Congratulations! You passed!

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Eigenvalues and eigenvectors

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1. This assessment will test your ability to apply your knowledge of eigenvalues and eigenvectors to some special cases.

   1 / 1 point

   Use the following code blocks to assist you in this quiz. They calculate eigenvectors and eigenvalues respectively:

```
# Eigenvalues
M = np.array([[0.1,0.7,0.1,0.1],
              [0.7,.1,.1,.1],
              [0.1,.1,.1,.7],
              [.1,.1,.7,.1]])
# M = np.array([[0,1,0,0],
#                [1,0,0,0],
#                [0,0,0,1],
#                [0,0,1,0]])
vals, vecs = np.linalg.eig(M)
vals
```

Run

Reset

```
# Eigenvectors - Note, the eigenvectors are the columns of the output.
M = np.array([[0.1,0.7,0.1,0.1],
              [0.7,.1,.1,.1],
              [0.1,.1,.1,.7],
              [.1,.1,.7,.1]])
# M = np.array([[0,1,0,0],
#                [1,0,0,0],
#                [0,0,0,1],
#                [0,0,1,0]])
vals, vecs = np.linalg.eig(M)
vecs

```

Run

Reset

To practice, select all eigenvectors of the matrix, $A = \begin{bmatrix} 4 & -5 & 6 \\ 7 & -8 & 6 \\ 3/2 & -1/2 & -2 \end{bmatrix}$.

- [ ] None of the other options.

- [x] $\begin{bmatrix} -3 \\ -3 \\ -1 \end{bmatrix}$

Correct

This is one of the eigenvectors.

- [x] $\begin{bmatrix} -2/\sqrt{9} \\ -2/\sqrt{9} \\ 1/\sqrt{9} \end{bmatrix}$

Correct

This is one of the eigenvectors. Note eigenvectors are only defined upto a scale factor.

- [ ] $\begin{bmatrix} 1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$

- [ ] $\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$

- [ ] $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

- [x] $\begin{bmatrix} 1/2 \\ -1/2 \\ -1 \end{bmatrix}$

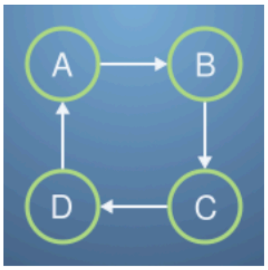
Correct

This is one of the eigenvectors. Note eigenvectors are only defined upto a scale factor.

2. Recall from the *PageRank* notebook, that in PageRank, we care about the eigenvector of the link matrix, L, that has eigenvalue 1, and that we can find this using *power iteration method* as this will be the largest eigenvalue.

1 / 1 point

PageRank can sometimes get into trouble if closed-loop structures appear. A simplified example might look like this,



With link matrix, $L = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

Use the calculator in Q1 to check the eigenvalues and vectors for this system.

What might be going wrong? Select all that apply.

- [ ] Some of the eigenvectors are complex.
- [ ] The system is too small.
- [ ] None of the other options.
- [x] Other eigenvalues are not small compared to 1, and so do not decay away with each power iteration.

> ✓ **Correct**
> The other eigenvectors have the same size as 1 (they are -1, i, $-i$)

- [x] Because of the loop, *Procrastinating Pat*s that are browsing will go around in a cycle rather than settling on a webpage.

> ✓ **Correct**
> If all sites started out populated equally, then the incoming pats would equal the outgoing, but in general the system will not converge to this result by applying power iteration.

3. The loop in the previous question is a situation that can be remedied by damping.

1 / 1 point

If we replace the link matrix with the damped, $L' = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.7 \\ 0.7 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.7 & 0.1 \end{bmatrix}$, how does this help?

- [x] The other eigenvalues get smaller.

Correct
So their eigenvectors will decay away on power iteration.

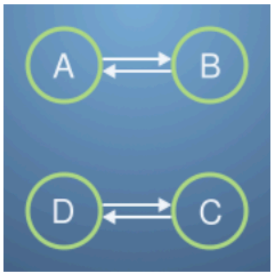
- [ ] The complex number disappear.
- [x] There is now a probability to move to any website.

Correct
This helps the power iteration settle down as it will spread out the distribution of Pats

- [ ] None of the other options.
- [ ] It makes the eigenvalue we want bigger.

4. Another issue that may come up, is if there are disconnected parts to the internet. Take this example,

1 / 1 point



with link matrix, $L = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

This form is known as block diagonal, as it can be split into square blocks along the main diagonal, i.e., $L = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$, with $A = B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ in this case.

What is happening in this system?

- [ ] None of the other options.
- [x] There are loops in the system.

Correct
There are two loops of size 2. (A ⇄ B) and (C ⇄ D)

- [x] There isn't a unique PageRank.

