Homework 9

1. State the decision rule (i.e., test) that would be used to test the following hypotheses *for the specific test statistic mentioned*. Then, make a decision using the data provided and write a conclusion. Assume the data come from a normal distribution with unknown μ and known σ. Include a picture (OK to draw by hand, doing this in R is inefficient) of the sampling distribution for the test statistic and label the critical region.

what we expect — alternative hypothesis — what you compare to critical value c

a. $H_0 : \mu = 20$, $H_1 : \mu < 20$, $n = 16$, $\sigma = 3$, and $\alpha = 0.06$. Test stat: $\overline{x}$. Data: $\overline{x} = 18.5$

pre-determined cut-off value

b. $H_0 : \mu = 20$, $H_1 : \mu < 20$, $n = 16$, $\sigma = 3$, and $\alpha = 0.06$. Test stat: $\dfrac{\overline{x} - 20}{\sigma/\sqrt{n}}$. Data: $\overline{x} = 18.5$

c. $H_0 : \mu = 10$, $H_1 : \mu \neq 10$, $n = 100$, $\sigma = 0.4$, and $\alpha = 0.12$. Test stat: $\overline{x}$. Data: $\overline{x} = 11$

d. $H_0 : \mu = 50$, $H_1 : \mu > 50$, $n = 60$, $\sigma = 4$, and $\alpha = 0.08$. Test stat: $3\overline{x}$. Data: $\overline{x} = 50.5$

a) If H_0 is true then $X \sim N(20, 3^2) \rightarrow \bar{X} \sim N(20, \frac{3^2}{16})$

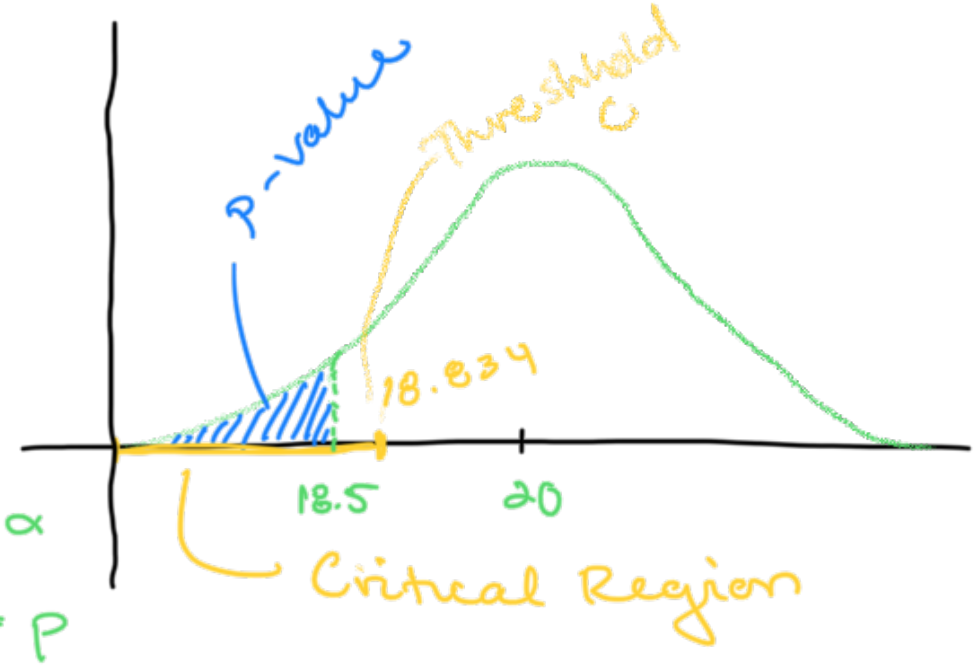
Sampling Distribution: $\bar{X} \sim N(20, (3/4)^2)$

