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Study of predicting aerodynamic heating for hypersonic boundary layer flow over a flat plate

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ABSTRACT

This work deals with how to use suitable formula of convective heat transfer coefficient in order to exactly calculate aerodynamic heating in high speed laminar flow over a flat plate. One formula of convective heat transfer coefficient of hypersonic flow was developed by modified Reynolds analogy, using reference temperature or reference enthalpy. The formula was checked basing on the Blasius similar solution of compressible flow. Due to similar solutions still exists in flat hypersonic boundary layer flow for variable specific heat, which is real gas effect resulting of high temperature, the heat flux could be calculated at isothermal wall condition and thus the convective heat transfer coefficient. In this paper, the computation is conducted in the flat hypersonic boundary layer flow, under isothermal condition with wall temperature ranging from 0.1 to 0.9 times the adiabatic temperatures, incoming flow Mach number 1 to 10. Both the results by the formula using reference enthalpy and that using Fourier's Law by Blasius solution of considering real gas effect are good agreement with each other, and the maximum relative error being less than 3%, which is much better than that by using reference temperature one. In addition, the formula was applied to real computation of supersonic flow passing one meter long flat plate with the wall being convective heat transfer. The time needed and wall temperature rising rule are studied for the wall temperature from the initial temperature to 0.9 times adiabatic temperature. The results indicated that whether isothermal or adiabatic condition in hypersonic flow chosen as the wall boundary condition is not suitable in real flow especially during the very long period flying.

1. Introduction

Accurately calculating the heat flux plays an important role in the design of thermal protection system especially for the vehicle in super- and hypersonic flying. It is a focal point in the field of aerospace. For the near space aerocraft, Mach number is always less than 20. Convection is usually the dominant form of heat transfer in gas. Therefore, the calculation of aerodynamic heating is composed mainly of convection heat transfer while radiation heat transfer accounts for a small proportion. Even for the flight vehicle at high Mach number when returning to the atmosphere, radiation heat transfer must be considered and convection heat transfer occupies a certain proportion. It is prerequisite that heat-transfer coefficient needs a suitable formula.

For hypersonic speed flying vehicle, aerodynamic heating occurs severely in the region close to the stagnation. Many studies of how to decide stagnation heat flux have been done [1–4] in both the experimental and computational areas, especially progresses have been made in application to thermal protection system. Qu et al. [5] propose a new scheme called AUSMPWM2 for hypersonic heating predictions. Zhang et al. [6] established a new correction model for current k-ω SST model which can improve the accuracy of aerothermal prediction. During hypersonic aircraft flies in very long period in the near space, severe aerodynamic heating also occurred near to its fuselage. Up to now very few studies have been done on this. When air flows along solid surface of hypersonic aerocraft, kinetic energy converted partly into heat energy due to the viscosity, which results in air temperature become very large. The air temperature near to the wall becomes so high that the wall will be heated even though the incoming air temperature is lower than the wall at the beginning. In engineering, convective heat transfer coefficient formula has been put forward for laminar compressible flow, which is based on modified Reynolds analogy, Blasius solution for the incompressible flow, using reference enthalpy or reference temperature [7–9] proposed by Eckert.

Convective heat transfer coefficient of compressible laminar flow over flat plate can be determined by the temperature gradient on the wall due to the similar solution of temperature profile exists in compressible laminar boundary layer flow. The research found

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that [10,11] the increasing of gas temperature attributed to shock waves and the reducing velocity near the wall of flat plate in the boundary layer. When the air temperature ranges from 800 K to 2500 K, the molecular vibration energy of the air is excited and the specific heat of gas is no longer a constant but relates to temperature. The real gas effect [12] on the solution of hypersonic boundary layer flow is studied and compared with the case of constant specific heat. It is found that the variable specific heat indeed has some effects on both temperature profile and adiabatic wall temperature. Although the boundary layer equation with the constant specific heat is different from variable specific heat one, the similar solution of temperature profile still exists in both cases. When the air temperature is higher than 2500 K or 4000 K, dissociation of oxygen and nitrogen molecules will take place respectively, in which whether similar solution exists or not needs further studying.

The real gas effect must be taken into consideration when calculating aerodynamic heat for flying vehicle in hypersonic speed in near space. Which one is better, whether using reference enthalpy or reference temperature, is worth researching. Because the similar solutions exist in hypersonic boundary layer flow for variable specific heat, the heat flux on isothermal wall may be calculated for any given wall temperature and can be served as a standard value.

The DNS or flow stability analysis is usually conducted for the wall boundary condition being selected as isothermal or adiabatic condition. Many new computational methods have also been put forward [13–15]. Schäfer et al. [16] investigated the effect of heat transfer on the stability of boundary layer by solving an extended version of the Orr-Sommerfeld equation. And we know that the wall surface temperature has indeed greatly influenced on the results, for example, disturbance amplified rate and transition position [17,18]. So how the wall surface temperature changes during real flying procedure is worth researching.

In this paper, the expression of heat-transfer coefficient is investigated by comparing the accuracy of the results when the fluid properties are introduced at a reference enthalpy or a reference temperature. Then, in order to know whether isothermal or adiabatic condition can be chosen as wall condition during flying, the wall temperature distribution of a flat plate is calculated under the wall condition taken as convective heat transfer for wall surface temperature from a given initial wall temperature to the recovery temperature.

2. The similar solution for laminar compressible boundary layer

In order to check the formula of convective heat transfer coefficient of compressible flow, the Blasius similar solution of compressible flow needed to be given firstly. Due to similar solutions still exists in flat hypersonic boundary layer flow for variable specific heat, which is one real gas effect taken into consideration result of high temperature, the heat flux could be calculated with isothermal wall condition and thus the convective heat transfer coefficient.

