CS 460/560: Kinematics Homework

Name: Eland Anthony 11/02/20

1. (6 points) Assume the robot has a velocity of (6 cm/s, 4 cm/s, 12 rad/s) in the global reference frame and is positioned at P and $\theta = \frac{\pi}{2}$ with respect to the global reference frame. What is the velocity with respect to the robot's local reference frame?

$\theta = \left(\frac{\pi}{2}\right)$ $V = (6 \text{ cm/s}, 4 \text{ cm/s}, 12 \text{ rad/s})$

$$\dot{\int}_R = R\left(\frac{\pi}{2}\right) \cdot \dot{\int}_I = \begin{bmatrix} \cos(\frac{\pi}{2}) & \sin(\frac{\pi}{2}) & 0 \\ -\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 4 \\ 12 \end{bmatrix} =$$

$$= \begin{bmatrix} 6\cos(\frac{\pi}{2}) + 4\sin(\frac{\pi}{2}) + 12 \cdot 0 \\ -6\sin(\frac{\pi}{2}) + 4\cos(\frac{\pi}{2}) + 12 \cdot 0 \\ 6 \cdot 0 + 4 \cdot 0 + 12 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 + 4 + 0 \\ -6 + 0 + 0 \\ 0 + 0 + 12 \end{bmatrix} =$$

$$= \begin{bmatrix} 4 \\ -6 \\ 12 \end{bmatrix}$$

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$\theta = \left(\frac{\pi}{2}\right)$ $V = (6\ \text{cm/s}, 4\ \text{cm/s}, 12\ \text{rad/s})$

$$\dot{\xi}_R = R\left(\frac{\pi}{2}\right)\cdot\dot{\xi}_I = \begin{bmatrix} \cos(\frac{\pi}{2}) & \sin(\frac{\pi}{2}) & 0 \\ -\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 4 \\ 12 \end{bmatrix} =$$

$$= \begin{bmatrix} 6\cos(\frac{\pi}{2}) + 4\sin(\frac{\pi}{2}) + 12\cdot 0 \\ -6\sin(\frac{\pi}{2}) + 4\cos(\frac{\pi}{2}) + 12\cdot 0 \\ 6\cdot 0 + 4\cdot 0 + 12\cdot 1 \end{bmatrix} = \begin{bmatrix} 0+4+0 \\ -6+0+0 \\ 0+0+12 \end{bmatrix} =$$

$$= \begin{bmatrix} 4 \\ -6 \\ 12 \end{bmatrix}$$

2. (6 points) Assume the robot has a velocity of (6 cm/s, 2cm/s, 18 rad/s) in the local reference frame and is positioned at P and $\theta = \frac{3\pi}{2}$ with respect to the global reference frame. What is the velocity with respect to the robot's global reference frame?

$\theta = \frac{3\pi}{2}$ $\quad v = (6\ cm/s, 2\ cm/s, 18\ rad/s)$

$$\dot{\xi}_I = R\left(\frac{3\pi}{2}\right)^{-1} \cdot \dot{\xi}_R = \begin{bmatrix} \cos(\frac{3\pi}{2}) & -\sin(\frac{3\pi}{2}) & 0 \\ \sin(\frac{3\pi}{2}) & \cos(\frac{3\pi}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 2 \\ 18 \end{bmatrix} =$$

$$= \begin{bmatrix} 6\cos(\frac{3\pi}{2}) - 2\sin(\frac{3\pi}{2}) + 18 \cdot 0 \\ 6\sin(\frac{3\pi}{2}) + 2\cos(\frac{3\pi}{2}) + 18 \cdot 0 \\ 6 \cdot 0 + 2 \cdot 0 + 18 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 + 2 + 0 \\ -6 + 0 + 0 \\ 0 + 0 + 18 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -6 \\ 18 \end{bmatrix}$$

3. (6 points) Assume the robot has a velocity of (1 cm/s, 3cm/s) in the local reference frame. What is the velocity with respect to the robot's global reference frame?

$V = (1\ cm/s, 3\ cm/s)$ $\theta = ?$

NOT SOLVEABLE

Not enough information

4. (6 points) A robot is positioned at a 90 degree angle ($\theta = \frac{\pi}{2}$) with respect to the global reference frame and has wheels with a radius of 6 cm. These wheels are 2 cm from the center of the chassis. The speed of wheel 1 is 8 rad/s and the speed of wheel 2 is 4 rad/s. What is the robot's velocity with respect to the global reference frame?

Right wheel (Φ_1) = 8 rad/s

Left wheel (Φ_2) = 4 rad/s

$\theta = \frac{\pi}{2}$

$r = 6$ cm

$l = 2$ cm

$$\dot{\xi}_I = R\left(\frac{\pi}{2}\right)^{-1} \cdot \begin{bmatrix} \frac{r\dot\Phi_1}{2} + \frac{r\dot\Phi_2}{2} \\ 0 \\ \frac{r\dot\Phi_1}{2l} + -\frac{r\dot\Phi_2}{2l} \end{bmatrix} =$$

$$\begin{bmatrix} \cos\left(\frac{\pi}{2}\right) & -\sin\left(\frac{\pi}{2}\right) & 0 \\ \sin\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{6 \cdot 8}{2} + \frac{6 \cdot 4}{2} \\ 0 \\ \frac{6 \cdot 8}{4} + \frac{-6 \cdot 4}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 36 \\ 0 \\ 12 \end{bmatrix} = \begin{bmatrix} 36 \cdot 0 + 0 \cdot -1 + 12 \cdot 0 \\ 36 \cdot 1 + 0 \cdot 0 + 12 \cdot 0 \\ 36 \cdot 0 + 0 \cdot 0 + 12 \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 36 \\ 12 \end{bmatrix}$$