolutionary fictionalism can be retained if we understand revolutionary fictionalism as holding that, *regardless* of what mathematicians actually mean by their assertions, the best interpretation of the assertions of mathematical theories is as literally false, and at most true only in some fictional sense.[4]

Whether or not Burgess is right that the default interpretation of the mathematical assertions of mathematicians is a literal one, revolutionary fictionalism remains a revolutionary theory. The revolutionary fictionalist

[2] This is not to denigrate mathematicians. Coherent metaphysical views are certainly not necessary for the practice of good mathematics, and may even be a hindrance to practising mathematicians.

[3] Alexander Paseau considers in more detail the issue of uncovering a default interpretation of mathematical assertions in his paper 'Does mathematics have a given interpretation?' [forthcoming] where he concludes that no default (given) interpretation of mathematical assertions can be assumed.

[4] In holding that mathematical assertions might be 'fictionally' true, the fictionalist should not be thought of as introducing some other species of truth. 'Fictional' truths are not truths at all, but are rather assertions that are correct within the context of the fiction. Mathematical assertions are correct if they follow (in some suitably rich sense) from the assumptions of mathematical theories. But merely following from assumptions does not amount to truth of any sort. Perhaps, then, the fictionalist ought to shun talk of truth altogether, and talk instead of mathematical assertions as being correct or incorrect.

does deny 'while doing philosophy what is asserted while doing mathematics' (Burgess [2004], p. 28), and this is revolutionary even if there is scope for interpreting *some* mathematicians as never having meant their mathematical assertions in the first place, or as not understanding what it is that they have asserted. Whether or not they take themselves to be asserting truths when they are doing mathematics, mathematicians do in fact regularly make statements that, taken at face value, assert the existence of mathematical objects. In denying the existence of such objects, the revolutionary fictionalist denies the truth of many claims made (wittingly or unwittingly) by mathematicians when doing mathematics. The revolutionary fictionalist claims that fictionalism is the correct attitude to mathematical assertions, regardless of whether any mathematician actually holds this attitude, and *even if* most or all mathematicians explicitly denied that they mean their assertions only fictionally.

Burgess's first response to this revolutionary form of fictionalism is incredulity at the gall of its proponents for daring to question the truth of assertions regularly made by mathematicians and scientists regarding the existence of mathematical objects. From Burgess's perspective, mathematicians and scientists are the big shots. Theirs are the successful disciplines—they are the ones whom we would trust to tell us that bridges will stay up, and that planes will not fall out of the sky when the wind blows the wrong way. Surely what they say goes? The spectacle of the lowly philosopher pulling on the coat-tails of these titans in order to let them know, 'Well, I hear what you're saying, but I'm not sure whether what you're saying is true' would be enough, suggests Burgess, to move anybody to laughter.

In place of the revolutionary fictionalist's outrageous philosophical arrogance, Burgess advocates a more modest approach. Wary of the many historical failures of philosophy, he suggests that

> on simple inductive grounds it seems extremely unlikely that philosophy can do better than mathematics in determining what mathematical entities exist, or what mathematical theorems are true, and much more likely that for the $(n+1)^{\text{st}}$ time, philosophy has got the nature of truth and existence wrong. (Burgess [2004], p. 30)

Burgess is not alone here—such a degree of modesty has been characteristic of many recent so-called 'naturalist' approaches to mathematics and science. Aside from David Lewis, to whom Burgess alludes as the spiritual leader of the modest philosophers' camp, a similar modesty stands behind Penelope Maddy's assertion that, 'if philosophy conflicts with successful practice, the philosophy must give' ([1997], p. 171). Indeed, Maddy herself suggests the label 'philosophical modesty' as an alternative to 'naturalism' when describing her own position (Maddy [1997], p. 161).

It is worth noting that our scepticism about literalism as the default interpretation of mathematicians weakens Burgess's position here, since it is not entirely clear that the revolutionary fictionalist is denying anything that mathematicians and scientists have sincerely asserted. But let us suppose for now that Burgess is right that revolutionary fictionalism arrogantly denies claims that are seriously asserted by many mathematicians. Is Burgess right to be moved to laughter at the thought of such denials?

