

18x0=0	19x0=0	20x0=0	21x0=0	22x0=0	23x0=0	24x0=0
18x1=18	19x1=19	20x1=20	21x1=21	22x1=22	23x1=23	24x1=24
18x2=36	19x2=38	20x2=40	21x2=42	22x2=44	23x2=46	24x2=48
18x3=54	19x3=57	20x3=60	21x3=63	22x3=66	23x3=69	24x3=72
18x4=72	19x4=76	20x4=80	21x4=84	22x4=88	23x4=92	24x4=96
18x5=90	19x5=95	20x5=100	21x5=105	22x5=110	23x5=115	24x5=120
18x6=108	19x6=114	20x6=120	21x6=126	22x6=132	23x6=138	24x6=144
18x7=126	19x7=133	20x7=140	21x7=147	22x7=154	23x7=161	24x7=168
18x8=144	19x8=152	20x8=160	21x8=168	22x8=176	23x8=184	24x8=192
18x9=162	19x9=171	20x9=180	21x9=189	22x9=198	23x9=207	24x9=216
18x10=180	19x10=190	20x10=200	21x10=210	22x10=220	23x10=230	24x10=240

2x0=0	3x0=0	4x0=0	5x0=0	6x0=0	7x0=0	8x0=0	9x0=0
2x1=2	3x1=3	4x1=4	5x1=5	6x1=6	7x1=7	8x1=8	9x1=9
2x2=4	3x2=6	4x2=8	5x2=10	6x2=12	7x2=14	8x2=16	9x2=18
2x3=6	3x3=9	4x3=12	5x3=15	6x3=18	7x3=21	8x3=24	9x3=27
2x4=8	3x4=12	4x4=16	5x4=20	6x4=24	7x4=28	8x4=32	9x4=36
2x5=10	3x5=15	4x5=20	5x5=25	6x5=30	7x5=35	8x5=40	9x5=45
2x6=12	3x6=18	4x6=24	5x6=30	6x6=36	7x6=42	8x6=48	9x6=54
2x7=14	3x7=21	4x7=28	5x7=35	6x7=42	7x7=49	8x7=56	9x7=63
2x8=16	3x8=24	4x8=32	5x8=40	6x8=48	7x8=56	8x8=64	9x8=72
2x9=18	3x9=27	4x9=36	5x9=45	6x9=54	7x9=63	8x9=72	9x9=81
2x10=20	3x10=30	4x10=40	5x10=50	6x10=60	7x10=70	8x10=80	9x10=90

25x0=0	26x0=0	27x0=0	28x0=0	29x0=0	30x0=0
25x1=25	26x1=26	27x1=27	28x1=28	29x1=29	30x1=30
25x2=50	26x2=52	27x2=54	28x2=56	29x2=58	30x2=60
25x3=75	26x3=78	27x3=81	28x3=84	29x3=87	30x3=90
25x4=100	26x4=104	27x4=108	28x4=112	29x4=116	30x4=120
25x5=125	26x5=130	27x5=135	28x5=140	29x5=145	30x5=150
25x6=150	26x6=156	27x6=162	28x6=168	29x6=174	30x6=180
25x7=175	26x7=182	27x7=189	28x7=196	29x7=203	30x7=210
25x8=200	26x8=208	27x8=216	28x8=224	29x8=232	30x8=240
25x9=225	26x9=234	27x9=243	28x9=252	29x9=261	30x9=270
25x10=250	26x10=260	27x10=270	28x10=280	29x10=290	30x10=300

10x0=0	11x0=0	12x0=0	13x0=0	14x0=0	15x0=0	16x0=0	17x0=0
10x1=10	11x1=11	12x1=12	13x1=13	14x1=14	15x1=15	16x1=16	17x1=17
10x2=20	11x2=22	12x2=24	13x2=26	14x2=28	15x2=30	16x2=32	17x2=34
10x3=30	11x3=33	12x3=36	13x3=39	14x3=42	15x3=45	16x3=48	17x3=51
10x4=40	11x4=44	12x4=48	13x4=52	14x4=56	15x4=60	16x4=64	17x4=68
10x5=50	11x5=55	12x5=60	13x5=65	14x5=70	15x5=75	16x5=80	17x5=85
10x6=60	11x6=66	12x6=72	13x6=78	14x6=84	15x6=90	16x6=96	17x6=102
10x7=70	11x7=77	12x7=84	13x7=91	14x7=98	15x7=105	16x7=112	17x7=119
10x8=80	11x8=88	12x8=96	13x8=104	14x8=112	15x8=120	16x8=128	17x8=136
10x9=90	11x9=99	12x9=108	13x9=117	14x9=126	15x9=135	16x9=144	17x9=153
10x10=100	11x10=110	12x10=120	13x10=130	14x10=140	15x10=150	16x10=160	17x10=170

