

4. Basic Visualisation and Exploratory Data Analysis

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1 Introduction

After preparing data for analysis, the first step is usually an “Exploratory Data Analysis” (EDA). This is the stage where the analyst summarises and/or visualises the data in order to assess the integrity of data (e.g. detect outliers or errors) and/or to generate interesting research questions. For details, see ([Chihara and Hesterberg, 2018](#), chapter 2) and ([Ruppert and Matteson, 2015](#), chapter 4). There are several excellent R packages for data visualisation. We will work with the `ggplot2` package, which is a part of the `tidyverse`.

2 Intended Learning Outcomes

By the end of this session, students should be able to

1. plot graphs of functions,
2. produce descriptive statistics (e.g. mean and standard deviation),and
3. produce and label basic plots (e.g. scatterplots and histograms).

3 Template of a ggplot Code

Wickham and Grolemund (2016) summarise that all `ggplot2` graphics are generated with the following coding template:

```
ggplot(data = <DATA>) +  
  <GEOM_FUNCTION>(  
    mapping = aes(<MAPPINGS>),  
    stat = <STAT>,  
    position = <POSITION>  
  ) +  
  <COORDINATE_FUNCTION> +  
  <FACET_FUNCTION>
```

The above is available [here](#). The examples and exercises covered in this session will clarify the above template. We start by loading the `tidyverse` package.

```
# Load the package `tidyverse` that includes ggplot2  
library(tidyverse)  
library(gridExtra)
```

4 Plotting Functions

4.1 Net Returns vs Log Returns

In financial analysis, we regularly work with returns on assets. Suppose the price of an asset at time $t - 1$ was P_{t-1} and the price today at time t is P_t . The **net return** on this asset for the given time interval is

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \quad (1)$$

The above shows that the return on an asset is basically the revenue as a fraction of the initial investment. We also use **log return**, which is computed as follows:

$$\log(1 + R_t) = \log\left(\frac{P_t}{P_{t-1}}\right) \quad (2)$$

We will now show that when net return R_t is close to 0, then

$$\log(1 + R_t) \approx R_t \quad (3)$$

The first step in creating a graph with the `ggplot2` package is to have a dataframe. Let us create a dataframe in order to check whether $\log(1 + R_t) \approx R_t$ when R_t is close to 0. We use the `tibble()` function to create a dataframe. It is sufficient for us to remember that `tibble` is a special kind of dataframe. We will use the `seq()` function to create a sequence of real numbers from -0.5 to 0.5.

```
# Create a column Rt, which is net return from -0.5 to 0.5.

# We have 500 observations of net returns

df1 <- tibble(Rt = seq(from = -0.5, to = 0.5, length.out = 500))

# Use mutate() to create logRt = log(1+Rt)

df1 <- df1 %>%
  mutate(logRt = log(1+Rt))
```

Before we proceed, let us view the first five rows in the dataframe that we have created.

```
head(df1, 5)
```

Rt	logRt
-0.500000	-0.6931472
-0.497996	-0.6891472
-0.495992	-0.6851631
-0.493988	-0.6811948
-0.491984	-0.6772423

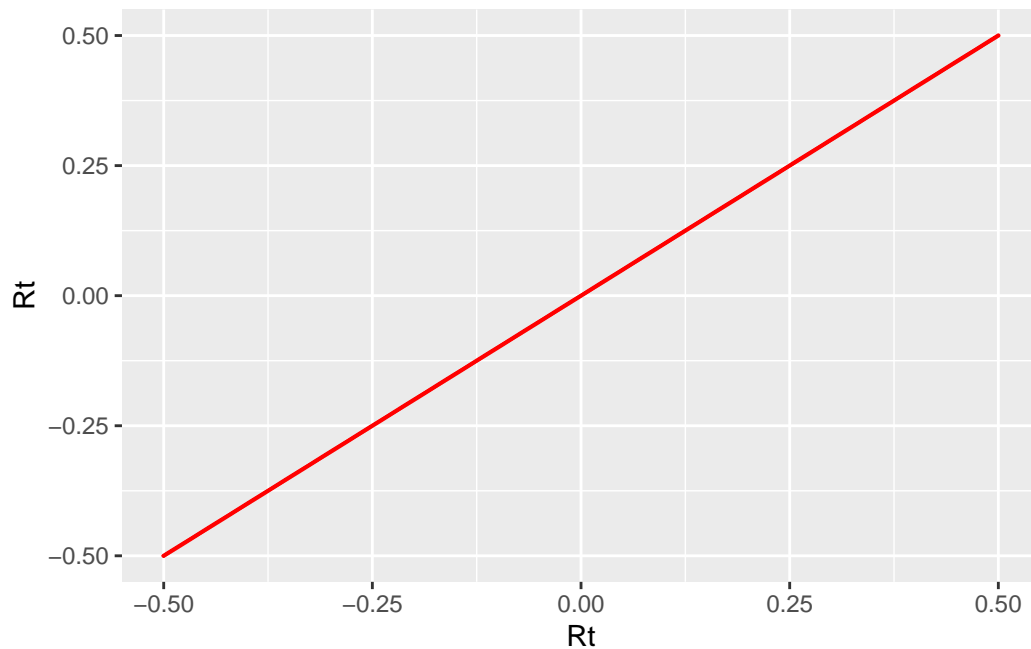
We now use `ggplot()` function to plot the R_t on both x-axis and y-axis. This, of course, is just a 45 degree

line.

```
fig1 <- ggplot(data = df1) +  
  geom_point(aes(x = Rt, y = Rt), size = 0.1, color = "red")
```

The above create a figure object `fig1`. It is important to compare the above code with the `ggplot2` template outlined earlier. Let us display our first plot.

