University of Waterloo
Department of Philosophy
PHIL 271—Special Topics
Mathematics and Philosophy
Puzzles, Foundations and Progress
Spring 2017

Instructor Information

Time:	10:00-11:20 Tuesday and Thursday
Place:	HH 334
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Philosophy and Mathematics

Academics in many disciplines like to tell stories about how people outside academia have no idea what they do. Philosophers get confused with psychiatrists ("Don't shrink my head, Man!"). Mathematicians are sometimes asked why mathematicians are needed anymore, now that everyone has calculators, computers and such. But philosophers and mathematicians have much more in common than being paid for doing things that many people find inscrutable. For one thing, maybe more so than other academic fields, philosophy and mathematics are deeply puzzling activities, *even to those who know them best*—questions such as "what is it about?" are genuine, difficult questions for both disciplines. For another, their history and practice are intertwined in interesting ways.

Indeed, the list of great thinkers who are known both as mathematicians and philosophers (and sometimes also as physical scientists, too) is long, and goes all the way back to the earliest days of both disciplines (at least in the Western tradition). It includes names like Pythagoras, Descartes, Leibniz, Newton, Pascal, Frege, Hilbert, Russell, Gödel and Tarski. But the list of great philosophers who were inspired by mathematics even though they are not known as great mathematicians themselves is also long. Often motivated by what they perceived as a stark contrast between rapid progress in the physical sciences and mathematics and the lack of progress in philosophy, which they thought had been spinning its wheels confronting the same problems again and again, many important philosophers (Plato, Spinoza, Kant, among others) have advocated adopting mathematical methods in philosophy as the surest way to remaking philosophy as a progressive discipline in its own right.

On the other hand, as we will see, mathematical progress has often come either from the need to overcome some surprising conceptual "crisis," which has forced mathematicians to rethink something they took to be well-understood and well-established, or by research programs that had an explicitly philosophical motivation—sometimes, for instance, they have been motivated by a claim that "classical" mathematics is fundamentally mistaken in some way and so mathematicians have to start doing things differently.

About this course

Fundamentally, this is a course in the Philosophy of Mathematics, but it will be a somewhat unconventional one. Traditional courses in Philosophy of Mathematics tend to focus on the most important philosophical views *about* mathematics (Platonism, Aristotelean moderate realism, nominalism; logicism, formalism, intuitionism; and so on). We will certainly encounter these positions, but will do so as part of a broader look at the relationship between philosophy and mathematics. We'll look at the historical reasons some have given for thinking that philosophy ought to adopt "the mathematical method." We'll also look at some specific, recent examples where people find reason for thinking that mathematical methods can provide illumination on philosophical questions. We'll look at some historical cases where philosophical concerns led to important developments in mathematics, and cases where surprising developments in mathematics have overthrown widely held philosophical commitments. We'll consider whether a philosophical argument ought ever to be taken to be grounds for thinking that a mathematical claim is mistaken. And we'll spend considerable time on one of the most important and multi-faceted philosophical questions about mathematics: what is its role in natural science? Is it, as some have argued, a sort of "miracle" that mathematical concepts and techniques discovered (or invented?) by some mathematician who simply thought them interesting turned out much later to be exactly the tool some scientist needed to formulate a physical theory?

The class will likely include both philosophy majors and student for whom this will be the only philosophy course they ever take. The course is therefore designed not only to acquaint students with some interesting philosophical issues and positions different philosophers have taken on them, but also to help students develop some of the skills required to do philosophy well. While it will be important to understand the content we cover, the class activities and assessments are intended to help students develop these analytical and argumentative skills. Doing so will obvious help students who intend to take many more philosophy courses, but such skills are useful in any field requiring a clear head.

Similarly, the class is likely to include many students with a lot of background in mathematics, and some with almost none. The course will not presume that students have any more than high school mathematics. Mathematical ideas and proofs will be explained as they are needed. Sometimes a bit of effort might be required for students who don't already know the topic in question to follow the explanation, but the relevant details will be provided, and students are invited to make use of office hours to seek clarification when it is needed.

Intended Learning Outcomes

Upon successful completion of this course, you should be able to:

- Articulate what is at issue in several philosophical debates about mathematics
- Discuss and assess claims about the value (in general, and with respect to certain specific examples) of using mathematical methods in philosophy
- Describe some of the influential philosophical views about mathematics, both historical and contemporary, that have been held by philosophers and mathematicians
- Implement the basic steps of the philosophical analysis of arguments, including recognizing argument structure and identifying premises, conclusions and presuppositions
- Successfully critique philosophical arguments, showing awareness of the role of the dialectical

context in which arguments are presented
- Formulate arguments defending a philosophical view

Required Readings, weekly schedule

The readings are all available as pdf files from the course Learn site, and are available to students registered in the course as "fair dealing" under copyright law. During the course of the term, additional notes and readings for specific topics may be added to the Learn site, so it is worth checking back frequently.