→ Prime Numbers 0-100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
									100

→ Prime Numbers 100-200

100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127	128	129
130	131	132	133	134	135	136	137	138	139
140	141	142	143	144	145	146	147	148	149
150	151	152	153	154	155	156	157	158	159
160	161	162	163	164	165	166	167	168	169
170	171	172	173	174	175	176	177	178	179
180	181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198	199

→ Squares & Cubes 1-30

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
0	0	0	10	100	1000	20	400	8,000
1	1	1	11	121	1331	21	441	9,261
2	4	8	12	144	1,728	22	484	10,648
3	9	27	13	169	3,197	23	529	12,167
4	16	64	14	196	2,744	24	576	13,824
5	25	125	15	225	3,375	25	625	15,625
6	36	216	16	256	4,096	26	676	17,576
7	49	343	17	289	4,913	27	729	19,683
8	64	512	18	324	5,832	28	784	21,952
9	81	729	19	361	6,859	29	841	24,389
						30	900	27,000

→ Percentage

Fraction	%	Fraction	%	Fraction	%	Fraction	%	Fraction	%
3/4	75%	5/4	125%	1/1	100%	1/6	16.66%	1/11	9.09%
4/5	80%	3/2	150%	1/2	50%	1/7	14.28%	1/12	8.33%
2/3	66.66%	1/16	6.25%	1/3	33.33%	1/8	12.5%	1/13	7.69%
5/6	83.33%			1/4	25%	1/9	11.11%	1/14	7.14%
6/5	120%			1/5	20%	1/10	10%	1/15	6.66%

→ Factorial

$$0! = 1 \quad 4! = 4 \times 3 \times 2 \times 1 = 24$$

$$1! = 1 \quad 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$2! = 2 \quad 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

$$3! = 3 \times 2 \times 1 = 6 \quad 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

g) divisible by 7 → difference between two alternate groups taking

3-digits at a time should either be zero or multiple of 7.

eg - $\underline{\begin{array}{r} \text{odd} & \text{even} & \text{odd} \\ 5 & 5 & 0 \\ + & 5 & 0 & 6 \\ \hline 5 & 5 & 6 \end{array}}$ → $(\underline{\text{odd}} + \underline{\text{even}}) - (\underline{\text{even}}) = 556 - 500 = 56 \nmid 7 = 0$

h) divisible by 13 → difference between two alternate groups taking 3-digits at a time should either be zero or

multiple of 13.

eg - $\underline{\begin{array}{r} \text{even} & \text{odd} & \text{even} & \text{odd} \\ 2 & 0 & 0 & 1 & 7 & 4 \\ - & 2 & 0 & 0 & 1 & 7 & 4 \\ \hline 2 & 6 \end{array}}$ → $26 \nmid 13 = 0$

i) divisible by 17 → multiply last digit by 5 & subtract that from the rest. If result $\nmid 17 = 0$, then $n \cdot 17 = 0$ too.

eg - $98 \underline{5} \rightarrow [98][5] \rightarrow 98 - 6 \times 5 = 98 - 30 = 68 \nmid 17 = 0 \Rightarrow 986 \nmid 17 = 0$

$87 \underline{6} \rightarrow [87][6] \rightarrow 87 - 6 \times 5 = 87 - 30 = 57 \nmid 17 = 0 \Rightarrow 876 \nmid 17 = 0$

divisible by 19 → multiply last digit by '2' & add it to the rest.

If sum $\cdot 19 = 0$, then $n \cdot 19 = 0$ too.

eg → $47 \underline{5} \rightarrow [47][5] \rightarrow 47 + 5 \times 2 = 47 + 10 = 57 \nmid 19 = 0 \Rightarrow 475 \nmid 19 = 0$

→ $57 \underline{5} \rightarrow [57][5] \rightarrow 57 + 5 \times 2 = 57 + 10 = 67 \nmid 19 = 0 \Rightarrow 575 \nmid 19 = 0$

Divisibility Rules

→ Division by ZERO is not possible

→ If two numbers are divisible by a number, then their sum & difference is also divisible by the number.