Burgess's response to revolutionary fictionalism rests on an appeal to 'philosophical modesty'. But what is the argumentative force of such an appeal? If Burgess's argument is simply that we should never trust the conclusions of philosophical arguments, since philosophers have been proved wrong many times before, then it looks like we should despair and give up philosophy altogether (although we might wonder how it is, except through philosophical argument, that we can prove previous philosophers wrong). However, looking more closely at the arguments of Burgess, Lewis, and Maddy, it appears that such extreme scepticism about the value of philosophy is not intended. Rather, Burgess, Lewis, and Maddy all stress that philosophy should defer to other more successful disciplines just on those occasions where a conflict occurs. Lewis and Maddy, in particular, seem to advocate modesty in cases where philosophical argument would otherwise lead philosophers to suggest a revision of successful *practices*. Thus Lewis famously states that he is 'moved to laughter' at the thought of '*telling the mathematicians that they must change their ways* . . . now that philosophy has discovered that there are no classes' (Lewis [1991], p. 59, my italics).

Certainly, there is something right in Lewis's thought that we should think twice before demanding that mathematicians stop what they're doing as a result of our philosophical scruples. As philosophers, part of the reason why we are interested in mathematics in the first place is because it is a successful discipline. Should we not, then, be concerned if philosophical theorizing leads us to the conclusion that mathematics is, in fact, a wrongheaded pursuit that should be abandoned? At any rate, as Lewis, Burgess, and Maddy are quick to note, we stand little chance in persuading mathematicians and scientists to change their ways and abandon their use of mathematics. Indeed, if we did manage to convince them to abandon all talk of mathematical objects, it is not at all clear what would be left of those disciplines.

There is something to be said, then, for a degree of modesty in providing philosophical accounts of successful enterprises. If our philosophical account of a successful practice suggests that practice to be wrongheaded, even though it is successful, it is right to wonder whether it is our philosophical argument that has gone awry, and not the practice we are criticizing. It is unclear, though, what consequence this degree of modesty has for the revolutionary fictionalist's position. The revolutionary fictionalist is not, after all, advocating the *abandonment* of mathematics, even though, according to fictionalism, the correct understanding of mathematical assertions is not as literal assertions of truth. Rather, the revolutionary fictionalist seeks to explain how it is that such literally false theories can nevertheless be useful. As such, the revolutionary fictionalist does not advocate the abandonment of any successful mathematical theory, but rather to account for the success of that theory under the assumption that that theory might not in fact be literally true.

Indeed, despite popular myth to the contrary, this is almost exactly the form of Hartry Field's approach in *Science without Numbers* [1980]. Field does not—chronically immodestly—advocate that scientists actually *stop* using mathematics when doing science. Rather, he seeks to *justify* the use of platonistic Newtonian physics by showing it to be a conservative extension of a nominalistically acceptable theory. The programme of nominalization is meant to provide scientists with nominalistically statable theories to believe. However, by giving representation theorems that link claims in the nominalistic theories with claims in their platonistic counterparts, and arguing that the platonistic counterparts are conservative over the nominalistic theories, Field's programme, if successful, would justify scientists in behaving *as if* the platonistic theories were true. Field's project is only revisionary to the extent that he hopes to revise scientists' own attitudes to their practices—he hopes that scientists will agree, once faced with nominalistic theories of which their platonistic theories are conservative extensions, that it is *those* theories that are justified by the usual scientific standards, and not their platonistically stated counterparts.

It is, we can accept, a mistake to advocate the *abandonment* of a successful discipline on the basis of philosophical scruples about, for example, the ontological assumptions of that discipline. To this extent, we may be modest. However, so long as we can explain why a practice that is in some respects misguided is nevertheless successful, we need not advocate the abandonment of that practice just because it falls short of some of our expectations. If 'philosophical modesty' gives a reason to abandon the revolutionary fictionalist's claim that many mathematical assertions are false though useful, Burgess has yet to make that argument.

There is room, then, for a weak form of modesty, according to which we hold back from advocating revisions in successful practices that we do not believe to be truth-stating, by seeking to explain how such practices may be successful even if practitioners are involved in making assertions that we do not believe are literally true. Such a position avoids the embarrassment of trying to tell scientists and mathematicians to stop what they are doing, while allowing philosophers the resources to evaluate the claims of these disciplines to consist of literal assertions of truth. A philosopher who holds back from taking this sort of 'revolutionary' approach to mathematics, for example, on grounds that her claims might still be laughed out of town, must surely lack the robust resistance to ridicule that her chosen discipline requires.