The readings here are quite varied. Some are summaries of historical developments. One is an extract from a biography. Some are articles from professional philosophical journals.

In some cases, there will be guidance in the lectures about which parts of a longer reading are particularly salient for our purposes in the course. But *in no case do the readings cover everything that we will be considering in the lectures and class discussions*. This is not a course that a student is likely to be able to do well in without attending class regularly. (The reading list is relatively short for a philosophy course; part of the trade-off for a shorter reading list is that more things must come up in class that do not get covered in the readings.)

Week of:

Section 1: Some History, Some Philosophy, and Some History of Philosophy

May 1:   Introduction to the course, and Early History: Pythagoras, Plato, Aristotle, Euclid
(Reading: Stewart Shapiro, "Plato's Rationalism, and Aristotle," *Thinking About Mathematics,* Oxford: Oxford University Press, 2000, pp. 49-71)

May 8:   Empiricism and Kant
(Reading: Stewart Shapiro, "Near Opposites: Kant and Mill," *Thinking About Mathematics,* Oxford: Oxford University Press, 2000, pp. 73-102)

Section 2: Puzzles, Crises, and Foundations: 150 years of progress?

May 15:   Monsters, Rigour, and Logicism
(Reading: Alberto Coffa*, "*Kant, Bolzano and the Emergence of Logicism,,**"** in *Frege's Philosophy of Mathematics*, William Demopoulos, ed., Cambridge, Mass.: Harvard University Press, 1994, pp. 29-40.)

May 22:   **No class on Tuesday May 23 (Monday schedule in effect that day)**
Russell's Paradox and the Crisis of Foundations
(Reading: W.V.O. Quine, "Paradox," *Scientific American*, **206** (1962) 84-100)

May 29:   **Test 1 on May 30**
Set Theory and Type Theory
(Reading: George Boolos, "The Iterative Conception of Set," *The Journal of Philosophy*, **68** (1971), pp. 215-231)

June 5: The Nature of Mathematical Existence: Formalism and Constructivism
(Reading: Constance Reid, "Gordon's Problem," chapter 5 of *Hilbert*, New York: Springer-Verlag (1996) pp.28-38. Originally published 1970)

Section 3: Mathematics, Philosophy, and Natural Science

June 12: **Building-blocks assignment due June 13**
Conventionalism, Quinean Realism, and Applied Mathematics
(Mark Colyvan, "The Miracle of Applied Mathematics," *Synthese* **127.3** (2001) pp.265-277)

June 19: Nominalism, Error Theory, and Fictionalism
(Reading: Mary Leng, "Revolutionary Fictionalism: A Call to Arms," *Philosophia Mathematica* **13.3** (Oct 2005) pp. 277-293)

Section 4: Proofs: what are they, and what are they good for?

June 26: **Test 2 on June 27**
Another go for Plato?
(Reading: Penelope Maddy, "Perception and Mathematical Intuition," *The Philosophical Review* **84** (1980) 163-196)

July 3 (and July 11)
Mathematical Explanations, Diagrams, and Computer Assisted Proofs
(Readings: Paolo Mancosu, "Mathematical Explanations: Problems and Prospects"; *Topoi* **20** (2001) 97-117; J.R. Brown, "Proofs and Pictures," *The British Journal for Philosophy of Science* **48,** 161-86)

Section 5: Mathematical methods in Philosophy?

July 13: Mathematics as a Model
(Reading: René Descartes, *Discourse on Method (parts 1-3)*, in Norman Kemp Smith, ed. and transl., *Descartes' Philosophical Writings*, London: MacMillan & Co., 1952.)

July 17: Philosophical Theorems? Truth and Provability
(Readings: Alfred Tarski, "The Semantic Conception of Truth and the Foundation of Semantics," in *Philosophy and Phenomenological Research* **4** (1944) 341-376; George Boolos, "Gödel's Theorem Explained in Words of One Syllable," *Mind* (New Series) **103** (1994) 1-3)

July 25: **Term Paper due July 25**
Review session July 25

Final exam: **Date TBD, during exam period**

Assessments

As can be seen by scanning the learning objectives for the course, the goal is to teach students some philosophy of mathematics, yes, but as importantly it is to provide an opportunity for students to develop their philosophical skills. Most of these skills involve the ability to analyze, evaluate and construct arguments, but clear expression of ideas is also a cardinal virtue in philosophy.