$\bar{X} = 18.5$

$\mu = 20$

$P(X < c) = \alpha$

$P(X < \bar{X}) = p$



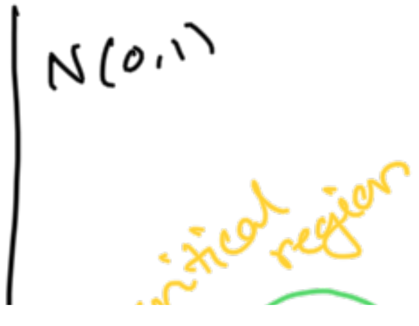
> qnorm(.06, 20, 3/4)

18.834

TEST: * If $\bar{x} \leq c$ then reject H_0.

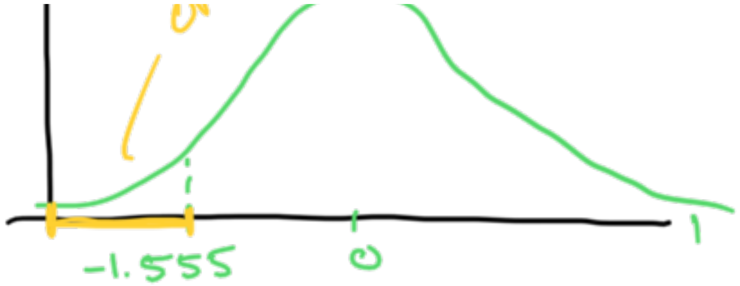
$18.5 < 18.834$ so reject H_0 in favor of H_1.

b) $X \sim N(20, 3^2)$ $\bar{X} \sim N(20, (\frac{3}{4})^2)$



qnorm(.06, 0, 1)

$-1.555 = c^*$



$P(X < -1.555) = .06$

* If $\dfrac{\bar{x} - 20}{\sigma/\sqrt{n}} \le c^*$ then reject H_0, favor H_1.

$\bar{x} = 18.5$

$\dfrac{\bar{x} - 20}{3/\sqrt{16}} < -1.555 \implies -2 < -1.555$

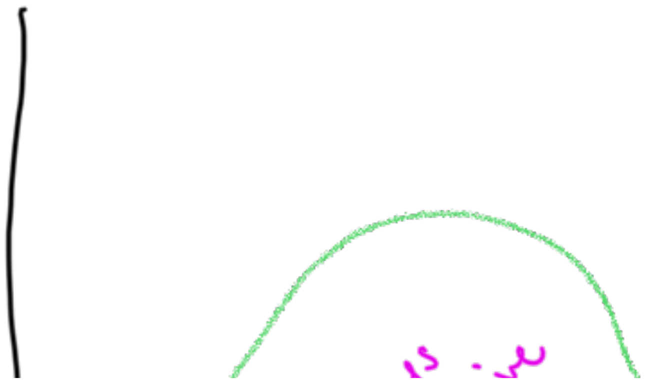
We reject H_0.

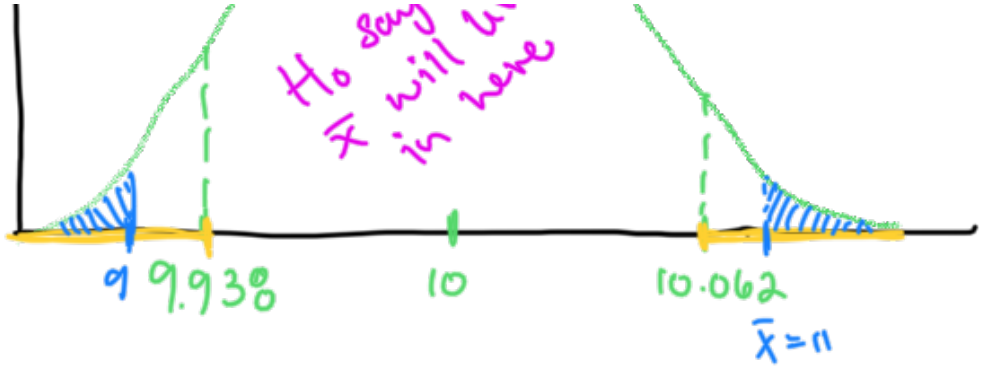
c) $X \sim N(10, .4^2)$ $\quad \bar{X} \sim N(10, \frac{.4^2}{100})$

We have $\alpha = 0.12$
but we need symmetric tails.

