8. Linear Regression and Event Study

Work in Progress - not proofread - still notes in it

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Contents

1 Introduction

On September 3, 2015, Volkwagen admitted that its software provided incorrect measurements of CO_2 emissions. On November 6, 2019, Apple raised over $2 billion by issuing **green bonds** to invest in energy efficienct products. Do events like these affect the value of firms? This session provides a brief introduction to the **event study**, a widely used method to examine the impact of events on the value of firms. We also describe how parameters of a simple linear regression model can be estimated with `R`. There are several excellent sources to learn event studies in more detail including (Benninga 2014, chapter 14), (Campbell et al. 1997, chapter 4), and (Kothari and Warner 2007).

```
library(tidyverse)
```

```
## -- Attaching packages ----------------------------------------------------------- tidyverse 1.3.0 --

## v ggplot2 3.3.0     v purrr   0.3.3
## v tibble  2.1.3     v dplyr   0.8.5
## v tidyr   1.0.2     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.5.0

## -- Conflicts -------------------------------------------------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

2 Intended Learning Outcomes

By the end of this session, students should be able to:

1. understand the application of the event study method in testing the Efficient Market Hypothesis;

2. understand the procedure of the event study method;

3. use R to carry out an event study.

3 Event Studies and the Efficient Market Hypothesis (EMH)

The efficient market hypothesis states that stock price fully reflects all relevant information. Consequently, price may change only when there is new information. For example on September 3, 2015 when Volkswagen admitted that the company programmed its sofware to provide inaccurate measurement of CO_2 emissions, investors may adjust their estimate of future return downward. This would lead to a downward movement in the stock stock price. As new information arrives randomly, stock prices are assumed to follow a *random walk*.

fama1970efficient presents the EMH in his fair game model as follow. Define Φ_t as a set of information available to investors at time t and $E(r_{j,t+1}|\Phi_t)$ as the expected return for stock j in period $t+1$ based on the information available at time t. The realised return for stock j in period $t+1$, is $r_{j,t+1} = E(r_{j,t+1}|\Phi_t) + z_{j,t+1}$. An investor may gain positive excess return if the realised return is higher than the expected return, that is $z_{j,t+1} > 0$ or negative excess return if $z_{j,t+1} < 0$. The efficient market hypothesis states that $z_{j,t+1}0$ is a fair game with respect to Φ_t if

$$E(z_{j,t+1}|\Phi_t) = 0 \tag{1}$$

Event studies have been used as a *semi-strong efficient market* test. In this context, Φ_t is defined as *present public information*, which is all past information and announcement(s) of new information. The market is said to be *semi-strong efficient* if it is not possible for investors to use information at the time of announcement t to **persistently** earn a return in excess of equilibrium expected return. Note that the EMH does not imply that investors cannot earn positive excess return at all. For a particular event, $z_{j,t+1}$ may be positive, negative or zero. However if the market is efficient, it is not possible for investors to use publicly available information from announcements to earn positive excess return all the time. In our example, we can use the event study method to test whether it is possible to invest in Volkswagen shares after the admission made public and experience superior return.

Event studies have also been used to measure the market reaction to a certain type of news. For example, there is a plethora of research using event studies to measure the impact of M&A announcements on bidders' stock prices. This research has established that on average bidders witness unfavourable market reactions in the form of significant negative abnormal returns. It should be noted that not all M&A announcements are associated with a drop in stock price. Some firms experience no change or even increase in stock price.

4 Structure of an Event Study

An event study usually contains 7 steps (Campbell et al. 1997, pp. 151-152).

4.1 Identifying Timing of an Event

Researchers must define the event and identify the time of the event being evaluated. In this session, we will examine the impact of Volkwagen's admission on September 3, 2015 that the company's sofware was programmed to provide inaccurate measurement of CO_2 emissions. A timeline of events leading up to Volkswagen's admission is avaiable at Reuters. It is important to note that a clear identification of event is rarely possible. One reason is that the exact timing of the event is rarely known. For example, even though Volkwagen's admission was announced on September 3, 2015, it is very likely that many analysts already knew about it. Thus, Volkswagen's stock price probably incorporated the adverse impact of the admission before September 3, 2015. In the timeline of this event, the announcement date can be called as t.

Figure 1: Timeline for an Event Study

4.2 Selection Criteria