`fig1`

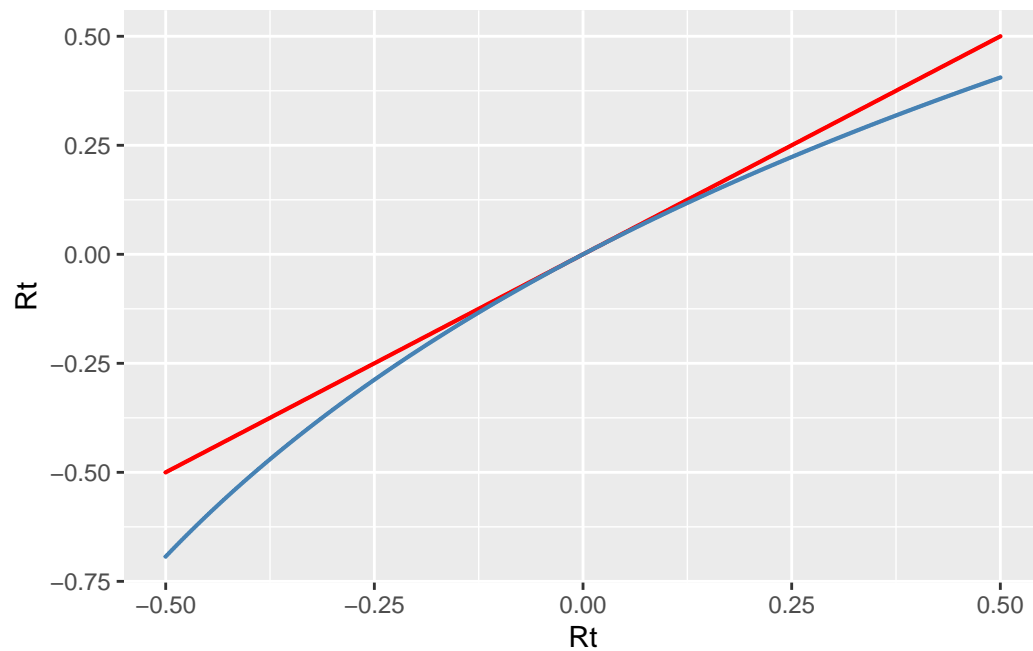


We now **add** another layer containing $\log(1 + R_t)$ to the object `fig1`. Notice that `fig1` object is stored in our R environment.

```
fig1 <- fig1 +  
  geom_point(aes(x = Rt, y = logRt), size = 0.1, color = "steelblue")
```

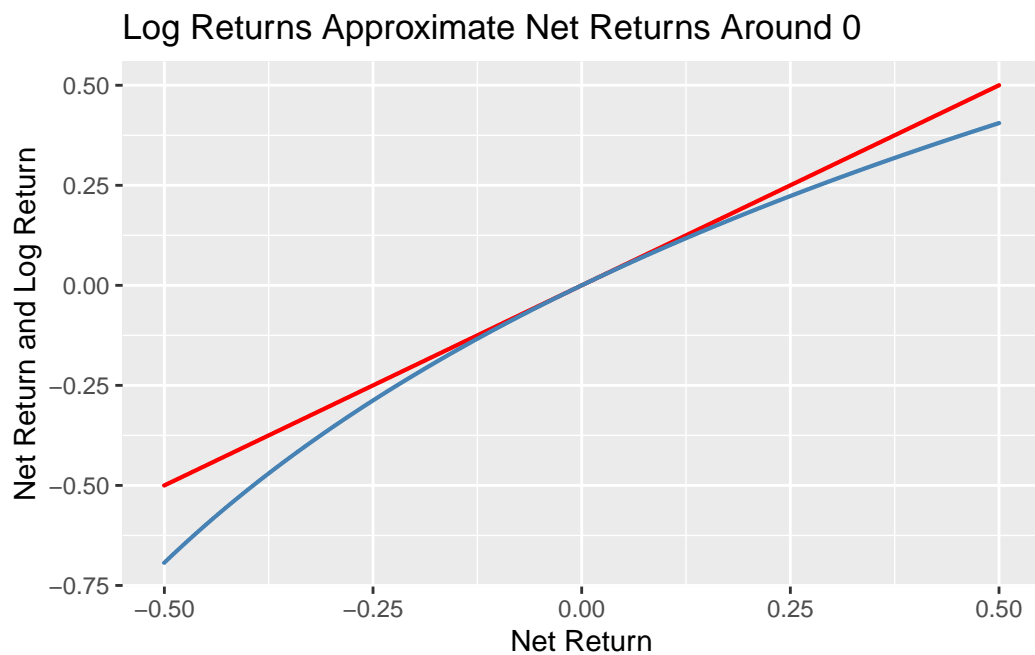
The object `fig1` now contains two layers: R_t and $\log(1 + R_t)$. Let us view this plot.

`fig1`



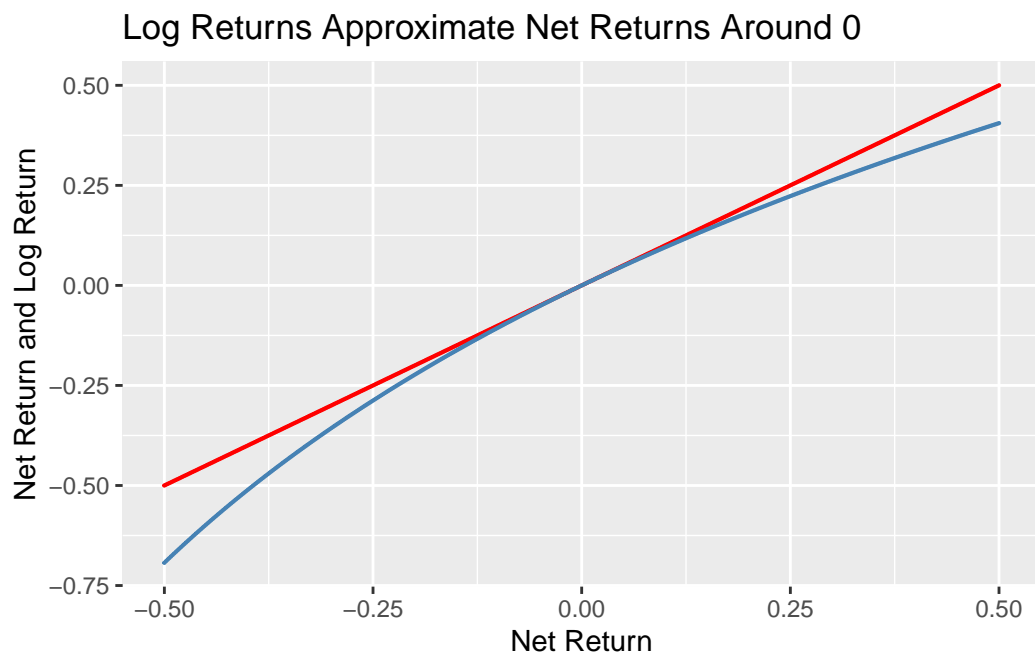
The above plot is OK but the label for y-axis is not informative because we show both R_t and $\log(1 + R_t)$. So, we can add another layer containing labels using the `labs()` function to the above plot as follows. We also add a title.

```
fig1 <- fig1 +  
  labs(title = "Log Returns Approximate Net Returns Around 0",  
        y = "Net Return and Log Return",  
        x = "Net Return")  
  
fig1
```



```
fig1 <- fig1 +  
  labs(title = "Log Returns Approximate Net Returns Around 0",  
        y = "Net Return and Log Return",  
        x = "Net Return")
```