Correct
The power iteration algorithm could settle to multiple values, depending on its starting conditions.

- [x] There are two eigenvalues of 1.

Correct
The eigensystem is degenerate. Any linear combination of eigenvectors with the same eigenvalue is also an eigenvector.

- [ ] The system has zero determinant.

5. By similarly applying damping to the link matrix from the previous question. What happens now?

1 / 1 point

- [ ] There becomes two eigenvalues of 1.
- [ ] The negative eigenvalues disappear.
- [ ] The system settles into a single loop.
- [ ] Damping does not help this system.
- [x] None of the other options.

Correct
There is now only one eigenvalue of 1, and PageRank will settle to it's eigenvector through repeating the power iteration method.

6. Given the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$, calculate its characteristic polynomial.

1 / 1 point

- ( ) $\lambda^2 + 2\lambda + \frac{1}{4}$
- ( ) $\lambda^2 + 2\lambda - \frac{1}{4}$
- ( ) $\lambda^2 - 2\lambda - \frac{1}{4}$
- (•) $\lambda^2 - 2\lambda + \frac{1}{4}$

Correct
Well done - this is indeed the characteristic polynomial of A.

7. By solving the characteristic polynomial above or otherwise, calculate the eigenvalues of the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$.

1 / 1 point

- ○ $\lambda_1 = -1 - \frac{\sqrt{5}}{2}, \lambda_2 = -1 + \frac{\sqrt{5}}{2}$
- ○ $\lambda_1 = -1 - \frac{\sqrt{3}}{2}, \lambda_2 = -1 + \frac{\sqrt{3}}{2}$
- ○ $\lambda_1 = 1 - \frac{\sqrt{5}}{2}, \lambda_2 = 1 + \frac{\sqrt{5}}{2}$
- ◉ $\lambda_1 = 1 - \frac{\sqrt{3}}{2}, \lambda_2 = 1 + \frac{\sqrt{3}}{2}$

✓ **Correct**

Well done! These are the roots of the above characteristic polynomial, and hence these are the eigenvalues of A.

8. Select the two eigenvectors of the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$.

1 / 1 point

- ○ $\mathbf{v_1} = \begin{bmatrix} 1 - \sqrt{3} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 + \sqrt{3} \\ 1 \end{bmatrix}$
- ○ $\mathbf{v_1} = \begin{bmatrix} 1 - \sqrt{5} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 + \sqrt{5} \\ 1 \end{bmatrix}$
- ○ $\mathbf{v_1} = \begin{bmatrix} -1 - \sqrt{5} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -1 + \sqrt{5} \\ 1 \end{bmatrix}$
- ◉ $\mathbf{v_1} = \begin{bmatrix} -1 - \sqrt{3} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -1 + \sqrt{3} \\ 1 \end{bmatrix}$

✓ **Correct**

These are the eigenvectors for the matrix A. They have the eigenvalues λ_1 and λ_2 respectively.

9. Form the matrix C whose left column is the vector $\mathbf{v_1}$ and whose right column is $\mathbf{v_2}$ from immediately above.

1 / 1 point

By calculating $D = C^{-1}AC$ or by using another method, find the diagonal matrix D.

- (selected) $\begin{bmatrix} 1+\frac{\sqrt{3}}{2} & 0 \\ 0 & 1-\frac{\sqrt{3}}{2} \end{bmatrix}$
- $\begin{bmatrix} -1-\frac{\sqrt{5}}{2} & 0 \\ 0 & -1+\frac{\sqrt{5}}{2} \end{bmatrix}$
- $\begin{bmatrix} 1-\frac{\sqrt{5}}{2} & 0 \\ 0 & 1+\frac{\sqrt{5}}{2} \end{bmatrix}$
- $\begin{bmatrix} -1-\frac{\sqrt{3}}{2} & 0 \\ 0 & -1+\frac{\sqrt{3}}{2} \end{bmatrix}$

Correct

Well done! Recall that when a matrix is transformed into its diagonal form, the entries along the diagonal are the eigenvalues of the matrix - this can save lots of calculation!

10. By using the diagonalisation above or otherwise, calculate A^2.

1 / 1 point

- $\begin{bmatrix} 11/4 & -1 \\ -2 & 3/4 \end{bmatrix}$
- $\begin{bmatrix} -11/4 & 1 \\ 2 & -3/4 \end{bmatrix}$
- $\begin{bmatrix} -11/4 & 2 \\ 1 & -3/4 \end{bmatrix}$
- (selected) $\begin{bmatrix} 11/4 & -2 \\ -1 & 3/4 \end{bmatrix}$

Correct

Well done! In this particular case, calculating A^2 directly is probably easier - so always try to look for the method which solves the question with the least amount of pain possible!