# Transversal achievement game on a square grid

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**Abstract**

This paper intends to solve the 3-color Ramsey number for n-cycles

## 1 Introduction

**Conjecture 1.1.** *Show that*

$$r(C_n, C_n, C_n) \leq 4n - 3.$$

## 2 Proof of Conjecture

Consider a cycle of length $n = 3$. Then we need to prove that there exists a clique $K_m$, such that $m \leq 9$. Consider $K_9$. Let's color it with three colors labeled $0, 1, 2$. According to the pigeonhole principle there exists a color that has been colored atleast 3 times. Without loss of generality, let it be 0. Label the three vertices as $u, v$ and $w$. Since it is a clique each vertex is connected to each other. Then we can find a $u - v - w$ cycle.

Assume true for all $n - 1$. Let's consider a clique of length $4n - 3$. The clique that you obtain by deleting any 4 vertices is a clique of length $4n - 7$ and when colored with $0, 1, 2$ contains a monochromatic cycle in it. Let these 4 vertices form the set $B$. Let all other vertices form the set $A$. Color all the sets with colors $0, 1, 2$. Then there exists atleast two vertices in $B$ such that they are monochromatic. Without loss of generality, let it be colored by 0.

1. *Case 1:* 2 vertices are colored with 0
   Then for all cycles of different color we have an extra vertex remaining. Join this vertex and it forms an $n$ cycle.

2. *Case 2:* 3 vertices are colored with 0.
   Assume the last vertex is colored with 1. Then for the 0 and 1 case of the cycles, we are covered. Assume the cycle that exists is colored with 2 and has length $n-1$. Now there are $3(n-1)-3 = 3(n-2)$ vertices remaining. Dividing it among the red and blue we get at least $\frac{3}{2}(n-2) \geq n-1 \forall n \geq 3$ vertices for atleast one vertex. This with the remaining blue and red vertex does the job

3. *Case 2:* All 4 vertices are colored with 0.
   Follow similar steps ices are colored with 0.