eg - $n_1 = 63 \cdot 1.9 = 0$ sum = $63 + 27 = 90 \nmid 9 = 0$

$n_2 = 27 \cdot 1.9 = 0$ diff = $63 - 27 = 36 \nmid 9 = 0$

$n_3 = 9$

a) divisible by 2 → unit place is even or zero. eg → 324

b) divisible by 3 → sum of digits is divisible by 3. eg - 324 → $3+2+4=9 \nmid 3=0$

divisible by 9 → sum of digits is divisible by 9. eg - 324 → $3+2+4=9 \nmid 9=0$

c) divisible by 4 → Last 2 digits divisible by 4 or be '00'. eg - 324 → $24 \nmid 4=0$

divisible by 8 → Last 3 digits divisible by 8 or be '000'. eg - 1088 → $088 \nmid 8=0$

d) divisible by 5 → unit digit is '5' or '0'. eg - 15 → $5 \cdot 5 = 0$

divisible by 10 → unit digit is '0'. eg - 100 →

divisible by 25 →

e) divisible by 6 → divisible by co-primes 2 & 3 → eg 324 → $4 \cdot 2 = 0$ $324 \rightarrow 3+2+4=9 \nmid 3=0$

f) divisible by 11 → difference between sum of ~~even~~ digits in odd & even places should be either be zero or divisible by 11.

eg $\underline{\begin{array}{r} 8 & 2 & 8 & 3 \\ + & 7 & 5 & 6 \\ \hline 15 & 7 & 9 & 9 \end{array}}$ → $(2+3)-(8+6) = 5-14 = 11 \cdot 1 \cdot 11 = 0$

Properties of divisibility

a) dividend = divisor \times quotient + remainder

$$D = [d \times q] + r$$

$$\begin{array}{r} \cancel{1} \cancel{5} \cancel{3} \cancel{2} \cancel{1} \\ \cancel{1} \cancel{5} \cancel{3} \cancel{2} \cancel{1} \\ \hline d \div D \quad q \\ \cancel{1} \cancel{5} \cancel{3} \cancel{2} \cancel{1} \\ \hline r = 1 \end{array}$$

b) Greatest 'm' digit number exactly divisible by a number

→ Subtract the remainder

eg - Greatest 3 digit number divisible by 13

$$\text{Greatest 3 digit num} = 999. \quad 999 \div 13 \rightarrow D=76, r=11$$

$999 - 11 = 988 \rightarrow$ Greatest 3 digit number divisible by 13

c) Least 'm' digit number exactly divisible by a number

→ Add (divisor - remainder) $[d - r]$

eg - Least 3 digit number divisible by 13

$$\text{Least 3 digit num} = 100. \quad 100 \div 13 \rightarrow D=7, r=9$$

$$100 + [13 - 9] = 100 + 4 = 104 \rightarrow \text{least 3 digit number divisible by 13}$$

→ Sieve of Eratosthenes : Let 'p' be a number & 'n' be the smallest counting number such that $n^2 \geq p$.

e.g. - To check 811 is prime or not $\rightarrow 29^2 = 841 > 811$

- now check if 811 is divisible by any of prime nos. below 29.

- none of prime nos below 29 divide 811 $\Rightarrow 811$ is prime number.

a) divisible by 2 \rightarrow unit place is even or '0'. eg - 324

b) divisible by 3 \rightarrow sum of digits is divisible by 3. eg $\rightarrow 324 \rightarrow 3+2+4 = 9 \cdot 1 \cdot 3 = 0$
divisible by 9 \rightarrow sum of digits is divisible by 9. eg $\rightarrow 324 \rightarrow 3+2+4 = 9 \cdot 1 \cdot 9 = 0$

c) divisible by 4 \rightarrow last 2 digits divisible by '4' or be '00'. eg - 324 $\rightarrow 24 \cdot 1 \cdot 4 = 0$

divisible by 8 \rightarrow last 3 digits divisible by '8' or be '000'. eg - 324 $\rightarrow 088 \cdot 1 \cdot 8 = 0$

divisible by 16 \rightarrow last 4 digits divisible by '16' or be '0000'. eg - 4096 $\rightarrow 4096 \cdot 1 \cdot 16 = 0$

d) divisible by 5 \rightarrow unit digit is '5' or '0' eg - 15

divisible by 10 \rightarrow unit digit is '0'. eg 100

divisible by 20 \rightarrow unit digit is '0' & tens digit is even. eg - 460 $\rightarrow 6 \cdot 1 \cdot 2 = 0$

e) divisible by 6 \rightarrow divisible by coprimes '2 & 3'. eg 324 $\rightarrow 4 \cdot 1 \cdot 2 = 0$. $324 \rightarrow 3+2+4 = 9 \cdot 1 \cdot 3 = 0$

divisible by 12 \rightarrow divisible by coprimes '3 & 4'

divisible by 14 \rightarrow divisible by coprimes '2 & 7'

divisible by 15 \rightarrow divisible by coprimes '3 & 5'

divisible by 18 \rightarrow divisible by coprimes '2 & 9'