2. Revolutionary Fictionalism and Nonsense Poetry

Fear of being laughed at is thus not reason enough to hold back from revolutionary fictionalism, according to which mathematicians and scientists, when involved in making assertions that imply the existence of mathematical objects, are making assertions that we have no reason to believe to be true.[5] However, Burgess has a second, more serious, concern about revolutionary fictionalism. This concern threatens to render the revolutionary's denial of the existence of mathematical objects if not meaningless, then at least untestable. If Burgess is right in this worry, then the revolutionary fictionalists should be ignored, not because their assertions are outrageous and obviously false, but rather because we are not really sure that they have said anything empirically significant at all.

Burgess's argument here takes its cue from Rudolf Carnap, in 'Empiricism, semantics, and ontology' [1950]. Here Carnap makes his famous distinction between internal and external questions regarding the existence claims of a discourse (linguistic framework). According to Carnap, internal (non-philosophical) existence questions are answerable by recourse to the rules of the discourse. In the case of mathematics, that numbers exist is established almost trivially by internal rules of the framework of arithmetic. However, when *philosophers* ask the question 'Do numbers exist?', they mean not to ask whether certain existence claims are warranted by the internal rules of the framework of arithmetic, but rather, whether certain assertions made within that framework are (as Burgess puts it) capital-R Really true. But, Carnap tells us, we have provided no grounds on which to answer such metaphysical external questions. The only 'external' question regarding a framework as a whole that we may ask is the practical one of whether it serves any useful purpose to incorporate such a framework into our language. Philosophers of a metaphysical bent who try to ask a further question regarding the capital-R Reality of the objects referred to in the framework have, according to Carnap, failed to give 'a formulation of their question in terms of the common scientific language'. 'Therefore,' Carnap continues,

> our judgment must be that they have not succeeded in giving to the external question and to the possible answers any cognitive

[5] I present the revolutionary fictionalist as stating that we have no reason to believe assertions implying the existence of mathematical objects to be true, rather than that we ought to believe them false, since the agnostic rather than atheistic version of fictionalism is more inclusive, and usually the version that fictionalists defend directly. However as Hartry Field has pointed out (Field [1989], p. 45), it is just a short step from the claim that we have no reason to believe that mathematical-existence assertions are true to the claim that we ought to believe them false, and a step that many fictionalists would be inclined to take.

> content. Unless and until they supply a clear cognitive interpretation, we are justified in our suspicion that their question is a pseudo-question, that is, one disguised in the form of a theoretical question while in fact it is non-theoretical; in the present case it is the practical problem whether or not to incorporate into the language the new linguistic forms which constitute the framework of numbers. ([1950], p. 209)

The claims of a revolutionary fictionalist, who agrees that numbers exist 'according to the framework' of arithmetic, but denies that numbers capital-R Really exist, are rendered by this view not outrageous and obviously false, but rather, simply '*meaningless*, a kind of nonsense poetry without the poetry' (Burgess [2004], p. 34).

Burgess does not want to follow Carnap completely in decrying the question of the capital-R Real existence of numbers as a meaningless pseudo-question, since he does not accept Carnap's 'empiricist criterion of meaningfulness'. Unlike Carnap, Burgess thinks that we *can* make *some* sense of the philosophically meant ontological questions. They can, *e.g.*, be recast as questions such as 'Did it or did it not happen, on one of the days of creation, that God said, 'Let there be numbers!' and there were numbers' (Burgess [2004], p. 34). However, although we might be able to *understand* the question regarding the Reality of the objects mentioned within a framework, this in no way implies that we have any way of answering these questions. If anything, Burgess thinks, the question of the Real existence of numbers, considered in this theological sense, should be answered in the negative, but not in the way the revolutionary fictionalist would like. The revolutionary fictionalist wants her denial of the existence of mathematical objects to be in contrast to her acceptance of the existence of other objects—tables and chairs, for example. However, to the extent that Burgess denies the Real existence of numbers, he would also deny the '*Real* existence of just about anything. For . . . everything we have learned about our processes of cognition points in the direction of the conclusion that even other intelligent creatures, to say nothing of an Omniscient Creator, would or might well have patterns of language and thought very different from ours, recognizing categories of objects very different from those we recognize' (Burgess [2004], p. 35). If these considerations do provide us with reasons to doubt the capital-R Reality of numbers, then that is only because they are grounds to doubt the capital-R Reality of *any* of the objects posited by our theories.