fig1



4.2 Exercise

Financial theory builds models of investors' demand for risky assets. An important element of the financial theory is the way it captures investors' attitude towards risk. Many models in finance begin with the assumption that investors are **risk averse**. To understand the basic idea, consider the following game. Suppose you are offered a lottery that pays \$100 with probability 0.5 and 0 with probability 0.5. On average, the lottery pays \$50. Will you pay \$50 to play this lottery?

It is very likely that you will pay far less than \$50 to play this lottery. How much less depends on your degree of risk aversion. Investors with high risk aversion would pay less (high risk premium) than investors with low risk aversion (low risk premium). In finance, we assume that investors' risk attitude can be captured in their **utility function**. One class of utility function represents **Constant Relative Risk Aversion (CRRA)**. Investors with CRRA utility function invest a fixed percentage of their wealth in risky assets.

Let W denote the wealth of an investor. A widely used CRRA utility function in finance is

$$U(W) = \frac{W^{1-\lambda}}{1-\lambda}, \lambda > 0 \text{ and } \lambda \neq 1 \quad (4)$$

For this exercise, suppose there are two investors: Jack and Jill. For Jack, $\lambda_{jack} = 0.5$. For Jill, $\lambda_{jill} = 0.7$. We seek to plot the utility functions of the two investors for wealth $W \in [0, 20]$. Both investors have the CRRA utility function shown in Equation (4). Draw a plot that shows utility functions for Jack and Jill. Comment on the difference between Jack's and Jill's utility function.

```
# Your code - Carefully look at the code below

# You will need to remove # and modify "???"

# Hints below

# Create a tibble

# df2 <- tibble(W = seq(???, ???, length.out = 500))

# Create new columns, for Jack's and Jill's utility function
```



```

# df2 <- df2 %>%
#   mutate(Jack = W^(1 - ???)/(1 - ???),
#   Jill = W^(1 - ???)/(1 - ???))

# Create ggplot object

# ggplot(df2) +
#   geom_point(aes(x = W, y = ???), color = "red", size = 0.1) +
#   geom_point(aes(x = W, y = ???), color = "steelblue", size = 0.1) +
#   labs(x = "Wealth",
#   y = "Utility Function for Jack and Jill")

```

We now clear our R environment.

```
rm(list = ls())
```

5 Descriptive Statistics

This section uses hypothetical financial data for 100 firms. Data contains 4 variables: Sales, CAPX (Capital Expenditures), DebtEquity (Debt to Equity Ratio), and Industry that the firm belongs to. We start by loading the data. Sales and CAPX are in millions of Pounds.

```
financialData <- read_csv("financialData.csv")
```

```

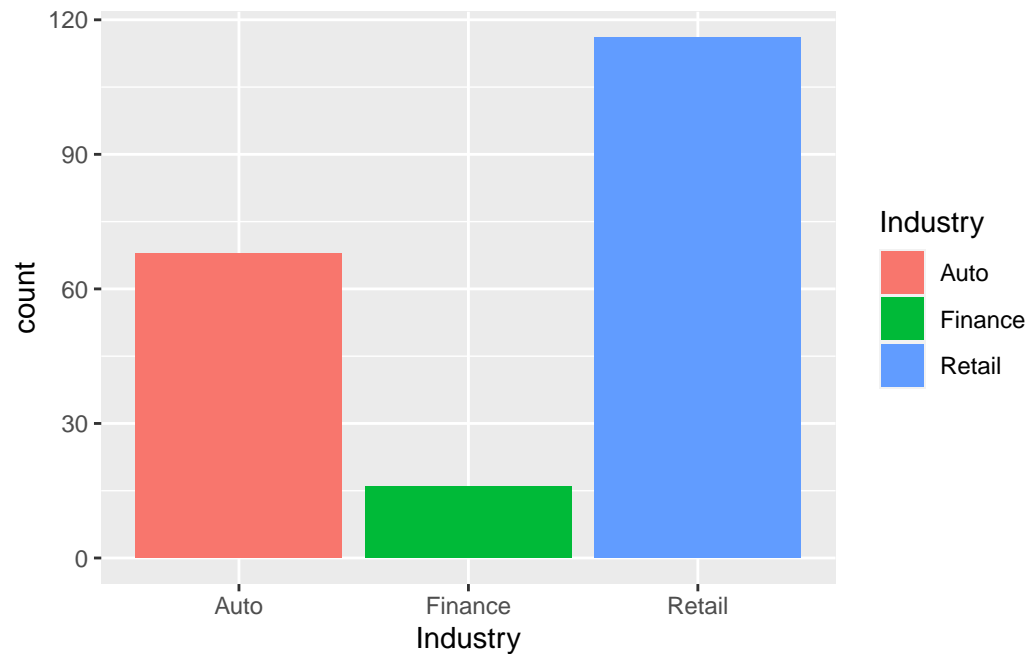
## Parsed with column specification:
## cols(
##   Sales = col_double(),
##   CAPX = col_double(),
##   DebtEquity = col_double(),
##   Industry = col_character()
## )