$\alpha/2 = 0.06$

qnorm(.06, 10, $\frac{.4}{10}$, lower = T)





9.938 = c_{low}

qnorm(.06, 10, $\frac{.4}{10}$, lower = F)

10.062 = c_{high}

$\bar{x}_{high}=11$ need $\bar{x}_{low}$ $\longrightarrow$ pnorm(11, 10, $\frac{.4}{10}$, lower = F)

$3.057\,e^{-138}$

qnorm(, 10, $\frac{.4}{10}$, lower = T)

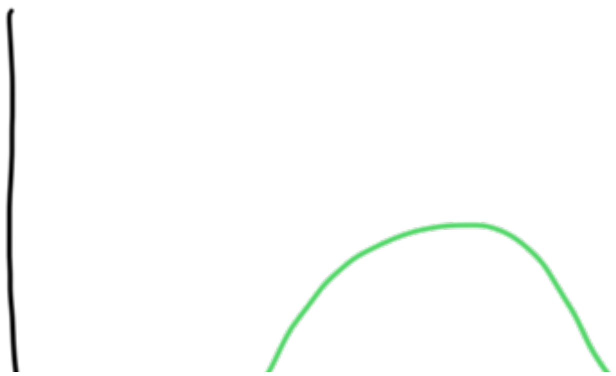
9

* if $\bar{x}_{low} < c_{low}$ or $\bar{x}_{high} > c_{high}$ reject H_0.

$9 < 9.938$ and $11 > 10.062$ so we reject H_0.

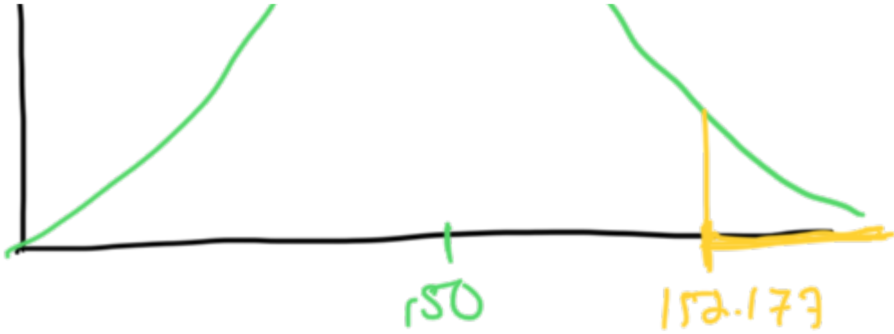
d) $X \sim N(50, 4^2)$ $\bar{X} \sim N\left(50, \frac{4^2}{60}\right)$

$3\bar{X} \sim N\left(150, 9\frac{4^2}{60}\right)$



qnorm(.08, 150, $\frac{12}{\sqrt{60}}$, lower = F)

152.177 = c



$3\bar{x}$? – test stat

3 *

* if $3\bar{x} \geq c$ then reject H_0

$\bar{x} = 50.5$ $3\bar{x} = 151.5$

We maintain H_0.

2. Calculate the P-values for problems 1b and 1c. Does using these P-values lead you to the same conclusions as the critical regions did?