Researchers justify their *selection criteria* that determines the firms that they wish to include in their analysis. In an event study, researchers normally examine movements in stock prices, or stock returns, of several firms that are exposed to similar events. For example, the impact of acquisitions is examined by evaluating stock returns of several firms that announce acquisition deals.

4.3 Expected Returns and Abnormal Returns

In an event study, researchers specify returns on a stock as a sum of expected return (sometimes referred to as normal return) and **abnormal return**. The expected return is the return should the event not happen whereas the abnormal return is considered as the part of return occured due to the event.

As we discuss in Session 5, returns are random variables and we rely on the probability theory to describe and quantify uncertainty associated with future returns. An important parameter of the probability distribution of returns is the *expected returns*. The expected return reflects the market's long-term expectation about the stock return given its specific risk characteristics. If we can observe all possible outcomes/returns and their associated probabilities we can use this formula to calculate the expected returns: $E(R_i) = \mu_i = \sum_{k=1}^{N} R_{ik}.p_k$. But what if we cannot observe all possible returns and their associated probabilities (we certainly can't)? How about using the mean of past returns, or sample mean, as an estimate of expected returns? This is fine if returns are *stationary*, i.e. although returns in one period can be different from the other and such fluctuations are random, the mean (and other properties) of returns in different periods is similar. But what if returns are not stationary? We have to rely on models such as the Market Model or the Capital Asset Pricing Model. In the next section we will see how the Market Model can be used to estimate expected returns.

4.4 Estimation Procedure

4.4.1 Estimation Window and Expected Returns

The estimation window can be used to estimate *expected returns* which are the returns should the event not happen, or the returns under the normal circumstances. The estimation window therefore is set to be prior to the event window. The common practice is to set the estimation window for 252 trading days (one trading year). Another widely used length is 180 trading days. In our example, we set the estimation window for 252 trading days, from August 4, 2014 to August 6, 2015. You can see that this window ends 20 days before the announcement date September 3, 2015. We assume that the expected return that we estimate using this estimation window is not affected by price movements relating to the event. In the timeline of this event where the event date is t, the estimation window $[T_0 T_1]$ is $(t - 273, t - 21)$. It should be noted that the expected returns should be estimated unconditional on the event but it may be conditional on other information. During the 252 trading days of the estimation window, Volkswagen may have released earnings forecasts, made important governnance announcements etc. but it did not make any announcement relating to the admission about inaccurate measurement of CO_2 emissions. Thus we can say that the expected returns that we estimate using this specified window are unconditional on the admission event.

The next step is to use the estimation window to estimate a model of the expected returns. The widely used model in the event study is the Market Model, which assumes a linear relationship between the return of a stock and the return of the market portfolio. The Market Model can be expressed as

$$r_{it} = \alpha_i + \beta_i r_{Mt} \tag{2}$$

where β_i measures the responsiveness of the the stock return on the market return and r_{it} and r_{Mt} are stock and market daily returns.

In implementating the Market Model, it is important to select an appropriate market portfolio. The common practice is to use the market index where the shares of the company of interest are listed on. In our example, we use DAX, a blue chip stock market index consisting of the 30 major German companies trading on the Franfurt Stock Exchange. Volkswagen is a component of this index. In some cases an industry index can be used instead of a market index.

The Market Model implies a linear relationship between stock returns and market returns. But you will see from the scatterplot in the next section, where returns for Volkswagen and DAX are plotted, the points representing the returns on a particular day do not sit on one straight line. To estimate the coefficients α_i and β_i we aim to find a *line of best fit* which is as close to our data as possible. Let us define the error term e_{it} as the difference (or deviation) between the actual return and the fitted return estimated by the Market Model. The simple linear regression model assumes that the error terms are independently and identically distributed with mean 0 and constant variance σ_e^2 (for full list of assumptions see **RuppertMatteson2015**). The Market Model that represents the relationship between stock returns and market returns becomes:

$$r_{it} = \alpha_i + \beta_i r_{Mt} + e_{it} \tag{3}$$