What the revolutionary fictionalist is looking for is some sense of 'real', according to which it is reasonable to assert the reality of some of the objects posited by our theories while denying the reality of other such objects. But if we *can* make sense of capital-R Real existence, *e.g.*, in terms of a question regarding the ultimate metaphysical furniture of the universe, then it seems pretty clear that either this question will remain forever unanswerable, or it will only be plausibly answerable in a way that lumps numbers and tables and chairs together as equally un-Real. If the revolutionary fictionalist's claim is a claim regarding the metaphysical Reality of tables and chairs as contrasted with the metaphysical un-Reality of numbers and functions, then her thesis is, if not nonsense, then either radically untestable, or testable but false. Best, then, to steer clear of claims about capital-R Reality.

Which leaves us where, exactly, as philosophers interested in ontology? Carnap clearly thought that we should pack up and go home. Once metaphysical questions regarding the Reality of the objects of a framework are abandoned, the only questions that remain are internal questions regarding which existence claims follow from the rules of a given framework (such questions are not always trivial to answer—take, for example, the question of whether there exists an even number greater than two that is not the sum of two primes), and practical questions regarding whether it is convenient to adopt a given framework in our scientific theorizing. But these are questions for practitioners to answer. In the case of mathematics, pure mathematicians can be left to answer the internal questions that arise regarding their theories, and natural scientists to answer the practical questions regarding whether to adopt these theories as part of our description of the world. In each case, on Carnap's view; a positive answer to an internal question within a given framework, or a decision to adopt a particular linguistic framework, suggests nothing of particular philosophical interest, at least regarding ontology. Hence we should have no scruples about leaving practitioners to decide which theoretical frameworks are most useful for their purposes:

> Let us learn from the lessons of history. Let us grant to those who work in any special field of investigation the freedom to use any form of expression which seems useful to them; the work in the field will sooner or later lead to the elimination of those forms which have no useful function. *Let us be cautious in making assertions and critical in examining them, but tolerant in permitting linguistic forms.* (Carnap [1950], p. 221)

In the light of Carnapian scepticism regarding the possibility of any meaningful philosophical project of ontology, the task of resurrecting at least some project of ontology as a going-concern for philosophers fell to W. V. Quine. But although it is often the disagreements between Quine and Carnap that are stressed, these disagreements are actually rather minor in comparison with the points of agreement between their views. Quine agrees with Carnap that 'metaphysical' ontological questions have not been given any significance. Quine also agrees that, for a given theory, we can make sense of the question 'What (according to that theory) is there?', and answer it by considering which existentially quantified assertions follow from the theory's background assumptions.[6] As Quine tells us, there is some terminological disagreement here, in that Quine is happy to talk of the existentially quantified claims of a theory as revealing the theory's 'ontological commitments', whereas Carnap (wary of the metaphysician's misuse of the world ontology in making meaningless assertions about the nature of Reality) shies away from this use. But the difference here is not a substantial one—Quine's use of the word 'ontology' by no means indicates a return to traditional metaphysics.[7]

The real difference between Carnap and Quine concerns whether the adoption of a discourse 'for practical purposes' indicates anything that could be thought of as commitment to the truth of that discourse. Seeing a sharp dividing line between the theoretical question of which assertions are justified according to the rules of a given discourse, and the practical question of whether it is useful to adopt that discourse, Carnap felt that we ought not to read anything substantial into a decision to adopt a particular linguistic framework. The 'ontological' commitments of a given framework do not reflect anything about *our* commitments incurred in choosing to use the framework, since our reasons for choosing to speak in a particular way reflect only our judgement that the language form we use is *convenient*, nothing more. In particular, we cannot conclude from our choice to adopt a particular framework that the assertions licensed by that framework are in any way more *justified* than assertions licensed by other potential frameworks which we have not chosen to adopt, but only that it is more convenient for us to use the framework that we have in fact adopted. Thus Carnap stresses that the question, 'Shall we introduce such and such forms into our language?' is 'not a theoretical but a practical question, *a matter of decision rather than assertion*' ([1950], p. 213, my italics).