→ AP : Arithmetic Progression

GP: Geometric Progression

→ AP: quantities increase/decrease by a common difference

a = first term ; d = common difference ; l = last term

→ General form = $a, a+d, a+2d, a+3d, \dots, a+(n-1)d$

→ Last term, $l = a + (n-1)d$

→ Sum of ' n ' terms = $\frac{n}{2} [2a + (n-1)d] = \frac{n}{2} [a + a(n-1)d] = \frac{n}{2} [a + l]$

$$n = \frac{l-a}{d} + 1$$

→ GP: quantities increase/decrease by a constant factor

→ a = first term ; r = common ratio ; l = last term

→ General form = $a, ar, ar^2, ar^3, \dots, ar^{n-1}$
 $= a r^0, a r^1, a r^2, a r^3, \dots, a r^{n-1}$

→ Last term, $l = a r^{(n-1)}$

→ Sum of ' n ' terms, $S_n = \frac{a(r^n - 1)}{r - 1}$

$$x^a \cdot x^b = x^{a+b}; \quad x^{\frac{a}{b}} = x^{\frac{a-b}{b}}; \quad (x^a)^b = x^{a \cdot b}$$

→ HCF (i) Factorization Method (ii) Difference Method

(*) HCF \leq the smallest of given numbers

→ LCM (i) Factorization Method (iii) LCM-HCF multiplication property

(*) LCM \geq the greatest of given numbers

→ (a) Product of two given numbers is equal to the product of their HCF & LCM $\Rightarrow A \times B = \text{HCF}(A,B) \times \text{LCM}(A,B)$

(b) If a, b, c are three numbers that divide a number ' n ' to leave the same remainder ' r ', the smallest value of ' n ' is
 $n = (\text{LCM of } a, b, c) + r$

→ HCF \rightarrow (1) Apply Rule/property/Method (2) Find HCF

LCM \rightarrow (1) Find LCM (2) Apply Rule/property/Method

→ In LCM problem, if difference is common/constant, then
Result = LCM - Common Difference.

→ LCM = LCM of ratio units \times HCF \Rightarrow HCF = $\frac{\text{LCM}}{\text{LCM of Ratio Units}}$

→ In fractions, LCM = LCM of Numerators
HCF of Denominators

& HCF = $\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$

→ Ages

Past

$$(x-15)$$

[age]
before

Present

$$(x)$$

[of/then]

$$\Rightarrow n \times \text{age}$$

Future

$$(x+15)$$

[after]
hence
later

Simple Average (Arithmetic Mean)

$$\rightarrow \text{Average}(A) = \frac{\text{Sum}(S)}{\text{Number}(n)} \Rightarrow \text{Avg} = \frac{\text{Sum}}{n}$$

$$\rightarrow \text{Sum}(S) = \text{Average}(A) \times \text{Number}(n) \Rightarrow S = A \times n$$

Weighted Average

$$\rightarrow \text{If values } y^1, y^2, y^3, \dots \text{ occur } w_1, w_2, w_3, \dots \text{ times then}$$

$$\text{Weighted Average} = \frac{w_1 y^1 + w_2 y^2 + w_3 y^3 + \dots}{w_1 + w_2 + w_3 + \dots}$$

→ Ratio & Proportion

$$a:b = a/b = a \div b$$

$$\rightarrow \text{If } a:b = c:d \text{ OR } \frac{a}{b} = \frac{c}{d} \text{ OR } a:b :: c:d$$

$$1) axd = bxc \Rightarrow \frac{axd}{bxd} = \frac{bxc}{bxd} = \frac{a}{b} = \frac{c}{d}$$

$$2) \frac{b}{a} = \frac{d}{c} \text{ (invertendo)} \Rightarrow b \times c = a \times d \Rightarrow axd = b \times c$$

$$3) \frac{a}{c} = \frac{b}{d} \text{ (Altendendo)} \Rightarrow a \times d = b \times c$$

$$4) \frac{a+b}{b} = \frac{c+d}{d} \text{ (componendo)} \Rightarrow ad + bd = bc + bd \Rightarrow axd = b \times c$$

$$5) \frac{a-b}{b} = \frac{c-d}{d} \text{ (dividendo)} \Rightarrow ad - bd = bc - bd \Rightarrow axd = b \times c$$