```

5.1 A Basic Barplot

We begin by visualising the number of firms in different industries. The graph below shows that majority of firms in our data are from the retail sector. Note the use of `fill = Industry` within the `aes()`.

```
ggplot(financialData) +  
  geom_bar(aes(Industry, fill = Industry))
```



We can count the

number of firms in each industry using `dplyr` functions as follows.

```
financialData %>%  
  group_by(Industry) %>%  
  summarise(Number = n()) %>% # n() is to count  
  arrange(Number) # arrange() is used to order the data
```

Industry	Number
Finance	16
Auto	68
Retail	116

5.2 Centre, Spread and Shape of Data

Data analysis almost always begins with basic **summary statistics** of data, where a statistic is a “numerical characteristic of data” (Chihara and Hesterberg, 2018, p. 6). As the data almost always have variations, we begin by looking at summary statistics that provide indication of the centre, spread and shape of data.

Two important measures of the centre of the distribution of a variable are *mean* and *median*. The mean \bar{X} of a variable X is simply

$$\bar{X} = \sum_{i=1}^{i=N} \frac{X_i}{N} \quad (5)$$

Median is obtained by ordering the data and then identifying the middle value if there are odd number of observations or the average of two middle values if there are even number of observations. In R, mean is obtained using `mean()` and median is computed using `median()`. Median is a better measure of the centre if there are *outliers* in the data. R also has function to compute **trimmed mean**, which ignores the extreme values in the data.

The spread of the distribution of a variable is usually summarised with the *standard deviation*. In R, standard deviation is computed with `sd()` function. The standard deviation of a variable is the average squared deviation from the mean of the variable. A variable with large standard deviation exhibit large variation around the mean value. Specifically, the standard deviation of a variable X , denoted by s_X is

$$s_X = \sqrt{\frac{\sum_{i=1}^{i=N} (X_i - \bar{X})^2}{N - 1}} \quad (6)$$

The shape of the distribution of a variable is usually summarised by *skewness* and *kurtosis*. Skewness determines whether the distribution is symmetrical or not, while kurtosis captures whether the distribution has heavy tails. We create a new data frame that contains summary statistics for the **Sales** variable. We use the **moments** package for skewness and kurtosis.

```
library(moments)

df_summary <- financialData %>%
  summarise(mean_Sales = mean(Sales),
```

```

med_Sales = median(Sales),
trim_mean = mean(Sales, trim = 0.05),
sd_Sales = sd(Sales),
skew_Sales = skewness(Sales),
kurt_Sales = kurtosis(Sales))

```

We now view the dataframe containing summary statistics. There is a big difference between mean and trimmed mean, which suggests the presence of some extreme observations.

```
df_summary
```

mean_Sales	med_Sales	trim_mean	sd_Sales	skew_Sales	kurt_Sales
10295.02	6655.614	6673.271	51276.48	14.03547	197.9979

5.3 Quantiles, Quartiles and Percentiles

Sometimes we would like to determine values of a variable that divide the data in different parts. For example, median is the middle value of ordered data. Thus, median divides data in two parts. Median is the 0.5 quantile, or the second quartile, or the 50th percentile of a data. We can also compute other quantiles. For example, quartiles divide data into four roughly equal groups. The first quartile is the 0.25 quantile or the 25th percentile of data, the third quartile is the 0.75 quantile or 75th percentile of data. Around 25% of the data is below the first quartile, around 50% of the data is below the second quartile (which is also the median), and around 75% of the data is below the third quartile.

The computation below compute three quantiles (i.e. 0.25, 0.50, and 0.75) for the variable **Sales** in our dataframe **financialdata**. Note that the median we computed earlier is equal to the 0.50 quantile.

```
quantile(financialData$Sales, c(0.25, 0.5, 0.75))
```

```
##      25%      50%      75%
## 6526.530 6655.614 6843.571
```

5.4 Exercise

1. Follow the above code and create a data frame that contains summary statistics for Debt to Equity ratio available in **financialData**.

```
# Your code
```

2. Compute the 0.05, 0.25, 0.50, 0.75 and 0.975 quantiles of `DebtEquity`.

```
# Your code
```

5.5 Summarising Multiple Columns

It is not ideal to summarise variables one by one. There are several variations of the `summarise()` function that enable analysts to apply functions to multiple columns in a dataframe. For example, we can use the `summarise_at()` to summarise several variables as follows. We will apply three functions `mean()`, `median()` and `sd()` to three variables below.

```
df_summary <- financialData %>%  
  summarise_at(c("Sales", "DebtEquity", "CAPX"),  
               list(mean, median, sd), na.rm = TRUE)  
  
df_summary[,1:4]
```

Sales_fn1	DebtEquity_fn1	CAPX_fn1	Sales_fn2
10295.02	1.128494	997.3256	6655.614

```
df_summary[,5:9]
```

