Properties of inner products

LATEST SUBMISSION GRADE

100%

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1. The function

1 / 1 point

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] not an inner product
- [x] an inner product

> ✓ **Correct**
> It's symmetric, bilinear and positive definite. Therefore, it is a valid inner product.

- [x] symmetric

> ✓ **Correct**
> Yes: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

- [x] positive definite

> ✓ **Correct**
> Yes, the matrix has only positive eigenvalues and $\beta(\mathbf{x}, \mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{0}$ and $\beta(\mathbf{x}, \mathbf{x}) = 0 \iff \mathbf{x} = \mathbf{0}$

- [ ] not bilinear
- [ ] not positive definite
- [ ] not symmetric
- [x] bilinear

> ✓ **Correct**
> Yes:
>
> - β is symmetric. Therefore, we only need to show linearity in one argument.
> - For any $\lambda \in \mathbb{R}$ it holds that $\beta(\mathbf{x} + \lambda\mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda\beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

2. The function

1 / 1 point

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] not symmetric
- [x] symmetric

Correct

Correct: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

- [ ] not bilinear
- [x] bilinear

Correct

Correct:

- β is symmetric. Therefore, we only need to show linearity in one argument.
- $\beta(\mathbf{x} + \lambda \mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda \beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

- [ ] positive definite
- [x] not positive definite

Correct

With $x = [1, 1]^T$ we get $\beta(\mathbf{x}, \mathbf{x}) = 0$. Therefore β is not positive definite.

- [x] not an inner product

Correct

Correct: Since β is not positive definite, it cannot be an inner product.

- [ ] an inner product

3. The function

1 / 1 point

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] symmetric
- [x] not symmetric

Correct

Correct: If we take $\mathbf{x} = [1, 1]^T$ and $\mathbf{y} = [2, -1]^T$ then $\beta(\mathbf{x}, \mathbf{y}) = 0$ but $\beta(\mathbf{y}, \mathbf{x}) = 6$. Therefore, β is not symmetric.

- [x] bilinear

Correct

Correct.

- [ ] not bilinear
- [ ] an inner product
- [x] not an inner product

Correct

Correct: Symmetry is violated.

4. The function

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{y}$$

is

1 / 1 point

- [ ] not an inner product
- [ ] not positive definite
- [x] an inner product

Correct
It is the dot product, which we know already. Therefore, it is also an inner product.

- [ ] not symmetric
- [x] symmetric

Correct
It is the dot product, which we know already. Therefore, it is symmetric.

- [ ] not bilinear
- [x] positive definite

Correct
It is the dot product, which we know already. Therefore, it is positive definite.

- [x] bilinear

Correct
It is the dot product, which we know already. Therefore, it is positive bilinear.

5. For any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ write a short piece of code that defines a valid inner product.

1 / 1 point

```python
import numpy as np

def dot(a, b):
  """Compute dot product between a and b.
  Args:
    a, b: (2,) ndarray as R^2 vectors

  Returns:
    a number which is the dot product between a, b
  """

  dot_product = np.inner(a, b)

  return dot_product

# Test your code before you submit.
a = np.array([1,0])
b = np.array([0,1])
print(dot(a,b))
```

Run

Reset

```
0
```

Correct

Good job!