For flat hypersonic boundary layer flow, the dimensionless compressible Navier-Stokes equations are written as follows [19], named compressible boundary layer equations.

$$\begin{cases} \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \\ \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{1}{Re} \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \\ \rho u C_p \frac{\partial T}{\partial x} + \rho v C_p \frac{\partial T}{\partial y} = \frac{u_e^2}{T_e Re} \mu \left(\frac{\partial u}{\partial y}\right)^2 + \frac{u_e^2}{T_e Re} \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) \end{cases} \tag{1}$$

where x the coordinate of flow direction, y the vertical direction. The Illingworth transformation is introduced as follows:

$$\begin{cases} \xi = \rho_e u_e \mu_e x \\ \eta = \sqrt{\frac{\rho_e u_e}{2\mu_e x}} \int_0^y \frac{\rho}{\rho_e} dy \end{cases} \tag{2}$$

In which, ρ_e, μ_e and u_e denote density value, viscosity coefficient value and velocity value. The subscript 'e' denotes the gas parameters corresponding to the outer edge of boundary layer. The similarity solutions of velocity and temperature can be written as follows:

$$\begin{cases} u(x,y) = u_e f'(\eta) \\ T(x,y) = T_e g(\eta) \end{cases} \tag{3}$$

By substituting (3) into the plate boundary Eq. (1), the dimensionless compressible boundary layer equations of the specific heat as a function of temperature are obtained as follows:

$$(C_1 f'')' + ff'' = 0 \tag{4}$$

$$(C_2 g')' + \frac{C_p}{C_{pe}} Pr_e fg' + (\gamma_e - 1) Ma_e^2 Pr_e C_1 f''^2 = 0 \tag{5}$$

When the specific heat being a constant value, the dimensionless energy Eq. (5) is changed into following form:

$$(C_2 g')' + Prfg' + (\gamma - 1) Ma_e^2 Pr C_1 f''^2 = 0 \tag{6}$$

where

$$C_1 = \frac{\rho\mu}{\rho_e\mu_e} = \left(\frac{T}{T_e}\right)^{1/2} \left(\frac{T + c_m}{T_e + c_m}\right),$$
$$C_2 = \frac{\rho\lambda}{\rho_e\lambda_e} = \left(\frac{T}{T_e}\right)^{1/2} \left(\frac{T + c_k}{T_e + c_k}\right),\ c_m = 110.4\ \mathrm{K},\ c_k = 194\ \mathrm{K}$$

Here, c_m and c_k is chosen the same as in Ref. [20]. Constant pressure specific heat can be calculated according to the specific enthalpy. When the air temperature range is from 600 K to 2500 K, the molecular vibration degrees of freedom of oxygen and nitrogen molecules in the air are excited, the specific enthalpy can be expressed as follows:

$$H = \frac{7}{2}\mathrm{R}T + \frac{\mathrm{R}T_{ve}}{e^{T_{ve}/T} - 1} \tag{7}$$

in which, R represents the gas constant, and T_{ve} denotes the vibration eigen temperature. In this paper, T_{ve} is taken as 3030 K [11]. So the constant pressure specific heat can be written as follows:

$$C_p = \frac{7}{2}\mathrm{R} + \mathrm{R}\left(\frac{T_{ve}}{T}\right)^2 \frac{e^{T_{ve}/T}}{\left(e^{T_{ve}/T} - 1\right)^2} \tag{8}$$

When calculating the basic flow, no-slip, adiabatic or isothermal wall boundary condition is often adopted. The isothermal boundary condition can be expressed as follows:

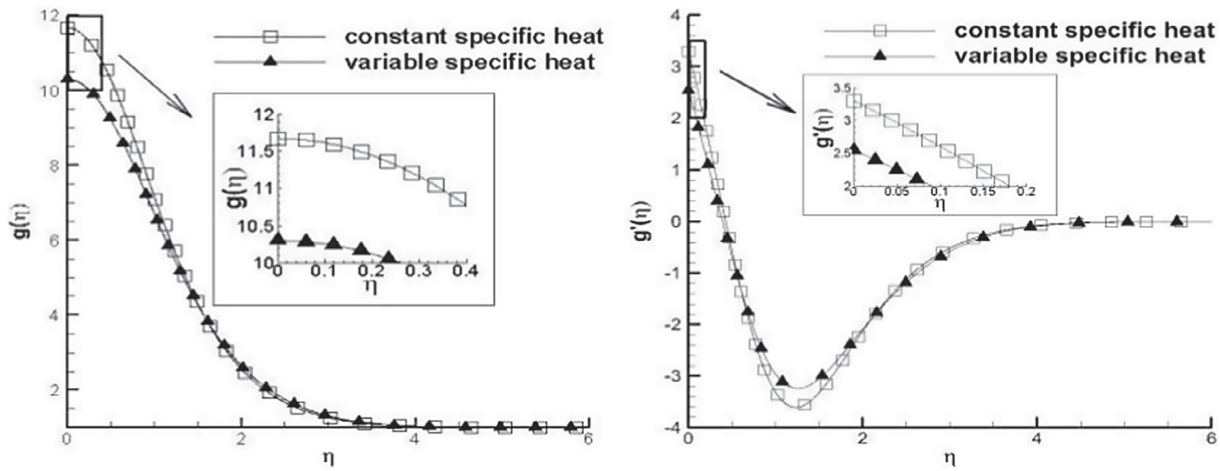
$$f(0) = f'(0) = 0, g(0) = \frac{T_w}{T_e}, f'(\infty) = 1, g(\infty) = 1,$$

The adiabatic boundary condition can be expressed as follows:

$$f(0) = f'(0) = 0, g'(0) = 0, f'(\infty) = 1, g(\infty) = 1$$

The similarity solutions are solved by the Runge-Kutta method. For free-stream flow Mach number 8, for the case of isothermal and adiabatic boundary condition, the comparison results between constant specific heat and variable specific heat are shown in Fig. 1. It shows that the difference is so large especially near to the wall for temperature or its derivative that the variable specific heat should be taken.

On one hand, the wall heat flux can be obtained by the formula of heat conduction or convection heat transfer as follows:



(a) Distribution of temperature on adiabatic condition (b) Distribution of temperature derivative on isothermal condition

Fig. 1. Comparison between constant specific heat and variable specific heat on $Ma = 8$.

$$q_w = -\lambda \frac{\partial T}{\partial y}\bigg|_w \tag{9}$$

$$q_w = h(T_w - T_r) \tag{10}$$

In which, q_w indicates the heat flux, h the heat-transfer coefficient, T_w the wall temperature, T_r the recovery temperature calculated by similar solution. The subscript 'w' denotes the gas parameters corresponding to the values on the wall surface.