$$6) \frac{a+b}{a-b} = \frac{c+d}{c-d} \text{ (componendo & dividendo)} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d} = \frac{a+b}{c-d} = \frac{c+d}{a-b}$$

extreme terms

$$\rightarrow \text{If } a:b = c:d, \text{ then Product of means} = \text{Product of extremes}$$

mean terms

$$\Rightarrow b \times c = a \times d$$

$$\rightarrow \text{If } a:b = b:c \text{ OR } a/b = b/c \text{ OR } a:b :: b:c$$

$$\text{then } b \times b = a \times c \Rightarrow b^2 = ac$$

→ Average for Consecutive Numbers

In case of consecutive numbers OR consecutive odd numbers OR consecutive even numbers, average is always the middle number

$$\text{eg} - 5, 6, 7, 8, 9 \rightarrow 7 \quad | 5, 6, 7, 8 \rightarrow 6.5 |$$

→ New value = old value + $(n \pm 1)$ (difference)

$(+) \rightarrow$ member added ; $(-) \rightarrow$ member removed ; diff = 1 old avg - new avg

→ Δ increase/decrease in avg = $n \times \text{change}$

total age value of replacement = value of removed member + Δ

$$\text{eg: } \Delta \text{ increase in age} = 16 \times 3y = 48y$$

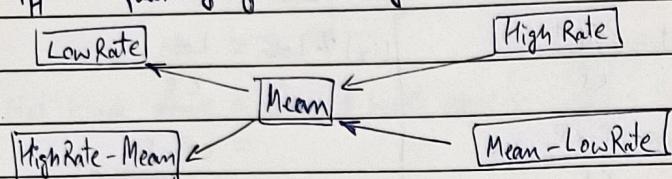
$$\text{total age of person replacing} = 27 + 48y = 75y$$

$$\rightarrow \text{If } x:y \text{ & } y:z, \text{ then } x:y:z = x:y:y:z$$

$$\left[\begin{array}{l} x:y \\ \text{or} \\ y:z \end{array} \right]$$

→ By Rule of Alligation,

$$Q_L = \frac{\text{Quantity of cheaper ingredient}}{\text{Quantity of dearer ingredient}} = \frac{d-m}{m-c}$$



$$\rightarrow \text{In Profit, } SP = CP + \text{Profit} \Rightarrow SP = CP \left(100 + \frac{\% \text{age Profit}}{100}\right)$$

$$(SP > CP) \quad \text{Profit} = SP - CP \Rightarrow \text{Profit \% age} = \frac{SP - CP}{CP} \times 100$$

$$CP = SP - \text{Profit} \Rightarrow CP = \frac{SP \times 100}{100 + \% \text{age Profit}}$$

$$\text{In Loss, } SP = CP - \text{Loss} \Rightarrow SP = CP \left(100 - \frac{\% \text{age loss}}{100}\right)$$

$$(CP > SP) \quad CP = SP + \text{Loss} \Rightarrow CP = SP \left(100 + \frac{\% \text{age loss}}{100}\right)$$

$$\text{Loss} = CP - SP \Rightarrow \text{Loss \% age} = \frac{CP - SP}{SP} \times 100$$

$$\rightarrow \text{Final concentration} = \text{Initial} \left(\frac{1-R}{\text{Initial}}\right)^n$$

where

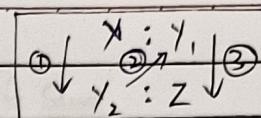
Final concentration = amount of concentration remaining after the process

'n' = no. of times process is done

'R' = replaced quantity

'Initial' = Initial concentration.

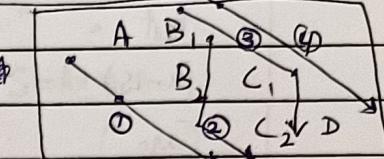
→ If $X : Y_1$ & $Y_2 : Z$
then $X : Y : Z = XY_2 : Y_2Y_1 : Y_1Z$



→ If ~~A : B, B : C, C : D~~

→ If $A : B_1, B_2 : C_1, C_2 : D$

$$\text{then } A : B : C : D = AB_2C : B_1B_2C_2 : BC_1C_2 : B_1C_1D$$



→ Share of 'a' = Total amount $\times \frac{a}{a+b+c}$; Total Amount = $a+x+b+y+c$

Any Share = Total Amount $\times \frac{\text{Related Ratio Term}}{\text{Sum of Ratio Terms}}$

Mixtures & Alligation

→ Alligation - Rule which enables us to find the ratio of ingredients at given price to be mixed to produce a mixture of desired price (mixing/blending).

