1. Most manufacturing processes create defective items. Often these are tolerated up to a certain point, after which machines must be replaced, a costly and time-consuming process. Suppose you are working at a company that will permit 6% of all items to be defective. Your boss is curious if things have gotten worse and asks you to inspect 230 random items.

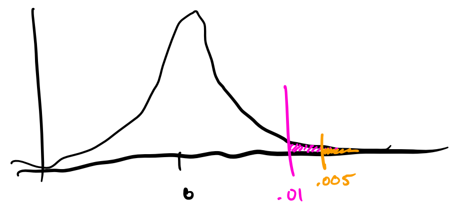
?

a. If you find 23 items are defective, what advice should you give your boss based on an approximate hypothesis test with $\alpha = 0.01$? As always, follow the steps from class.

a) Let p be proportion of all defective items.

H_0 : p is 6% of 100% $\Rightarrow$ $p = \frac{6}{100}$

H_1 : more than 6% are defective $\Rightarrow$ $p > \frac{6}{100}$



Assuming H_0 is true... $Z = \frac{\hat{p} - p}{\sqrt{pq/n}}$ $\approx N$ if $np, nq \geq 10$

$n = 230 \rightarrow np = 69/5 > 10$

$nq = 1081/5 > 10$ $\quad \hat{p} = \frac{23}{230}$

$\rightarrow Z = \dfrac{\frac{23}{230} - \frac{6}{100}}{\sqrt{6/100\,(1 - 6/100)/230}}$ $\quad Z = 2.554$

> pnorm (2.554, lower = F)

P = .005

$\alpha = .01$

$.005 < .01$

As the p-value is less than α we will suggest to the boss that things have gotten worse... $p > 6\%$

Machines should be replaced because they are permitting more than 6% defective items.

b. In R, you can conduct the test quite easily with the prop.test command. Read the documentation for this and write a single line of code that reproduces your results from part a. (Note: Set the "continuity correction" to false.)

$1 - \alpha = 1 - .01 = .99$ $\quad CL = .99$

```
Console   Terminal   Jobs
R 4.1.3 · ~/
> prop.test(23,230,.06, alternative = c("greater"), correct = F)

        1-sample proportions test without continuity correction

data:  23 out of 230, null probability 0.06
X-squared = 6.5248, df = 1, p-value = 0.005319
alternative hypothesis: true p is greater than 0.06
95 percent confidence interval:
 0.07197016 1.00000000
sample estimates:
  p
0.1
```

p = .005

again, $p < \alpha$

We reject H_0.

c. Your boss is wondering if an exact test for this situation would give different results. Find the P-value based on an exact test without using binom.test and then with binom.test in R (you'll get the same answer).

Exact Binomial Test

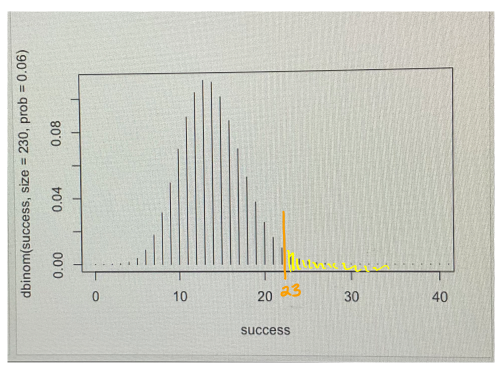
$X \sim Bin(n = 230,\ p = 6/100)$

$P(X \geq 23) = 1 - P(X \leq 22)$

> 1 - pbinom(22, 230, .06)

.012

$\boxed{\text{P-value} = .012}$



> binom.test(23, 230, .06, alturnitive = "greater")

```
Console   Terminal   Jobs
R 4.1.3 · ~/
> binom.test(23,230,.06,alternative = "greater")

        Exact binomial test

data:  23 and 230
number of successes = 23, number of trials = 230, p-value = 0.01183
alternative hypothesis: true probability of success is greater than 0.06
95 percent confidence interval:
 0.06931378 1.00000000
sample estimates:
probability of success
                   0.1
```

p-value = .012 ✓ > α

We keep H_0.

2. Does the idea known as "home-field advantage" (HFA) actually exist? HFA suggests that in a given sport, the home team will beat the away team more often than half the time. Several theories have been offered for why this might occur: familiarity with your arena/playing space, support of the home crowd, refereeing that favors the home team, etc. To explore HFA, researchers looked at 1000 random NFL games in the last 40 years and found that in 574 cases, the home team won.

a. Draw a conclusion about the idea of HFA using an approximate test with $\alpha = 0.02$, and show that you have met the conditions necessary for using this test.

Assuming H_0 is true… What is sampling distribution

$np = 1000\left(\frac{1}{2}\right) = 500 > 10$ ✓

$nq = 500 > 10$ ✓

Let p be proportion of home team wins.