The procedure is to find the straight line that minimises the sum of the squared deviations (think about why we need to minimise the sum of the squared deviations but not the sum of the deviations). That is, we estimate $\hat{\alpha}_i$ and $\hat{\beta}_i$ to minimise:

$$\Sigma_{t=1}^{T} \hat{e}_{it}^2 = \Sigma_{t=1}^{T} (r_{it} - \hat{\alpha}_i - \hat{\beta}_i r_{Mt})^2 \tag{4}$$

$\hat{\alpha}_i$ and $\hat{\beta}_i$ are estimates of α_i and β_i whereas $\hat{e}_{it}$ is the difference between actual r_{it} and the return predicted by $\hat{\alpha}_i$ and $\hat{\beta}_i$. r_{it} is called the dependent variable, or predicted variable. r_{Mt} is called the independent variable, or the predictor variable.

We use the OLS (Ordinary Least Squares) estimates as below (See **RuppertMatteson2015** for more details).

$$\hat{\beta}_i = \frac{\Sigma_{t=1}^{T}[(r_{it} - \bar{r}_{it})(r_{Mt} - \bar{r}_{Mt})]}{\Sigma_{t=1}^{T}(r_{Mt} - \bar{r}_{Mt})^2} \tag{5}$$

and

$$\hat{\alpha}_i = r_{it} - \hat{\beta}_i r_{Mt} \tag{6}$$

The residual standard error (also called the standard error of the regression) $\hat{\sigma}_i$ estimates the standard deviation in the predicted variable (in our example, stock returns of Volkswagen) after the effect of the predictor variable (returns of DAX) has been taken out.

$$\hat{\sigma}_i^2 = \frac{1}{(T-2)} \Sigma_{t=1}^{T} \hat{e}_{it}^2 \tag{7}$$

The residual standard error $\hat{\sigma}_i$ can also be used to estimate the standard error of $\hat{\beta}_i$ to see how close or how far our estimate $\hat{\beta}_i$ is from β_i.

$$se(\hat{\beta}_i) = \hat{\sigma}_i / \sqrt{\Sigma_{t=1}^{T}(r_{Mt} - \bar{r}_{Mt})^2} \tag{8}$$

Similarly if you want to know how precise $\hat{\alpha}_i$ is you need to estimate its standard error of $se(\hat{\alpha}_i)$.

4.4.2 Event Window and Abnormal Returns

Once we have estimated α_i and β_i we can then calculate the abnormal returns during the event window. The event window commonly begins a few days before the event date and ends a few days after. In our example, we set the event window to be during August 13, 2015 to September 24, 2015. This means $[T_1 + 1, T_2]$ is $(t - 15, t + 15)$. You can use other windows such as $(t - 10, t + 10)$, $(t - 5, t + 5)$, $(t, t + 5)$ or $(t - 1, t + 2)$.

We calculate the *abnormal return* for each day during the event window, which is the difference between the realised return (also called actual return) and the expected return (also called normal return, or predicted return) as follows:

$$AR_{it} = r_{it} - E(r_{it}) = r_{it} - (\hat{\alpha}_i + \hat{\beta}_i r_{Mt}) \tag{9}$$

For example, the abnormal return one day after the event date is $AR_{i1} = r_{i1} - (\hat{\alpha}_i + \hat{\beta}_i r_{M1})$. This measures the difference in the return due to the event compared to the predicted return asuming that the event did not take place.

Now we can calculate the cumulative abnormal return by summing all the abnormal returns during the event window:

$$CAR_i = \Sigma_{t=1}^{T} AR_{it} \tag{10}$$

4.5 Testing Procedure, Time-Series Aggregation and Test Statistics of Abnormal Returns

In examining the market reaction to the event, we are interested in knowing if there is any change to shareholder value following the event. We can test the null hypothesis that abnormal returns in the event window are not systematically different from zero. The statistical significance of abnormal returns can be measured using the ratio of the abnormal return divided by the residual standard error estimated in the market model.

$$\frac{AR_{it}}{\hat{\sigma}_i^2} \tag{11}$$

The regression residuals, and consequently the abnormal returns and their test statistic are random variables. This is because the expected returns are measured with errors (read the statistical properties of the OLS in Ruppert and Matteson for mroe details). If the regression residuals are assumed to be normally distributed, an abnormal return is statistically and significantly different from 0 at the standard 5% significance level if its test statistic is greater than 1.96. (Kothari and Warner 2007) point to factors that are unrelated to the event but might affect the abrnomal returns during the event window as another source of errors in measuring abnormal returns.

Examining the abnormal returns is also informative about how fast the market incorporates information revealed in the event into price, i.e. speed of adjustment. For example, nonzero abnormal returns in the days prior to the event date provide some evidence of pre-event leakage of information whereas systematically different from zero abnormal returns in the many days after the event date are inconsistent with the EMH as it seems investors are slow in reacting to the information.

We can test the null hypothesis that the cumulative abnormal return in the event window are not systematically different from zero using the ratio of the cumulative abnormal return divided by an estimare of its standard deviation.

$$\frac{CAR_{i(t_1 t_2)}}{[L\sigma^2(AR_{i(t_1 t_2)})]^{1/2}} \tag{12}$$

where L is the length of the event window and $\sigma^2(AR_{i(t_1 t_2)})$ is the variance of the abnormal returns in the event window.

5 An Event Study - The Case of Volkswagen

5.1 Prices and returns

To estimate the expected return for Volkswagen, we first open the files containing daily price data of Volkswagen:

```
volks <- read_csv("volks.csv")
```

```
## Parsed with column specification:
## cols(
##   date = col_date(format = ""),
##   VOW.DE = col_double()
## )
```

and the value of DAX:

```
dax <- read_csv("GDAXI.csv")
```

```
## Parsed with column specification:
```

```
## cols(
##   date = col_date(format = ""),
##   GDAXI.Adjusted = col_double()
## )
```

We now join the two files as follow:

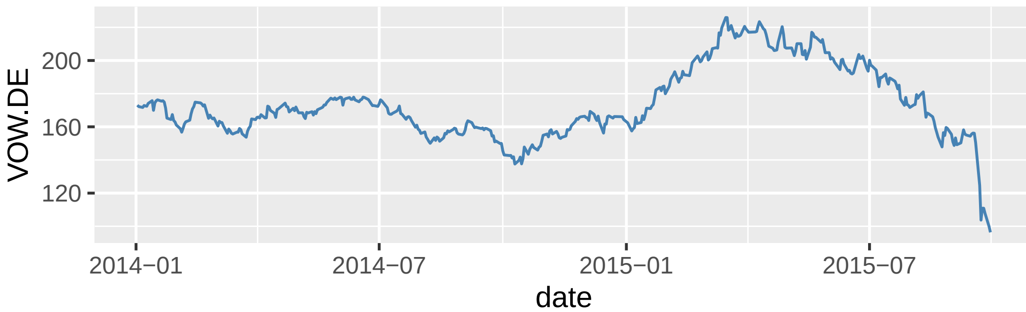
```r
volks <- left_join(volks, dax, by = "date")
```

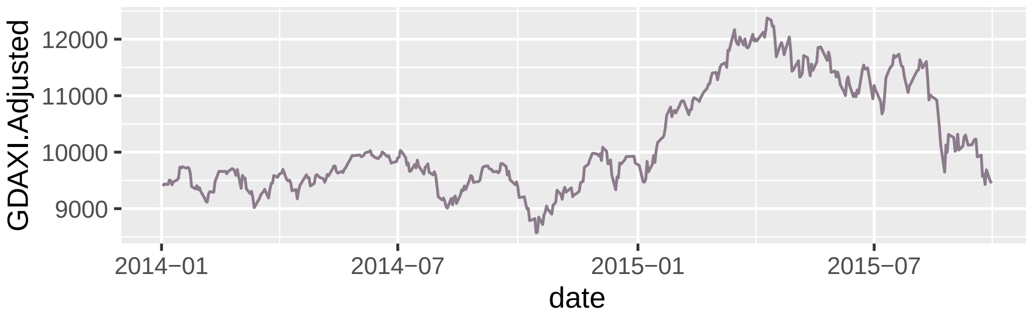
We use `ggplot()` function to plot the price movements of Volkswagen and DAX. You can see how closely the stock and the index move together.

```r
volksfig <- ggplot(volks) +
  geom_line(aes(date, VOW.DE), color = "steelblue")

daxfig <- ggplot(volks) +
  geom_line(aes(date, GDAXI.Adjusted), color = "thistle4")

library(gridExtra)
```

```
##
## Attaching package: 'gridExtra'

## The following object is masked from 'package:dplyr':
##
##     combine
```

```r
grid.arrange(volksfig, daxfig)
```





Next we need to compute the daily returns for Volkswagen and DAX as follows:

```
# compute return


volks <- volks %>%

  mutate(volksRt = log(VOW.DE/(lag(VOW.DE))),

         daxRt = log(GDAXI.Adjusted/(lag(GDAXI.Adjusted)))) %>%

  drop_na() %>%

  select(date, volksRt, daxRt)
```

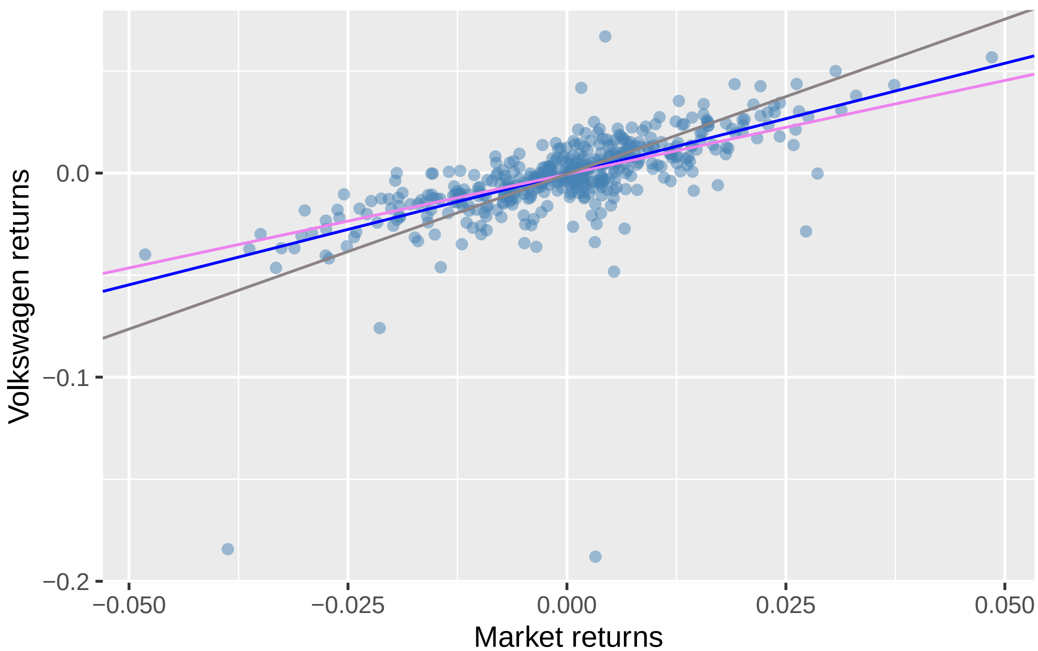
and then set the estimation window of 252 trading days:

```
# Create estimation window


est_window <- volks %>%

  filter(date <= "2015-08-06" & date >= "2014-08-04")
```

5.2 The Market Model and the Simple Linear Regression Model

We can plot the returns of Volkswagen and DAX like this.

```r
# Scatterplot

volksscat <- ggplot(volks) +
  geom_point(aes(daxRt, volksRt), color = "steelblue", alpha = 0.5) +
  xlab("Market returns") +
  ylab("Volkswagen returns") +
  geom_abline(intercept = -0.000478, slope = 1.08528, color = "blue") +
  geom_abline(intercept = -0.000562, slope = 0.918528, color = "violet") +
  geom_abline(intercept = -0.000502, slope = 1.518528, color = "lavenderblush4")

volksscat
```



Each point in the scatterplot represents the returns of Volkswagen and of DAX on a particular day. Next we want to fit a straight line which best represents the data on this scatterplot. The intercept of this line would be our best estimate of α_i and the slope of this line would be our best estimate of β_i for the relationship between Volkswagen and DAX over the estimation window.

As you can see, it looks like more than one line that could serve as candidates for this *line of best fit*. Although these lines may pass through some of the points, no line passes through all the points. This indicates that the stock return and market return are not perfectly linearly related. But which line best represents our data?

We can use the simple linear regression model discussed above to estimate α_i and β_i in the Market Model for the estimation window $(t-273, t-21)$ with the following code in R:

```
# Estimating beta


market_model <- lm(volksRt ~ daxRt, data = est_window)
```

To see the results:

```
summary(market_model)
```

```
##
## Call:
## lm(formula = volksRt ~ daxRt, data = est_window)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.033887 -0.005534  0.000689  0.005275  0.023286
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0004783  0.0006164  -0.776    0.438
## daxRt        1.0852813  0.0482739  22.482   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.009799 on 252 degrees of freedom
## Multiple R-squared:  0.6673, Adjusted R-squared:  0.666
## F-statistic: 505.4 on 1 and 252 DF,  p-value: < 2.2e-16
```

The OLS regression line relating stock returns of Volkswagen to return of DAX is $\hat{r} = -0.0005 + 1.085 r_M$. You can also see that the Multiple R-squared: 0.6673. This figure summarises how well the estimated OLS regression line fits the data of the estimation window. The value of the R-squared indicates that 66.73% of the variations in stock returns of Volkswagen can be explained by market (DAX) returns and that the Market Model cannot explained 33.27% of the variations.

As mentioned above, besides the Market Model, several other expected return models such as the Capital Asset Pricing Model (CAPM) have been used in event studies. The expected returns (as well as abnormal returns) estimated from each model differ from one another (see Kothari and Warner (2007) for more discussion on the bias, precision and other properties of expected returns and abnormal returns in different models).

5.3 Event Window and Abnormal Returns

In our example, we set the event window to be from August 13, 2015 to September 24, 2015, i.e. $(t-15, t+15)$.

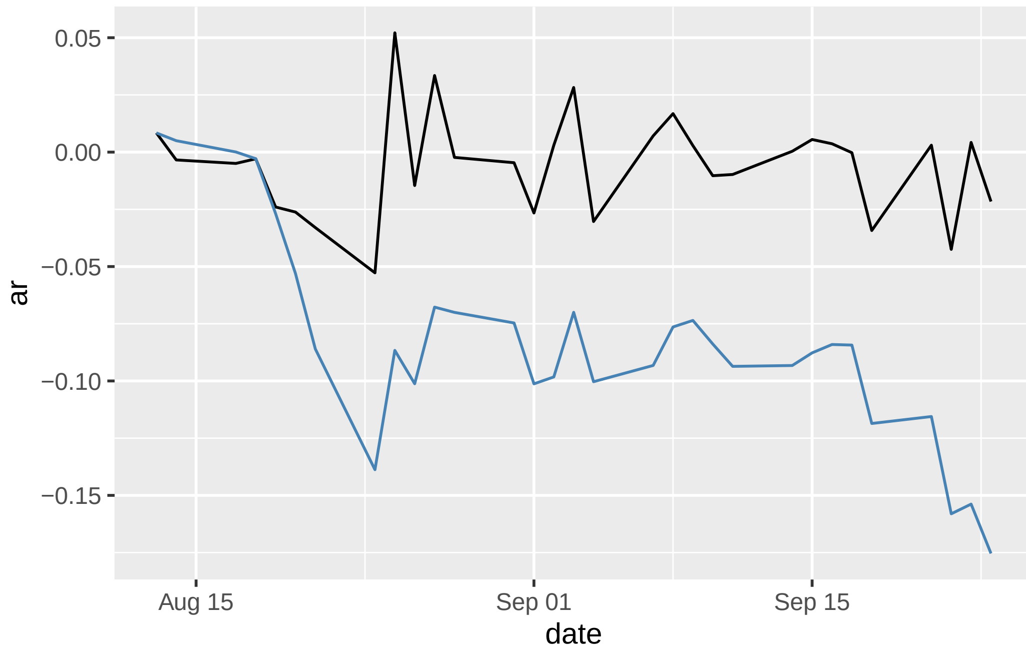
```
# Create Estimation and Event Window

event_window <-  volks %>%
  filter(date >= "2015-08-13" & date <= "2015-09-24")
```

Using the estimated $\hat{\alpha}_i$ and $\hat{\beta}_i$ in the previous step, we can now calculate the abnormal returns (AR) and the cumulative abnormal returns (CAR) during this event window.

```
# Calculate Abnormal Returns and Cumulative Abnormal Returns duing the event window

ar <- event_window %>%
  mutate(ar = -0.0005 + 1.085*daxRt,
         t = ar/0.009799,
         car = cumsum(ar))
```

Negative abnormal returns from day 12 preceding the event date are evidence that market partially anticipated the information prior to the formal announcement. We have some weak evidence of value destruction here as only twelve abnormal returns during the event window are statistically different from zero.

```
# Plot Abnormal and Cumulative Abnormal Returns

ggplot(ar, aes(date, ar)) + geom_line() +
  geom_line(aes(date, car), color = "steelblue")
```



We can now calculate the test statistic for CAR. The test statistic is -1.416, which is not significant at 5% level. This means although the CAR(-15,15) for the event is -17.5% which might suggest that the market reacted negatively to the admission by Volkswagen, statistically we cannot reject the null hypothesis that this CAR is zero.