→ Mean Price - Cost Price (C.P.) of a unit quantity of mixture

→ Dearer - most expensive ingredient.

→ Rule of Alligation - Always maintain the order in which problem is given, else answer gets changed

→ CP of cheaper (c) (d) CP of dearer ingredient (-) (-) ingredient

Mean Price (m)

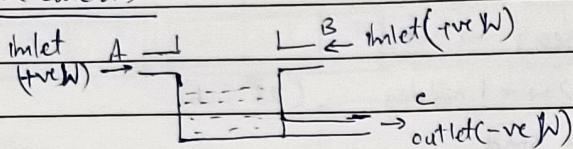
(d-m)

(m-c)

CP of dearer - Mean
ingredient price

Mean - CP of cheaper
ingredient

Pipes & Cisterns



→ Net Work done = Sum of Work done by inlets - Sum of work done by outlets

Chain Rule

→ Work Done = No. of Men × Days × Hrs/day

$$W = M \times D \times H$$

$$\rightarrow W_1 = M_1 \times D_1 \times H_1 ; W_2 = M_2 \times D_2 \times H_2$$

$$\frac{W_1}{W_2} = \frac{M_1 \times D_1 \times H_1}{M_2 \times D_2 \times H_2}$$

Probability

→ How likely an event is supposed to happen

→ Probability = Favourable Outcome

Total Number of Outcomes

→ AND → Multiply eg → 1 Green AND 1 Blue ball in a box

→ OR → Add eg → 1 red OR 1 blue ball in a box

$$\begin{aligned} [3] &\leftarrow \text{Total No. of things} & 3 & \quad \begin{matrix} 1^{\text{st}} \\ 2^{\text{nd}} \end{matrix} \\ \rightarrow & C & C_2 = \frac{3 \times 2}{2 \times 1} = 3 \end{aligned}$$

[2] selected / drawn / picked up

→ 0 < probability < 1

→ at least : min to max ; at most : max to min

$$\rightarrow {}^n C_r = \frac{n!}{r!(n-r)!} = \frac{(n) \times (n-1) \times \dots \times (n-r+1)}{r!(n-r)!}$$

Profit & Loss

→ ($SP > CP$) ($CP > SP$)

→ (i) Profit = $SP - CP$

→ (ii) % Profit = $\frac{\text{Profit}}{CP} \times 100$

$$= \frac{SP - CP}{CP} \times 100$$

(iii) Loss = $CP - SP$

(iv) % Loss = $\frac{\text{Loss}}{CP} \times 100$

$$= \frac{CP - SP}{CP} \times 100$$

→ Multipliers to find SP

for Profit, $SP = CP \times \frac{(100 + \% \text{ profit})}{100}$

for Loss, $SP = CP \times \frac{(100 - \% \text{ loss})}{100}$

Time & Work

→ Work (Effort) = Manpower × time

Total Work = Efficiency × Total time

→ Total Work = LCM \Rightarrow If A, B, C, Total Work = LCM(A, B, C)

→ Efficiency = $\frac{\text{Total Work}}{\text{Total Time}} \Rightarrow A = \frac{TW}{t}$

→ OR, Total Work = Efficiency × Total time $\Rightarrow A \cdot TW = A \times t$

→ Efficiency = capacity to work

→ Efficiency $\propto \frac{1}{\text{Time}}$; Efficiency $\propto \text{Work}$

For Months

J	F	M	A	M	J	J	A	S	O	N	D
0	3	3	6	1	4	6	2	5	0	3	5

For Years

1600 - 1699	6	
1700 - 1799	4	
1800 - 1899	2	
1900 - 1999	0	
2000 - 2099	6	

For calendar re-use

(any $y \mod 4$, then only possible remainders are 0, 1, 2, 3)

$$\text{If } \text{rem} = 0, \quad y \mod 28$$

$$\text{If } \text{rem} = 1, \quad y \mod 6$$

$$\text{If } \text{rem} = 2/3, \quad y \mod 11$$

← 26 Jan 1947

Last 2 digit of YYYY = 47
$Y \mod 4 = 47 \mod 4 (9) = 11$
odd day for MM = 0
Date as it is = 26
Odd day for YY00(1900) = 0
<u>Total</u> <u>84.7.4 = 0</u>
<u>If year is leap then (-1)</u> <u>-0</u>
<u>0 → Sum</u>