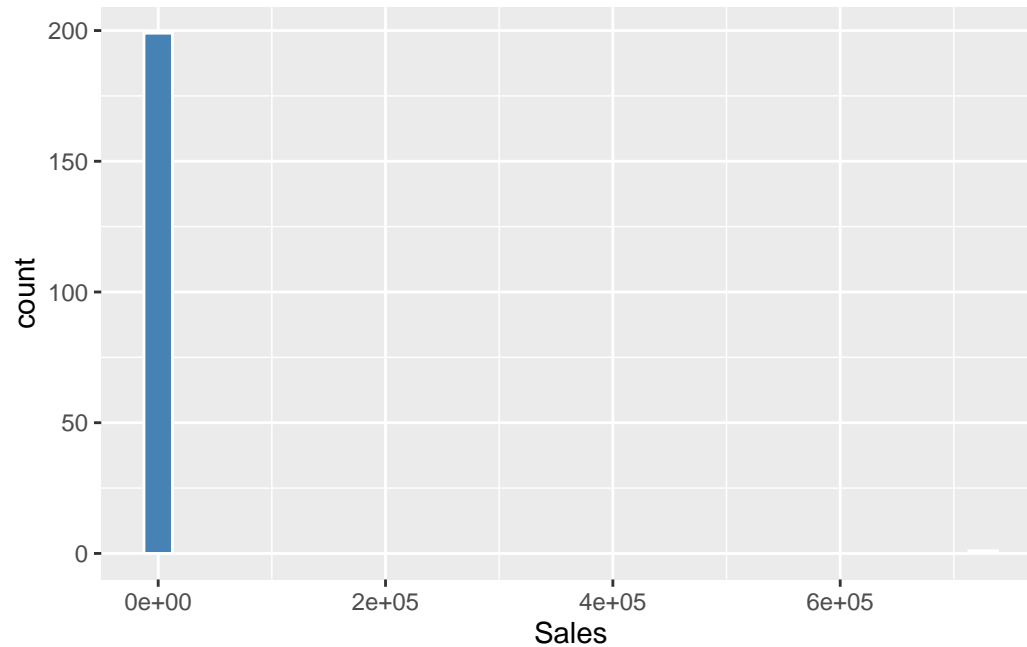
DebtEquity_fn2	CAPX_fn2	Sales_fn3	DebtEquity_fn3	CAPX_fn3
1.197175	999.3233	51276.48	0.5247542	92.3662

6 Basic Plots

6.1 Histogram

Histogram is a widely used device to visualise the variations in observed values of a variable. To draw a histogram, we divide the x-axis into equally spaced intervals (called bins) and then show the frequency of observations falling in each bin as bars. We plot the histogram for `Sales` below and leave histogram of other variables for practice.

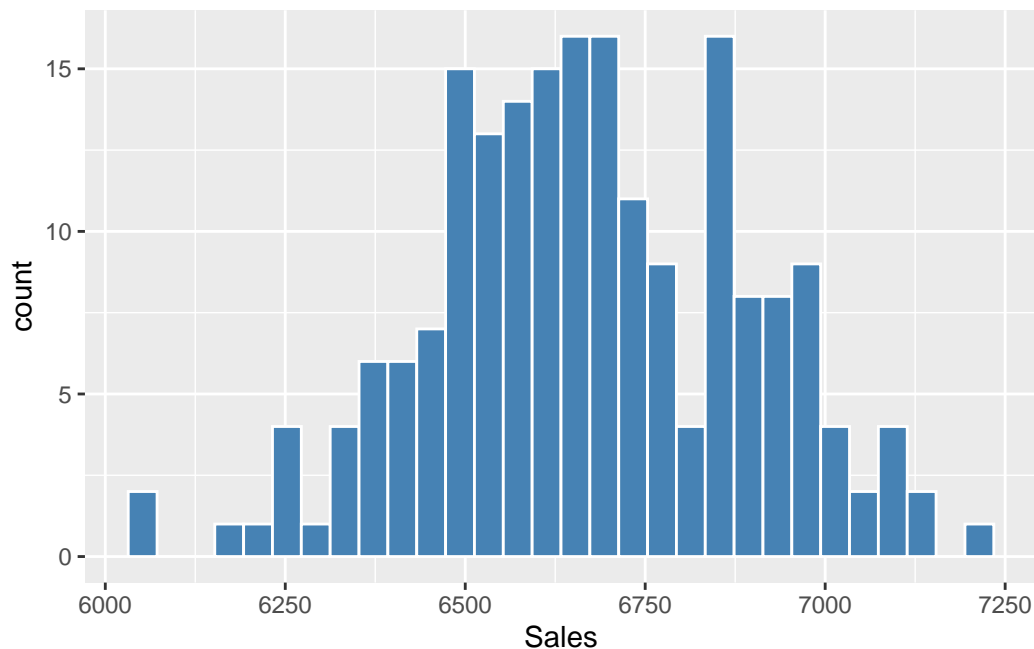
```
ggplot(financialData) +
  geom_histogram(aes(Sales), bins = 30,
                 color = "white",
                 fill = "steelblue")
```



The above histogram suggest that there seems to be a problem with our data. If you go back to your summary statistics, you will notice a big difference between the mean and the median values of Sales. This is because there is one extremely large value of Sales. The largest value of sales is \$731821.7, while the second largest value is only \$7201.2. The largest value seems to be an **outlier** (i.e. an exceptionally large value compare to other observations). We can use the `filter()` function to remove this from our data and then plot the histogram again.

```
# Keep observations where Sales < 9000 million.
financialData <- financialData %>%
  filter(Sales <= 9000)
```

```
ggplot(financialData) +
  geom_histogram(aes(Sales), bins = 30,
                 color = "white",
                 fill = "steelblue")
```



6.2 Exercise

Plot histograms CAPX and DebtEquity in financialData.

```
# Your code
```

6.3 Boxplot

Boxplot is a very useful tool to visualise the distribution and summary statistics of a variable. A boxplot shows the median, the first quartile, the third quartile and 1.5 times the interquartile range (IQR). The plot also shows possible outliers. Note that quartiles divide the data into four parts. IQR is the difference between the third and the first quartile. A boxplot for `Sales` is given below in Figure 1.

```
box_sales <- ggplot(financialData) +
  geom_boxplot(aes(Sales), fill = "thistle") +
  coord_flip()
box_sales
```