$H_0: p = \frac{1}{2}$ $H_1: p > \frac{1}{2}$

$\hat{p} = 574/1000$

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}} \approx 4.680$$

Where is c?

$\alpha = .02$

>qnorm(.02, lower = F)

2.054

$c = 2.054$ $z = 4.680$

Since $z > c$ we reject H_0

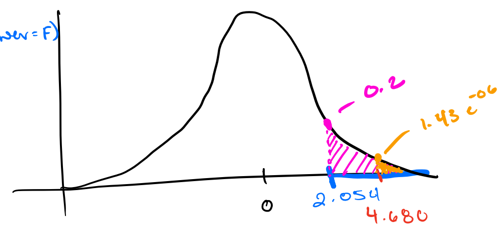
P-value?

>pnorm(4.68, lower = F)

$1.43e^{-06}$

$p < \alpha = .02$

Reject H_0.



It seems as though there is a home field advantage.

b. After publication of the findings from part a, you read on an NFL blog that "Data show the existence of HFA, likely the result of biased refereeing." Respond to this claim from a statistical perspective.

From a statistical perspective, we can not make any claims about why a HFA exists. We have not any data to suggest biased referees, only that the advantage exists.

3. People often look down on machine learning because in some settings, it can only improve things in small increments. While this might be true, in many settings a slight change can have a huge impact. As an example, Americans spend about 3 trillion dollars per year spread across 30 billion credit card transactions. Suppose that 0.4% (0.004, as a decimal) of these transactions are fraudulent, and hence, credit card companies lose money reimbursing their users. If machine learning could help reduce the percentage of fraudulent claims even slightly, this would save companies billions of dollars! Researchers at Visa have designed a new algorithm to predict fraud and are curious if it has reduced illegal card usage. You are tasked with determining whether this claim is statistically reasonable using an approximate test with $\alpha = 0.03$.

a. What is the smallest number of transactions you could look at and meet Larsen & Marx's requirements for using the approximate test?

Larson & Marx req: $0 < np - 3\sqrt{npq} < np + 3\sqrt{npq} < n$

here $n = ?$ $p = .004$ $q = .996$

$0 < np - 3\sqrt{npq}$

$0 < n(.004) - 3\sqrt{n}\sqrt{.004}$

$0 < n^{1/2}(n^{1/2}.004 - 3\sqrt{.004})$

$3\sqrt{.004} < \sqrt{n}\,.004$

$47.434 < \sqrt{n}$

$n > 2241$

$np + 3\sqrt{npq} < n$

$3\sqrt{n}\sqrt{pq} < n(1-p)$

$\frac{3\sqrt{pq}}{(1-p)} < \sqrt{n}$

$n > .036$

$\cancel{np} - 3\sqrt{npq} < \cancel{np} + 3\sqrt{npq}$

$-3\sqrt{n}\sqrt{pq} < 3\sqrt{n}\sqrt{pq}$

$0 < 6\sqrt{n}\sqrt{pq}$

$0 < n$

For any n

So for Larson Marx's inequality to hold true we must look at at least 224 transactions.

b. Suppose you end up looking at 2400 claims. What is the largest number of fraudulent claims that could appear among those 2400 claims that would cause a move to the alternative hypothesis? (Continue to think about the Larsen & Marx criterion as in part a.)

$\alpha = .03$

$H_0: p = .004 \qquad H_1: p < .004$

We want the proportion of fraudulent claims $p = \frac{x}{2400}$

$$\frac{\hat{p} - p}{\sqrt{pq/n}} = z \implies \frac{\frac{x}{2400} - .004}{\sqrt{.004(1-.004)/2400}} = z$$

qnorm(.03)

Lower bound

-1.881

$$\left(\frac{x}{2400}\right) - .004 = (-1.881)\sqrt{.004(1-.004)/2400}$$

$$x = 3.784$$

When $x = 3 \longrightarrow z = -2.134$ >pnorm(−2.134) $\longrightarrow$ p-val = .016 $< \alpha$ ✓

When $x = 4 \longrightarrow z = -1.811$ >pnorm(−1.811) $\longrightarrow$ p-val = .035 $> \alpha$ ✗

So the max frauduleut claims is 3

4. What hypothesis test has duality with the CI: $(-\infty, \overline{X} + 1.3\sigma_{\overline{X}})$? Assume $X \sim N(\mu, \sigma^2)$ with σ known.

$$\mu_0 \in \left(-\infty, \bar{X} + 1.3\sigma_{\bar{X}}\right)$$

$$\iff \quad \bar{X} - 1.3\sigma_{\bar{X}} > \mu_0$$

$$\iff \quad z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{X}}} > 1.3$$

$$\text{Keep } H_0 \iff z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{X}}} > 1.3$$

5. Suppose you are studying a phenomenon that is well-modeled by $N(\mu, 3^2)$. Existing research claims that $\mu = 20$, but you think μ might be lower based on recent changes in society. You'd like to collect some data to verify your claim and plan to use a sample of size 60. If μ is actually 19, what should you set α to in your hypothesis test if you want a Type II error rate of 0.08? Include a beautiful picture in your answer.