[6] More accurately, as Quine tells us ([1951], pp. 128–129), it is Carnap here who is agreeing with Quine's approach of uncovering existential commitments by tracking the existentially quantified claims of the theory.

[7] As is made clear by Quine's own discussion of his usage:

> I might say in passing, though it is no substantial point of disagreement, that Carnap does not much like my terminology here. Now if he had a better use for this fine old word 'ontology', I should be inclined to cast about for another word for my own meaning. But the fact is, I believe, that he disapproves of my giving meaning to a word which belongs to traditional metaphysics and should therefore be meaningless. Now my ethics of terminology demand, on occasion, the avoidance of a word for a given purpose when the word has been preempted in a prior meaning; meaningless words, however, are precisely the words which I feel freest to specify meanings for. But actually my adoption of the word 'ontology' for the purpose described is not as arbitrary as I make it sound. Though no champion of traditional metaphysics, I suspect that the sense in which I use this crusty old word has been nuclear to its usage all along. ([1951], pp. 126–127)

Quine's response to Carnap is, of course, to blur the distinction between theoretical questions about what is assertible given the assumptions of a discipline and practical/conventional questions concerning whether to adopt those assumptions. As a result, in our scientific theorizing, practical reasons to adopt a particular language form in describing, explaining, and predicting our experience are also considered reasons to *believe* the assertions thereby warranted. This is not, however, to resurrect discredited 'metaphysical' theses concerning the capital-R Reality of the objects quantified over in the resulting theory. Rather, it is a thesis concerning which theoretical claims are warranted by scientific evidence. According to Quine, then, we should believe in the small-r reality of the world described by our scientific theories. Or, avoiding talk of reality altogether, Quine's conclusion is that we have reason to believe claims made in the context of scientific theorizing. To take an attitude to these claims that falls short of belief on the grounds that mere 'practical' reasons cannot carry any epistemic clout is just, on Quine's view, to misunderstand the nature of theory and evidence.

Although Burgess has some qualms about the full Quinean picture, it is clear that he accepts Quine's identification of the ontological commitments of an adopted theoretical framework with *our* commitments incurred in choosing to use that framework. Burgess calls the small-r realist position that results 'anti-anti-realism': 'the refusal to apologize while doing philosophy for what is said while doing mathematics and science' ([2004], p. 19). With this in mind, we can consider Burgess's detour via Carnapian considerations as presenting the mathematical fictionalist with a dilemma. The revolutionary fictionalist notes that mathematicians and scientists do make many assertions that commit them (on the assumption that they ought to believe what they say) to the existence of mathematical objects. But the fictionalist proceeds to deny that mathematicians ought to believe these assertions, since they do not think that we have reason to believe in the existence of mathematical objects. Now, we may ask, on what grounds does the fictionalist deny that mathematicians and scientists ought to believe the claims made within their disciplines?

On the one hand, the fictionalist's denial might be based on a denial of the metaphysical Reality of mathematical objects. If so, then as Carnap has shown, we have good reasons to suspect that the revolutionary fictionalist's thesis is empirically meaningless. On the other hand, the fictionalist might be attempting to deny the small-r reality of the mathematical objects posited by our theories, while asserting the reality of (at least some of) the physical objects that our theories posit. In this case, the Quinean line comes into play. The fictionalist who thinks we have no reason to believe in the existence of mathematical objects is in this case simply misunderstanding the nature of evidence. That we find it convenient to present our best theory of the empirical world in mathematical terms *just is* reason to believe in the objects to which our theory is thereby committed. Of course we have evidence for the existence of the mathematical objects posited by our empirically successful scientific theories—that's just what evidence *is*.[8]

One might think, then, as Burgess clearly does, that the case is closed against the revolutionary fictionalist. And certainly, if revolutionary fictionalism is understood as a claim about the nature of capital-R Reality, rather than a claim about what we ought to believe given empirical evidence, the case for revolutionary fictionalism looks decidedly shaky. However, as a thesis about what we have reason to believe, there is perhaps some room for a revolutionary fictionalist response to the Quinean line, based not on heavy-duty metaphysical principles, but rather on more down-to-earth thoughts regarding the nature of evidence. Indeed, this is precisely how many recent doubts about Quine's indispensability argument have been cast. Witness the objections to the indispensability argument presented by Maddy [1992], Elliott Sober [1993], and, to some extent, Mark Balaguer [1998]. All of these authors base their attacks on Quine's argument for mathematical realism on concerns about his claim that the confirmation our theories receive extends to all of their assertions equally. Furthermore, these concerns do not stem from misguided metaphysical scruples, but rather from ground-level analysis of scientific standards of evidence. It is not my intention to comment on the success of these arguments here, but only to note that they suggest that there is room for an argument on *empirical* (*i.e.*, scientific) grounds against belief in the small-r reality of numbers, as contrasted with belief in the reality of some of the physical posits of our theories.