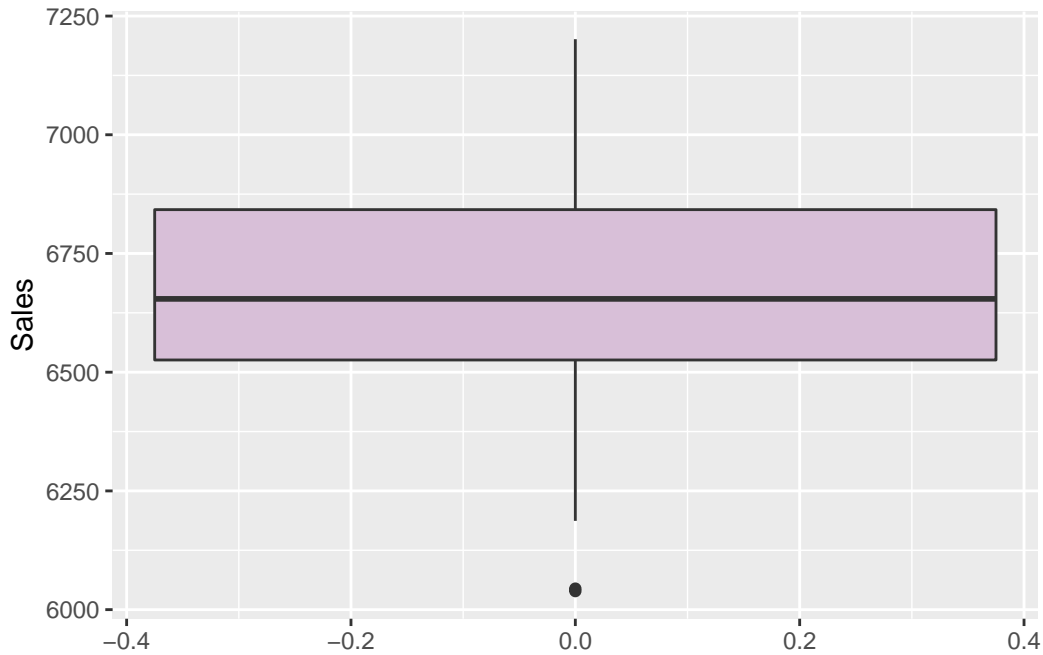


Figure 1: Boxplot for Sales

6.4 Exercise

Show boxplots for CAPX (call it `box_capx`) and DebtEquity(call it `box_de`).

Your code

We can put the three boxplots together using the `gridExtra()` package (you will need to install and load this). This is shown in Figure 2.

```
library(gridExtra)
```

```
grid.arrange(box_sales, box_capx, box_de, nrow = 1)
```

We now clear our R environment.

```
rm(list = ls())
```

7 Plots of Stock Prices

Financial modelling and empirical analysis relies very heavily on data from stock markets. This section looks at stock prices and returns for four companies: Apple (AAPL), Amazon (AMZN), Netflix (NFLX) and

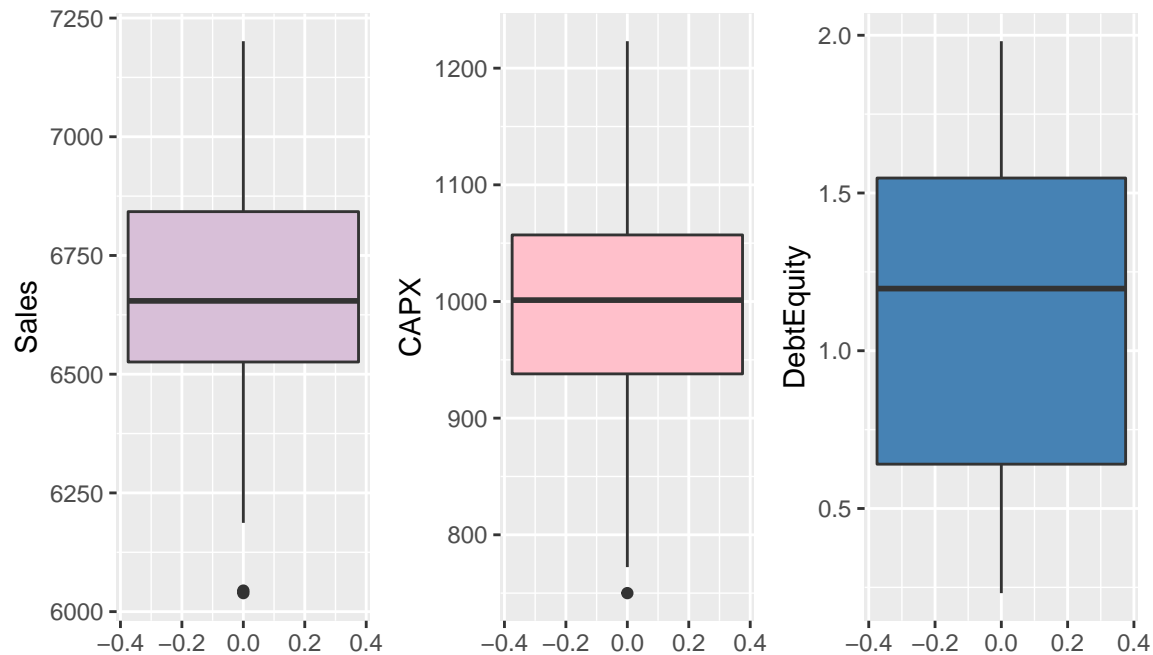


Figure 2: Boxplots for Financial Data

Microsoft (MSFT). The data contains daily closing prices from 2000-12-01 to 2020-03-27.

7.1 The Evolution of Asset Prices

We begin by loading our data, available in the `csv` format.

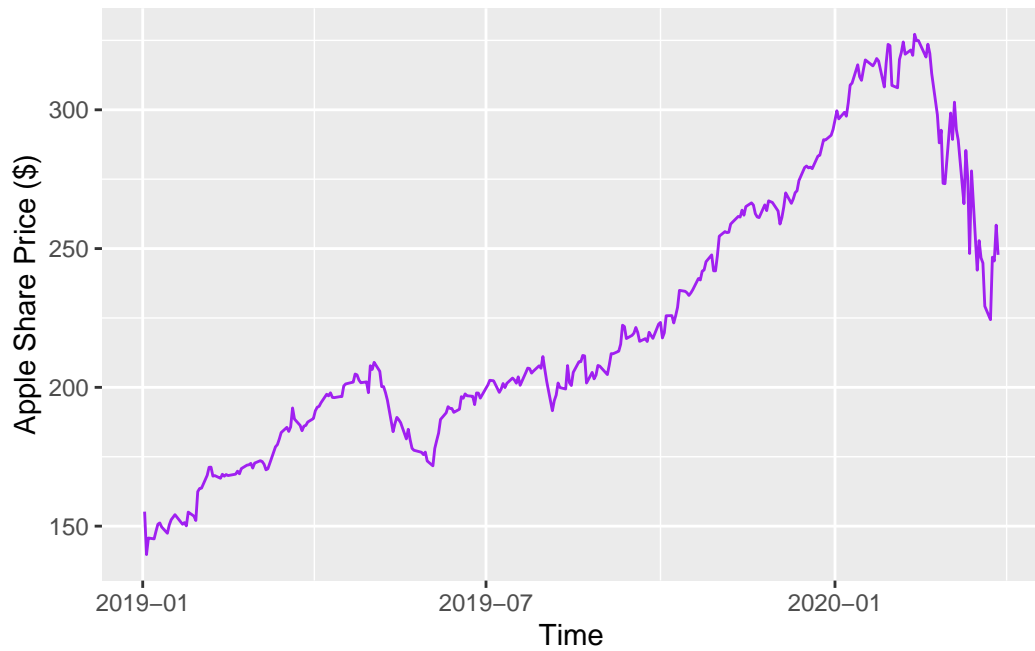
```
df_stocks <- read_csv("assets.csv")
```