On the other hand, the heat flux can also be calculated by basic flow from similar solution. By substituting the results of similar solution into (9), the heat flux is obtained as follows:

$$q_w = -\lambda \frac{\partial T}{\partial y}\bigg|_w = -\lambda \frac{\partial \eta}{\partial y}\bigg|_w \frac{\partial T}{\partial \eta}\bigg|_w = -\lambda T_e \frac{\rho_w u_e}{\sqrt{2\rho_e u_e \mu_e x}} g'(0) \tag{11}$$

By substituting the results of heat flux into (10), the formula of the heat-transfer coefficient becomes

$$h = -\frac{\lambda T_e}{(T_w - T_r)} \frac{\rho_w u_e}{\sqrt{2\rho_e u_e \mu_e x}} g'(0) \tag{12}$$

Formula (12) can be served as the standard to examine the accuracy by using the heat-transfer coefficient formula in engineering.

3. The heat-transfer coefficient in engineering

In engineering, there are many different empirical equations to calculate heat-transfer coefficient. The heat-transfer coefficient can be expressed for a fluid with a different temperature dependence of the properties when they are introduced at a reference enthalpy or a reference temperature. The reference enthalpy H^* and the reference temperature T^* are described by the equations as follows [7–9]:

$$\frac{H^*}{H_e} = A + B\frac{H_w}{H_e} + C\frac{H_r}{H_e} \tag{13}$$

$$\frac{T^*}{T_e} = A + B\frac{T_w}{T_e} + C\frac{T_r}{T_e} \tag{14}$$

In which, T_r, H_r denote recovery temperature and enthalpy calculated by empirical formulas as follows:

$$T_r = \left(1 + r\frac{\gamma - 1}{2} Ma^2\right) T_e \tag{15}$$

$$H_r = H_e + r\frac{U_e^2}{2} \tag{16}$$

Here, the recovery factor r is taken as $\sqrt{Pr}$. There are three sets of A, B and C [7–9]:

(i): $A = 0.36, B = 0.45, C = 0.19$
(ii): $A = 0.23, B = 0.58, C = 0.19$
(iii): $A = 0.28, B = 0.50, C = 0.22$

When the properties of fluid are introduced at reference enthalpy or reference temperature, wall friction coefficient of compressible plate boundary layer can be calculated. The wall frictional coefficient of incompressible fluid C_f, compressible fluid C_{fc}, and incompressible fluid under reference temperature C_f^* are written as follows:

$$Re_x = \frac{\rho_e u_e x}{\mu_e},\ Re_x^* = \frac{\rho^* u_e x}{\mu^*},\ \rho^* = \frac{\rho_e T_e}{T^*},\ \mu^* = \left(\frac{T^*}{T_e}\right)^{3/2}\left(\frac{T_e + c_m}{T^* + c_m}\right)\mu_e$$

$$C_f = \frac{0.664}{\sqrt{Re_x}},\ C_f^* = \frac{0.664}{\sqrt{Re_x^*}} = \frac{0.664}{\sqrt{Re_x}}\left(\sqrt{\frac{\rho_e \mu^*}{\rho^* \mu_e}}\right) = \frac{0.664}{\sqrt{Re_x}}\left(\sqrt{\frac{T^* \mu^*}{T_e \mu_e}}\right)$$

$$C_{fc} = \frac{2\tau_w}{\rho_e u_e^2} = \frac{2\tau_w}{\rho^* u_e^2}\frac{\rho^*}{\rho_e} = C_f^*\frac{\rho^*}{\rho_e} = \frac{0.664}{\sqrt{Re_x}}\left(\sqrt{\frac{\rho^* \mu^*}{\rho_e \mu_e}}\right)$$

In which, the superscript '$*$' denotes the gas parameters corresponding to reference enthalpy or reference temperature.

Then, with the Reynolds analogy, the relationship between wall frictional coefficient and Stanton number can be obtained. For the compressible flow, the compressible modified Reynolds analogy is written as follow:

$$St = \frac{C_{fc}}{2Pr^{2/3}}$$

The heat flux formula is derived as follows:

$$\begin{aligned} q_w &= h_i(H_w - H_r) = \rho_e u_e St(H_w - H_r) \\ &= \frac{0.332}{Pr^{2/3} Re_x^{1/2}} \rho_e u_e \left(\frac{\rho^* \mu^*}{\rho_e \mu_e}\right)^{1/2} (H_w - H_r) \\ &= \frac{0.332}{Pr^{2/3}} \left(\frac{\rho^* \mu^* u_e}{x}\right)^{1/2} (H_w - H_r) \end{aligned} \tag{17}$$

By substituting (17) into (10), the heat-transfer coefficient is obtained as follow:

$$h = \frac{0.332}{Pr^{2/3}} \left(\frac{\rho^* \mu^* u_e}{x} \right)^{1/2} \frac{(H_w - H_r)}{(T_w - T_r)} \tag{18}$$

When the specific heat being a constant value, reference temperature can be used to replace reference enthalpy and the simplified equation of local heat-transfer coefficient is written as follow:

$$h = \frac{0.332}{Pr^{2/3}} \left(\frac{\rho^* \mu^* u_e}{x} \right)^{1/2} c_p \tag{19}$$

In order to compare the results by using these three set, the heat-transfer coefficients is calculated, the free-stream parameters are taken as the air corresponding to the height 20 km:

$$T_\infty = 216.65\ \text{K},\ c_\infty = 295.07\ \text{m/s},\ P_\infty = 5.5293 \times 10^3\ \text{Pa}$$

$$\rho_\infty = 8.891 \times 10^{-2}\ \text{kg/m}^3,\ \mu_\infty = 1.4216 \times 10^{-5}\ \text{kg/(m} \cdot \text{s}^2)$$

At any location such as one meter away from the leading edge of the plate, the distributions of the heat-transfer coefficients calculated in engineering formula are shown in Fig. 2. In Fig. 2, (i), (ii) and (iii) stand for A, B, C in (i, ii, iii) respectively. As shown in Fig. 2, the deviation of convective heat-transfer coefficient calculated among the three sets is small. The set (iii) proposed by Eckert [21] is widely used and to be analyzed its application scope and properties below.