Such an argument would be based on the idea that, whatever confirmation our empirical theories receive, this confirmation does not extend to confirmation of the existence of the mathematical objects posited by these theories. The evidence for our empirical theories would therefore be considered not as evidence that those theories, as a whole, are *true*, but rather, as evidence that those theories are correct in some other respect.[9]

[8] Note that this response depends on the theoretical framework in question being incorporated into our scientific theories. The Quinean line cannot be used to support the claim that we have reason to believe the claims of *pure* mathematical theories independent of reasons that come via their incorporation into empirical scientific theories. Quine, in contrast with naturalist philosophers of a more modest bent, is happy to think of mathematical theories that are pursued independently of any applications, and are not incorporated into the framework of empirical science, as 'mathematical recreation, without ontological rights' (Quine [1986], p. 400). To this extent, even Quine can be thought of as a revolutionary fictionalist regarding the assertions of mathematicians working in these unapplied areas.

[9] Precisely how to characterize the relation of evidence to theory, if one accepts that the theory as a whole is not confirmed as true, presents some difficulty. Balaguer [1998] has suggested that what is confirmed by the success of our platonistically stated theories is not their truth but rather, that they are correct in their presentation of physical reality. Balaguer argues that, for platonistically stated empirical science to be true, the physical world would have to be a particular way and the mathematical realm would also have to be

In particular, such a defence of revolutionary fictionalism with respect to the mathematical objects posited by our theories would try to show that there is some principled difference between the mathematical posits of our theories as compared with other theoretical posits, such that the role they play in our successful theories does not rely on the truth of the claims in which they appear. As such, we should not expect that empirical confirmation of our theories will lead to confirmation of the existence of mathematical objects.[10]

Whether this kind of approach can be defended remains to be seen. What is interesting, however, is that considerations Burgess himself presents *against* revolutionary fictionalism themselves suggest a disanalogy between the role of mathematical and non-mathematical posits in our empirical theories. Furthermore, this is precisely the kind of disanalogy that the revolutionary fictionalist wants to stress.

Burgess points to this disanalogy in arguing that fictionalism is a non-empirical doctrine. He asks us to compare assumptions about mathematical objects made in the context of our scientific theories with assumptions about more down-to-earth objects and events made in the context of other works whose status as fact or fiction have been put in question. Burgess's examples of such controversial texts are the anthropological writings of Carlos Castaneda and the autobiography of Rigoberta Menchú. Mathematical assumptions differ from assumptions about characters in, say, Menchú's book, Burgess tells us, because we 'know very well what it would have *looked* like for the events in Menchú's book to have occurred', whereas, in the mathematical case, 'we have absolutely no idea of what difference it would make to how things *look*' for these assumptions to be

a particular way. What is confirmed by the success of our empirical theories is, according to Balaguer, not this full physical/mathematical package, but rather, that 'the physical world holds up its end of the "empirical-science bargain"' ([1998], p. 134). We might think of Balaguer's account as suggesting that empirical evidence confirms our theories to be at most *nominalistically adequate* (in analogy with Bas van Fraassen's [1980] notion of empirical adequacy).