The Assessments are designed to require not only that you show that you've absorbed material, but also that you practice certain philosophical skills.

"Class Participation" (10%): During the course of the term, there will be a variety of short, in-class activities that will be handed in for assessment. You will have to be in class to do them, and in some cases having done the reading before the class may make a significant difference in your ability to do them well. When they will happen will not be announced in advance. There will be six such activities, and your best five will count.

In class tests, May 30 (10%) and June 26 (15%): Students will have 75 minutes to complete these tests. My goal on these tests is to make them less an exercise in memorizing what we've covered, and more a test of how well you've understood it, so the questions will normally ask you to do things with the ideas we've covered, not merely state or describe them.

Building-block assignments due June 13 (15%): In some ways, doing philosophy is like dancing. To do it well requires doing a number of different things in a well-coordinated way. Also like dancing, it is often much easier to learn the steps by doing them in isolation before you try to do them all at once. The building-block assignment will ask you to practice skills essential to doing good philosophical work.

Term paper due July 25 (25%): This assignment will require that you write a short philosophical paper. In other words, it's time to dance: you will put together the skills you've practiced earlier, producing a paper in which you present your own argument defending a particular philosophical conclusion.

Final exam (25%): This will be, you know, a final exam.

Rescheduling of quizzes, tests and exams

If a student cannot write a test or exam at the scheduled time, or cannot meet the deadline for an assignment, s/he should be prepared to supply documentation (ideally the UW Verification of Illness form) that establishes a legitimate excuse, or a mark of zero will be given for that test/exam/assignment. For medical excuses, the rule of thumb will be that if you are too sick to write or meet a deadline, then you are ill enough to see a medical professional, so a note *from a doctor or other suitable medical professional* will be expected. In the case of students who have legitimate excuses, appropriate alternative arrangements will be worked out between the instructor and the student. **In all such cases, students should be in touch *before the test/exam or assignment deadline* to make special arrangements unless there are compelling reasons for not doing so.**

Getting in touch---email policy

You are always welcome to come and see me during office hours---that is what they're for! But you are also welcome to send me questions by email. I will try to respond promptly—that is, I will almost always reply to an email by the end of the next business day, and usually sooner than that. If you write on the

weekend, I will reply by the end of Monday. Please send your messages from your uwaterloo.ca email address. Please also check the Learn site for the course regularly for news and updates.

Cross-listed course

Please note that a cross-listed course will count in all respective averages no matter under which rubric it has been taken. For example, a PHIL/PSCI cross-list will count in a Philosophy major average, even if the course was taken under the Political Science rubric.

Academic Integrity

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo are expected to promote honesty, trust, fairness, respect and responsibility. See the UWaterloo Academic Integrity webpage and the Arts Academic Integrity webpage for more information.

Discipline: A student is expected to know what constitutes academic integrity, to avoid committing academic offences, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. When misconduct has been found to have occurred, disciplinary penalties will be imposed under Policy 71 – Student Discipline. For information on categories of offenses and types of penalties, students should refer to Policy 71 - Student Discipline. For typical penalties check Guidelines for the Assessment of Penalties.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4. When in doubt, please be certain to contact the department's administrative assistant who will provide further assistance.

Appeals: A decision made or penalty imposed under Policy 70 - Student Petitions and Grievances (other than a petition) or Policy 71 - Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72 - Student Appeals.

Accommodation for Students with Disabilities

Note for students with disabilities: The AccessAbility Services office, located on the first floor of the Needles Hall extension (1401), collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AS office at the beginning of each academic term.

Mental Health Services

Mental Health Services aim is to provide holistic programming and services to help you lead a healthy and balanced life. We strive to provide a secure, supportive environment for students of all orientations and backgrounds.

Students suffering from problems with anxiety, depression, problems with sleep, attention, obsessions or compulsions, relationship difficulties, severe winter blues, etc., may make an appointment by phone

or in person. Appointments are usually available within two days of initial contact with one of our medical doctors. All contacts are completely confidential.

Territorial Acknowledgement

We acknowledge that we are living and working on the traditional territory of the Attawandaron (also known as Neutral), Anishinaabe and Haudenosaunee peoples. The University of Waterloo is situated on the Haldimand Tract, the land promised to the Six Nations that includes six miles on each side of the Grand River.