```
# Calculate test statistics for Cumulative Abnormal Returns


ar$car[ar$date == "2015-09-24"]/((31*var(ar$ar))^(0.5))
```

```
## [1] -1.41678
```

6 Event Studies in Finance Research

The discussion above can be referred to as time-series aggregation of abnormal returns in response to an event. Some event studies focus on the post-event window (ranging from three months to five years) to measure the long-term impact of the event. A popular application of event studies is to measure the impact of a particular type of event on a set of stocks, also called cross-sectional aggregation. For example, we can study the impact of the issuance of bonds for environmental projects on firm value. We know that on November 6, 2019, Apple issued $2 billion green bond. Before that date, Apple already issued $1.5 billion green bond on February 18, 2016 and then $1 billion green bond on June 13, 2017. There have been many other bond issuance, for example on March 19, 2014 Unilever issued \$250 million green bond and on October 7, 2019 PepsiCo Inc. issued \$1 billion green bond. One may ask if issuing debt instruments for envinronmentally projects creates value for shareholders?

To examine if the market reacts favourably to the announcement of green bond issuance, we could test if the cross-sectional distribution of returns at the time of issuing a green bond is systematically different from the distribution of predicted returns. The common practice is to test the null hypothesis that the mean abnormal return is equal to zero. We first need to identify green bond events. To make the task more manageable researchers may select a particular type of issuers and a pre-specified period of time, e.g. US firms during 2014-2020, or European firms during 2010-2020. The event study procedure described above needs to be applied to each of the event to calculate the abnormal returns and cumulative abnormal returns for each event. The estimation, event and post-event window should be the same for all events. For each event, the abnormal returns in the event window are calculated as before:

$$AR_{it} = r_{it} - (\hat{\alpha}_i + \hat{\beta}_i r_{Mt}) \tag{13}$$

For a sample of N events, the cross-sectional mean abnormal return and mean cumulative abnormal return for any period t is:

$$AR_t = \frac{1}{N} \Sigma_{i=1}^{N} AR_{it} \tag{14}$$

and

$$CAR_{t_1 t_2} = \frac{1}{N} \Sigma_{i=1}^{N} CAR_{i(t_1 t_2)} \tag{15}$$

For more details about test statistics and properties of the (cumulative) abnormal returns see (Kothari and Warner 2007).

7 Exercise

You will now evaluate the market reaction to the news that Apple raised over $2 billion by issuing green bond on November 6, 2019 using the event study precedure described in this session.

7.1 Prices and returns

First open the files containing daily price of Apple and value of NASDAQ and then join the two files:

```
apple <- read_csv("apple.csv")
```

```
## Parsed with column specification:
## cols(
##   date = col_date(format = ""),
##   AAPL = col_double()
## )
```

```
nasdaq <- read_csv("nasdaq.csv")
```

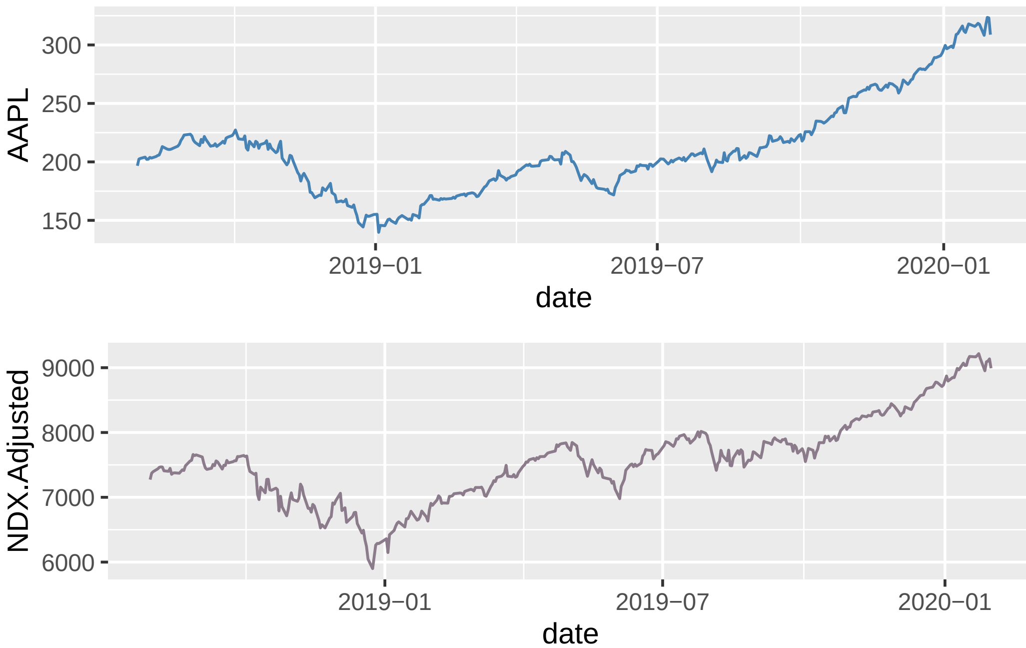
```
## Parsed with column specification:
## cols(
##   date = col_date(format = ""),
##   NDX.Adjusted = col_double()
## )
```

```
apple <- left_join(apple, nasdaq, by = "date")
```

Use `ggplot()` function to plot the price movements of Apple and NASDAQ.