4. Results

4.1. Accuracy and scope of application analysis of heat-transfer coefficient

When calculating the basic flow, the free-stream parameters are taken as the air corresponding to the height 20 km. For the cases of free-stream flow Mach number from 1 to 10 and isothermal wall condition adopted, considering effect of real gas, the heat-transfer coefficients are calculated by the Blasius similar solution. The heat-transfer coefficients calculated based on the Blasius similar solution should be taken as one standard scale to examine the accuracy of the results with engineering heat-transfer coefficient formula.

The distribution of the heat-transfer coefficients got from Eckert formula by reference enthalpy and reference temperature, respectively, are compared with the standard values from Blasius similar solution stood for by solid line ($T_w = 0.5T_r$) as shown in Fig. 3. In Fig. 3, it shows that for all Mach number, heat-transfer coefficient calculated using reference enthalpy agree well with that from Blasius similar solution, but the deviations are becoming large for Ma greater than 3 for the results by reference temperature.

The quantitative relative error of the coefficients above compared to the standard values is shown in Table 1. It is found that the results calculated using reference enthalpy is close to the standard values and the maximum relative error is less than 3%. And the relative error of coefficients calculated using reference temperature is significant with increasing Mach number. For example, the maximum relative error is more than 20% at Mach number 10.

Hence, only the results were presented using reference enthalpy below. In order to investigate what factors influence the heat-transfer coefficients, the curves of heat-transfer coefficients versus x, Ma, T_w are shown in Figs. 4–6, respectively.

It can be seen from Figs. 4–6 that as Ma increases, x and T_w decreases, heat-transfer coefficient increases.

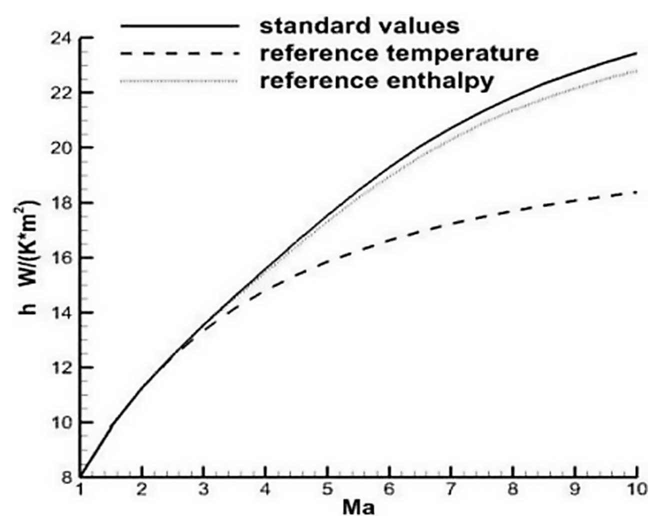


Fig. 3. Comparison of heat-transfer coefficient.

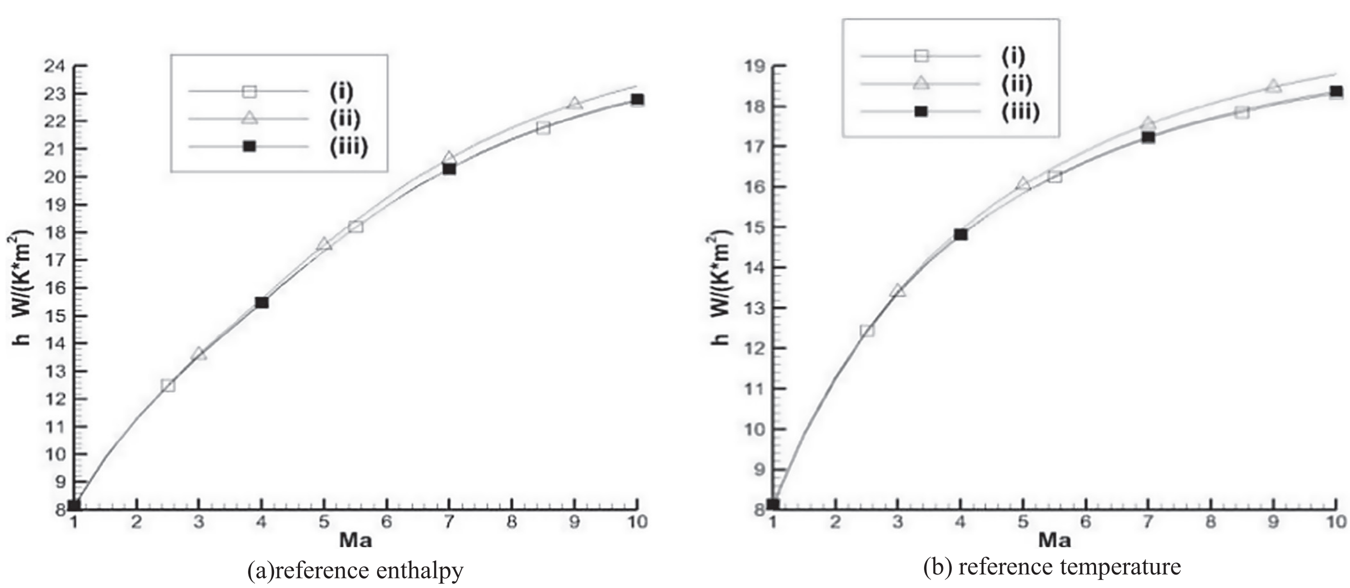


Fig. 2. Comparison of heat-transfer coefficient based on reference enthalpy and temperature.

Table 1
Relative error of heat-transfer coefficient.