Time & Distance

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} \Rightarrow S = \frac{D}{t} \Rightarrow D \propto t$$

$$\text{Distance} = \text{Speed} \times \text{time} \Rightarrow D = S \times t \Rightarrow D \propto t$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \Rightarrow t = \frac{D}{S} \Rightarrow D \propto \frac{1}{t}$$

$$1 \text{ km/hr} = \frac{5}{18} \text{ m/s} \left[\frac{1000 \text{ m}}{3600 \text{ s}} \right] ; 1 \text{ m/s} = \frac{18}{5} \text{ km/hr} \left[\frac{3600 \text{ m}}{1000 \text{ s}} \right]$$

Calendar

→ Non-Leap Year $\rightarrow 365d$

$$1y = 52w + 1 \text{ odd day} \quad (28^{\text{th}} \text{ Feb})$$

→ Leap Year $\rightarrow 366d$

$$1y = 52w + 2 \text{ odd days} \quad (29^{\text{th}} \text{ Feb})$$

→ Century Leap year \rightarrow exactly divisible by 400 (4 & 100)

eg - 1600 & 2000 were century Leap year

→ years 1700, 1800, 1900 were not century leap years

→ 01/Jan/2001 A.D. (Anno Domini) was a Monday

→ In a century $\rightarrow 24 \text{ leap years}$

24 leap years 76 non-leap years

$$24 \times (2 \text{ odd days})$$

$$76 \times (1 \text{ odd day})$$

Years No. of odd

$$\frac{48}{7} \rightarrow \text{remainder } 6 + 6$$

$$12 \times 7 = 5 \leftarrow \text{remainder}$$

Normal year 1

Leap 2

100y 5

200y 3

300y 1

400y 0

$$100y = 5 \text{ odd/extra days.}$$

$$200y = 10 \times 7 = 3 \text{ odd days}$$

$$300y = 15 \times 7 = 1 \text{ odd day.}$$

$$400y = 0 \text{ odd day (leap century leap year)}$$

Day of week No. of odd

Month Remainder Sum 0

Jan, Mar, May, Jul 31%7 3 Mon 1

Aug, Oct, Dec Tue 2

Feb (28%7) 0 or (29%7) 0 (Non-Leap) Wed 3

1 (Leap) Thu 4

Apr, Jun, Sep, Nov 30%7 2 Fri 5

5 Sat 6

Boats & Streams

- $\frac{1}{2}$, speed of boat in still water = x kmph & speed of the stream = y kmph
- Speed of boat going downstream, $S_{DN} = (x+y)$ kmph
 - Speed of boat going upstream, $S_{Up} = (x-y)$ kmph
 - $S_{DN} + S_{Up} = 2x$; $S_{DN} - S_{Up} = 2y$
 - \Rightarrow boat, $x = \frac{S_{DN} + S_{Up}}{2}$; stream, $y = \frac{S_{DN} - S_{Up}}{2}$

$$\rightarrow \frac{x}{y} = \frac{n+1}{n-1}; n = \text{factor, } 5 \text{ times the time to row up than to row down}$$

$$\Rightarrow n=5$$

Clocks

- 1 clock = 360° ; $60\text{ min} = 6^\circ$ each; $12\text{ hrs} = 30^\circ$ each
- In 60 min , minute hand gains 55 min on the hour hand.
- In 1 hr (60 min), both hands coincide once & make right angle twice
- Hands are in straight line while coincident or opposite (1) (1)
- Two hands at right angle, they are 15 min spaces apart (2) (2)
- Two hands at 180° or opposite, they are 30 min spaces apart. (1) (1)
- Every hour, hands coincide once after 65 min (11-12 o'clock, \Rightarrow it coincides at 12°)
- 11 times in 12 hrs \Rightarrow 22 times in 24 hrs

Two hands coincide, 11 times in 12 hrs \Rightarrow 22 times in 24 hrs

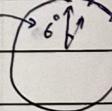
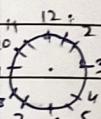
In opposite, 11 times in 12 hrs \Rightarrow 22 times in 24 hrs

Every hour, hands make right angle off two times in 65 min .
right angle \Rightarrow , 22 times in 12 hrs \Rightarrow 44 times in 24 hrs

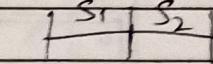
Speed of min hand = $60\text{ min sp}/\text{hr} = \frac{1 \text{ min space}}{\text{min}}$

Speed of hr hand = $5 \frac{\text{min sp.}}{\text{hr}} = \frac{5}{12} \frac{\text{min spaces}}{\text{min}}$

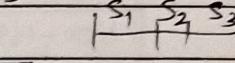
Relative speed of min hand & hr hand = $\frac{1}{1} - \frac{1}{12} \frac{\text{min sp.}}{\text{min}} = \frac{11}{12} \frac{\text{min spaces}}{\text{min}}$