(1b)

>pnorm(18.5, 20, 3/4, lower = T)

.023

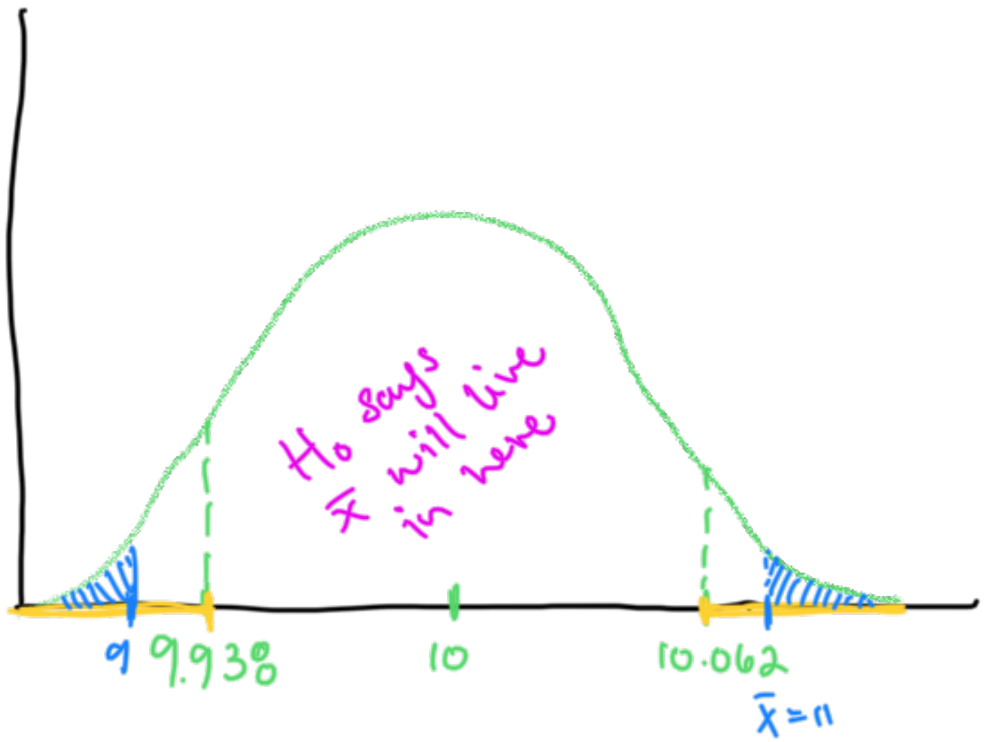
p-value = .023

If $p < \alpha$ then reject H_0.

$p = .023 \Rightarrow .023 < .06$ so we reject $H_0 = 20$.

We also rejected with critical region.

(1c) $X \sim N(10, .4^2)$ $\bar{X} \sim N(10, \frac{.4^2}{100})$



2 * pnorm(11, 10, 4/10)

6.113e-138

$\alpha = .12$

if ... $< \alpha$ reject H_0

$p_{lower} + p_{upper} < \alpha$ reject H_0

$6.113e^{-138} < .12$ so we reject.

As we did using critical regions.

3. Suppose you wanted to alter problem 1a so that the P-value, when calculated, would equal 0.04. If you could only change σ, what value would it need to equal to get the P-value to be 0.04?

$\mu = 20 \quad n = 16 \quad \sigma = 3$

Need Z for given p-value

> qnorm (.04, 0 1)

