Knowledge Representation Techniques

Propositional logic in AI

- Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions.
- A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

  a) It is Sunday. (True proposition)

  b) The Sun rises from West (False proposition)

  c) 3+3= 7(False proposition)

  d) 5 is a prime number. (True proposition)

Propositional logic forms the basis for many AI systems, including expert systems, rule-based systems, and natural language processing.

Propositional logic in AI

- Propositional logic is a formal language that uses symbols to represent propositions and logical connectives to combine them.
- The symbols used in propositional logic include
  - letters such as p, q, and r, which represent propositions, and
  - logical connectives/operators such as $\wedge$ (conjunction/and), $\vee$ (disjunction/or), and $\neg$ (negation/not), which are used to combine propositions.
- **Tautology**: a proposition formula which is always true and is also called a valid sentence.
- **Contradiction:** a proposition formula which is always false.
- Statements which are questions, commands, or opinions are not propositions such as "Where is Rohini", "How are you", "What is your name", are not propositions.

Types of Propositional

Atomic Proposition: Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

P: The sky is blue.   Q: The grass is green.

R: 2+2=4.   S: The Earth orbits the Sun.

Compound proposition: Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

a) "It is raining today, and street is wet."

b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives

- Logical connectives are used to connect two simpler propositions or representing a sentence logically.
- We can create compound propositions with the help of logical connectives.
- There are mainly five connectives, which are given as follows:
  - Negation
  - Conjunction
  - Disjunction
  - Implication
  - Biconditional

Logical Connectives

- Negation
  - A sentence such as $\neg P$ is called negation of P.
  - A literal can be either Positive literal or negative literal.
  - For example: $\neg P$: The sky is not blue.
- Conjunction:
  - Conjunction of two propositions P and Q is denoted by $P \wedge Q$ and is read as “P and Q's".
  - The conjunction is true only if both P and Q are true.
  - For example: $P \wedge Q$: The sky is blue and the grass is green.
  - Example: Rohan is intelligent and hardworking.

    p= Rohan is intelligent, q= Rohan is hardworking. $\rightarrow p \wedge q$.

Logical Connectives

- Disjunction:
  - Disjunction of two propositions P and Q is denoted by $P \vee Q$ and is read as “P or Q".
  - The disjunction is true if at least one of P and Q is true.
  - For example: $P \vee Q$: The sky is blue or the grass is green.
  - For example: "Ritika is a doctor or Engineer",

    P= Ritika is Doctor. Q= Ritika is Doctor, so we can write $P \vee Q$.

Logical Connectives

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

For disjunction:

P	Q	P ∨ Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

Logical Connectives

- Implication:
  - Implication of two propositions p and q is denoted by $p \rightarrow q$ and is read as "if p then q".
  - For example, Let P= It is raining, and Q= Street is wet,

    If it is raining, then the street is wet, is written as $P \rightarrow Q$
  - Let, P: Ram goes to school daily, Q: Ram gets good marks.

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

  - The implication is false only if p is true and q is false.

Logical Connectives

- Biconditional:
  - A sentence such as P $\Leftrightarrow$ Q is a Biconditional sentence,
  - Examples,
  - If I am breathing, then I am alive
    - P = I am breathing, Q = I am alive, represented as P $\Leftrightarrow$ Q.
  - I will eat lunch if and only if my mood improves.

For Biconditional:

P	Q	P$\Leftrightarrow$ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Logical Connectives

Connective symbols	Word	Technical term	Example
$\wedge$	AND	Conjunction	$A \wedge B$
$\vee$	OR	Disjunction	$A \vee B$
$\rightarrow$	Implies	Implication	$A \rightarrow B$
$\Leftrightarrow$	If and only if	Biconditional	$A \Leftrightarrow B$
$\neg$ or $\sim$	Not	Negation	$\neg A$ or $\neg B$

Precedence of connectives

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AND)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

Properties of Operators

- **Commutativity:**
  - $P \wedge Q = Q \wedge P$, or
  - $P \vee Q = Q \vee P$.
- **Identity element:**
  - $P \wedge \text{True} = P$,
  - $P \vee \text{True} = \text{True}$.
- **DE Morgan's Law:**
  - $\neg (P \wedge Q) = (\neg P) \vee (\neg Q)$
  - $\neg (P \vee Q) = (\neg P) \wedge (\neg Q)$.
- **Double-negation elimination:**
  - $\neg (\neg P) = P$.
- **Associativity:**
  - $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$,
  - $(P \vee Q) \vee R = P \vee (Q \vee R)$
- **Distributive:**
  - $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$.
  - $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$.

Translations

Denote:

- p = It is sunny this afternoon
- q = it is colder than yesterday
- r = We will go swimming
- s= we will take a canoe trip
- t= We will be home by sunset

Assume the following sentences:

- It is not sunny this afternoon and it is colder than yesterday.
- We will go swimming only if it is sunny.
- If we do not go swimming then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

Translations

Denote:

- p = It is sunny this afternoon
- q = it is colder than yesterday
- r = We will go swimming
- s= we will take a canoe trip
- t= We will be home by sunset

Assume the following sentences:

- It is not sunny this afternoon and it is colder than yesterday. $\neg p \rightarrow q$
- We will go swimming only if it is sunny. $p \rightarrow r$
- If we do not go swimming then we will take a canoe trip. $\neg r \rightarrow s$
- If we take a canoe trip, then we will be home by sunset. $s \rightarrow t$

Satisfiability and Validity

- A preposition is **satisfiable** if it is possible to find an interpretation (model) that makes the preposition true.
  - There is at least one interpretation under which the preposition can evaluate to True.
- A preposition is **contingent** if and only if there is some interpretation that satisfies it and some interpretation that falsifies it.
  - For example, the preposition $(p \land q)$ is contingent. If p and q are both true, it is true. If p and q are both false, it is false.
- A sentence is **unsatisfiable** if and only if it is not satisfied by any truth assignment.
  - For example, the sentence $(p \land \neg p)$ is unsatisfiable. No matter what truth assignment we take, the sentence is always false.

Satisfiability and Validity

- A preposition is **valid** if and only if all interpretations make the preposition true.
  - For example, the sentence $(p \vee \neg p)$ is **valid**. If a truth assignment makes p true, then the first disjunct is true and the disjunction as a whole true. If a truth assignment makes p false, then the second disjunct is true and the disjunction as a whole is true.
- A preposition is **valid** iff its negation is **not satisfiable**.
- A preposition is **satisfiable** if it is either **contingent** or **valid.**

Satisfiability and Validity

Check if following are unsatisfiable, contingent or valid

1. $P \vee Q$
2. $(P \vee Q) \wedge \neg Q$
3. $(P \vee Q) \wedge \neg Q \rightarrow P$

Satisfiability and Validity

Check if following are unsatisfiable, contingent or valid

1. $P \vee Q$
2. $(P \vee Q) \wedge \neg Q$
3. $(P \vee Q) \wedge \neg Q \rightarrow P$

P	Q	$P \vee Q$	$(P \vee Q) \wedge \neg Q$	**valid sentence ?** $((P \vee Q) \wedge \neg Q) \Rightarrow P$
True	*True*	*True*	*False*	*True*
True	*False*	*True*	*True*	*True*
False	*True*	*True*	*False*	*True*
False	*False*	*False*	*False*	*True*