[10] Would this provide a positive argument *for* revolutionary fictionalism, rather than simply a defence of fictionalism against refutation via the indispensability argument? This rather depends on one's attitude to the 'naturalist' ontological premise of that argument, according to which we should trust science and science alone to tell us what there is. If one accepts, as Burgess clearly does, the premise that ontology is an empirical scientific matter, then the lack of empirical confirmation of the mathematical posits of our scientific theories would tell us that we have no reason to believe in the existence of mathematical objects. Hence the lack of empirical confirmation for the existence of mathematical objects supports taking a fictionalist attitude to mathematical theories. Ontological naturalists therefore have a response to ontological sceptics such as Mark Balaguer, who accept that the indispensibility argument gives no reason for rejecting fictionalism, but argues that, nevertheless, we have no way of deciding between fictionalism and rival ontological theories. If ordinary scientific standards of evidence give no reason for believing in the existence of objects, then ontological naturalism tells us we ought not continue to believe that such objects exist.

true, 'or rather, we have a very strong suspicion that it would make no difference at all' ([2004], p. 35).[11]

The mathematical fictionalist will agree with this, but note further that, as the contested status of Burgess's preferred examples make clear, the disanalogy is not just with more down-to-earth fictional claims, but with more down-to-earth factual claims too. Just as there is a marked disanalogy between mathematical objects and more common-or-garden fictional objects, likewise there is a disanalogy between mathematical objects and more standard real objects, such as the physical objects posited by our physical theories. We tend to think that our experience would be vastly different if the unobservable physical entities posited by our theories did not exist. This is why van Fraassen's constructive empiricism seems to many to be implausible. But, as Burgess himself points out, we are just not sure what difference the existence of the mathematical objects posited in our theories would make to our experience. Granted, our scientific theories would be false if the mathematical posits mentioned in them did not exist, but would they be any less successful?[12]

It is this contrast that suggests to the fictionalist that there is something wrong in the Quinean account of confirmation, according to which the existence of mathematical entities is confirmed by their presence as

[11] It is interesting to note that the disanalogy disappears in the version of fictionalism defended by Sarah Hoffman in Hoffman [2004]. Following Philip Kitcher [1984], Hoffman proposes to rewrite our mathematical theories as theories concerning the activities of an ideal agent. By reconstruing mathematical theories as stories about the abilities of an agent endowed with super-human collecting and segregating abilities, Hoffman has given a sense in which our theories can be thought of as clearly fictional—we know something of what it would be like for there to be such an agent, and as a result we can be pretty sure that no such agent exists. Hoffman's account presents an interesting alternative to more standard 'object' fictionalism, and one which makes more active use of the analogy with fiction than is made by more traditional versions. However, I will not pursue this alternative here, except to note that its success appears to depend on our ability to reconstrue *all* of our mathematical theories as stories about an ideal agent. Object fictionalism, by contrast, avoids any controversial project of reconstrual, and is arguably preferable for precisely that reason.

[12] To be fair to Burgess, when presenting the contrast between mathematics and fact/fiction, the contrast he has in mind is one between the assumption of the metaphysical Reality of numbers as compared with the reality of the characters and events described by Menchú and Castaneda. This is intended as part of his argument that the ultimate metaphysical question of the Reality of numbers is empirically meaningless. However, given his earlier rejection of empirical meaning for *any* claims about the nature of metaphysical Reality, he should be no more clear on what difference the *metaphysical* Reality of the events and characters mentioned in the books would make to our experience than he is on what difference the metaphysical Reality of numbers should make. If the contrast is of any interest, it is because it works at the ground level, as a contrast between the question of the small-r reality of numbers as compared with people. Unlike the case with fictional characters and events, we have no idea what difference the ordinary existence of mathematical objects, or the truth of our assertions about them, might make to our day-to-day experience.

posits in our best scientific theories. For, if we strongly suspect that the truth or falsity of the mathematical claims of our theories would make *no difference* to the empirical success of those theories, then what grounds do we have to infer the truth of our theories in the light of their empirical success? This is not a misplaced metaphysical worry about whether our mathematized scientific theories 'correspond to Reality', but rather, a scientifically motivated request for an *explanation* of the role of mathematical assumptions in our physical theories. We need to know why assertions that imply the existence of mathematical objects are so crucial to our scientific theories, given our suspicion that the existence of the mathematical objects posited would make no difference to our successful theorizing.

The revolutionary fictionalist's suspicion is that such an explanation will show how the mathematical claims in our theories may serve their function for reasons that do not depend on the truth of those claims. If this is right, then the fictionalist has grounds to advocate, if not a revolution in our theoretical practice, then at least a revolution in our understanding of that practice. We may take back, while doing philosophy, what is asserted by our scientific theories, just so long as we can explain why the success of our theorizing does not depend on the truth of the assertions we make. The result is a bloodless revolution, since mathematical and scientific practice is left undisturbed, but a revolution nonetheless.

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