EECS16B Notes

2021/9/2

Note 1

Transistors 晶体管

- Switch that changes state based on voltage applied at gates

- 3 Gates
  - Source Terminal (S)
  - Drain Terminal (D)
  - Gate Terminal (G)

  ] will be connected if switch is ON.

- Gate Voltage $V_{GS} = V_G - V_S$

CMOS Circuits

NMOS Trans

On if $V_{GS} > V_{th}$

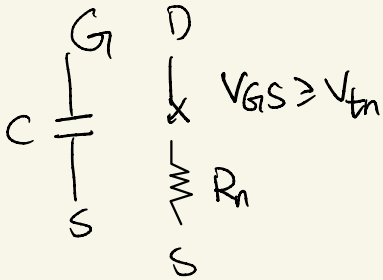
threshold voltage

PMOS Trans

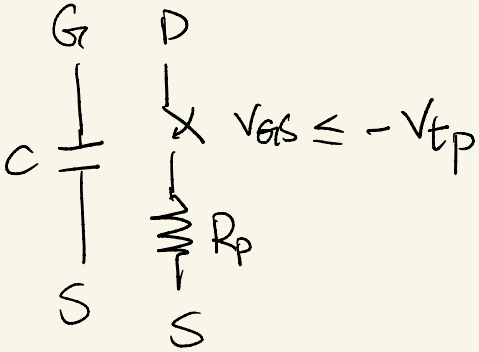
On if $V_{GS} < -V_{tp}$

negative threshold

NMOS



PMOS



It will take time for V_{GS} to charge.

Differentiation Formulas:

1. $\frac{d}{dx}(x) = 1$
2. $\frac{d}{dx}(ax) = a$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}(\cos x) = -\sin x$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\tan x) = \sec^2 x$
7. $\frac{d}{dx}(\cot x) = -\csc^2 x$
8. $\frac{d}{dx}(\sec x) = \sec x \ \tan x$
9. $\frac{d}{dx}(\csc x) = -\csc x(\cot x)$
10. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
11. $\frac{d}{dx}(e^x) = e^x$
12. $\frac{d}{dx}(a^x) = (\ln a)a^x$
13. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integration Formulas:

1. $\int 1\,dx = x + C$
2. $\int a\,dx = ax + C$
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
4. $\int \sin x\,dx = -\cos x + C$
5. $\int \cos x\,dx = \sin x + C$
6. $\int \sec^2 x\,dx = \tan x + C$
7. $\int \csc^2 x\,dx = -\cot x + C$
8. $\int \sec x(\tan x)\,dx = \sec x + C$
9. $\int \csc x(\cot x)\,dx = -\csc x + C$
10. $\int \frac{1}{x}dx = \ln|x| + C$
11. $\int e^x dx = e^x + C$
12. $\int a^x dx = \frac{a^x}{\ln a} + C\ a > 0,\ a \neq 1$
13. $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$
14. $\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$
15. $\int \frac{1}{|x|\sqrt{x^2-1}}dx = \sec^{-1} x + C$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{g(x)^2}$$

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Note J Complex Numbers

$z = a + bj$

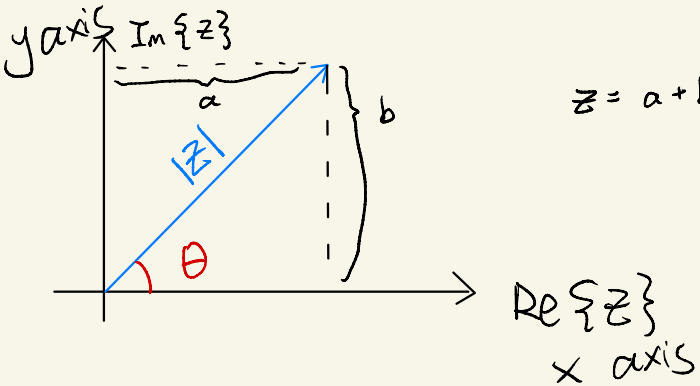
$\bar{z} = z^* = \overline{(a+jb)} = a - jb$ ⇐ complex conjugate 共轭复数.

$|z| = \sqrt{a^2+b^2}$ ⇐ magnitude

$\theta = \angle z = \operatorname{atan2}(b, a)$ ⇐ atan2 is the function that returns the angle from positive x axis to vector from origin to the point (x, y)

Normal tangent only gives range $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, but atan2 covers all quadrants.