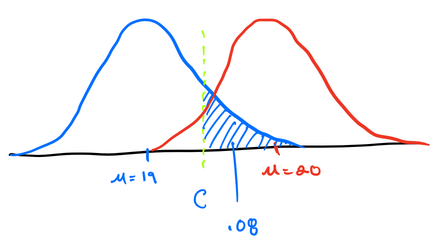
$x \sim N(\mu, 3^2)$ $\quad H_0: \mu = 20 \quad H_1: \mu < 20 \quad n = 60$

True $\mu = 19$

Type II Error: H_1 is true

$\beta = .08$

Sampling distribution $\bar{X}$ assuming H_0 true $N(20, 3^2/60)$



$>$qnorm$(\beta, 19, 3/\sqrt{60}$, lower=F)

19.544

$C = 19.544$

$>$pnorm$(C, 20, 3/\sqrt{60}$, lower = T)

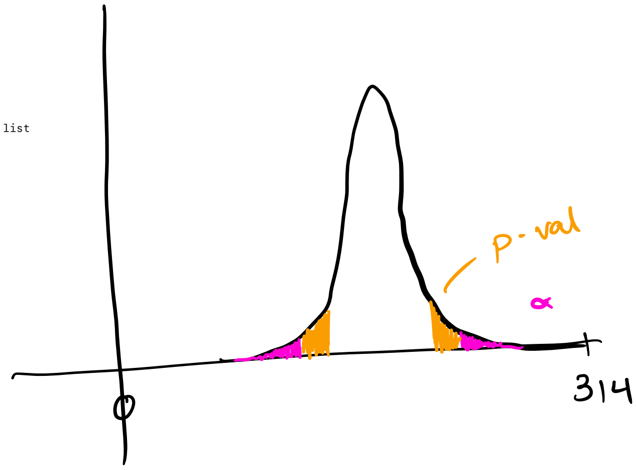
.12

$\alpha = .12$

6. A real number is said to be "normal" if, when written in any base b, the digits $0, 1, 2, \ldots, b-1$ all appear with equal frequency. That is, there should be an equal proportion of 0s, and 1s, and 2s, etc. One of the strangest results in mathematics is this: It can be shown that nearly all real numbers are normal, and yet, proving any particular number is normal is very difficult. Indeed, mathematicians do not know if π or e are normal!

On Canvas/TritonEd, you can find the file pibinary.csv which contains the first 10242 digits of pi in binary (base 2). Notice it starts with 11, which means 3 in binary. If π really is normal, what percentage of 1s do you expect in its binary expansion? Assuming the first 10242 digits are representative of all of π, conduct a hypothesis test on the proportion of 1s in π by randomly selecting 314 digits without replacement. You should set up variable(s) and hypotheses, write code to select the digits and get the sample proportion, draw a picture of a sampling distribution, shade an area, calculate a P-value, and reach a conclusion about your hypotheses using $\alpha = 0.05$.

```
#Pi Problem
#What percentage of 1s to expect?

#Converting list of pi digits into data frame
df <- as.data.frame(pibinary)
#Randomly sampling 314 digits from df without replacement
samples<-sample(df$digits,314,replace = FALSE) #df$digits -goes into "digit" list

# p is the proportion of 1s in the binary expansion of pi

#We expect half the digits to be 1s if pi is normal
p = .5
# H_o: p=.5 AND H_1: P does not =.5
n = length(samples)
successNum = sum(samples)
p_hat = successNum/n
z=(p_hat-p)/(sqrt(p*(1-p)/n))
cat("Z = ",z,"\n")
#Now lets solve for a P-Value
#Double sided alternative
if(p_hat > p) #choose which side to use
{
  p_val = 2*pnorm(z,lower=F)
}else{
  p_val = 2*pnorm(z,lower=T)
}

#Plotting sample distribution
success <- 0:n
plot(success, dbinom(success, size=n, prob=p),type='h')

#To verify our calculations:
res = prop.test(successNum,n,p,alternative = "two.sided",correct = F)
cat("P-value = ",p_val,"and to check: PropTest",res$p.value, "\n")


#Compare to alpha
alpha = .05
cat("P-value, (",p_val, ") is greater than alpha (", alpha, "). So we keep H_o.")
```



Script Output:

```
> source("~/Documents/Spring 22/RCode/piProblem.R")
Z =  -1.241532
P-value =  0.2144093 and to check: PropTest 0.2144093
P-value, ( 0.2144093 ) is greater than alpha ( 0.05 ). So we keep H_o.S
>
```