Model	Err (%) *Ma*									
	1	2	3	4	5	6	7	8	9	10
Reference temperature	0.604	0.124	1.426	5.068	9.705	13.79	16.84	19.00	20.51	21.58
Reference enthalpy	0.608	0.235	0.268	0.807	1.296	1.703	2.033	2.305	2.534	2.732

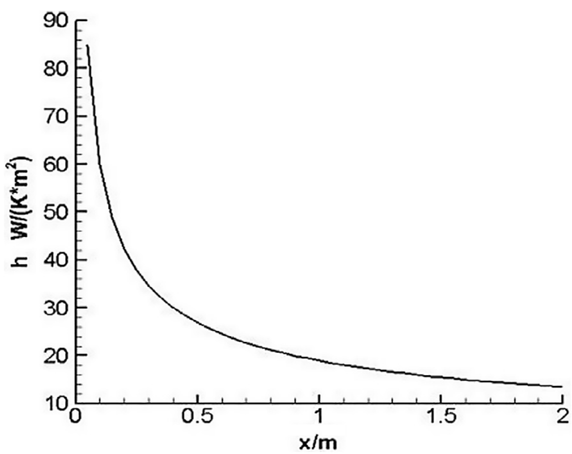


Fig. 4. $Ma = 6$, $T_w = 0.5T_r$, heat-transfer coefficient versus x.

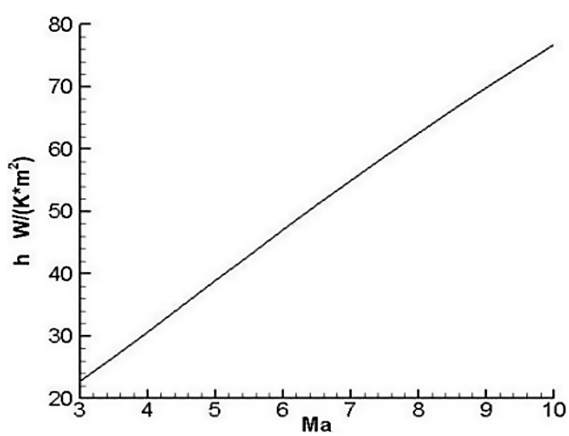


Fig. 5. $T_w = 500$ K, $x = 0.5$ m, heat-transfer coefficient versus Ma.

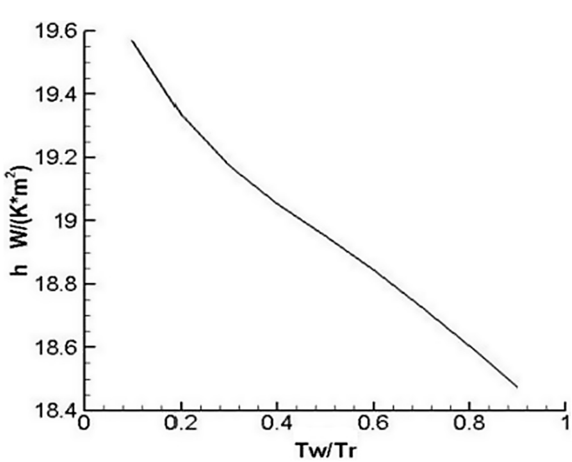


Fig. 6. $Ma = 6$, $x = 0.5$ m, heat-transfer coefficient versus T_w.

4.2. Application of convective heat transfer boundary conditions

4.2.1. Calculation model

With the verified formula of heat-transfer coefficient, the computation is conducted for supersonic flow passing one meter long flat plate with the wall being convective heat transfer. The free-stream flow Mach number is taken to be 4.5. Besides the parameters of 20 km altitude, the parameters of 10 km and 30 km altitude are chosen as the free-stream parameters in order to analyze the influence of flight altitude.

The gas parameters corresponding 10 km height are:

$$T_\infty = 223.25\ \mathrm{K}, c_\infty = 299.53\ \mathrm{m/s}, P_\infty = 2.65 \times 10^4\ \mathrm{Pa}$$

$$\rho_\infty = 0.41351\ \mathrm{kg/m^3},\ \mu_\infty = 1.4577 \times 10^{-5}\ \mathrm{kg/(m \cdot s^2)}$$

The gas parameters corresponding30 km height are:

$$T_\infty = 226.51\ \mathrm{K}, c_\infty = 301.71\ \mathrm{m/s}, P_\infty = 1.197 \times 10^3\ \mathrm{Pa}$$

$$\rho_\infty = 1.8410 \times 10^{-2}\ \mathrm{kg/m^3},\ \mu_\infty = 1.4753 \times 10^{-5}\ \mathrm{kg/(m \cdot s^2)}$$

The subscript '∞' denotes the gas parameters corresponding to the free-stream flow. Air flows over an infinitely width metal flat plate with 1 m long and 2 mm thick. The metal density, specific heat capacity and thermal conductivity are shown as follows:

$$\rho = 4.7 \times 10^3\ \mathrm{kg/m^3}, C = 520\ \mathrm{J/(kg \cdot K)}, \lambda = 10\ \mathrm{W/(m \cdot K)}$$

The initial wall surface temperature is equal to free-stream one. The convective heat transfer boundary condition is obtained from formula (17). The effect of shock wave was neglected due to thin plate although shock might appear from the leading edge. An assumption is made that ignore heat conduction inner the thin flat plate along stream-wise and considering only air-to-wall convective heat transfer along the normal direction.

In general, the compressible flow will calculate by DNS with variable specific heat. Based on the calculated flow field, the wall temperature and heat flux will be calculated and analyzed. However, with the wall being convective heat transfer, the heat flux can be calculated according to the parameters of the wall surface and free-stream with Eq. (17). The wall temperature changing with time is only related to the wall heat flux. Therefore, the calculation of wall temperature and heat flux is not necessary to use the DNS to calculate the whole flow field. Before introducing the numerical method, lumped capacitance should be introduced first.