Real Plane



$z = a + bj$

Multiplication of Multiplications

$Z = 1 + 0j \qquad Z_1 = 1 + j$

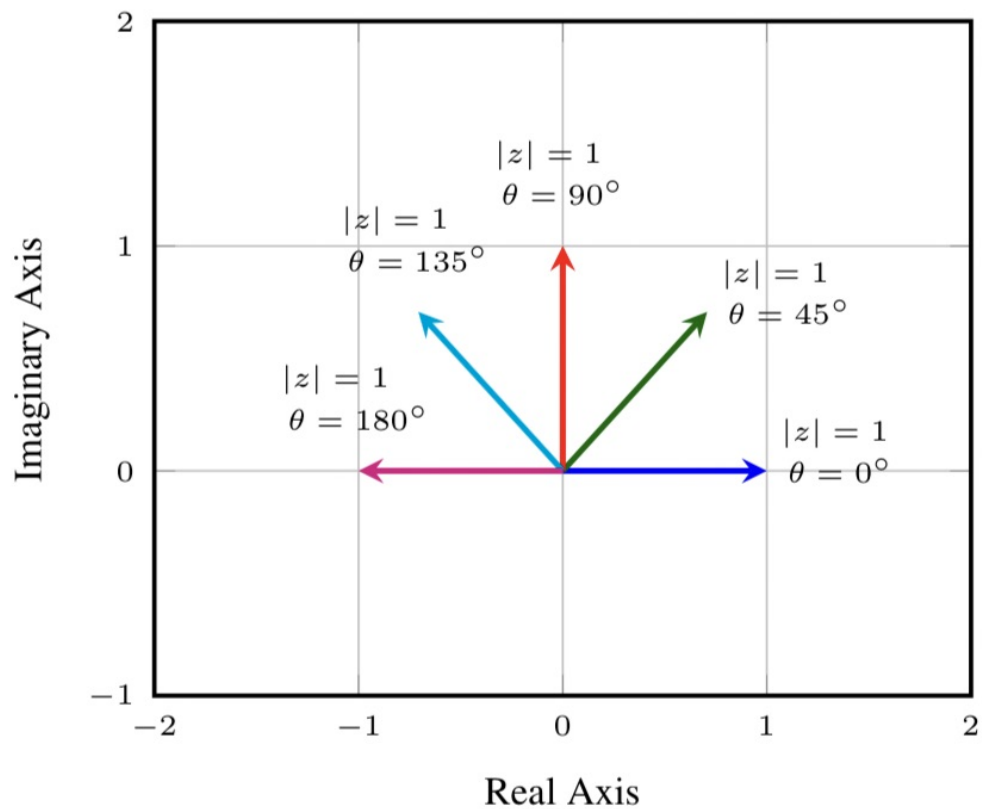


Figure 3: Multiplying z (in blue) by z_2 repeatedly

Try to make multiplication an "addition"

$$Z = |Z| \underbrace{e^{j\angle Z}}$$

expand using Taylor's expansion around 0.

$$= |Z| \left(1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \cdots + \frac{(j\theta)^{2n}}{2n!} + \frac{(j\theta)^{2n+1}}{(2n+1)!} + \cdots \right)$$

$$= |Z| \left(1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \cdots + (-1)^n \frac{\theta^{2n}}{2n!} + j(-1)^n \frac{\theta^{2n+1}}{(2n+1)!} + \cdots \right)$$

$$= |Z| \left[\left(1 - \frac{\theta^2}{2} + \frac{\theta^4}{4} - \cdots + (-1)^n \frac{\theta^{2n}}{2n!} + \cdots \right) + j\left(\theta - \frac{\theta^3}{3} + \cdots + (-1)^n \frac{\theta^{2n+1}}{(2n+1)!} + \cdots \right) \right]$$

$$= |Z| \left(\cos(\theta) + j\sin(\theta) \right) \qquad \theta = \angle Z \text{ in radians}$$

j^k patterns

```
switch (k % 4){
    case 0:
        return 1
    case 1:
        return j
    case 2:
        return -1
    case 3:
        return -j
}
```

$$\because Z = \overbrace{|Z|\, e^{j\angle Z}}^{\text{Polar Form}} = |Z|\,(\cos(\theta) + j\sin(\theta))$$

and $Z = a + bj$.

$$\therefore a = |Z|\cos(\theta), \quad b = |Z|\sin(\theta)$$

Euler's equation

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{-j\theta} = \cos(-\theta) + j\sin(-\theta) = \cos(\theta) - j\sin(\theta)$$

$\therefore$

$$\left.\begin{aligned} 2\cos(\theta) &= e^{j\theta} + e^{-j\theta} \Rightarrow \cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \\ 2j\sin(\theta) &= e^{j\theta} - e^{-j\theta} \Rightarrow \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j} = \frac{-je^{j\theta} + je^{-j\theta}}{2} \end{aligned}\right\} e^{j\pi} + 1 = 0$$

Polar Form

$$z = a + bj = |z| e^{j\angle z}$$

$$\bar{z} = |z| e^{-j\angle z} \qquad \because \sin(-\theta) = -\sin(\theta) \text{ and } \cos(-\theta) = \cos(\theta)$$

$$z_1 \cdot z_2 = \left(|z_1| e^{j\angle z_1}\right) \cdot \left(|z_2| e^{j\angle z_2}\right) = \left(|z_1| \cdot |z_2|\right) e^{j(\angle z_1 + \angle z_2)}$$