```
applefig <- ggplot(apple) +
  geom_line(aes(date, AAPL), color = "steelblue")

nasdaqfig <- ggplot(apple) +
  geom_line(aes(date, NDX.Adjusted), color = "thistle4")

library(gridExtra)
```

```r
grid.arrange(applefig, nasdaqfig)
```



Next compute the daily returns for Apple and NASDAQ:

```r
# compute return


apple <- apple %>%
  mutate(appleRt = log(AAPL/(lag(AAPL))),
         nasdaqRt = log(NDX.Adjusted/(lag(NDX.Adjusted)))) %>%
  drop_na() %>%
  select(date, appleRt, nasdaqRt)
```

and then set the estimation window and event window for 252 trading days:

```r
# Create Estimation and Event window


event_window <-  apple %>%
  filter(date >= "2019-10-16" & date <= "2019-11-27")


est_window <- apple %>%
  filter(date <= "2019-10-15" & date >= "2018-10-13")
```

Now run the regression for the market model:

```r
# Estimating beta

market_model <- lm(appleRt ~ nasdaqRt, data = est_window)

summary(market_model)
```

```
##
## Call:
## lm(formula = appleRt ~ nasdaqRt, data = est_window)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.063163 -0.005493  0.000271  0.005395  0.052765
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0002120  0.0006919  -0.306    0.759
## nasdaqRt     1.2156016  0.0499477  24.338   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01098 on 250 degrees of freedom
## Multiple R-squared:  0.7032, Adjusted R-squared:  0.702
## F-statistic: 592.3 on 1 and 250 DF,  p-value: < 2.2e-16
```

Set the event window and calculate the abnormal returns and test statistics:

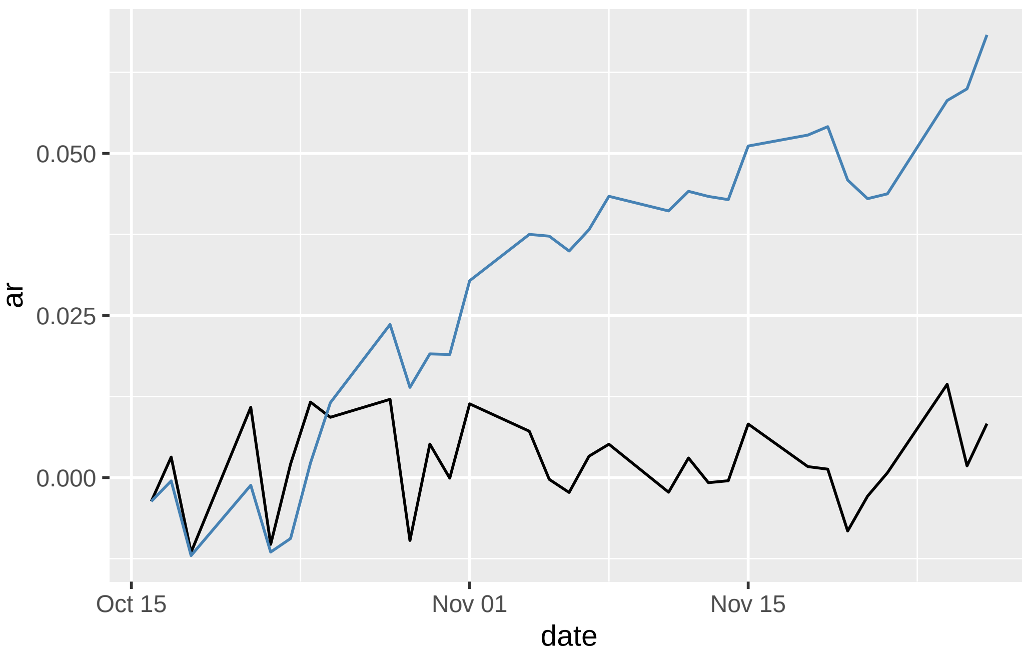
```r
# Calculate Abnormal Returns and Cumulative Abnormal Returns duing the event window

ar <- event_window %>%
  mutate(ar = -0.0002 + 1.2156*nasdaqRt,
         t = ar/0.01098,
         car = cumsum(ar))
```

```r
# Calculate test statistics for Cumulative Abnormal Returns

ar$car[ar$date == "2019-11-27"]/((31*var(ar$ar))^(0.5))
```

```
## [1] 1.801559
```

You can also plot the abnormal returns and cumulative abnormal returns for the event window.

```r
# Plot Abnormal and Cumulative Abnormal Returns

ggplot(ar, aes(date, ar)) + geom_line() +
  geom_line(aes(date, car), color = "steelblue")
```



What does the result from your event study tell you about the market reaction to Apple's issuance of green bond?

References

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