given p-value

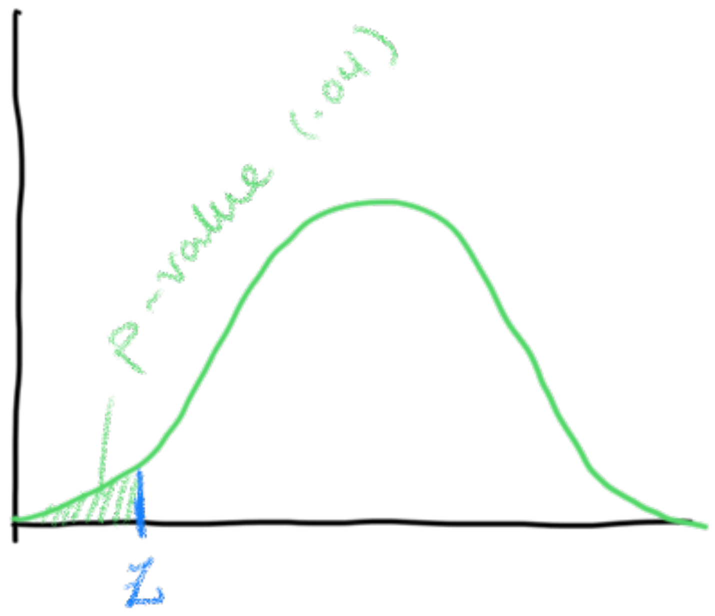
≈ 1.751 = Z value

$\mu = 20 \quad n = 16 \quad \sigma = ? \quad w/ \ \bar{x} = 18.5$

$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = Z \implies \frac{18.5 - 20}{\sigma/4} = 1.751$$

$$\bar{x} \xrightarrow{\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}} z$$

$$\boxed{\text{So } \sigma = 3.427}$$



4. In December 2017, the J-RPG Xenoblade Chronicles 2 was released for Nintendo Switch. The game is epic in its number of main quests and side quests. Those that try to finish every aspect of the game are known as "completionists". What is the average time for all completionists in the world (currently)? Assume completion times are normally distributed with unknown mean and standard deviation 50 hours (a reasonable estimate for RPGs). Before you collect data, your friend claims this average time is 250 hours (based on her personal experience). You think the value is something different and go to HowLongToBeat.com to find some data. Based on when I looked at this page (don't use more recent data!): 80 completionists had submitted their times for an average of ~~258 hours and 49 minutes~~. Define parameter(s), write hypotheses, draw a sampling distribution, and decide which hypothesis to support using $\alpha = 0.01$ (and any one of the three methods shown in class). [For those curious, my completion time was around ~~225 hours~~, and my current play time is around 600 hours because of expansion pass content!]

$H_0: \mu = 250 \quad H_1: \neq$

data!

μ is the averge completion time of all "completionists" in the world.