\rightarrow Same distance travelled by speeds S_1 & S_2 ,
then Average Speed, $S_{avg} = \frac{2 \times (S_1 \times S_2)}{(S_1 + S_2)}$



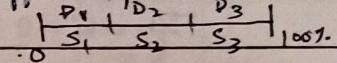
\rightarrow same distance travelled by speeds S_1 , S_2 & S_3
then, average speed, $S_{avg} = \frac{3 \times (S_1 \times S_2 \times S_3)}{(S_1 S_2) + (S_2 S_3) + (S_3 S_1)}$



\rightarrow same direction, Relative speed = $v_1 - v_2$ (A) (B)
Relative speed = $v_2 - v_1$

\rightarrow opposite direction, relative speed = $v_1 + v_2$ (A) \rightarrow (B)

\rightarrow different distance D_1 , D_2 , D_3 travelled at different speeds
 S_1 , S_2 , S_3 then



Average Speed, $S_{avg} = \frac{D_1 + D_2 + D_3}{(S_1 + S_2 + S_3)}$

Trains

S_1 = speed of train S_2 = speed of object

L_1 = length of train L_2 = length of object

t = time taken by train to completely pass the object

\rightarrow Case A: stationary object without considerable length ($L_2 = 0$) ($S_2 = 0$)

$$L_1 = S_1 \times t$$

\rightarrow Case B: stationary object with considerable length ($S_2 = 0$)

$$L_1 + L_2 = S_1 \times t$$

\rightarrow Case C: Moving object without considerable length ($L_2 = 0$)

$$L_1 = (S_1 \pm S_2) t$$

(same dir \rightarrow +ve, opp dir \rightarrow -ve)

\rightarrow Case D: Moving object with considerable length ($L_2 = V$) ($S_2 = V$)

$$L_1 + L_2 = (S_1 \pm S_2) t$$

→ If the difference b/w CI & SI is of 3 yrs, then

$$\text{Difference} = 3 \times P(R)^2 + P\left(\frac{R}{100}\right)^3$$

Partnership

$$\frac{C_1 T_1}{C_2 T_2} = \frac{P_1}{P_2}$$

$C_1, C_2 \rightarrow \text{Capital}; T_1, T_2 \rightarrow \text{Investment period}$
(in months)

$$P \propto C; P \propto C; C \propto \frac{1}{T}$$

Permutation & Combination

→ Permutation - No. of ways a group of things can be arranged.
eg A, B, C → ABC, ACB, BAC, CAB, CBA → 6

$${}^n P_r = \frac{n!}{(n-r)!} \rightarrow \text{order/sequence matters here}$$

$${}^n C_r = \frac{n!}{r!(n-r)!} \rightarrow \text{order/sequence does NOT matter here}$$

$${}^n C_r = {}^n C_{n-r}$$

→ for repetition, divide by $n!$, $n \Rightarrow$ no. of repetition.

$$\rightarrow \text{Angle} = \left| 30^\circ - \frac{11}{2} M \right| \Leftrightarrow \text{Angle} = \left| 30^\circ - 5.5 M \right|$$

Percentage

$$\frac{x}{y} \times 100$$

$$\rightarrow \% \text{ net change} = -\left(\frac{x}{100}\right)^2$$

→ If two step change in Percentage → If number is changed by a % and the result is again changed by b%, then net
Net % change in number = $\frac{a+b+ab}{100}$ (inc → +ve; dec → -ve)

$$\rightarrow \text{Expenditure} = \text{Price} \times \text{Consumption} \Rightarrow \text{Price} \propto \frac{1}{\text{Consumption}}$$

$\Rightarrow \text{Price} \propto \text{Expenditure}$

Interest

P=Principal, R=Rate of interest, N=Time in years, I=interest, A=amount

$$\rightarrow A = P + I$$

$$\rightarrow \text{Simple Interest}, SI = \frac{P \times R \times N}{100} \Rightarrow SI \text{ is an AP}$$

Basic Principal is constant

$$\rightarrow \text{In compounding interest}, A = \left(1 + \frac{R}{100}\right)^T \quad T = \text{periods of compounding}$$

$CI = A - P \quad R = \text{rate for compounding period}$

Basic principal keeps on increasing, because of interest on interest
 $\Rightarrow CI$ is an G.P.

→ If diff b/w CI & SI is of 2 yrs, then

$$\text{Difference} = \frac{P(R)^2}{10000} = \frac{P(R)^2}{(10)^2} \quad P = \text{Principal}$$

$R = \text{Rate of interest}$