4.2.2. Lumped capacitance

A measure of the relative importance of the thermal resistance within a solid body is the Biot number Bi, which is the ratio of the internal to the external resistance and can be defined by the equation

$$Bi = \frac{R_{\mathrm{internal}}}{R_{external}} = \frac{h\delta}{\lambda}$$

where δ represents half thickness of the flat plate, $\mathrm{R_{internal}}$ and $\mathrm{R_{external}}$ denote the internal and external resistance. In bodies whose shape resembles a plate, the error introduced by the assumption that the temperature at any instant is uniform will be less than

5% when the internal resistance is less than 10% of the external surface resistance, that is, when Bi < 0.1. A transient heat conducting system in which Bi < 0.1 is often referred to as a lumped capacitance, and this reflects the fact that its internal resistance is very small or negligible.

At 30 km flight altitude, the calculation results of Biot number is less than 0.1 on any location of the flat plate. According to lumped capacitance, temperature is uniform along the thickness of the flat plate at any instant and the wall temperature is only a function of time. With the lumped capacitance, the numerical method is introduced as following.

4.2.3. Numerical method and results

At the initial period of flying, the wall surface temperature is equal to free-stream one and the wall heat flux can be obtained with Eq. (17). Using lumped-heat-capacity method, the increasing amount of wall temperature within the time can be written as follows:

$$\Delta T = \frac{q_w \cdot A \cdot \mathrm{d}t}{C \cdot m} = \frac{q_w \cdot \mathrm{d}t}{C \cdot \rho \cdot \delta} \tag{20}$$

In which, C, ρ, δ denote the metal density, specific heat capacity, half thickness of metal plate, ΔT and t represents the increasing amount of wall temperature and time, respectively.

Therefore, the relationship between wall temperature and time can be written as:

$$T_w = T_\infty + \int_0^t \frac{q_w}{C \cdot \rho \cdot \delta}\mathrm{d}t \tag{21}$$

Considering hypersonic boundary layer flow over a flat plate with the wall being convective heat transfer, wall temperature distribution at different time can be obtained by Eq. (21).

Fly in the 30 km, for example, the wall temperature distributions at different time are shown in Fig. 7. It shows that, wall temperature increases with time for any same location, whereas decreases along the flow direction for the same time. During long period flying, the wall surface temperature distribution is not satisfied with the isothermal condition. The time required for the wall temperature rising to 0.9 times the recovery temperature is 650 s at x = 1 m. When considering the real situation, the flat plate becomes thicker and longer. It needs more time for the wall temperature rising to 0.9 times the recovery temperature. If the flat plate thickness is 2 cm, for example, the time required may be 108 min at x = 1 m and 216 min at x = 4 m. The results indicated that whether isothermal or adiabatic condition in hypersonic flow chosen as the wall boundary condition is not suitable in real flow especially during long period flying.

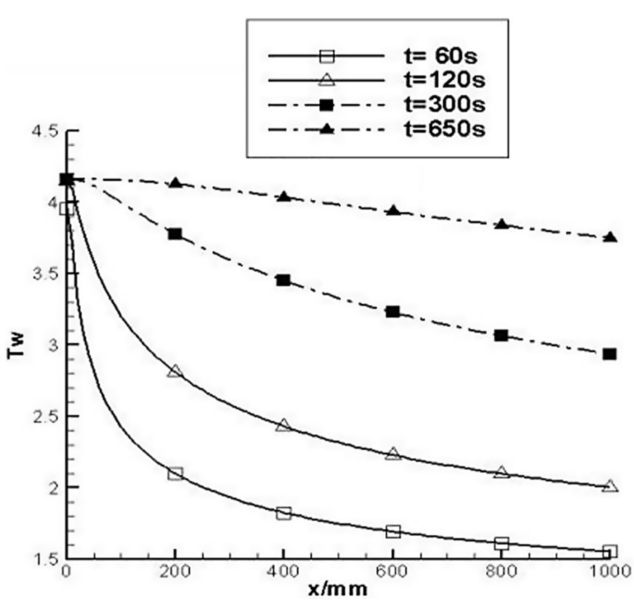


Fig. 7. At 30 km flight altitudes, dimensionless wall surface temperature distributions at different time.

4.2.4. Estimation method and results

The influence of flight altitudes is analyzed. At x = 0.5 m apart from the leading edge, for example, the heat flux distributions with wall temperature at different high altitude are shown in Fig. 8. As shown in Fig. 8, the heat flux is linear with wall temperature. It means that there is a quantitative relationship between heat flux for any same location at different heights.

According to Eq. (20), the relationship between wall temperature and time can also be written as:

$$t = \int_{T_\infty}^{T_w} \frac{C \cdot \rho \cdot \delta}{q_w}\mathrm{d}T_w \tag{22}$$

It assumes that H_1 and H_2 represent different heights. Using free-stream temperature as the reference temperature scale to make wall temperature dimensionless, t_1 and t_2 represents required time for dimensionless wall temperature T'_w rising to another dimensionless temperature T_p at H_1 and H_2, respectively. With Eq. (13), dimensionless temperature for the reference enthalpy is the same for any same dimensionless wall temperature. According to Eqs. (22) and (17), the required time ratio of t_1 to t_2 can be expressed as following

$$\begin{aligned} R_H &= \frac{t_1}{t_2} = \frac{\int_1^{T_p} \frac{Pr^{2/3}}{0.332}\left(\frac{x}{\rho_1^* \mu_1^* u_e}\right)^{1/2} \frac{C \cdot \rho \cdot \delta}{(H_w - H_r)/T_{e1}} \mathrm{d}T'_w}{\int_1^{T_p} \frac{Pr^{2/3}}{0.332}\left(\frac{x}{\rho_2^* \mu_2^* u_e}\right)^{1/2} \frac{C \cdot \rho \cdot \delta}{(H_w - H_r)/T_{e2}} \mathrm{d}T'_w} \\ &= \left(\frac{\rho_2 \mu_2}{\rho_1 \mu_1}\right)^{1/2} \frac{\int_1^{T_p} \left(\frac{T_{e1}}{T_1^o}\right)^{3/4} \left(\frac{T_1^o/T_{e1} + c_m/T_{e1}}{1 + c_m/T_{e1}}\right)^{1/2} \frac{1}{(H_w - H_r)/T_{e1}} \mathrm{d}T'_w}{\int_1^{T_p} \left(\frac{T_{e2}}{T_2^o}\right)^{3/4} \left(\frac{T_2^o/T_{e2} + c_m/T_{e2}}{1 + c_m/T_{e2}}\right)^{1/2} \frac{1}{(H_w - H_r)/T_{e2}} \mathrm{d}T'_w} \end{aligned}$$

In which, T^o the temperature for reference enthalpy and R_H the required time ratio of t_1 to t_2. The subscript '1' and '2' denotes the gas parameters corresponding to the free-stream flow at H_1 and H_2, respectively. The superscript '′' denotes the dimensionless parameters, which used free-stream temperature as the reference temperature scale.