Let us plot the four prices for the period 2019-01-01 to 2020-03-27 to see how stock prices evolved during the period. We create plots for AAPL and AMZN. Plots for NFLX and MSFT are left for you to practice.

Note how we use the filter() to obtain the relevant time period.

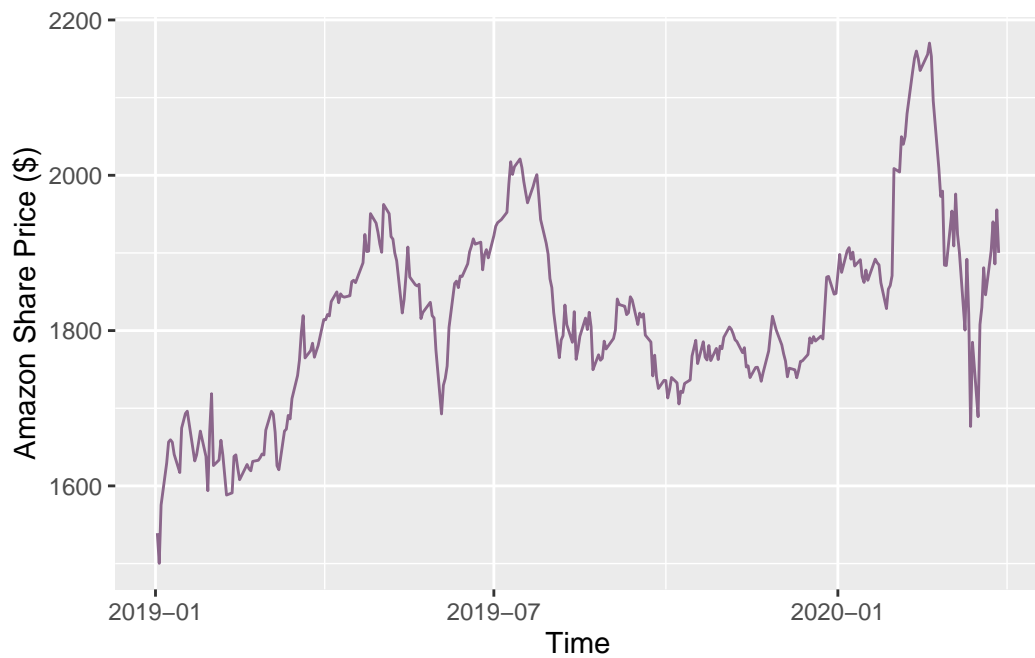
```
fig_APPL <- ggplot( data = filter(df_stocks, date >= "2019-01-01")) +
  geom_line(aes(x = date, y = AAPL), color = "purple") +
  labs(x = "Time",
       y = "Apple Share Price ($)")
```

```
fig_APPL
```



```
fig_AMZN <- ggplot( data = filter(df_stocks, date >= "2019-01-01")) +
  geom_line(aes(x = date, y = AMZN), color = "plum4") +
  labs(x = "Time",
       y = "Amazon Share Price ($)")
```

fig_AMZN



7.2 Exercise

Use `df_stocks` to plot the share prices of Netflix (call it `fig_NFLX`) and Microsoft (call it `fig_MSFT`) for the period 2019-01-01 to 2020-03-27.

```
# Your code
```

```
# There are many colours available.
```

```
# You may choose from the following colours:
```

```
# "steelblue", "palegreen4", "thistle3", "bisque4", "red", "green", "slategray".
```

Now, we use the `grid.arrange()` to put the four figures together as follows.

```
grid.arrange(fig_AMZN,fig_APPL,fig_MSFT,fig_NFLX, nrow = 2)
```