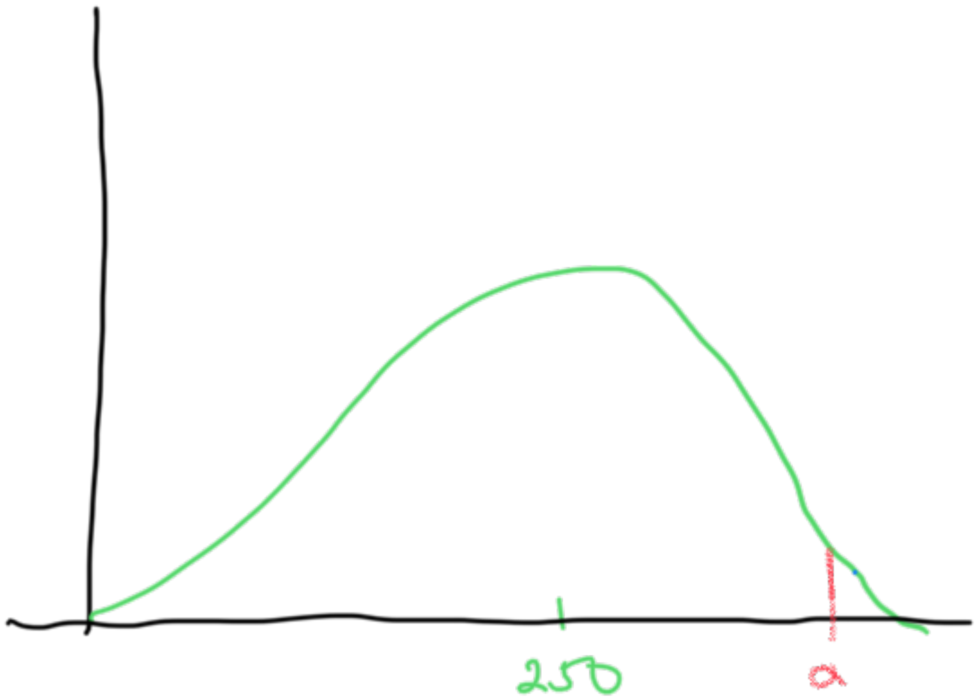
$X \sim N(\mu, 50^2)$

$H_0 : \mu = 250$ $\quad H_1 : \mu \neq 250$

$n = 80$

49min = .81hr $\longrightarrow$ $\bar{x} = 258.81$

$\alpha = .01$



$\bar{X} \sim N\left(250, \frac{50^2}{80}\right)$

* if $p < \alpha$ reject H_0

$> 2 \cdot \text{pnorm}\left(250.81, 250, \frac{50^2}{\sqrt{80}}\right)$

0.115

$0.115 \nless .01$

We keep H_0.

5. Students often wonder what to do if you get a P-value of exactly 0.05 when $\alpha = 0.05$. In truth, it doesn't matter if you suggest rejecting H_0 or keeping it, because the probability your P-value exactly equals α is a 0 probability event (the P-value is also a random variable). Let's say you wanted to be evil and make a problem for your next statistics exam where the P-value would exactly equal 0.05. You plan to make a problem where we study μ from $X \sim N(\mu, 3^2)$ with data $\overline{x} = 7$ and $n = 28$. What value(s) should you have students use for the null hypothesis to get your P-value to be 0.05 assuming a two-sided H_1?

$H_0: \mu = ?$

$X \sim N(\mu, 3^2)$

$\bar{x} \sim N(\mu, \frac{3^2}{\sqrt{28}})$

$\bar{x} = 7 \quad n = 28$

$\alpha = .05 \rightarrow \alpha/2 = .025$

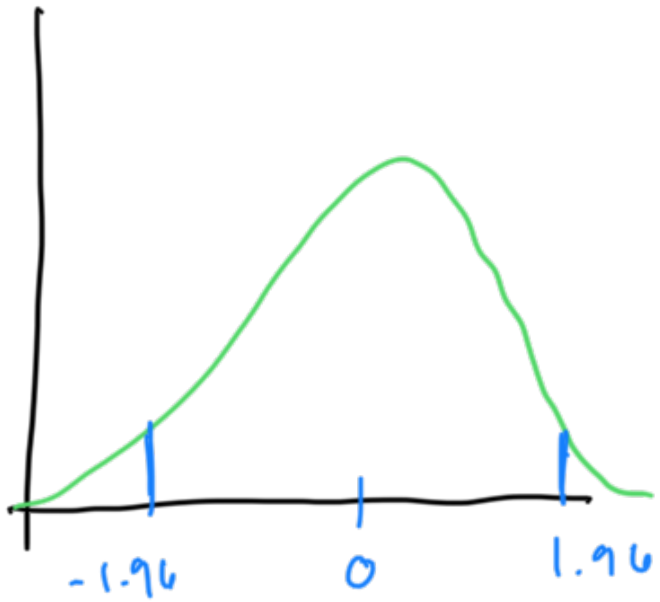
qnorm(.05/2) = ±1.96

We want 2pnorm$(7, \mu, \frac{3^2}{\sqrt{28}}) = .05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{7 - \mu}{3/\sqrt{28}} = \pm 1.960$$

Solve for $\mu = 5.889$

$H_0 = 5.889$



b

6. Change is Coming!

a. Suppose a problem has $H_0 : \mu = \mu_0$ and $H_1 : \mu \neq \mu_0$. If a given data set causes us to reject H_0 when $\alpha = 0.02$, would the same data force us reject H_0 if $\alpha = 0.05$?

b. Suppose a problem has $H_0 : \mu = \mu_0$ and $H_1 : \mu > \mu_0$. If a given data set causes us to reject H_0 for some α, would the same data force us to reject H_0 if we change H_1 to $\mu \neq \mu_0$? Assume α remains the same.

a) $H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_1$ (double sided alternitive Hypothesis)

given same $\bar{x}$ ... when $\alpha = .02$, reject H_0

when $\alpha = .05$? $2p < \alpha$

or $p < \frac{1}{2}\alpha$

So if α gets larger and the p value stays the same... Then we still have a p value less then $\frac{1}{2}\alpha$.