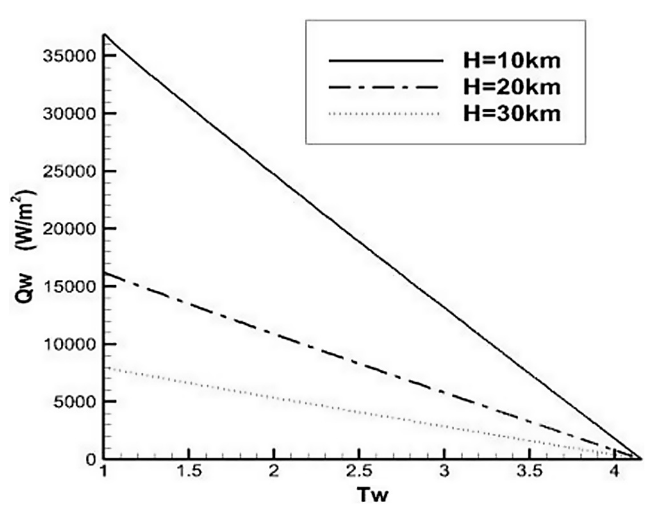


Fig. 8. At x = 0.5 m, heat flux distributions with dimensionless wall temperature at different high altitude.

Table 2
Relative error of integration result.

T_e (K)	Err (%) T_w						
	2	2.5	3	3.5	4	4.5	5
210	0.26	0.28	0.31	0.33	0.34	0.33	0.33
220	0.54	0.59	0.64	0.67	0.69	0.65	0.65
230	0.84	0.91	0.98	1.02	1.03	0.95	0.94
240	1.15	1.24	1.32	1.36	1.36	1.22	1.21
250	1.47	1.58	1.66	1.70	1.68	1.48	1.46
260	1.80	1.91	2.00	2.04	1.99	1.71	1.69

With different free-stream temperature at different height, the integration value is different. The relative error of the integration result at different free-stream temperature compared to the integration result with the free-stream temperature of 200 K is shown in Table 2. It is found that relative error is not significant, and the maximum relative error is about 2%. When the difference of free-stream temperature is less than 20 K, the relative error is less than 1%. In this case, the required time ratio of t_1 to t_2 can be expressed as following

$$R_H = \frac{t_1}{t_2} = \left(\frac{\rho_2\mu_2}{\rho_1\mu_1}\right)^{1/2} \tag{23}$$

This is the quantitative relationship between different heights. With the calculation results at 30 km high altitude, for example, the wall temperature distribution at another height can be estimated by using the ratio R_H. At 10 km and 20 km flight altitudes, estimation and calculation of the wall temperature distributions at 60 s and the time required for wall temperature reaching 3.7 times free-stream temperature are shown in Fig. 9.

As shown in Fig. 9(a), wall temperature decreases with flight altitude for any same location at the same time. It can be seen from Fig. 9(b), the time required for wall temperature rising to 3.7 times free-stream temperature increases with flight altitude for any same location. The results show that the estimated values agree well with the calculation results.

In this thin plate model, the wall temperature relationship between different locations is also analyzed. It assumes that x_a and x_b represent different locations, t_a and t_b represent the required time for wall temperature rising to another temperature T_q at x_a and x_b, respectively. At the same wall temperature with Eq. (13), the temperature for reference enthalpy is the same. According to Eqs. (22) and (17), the required time ratio of t_a to t_b can be expressed as following

$$\begin{aligned} R_x &= \frac{t_a}{t_b} = \frac{\int_{T_\infty}^{T_q} \frac{Pr^{2/3}}{0.332}\left(\frac{x_a}{\rho^*\mu^* u_e}\right)^{1/2} \frac{C\cdot\rho\cdot\delta}{(H_w-H_r)}\mathrm{d}T_w}{\int_{T_\infty}^{T_q} \frac{Pr^{2/3}}{0.332}\left(\frac{x_b}{\rho^*\mu^* u_e}\right)^{1/2} \frac{C\cdot\rho\cdot\delta}{(H_w-H_r)}\mathrm{d}T_w} \\ &= \left(\frac{x_a}{x_b}\right)^{1/2} \frac{\int_{T_\infty}^{T_q} \left(\frac{1}{\rho^*\mu^* u_e}\right)^{1/2} \frac{1}{(H_w-H_r)}\mathrm{d}T_w}{\int_{T_\infty}^{T_q} \left(\frac{1}{\rho^*\mu^* u_e}\right)^{1/2} \frac{1}{(H_w-H_r)}\mathrm{d}T_w} = \left(\frac{x_a}{x_b}\right)^{1/2} \end{aligned} \tag{24}$$

In which, R_x the required time ratio of t_a to t_b. This is the quantitative relationship between different locations. Therefore, with the calculation results at x = 0.5 m, for example, the wall temperature distribution on the rest of the plate for the same time can be estimated by using the ratio R_x. At 147.3 s, the wall temperature distribution of calculation and estimation is shown in Fig. 10. It can be seen from Fig. 10, the values of estimation agree well with the calculation results.

Therefore, in the calculation of wall temperature changing with time for supersonic flow passing a flat plate with the wall being

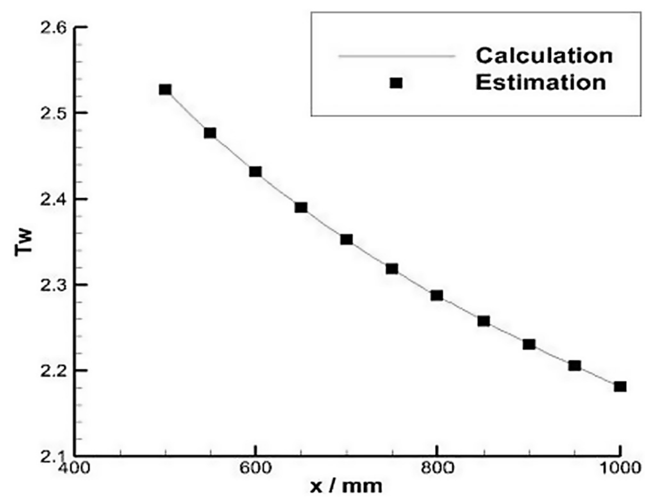
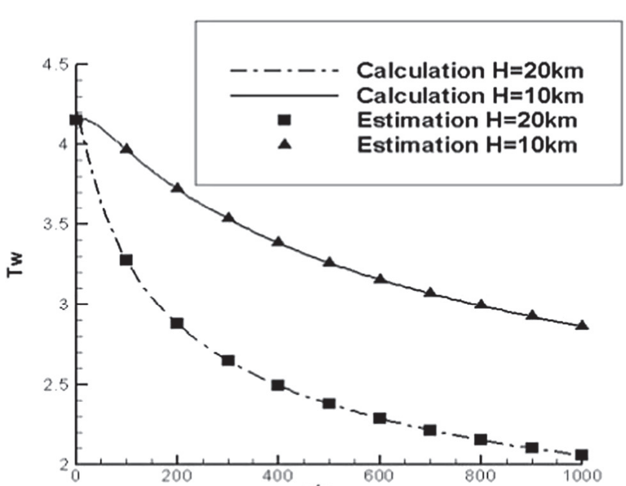
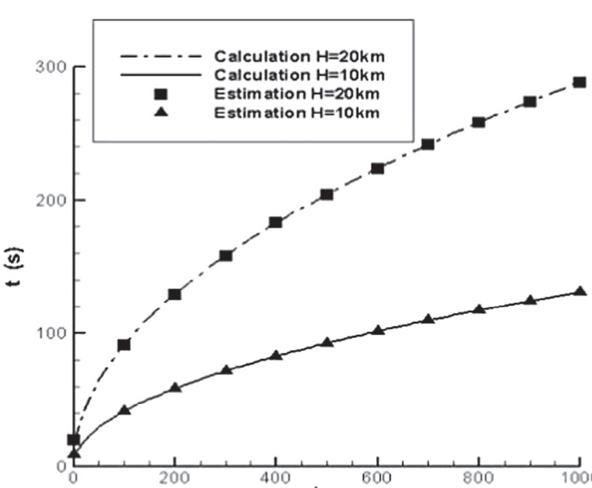


Fig. 10. Comparison between estimation and calculation results at 30 km high altitude.



(a) Dimensionless wall surface temperature distributions at 60s.



(b)The time required for wall temperature reaching 3.7 times free-stream temperature.

Fig. 9. The results of estimation and calculation at 10 km and 20 km high altitude.

convective heat transfer, we do not need to calculate in each height, and even do not need to calculate on all locations. The wall temperature on a section of length starting from the leading edge of the plate at a certain height should be calculated first. Then, the wall temperature distribution on the rest of the plate for the same time can be obtained by using the ratio R_x. According to the calculation results at the height, temperature distribution at another height can be obtained by using the ratio R_H. In this way, the accurate temperature distribution can be obtained in the case of reducing computing time.

The calculation formulas of wall temperature and heat flux are applied to the assumption conditions. The formula is verified only under this thin flat plate model that Bi < 0.1. When the model is changed into a thick flat plate model that Bi > 0.1, the applicability of the formula remains to be verified. When laminar boundary layer is transformed into turbulent flow, the formula of heat flux needs to be improved.

5. Conclusions

Considering hypersonic boundary layer flow over a flat plate, the paper quantitative analyses the properties and application scope of the heat-transfer coefficient formula in engineering and obtains mathematical expressions suitable for convective heat transfer boundary condition. This provides the basis for further direct numerical simulation or flow stability analysis.

The variation of temperature and heat flux along the wall with time is studied. Our computational results indicated that whether isothermal or adiabatic condition in hypersonic flow chosen as the wall boundary condition is not suitable in real flow especially during the initial long period flying. There is a appropriate formula to calculate the wall temperature and heat flux under the assumption of thin flat plate. The wall temperature condition at different time can be convenient assumed to be convective heat transfer condition with the formula. How to get the specific formula applicable to the real situation is a worth researching. The main conclusions are as follows:

(1) Basing on the heat-transfer coefficients calculated by virtue of the similar solutions which real gas effect is taken into consideration in hypersonic boundary layer flow, Eckert formula by reference enthalpy is confirmed that it may provide with good results for Mach number less than 10 and the maximum relative error is less than 3%. Whereas Mach number greater than 3 reference temperature could not get satisfactory results yet and the maximum relative error is more than 20%.

(2) The results indicated that isothermal wall condition is proper for only in the very beginning period and adiabatic condition is almost unsuitable in real flying period. Whether isothermal or adiabatic condition in hypersonic flow chosen as the wall boundary condition is not proper in real flow especially for most time during long period flying. As we have known that the wall surface temperature had indeed greatly influenced on the transition location. Therefore, convective heat transfer boundary condition should be taken in the computing flow field of hypersonic flow.

(3) When wall temperature distribution is calculated in this thin flat plate model that Bi < 0.1, a calculation method using lumped-heat-capacity method replace the DNS and the accurate temperature distribution can be obtained in the case of reducing computing time. The quantitative relationship between different heights and different locations for same height are studied, respectively. If known the ratio R_H and R_x, at any time, the wall temperature at every location will be known by the estimation method. In the actual case of the thick plate that Bi > 0.1, how the computational method and the quantitative relationship needed detailed research then.

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