Now in top 2% of strangest data

b) $H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$

given same $\bar{x}$ ... we reject H_0 $p < \alpha$

Now if $H_1 : \mu \neq \mu_0$ $\longrightarrow$ $p < \frac{1}{2}\alpha$

Both $\bar{x}$ and α remain same

We might not still reject H_0.

7. You've just made the best app ever! You plan to upload it to the app store and are curious how many reviews you might get from users. The histogram of review counts for various apps in the Apple store is very right-skewed: most apps get a small number of reviews, but some apps – like Pandora, PayPal, and LinkedIn – get millions. It turns out that ln(review count) is roughly normally distributed with $\sigma = 2.6$ (for those apps with more than 5 reviews). In this problem, we'll explore $Y \sim N(\mu, 2.6^2)$ where $Y = \ln X$ is the natural log of the review counts. Your friend claims that $\mu = 6.5$, but you think it's higher: people love rating stuff in the modern era! Using the data set AppleStore.csv (found on Canvas/TritonEd in the Homework folder), conduct a hypothesis test to determine whom to momentarily believe in life. This data set contains information on 7197 random apps from Apple's app store. Load this into R using the "Import Dataset" button in the upper right window of R studio. Make sure to remove rows with 5 or fewer reviews. Your answer will be a mix of R code and written work. Use $\alpha = 0.01$. The rating_count_tot column lists how many times a given app has been rated/reviewed by users.

X is number of reviews on given app

$Y = \ln(X)$

$Y \sim N(\mu, 2.6^2)$

$H_0 : \mu = 6.5$ $H_1 : \mu > 6.5$

$n = 7197$ $\alpha = 0.01$

new $n = 6026$ (after filter)

Test: If $\bar{x} < C$ we will reject H_0 and favor H_1.

$\bar{x} = 6.564$

$\bar{Y} \sim N\left(6.5, \frac{2.6^2}{6026}\right)$

$6.564 > C$ so we maintain null hypothesis.

AppleStore | AppleStoreProblem.R

```r
filteredDF = AppleStore[filterForData,]

#verifying all elements are greater than 5
minEl = min(filteredDF$rating_count_tot)
cat("minimum element is: " ,minEl,"\n")

logList = log(filteredDF$rating_count_tot)

hist(logList)
x_bar = mean(logList)
cat("X_bar = ",x_bar,"\n")
n = length(logList)
cat("n = ",n,"\n")
#find critcal value using the upper region of the Normal
criticalValue = qnorm(0.01,6.5,(2.6)/sqrt(n),lower = F)
cat("critical value = ",criticalValue,"\n")
#a critical was generated in the upper tail
if(x_bar < criticalValue)#x_bar lives outside critical region
{
  cat("Keep H_0")
}else
{
  cat("Reject H_0")
}
```

Console

```
R 4.1.3 · ~/
> source("~/Downloads/AppleStoreProblem.R")
minimum element is:  6
X_bar =  6.563858
n =  6026
critical value =  6.577917
Keep H_0
>
```

Environment — Global Environment

sd	14
sdError	3.29448761325276
setOfMeans	num [1:1000] 127.2 73.1 191.1 151 142 ...
setOfMu	num [1:1000] 112.2 109 122 150.2 93.1 ...
sigma	10.7055123867472
sigma_sqrd	114.607995462799
trueMean	106
trueVar	196
varError	68902.7995619441
x_bar	6.56385752359468