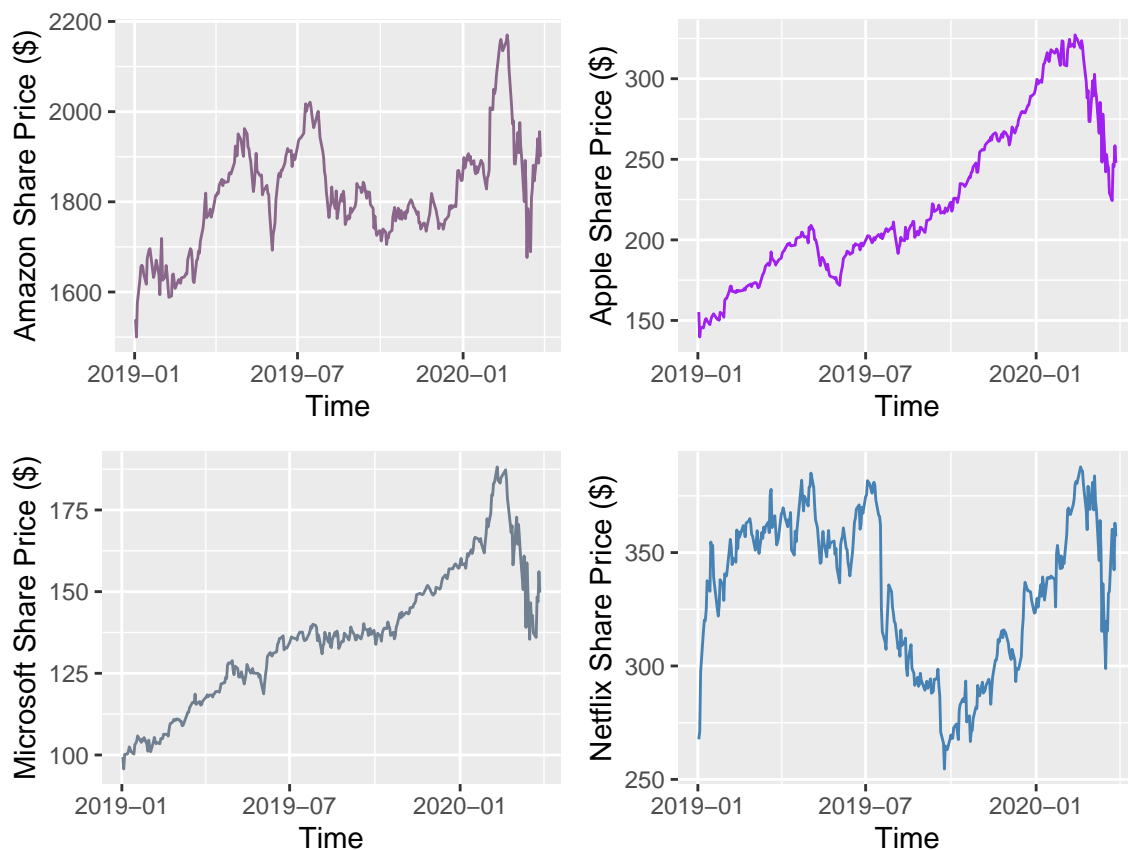


Figure 3: Prices of Four Stocks

8 Plots of Returns

8.1 Data in Long and Wide Format

For this section, we create a new dataframe that contains returns for the four stocks. Note that we have four stocks. We could compute return for each assets by applying `mutate()` to each price column. This is fine for four assets. But if we have many assets, then such repetitive computation would be very time consuming. We highly recommend learning the `purrr` package to perform repetitive tasks.

With experience, you would be able to write your own functions and use tools that will enable you to perform repetitive task efficiently. For our session, we use the `gather()` function to facilitate the computation of log returns for four assets. The `gather()` function transform our data from **wide** to **long** format. We then use `group_by()` and `mutate()` to compute returns. We explain our procedure with a simple data containing prices of two assets. You will practice with `df_stocks` in the exercise.

8.1.1 Step 1: Create a Tibble

```
simple_df <- tibble(date = c("2020-01-01", "2020-01-02", "2020-01-03", "2020-01-04"),
                    Asset1 = c(3,4,3,5),
                    Asset2 = c(7,5,6,7))
```

simple_df

date	Asset1	Asset2
2020-01-01	3	7
2020-01-02	4	5
2020-01-03	3	6
2020-01-04	5	7

8.1.2 Step 2: Change to Long Format

```
simple_df <- simple_df %>%
  gather(Stocks, Price, -date)
```

simple_df

date	Stocks	Price
2020-01-01	Asset1	3
2020-01-02	Asset1	4
2020-01-03	Asset1	3
2020-01-04	Asset1	5
2020-01-01	Asset2	7
2020-01-02	Asset2	5
2020-01-03	Asset2	6
2020-01-04	Asset2	7

8.1.3 Step 3: Compute Return for Each Asset

```
simple_df <- simple_df %>%  
  group_by(Stocks) %>%  
  mutate(Return = log(Price) - log(lag(Price)))
```

8.1.4 Step 4: Remove NAs

```
simple_df <- simple_df %>%  
  drop_na()
```

simple_df

date	Stocks	Price	Return
2020-01-02	Asset1	4	0.2876821
2020-01-03	Asset1	3	-0.2876821
2020-01-04	Asset1	5	0.5108256
2020-01-02	Asset2	5	-0.3364722
2020-01-03	Asset2	6	0.1823216
2020-01-04	Asset2	7	0.1541507

8.2 Exercise

Use the steps outlined above to compute returns for the four stocks in `df_stocks`.

```
# Your code

# Hint: Incomplete code provided below.
# Replace ??? with appropriate code

# df_returns <- ??? %>%
#   gather(???, ???, -date) %>%
#   group_by(Stocks) %>%
#   mutate(Return = log(Price)-log(lag(Price))) %>%
#   select(date, ???, ???) %>%
#   drop_na()
```

9 Summary Statistics for Returns

We can use the `summary()` function, combined with the `filter()` function, to obtain summary statistics for our return data for each stock.

```
# Apple
summary(filter(df_returns, Stocks == "AAPL"))
```

```
##           date           Stocks           Return
##  Min.      :2000-12-04   Length:4858   Min.      :-0.1974703
##  1st Qu.:2005-10-05   Class :character 1st Qu.: -0.0098001
##  Median :2010-08-03   Mode  :character Median : 0.0008947
##  Mean     :2010-08-03                      Mean    : 0.0011231
##  3rd Qu.:2015-06-01                      3rd Qu.: 0.0123645
##  Max.     :2020-03-27                      Max.     : 0.1301938
```

```
# Amazon
summary(filter(df_returns, Stocks == "AMZN"))
```

```
##      date           Stocks      Return
##  Min.    :2000-12-04   Length:4858   Min.     :-0.2845678
##  1st Qu.:2005-10-05   Class :character 1st Qu.   :-0.0113602
##  Median :2010-08-03   Mode  :character Median    : 0.0005509
##  Mean   :2010-08-03                      Mean     : 0.0008946
##  3rd Qu.:2015-06-01                      3rd Qu.  : 0.0134881
##  Max.    :2020-03-27                      Max.     : 0.2961811
```

```
# Netflix
summary(filter(df_returns, Stocks == "NFLX"))
```

```
##      date           Stocks      Return
##  Min.    :2002-05-24   Length:4492   Min.     :-0.5260487
##  1st Qu.:2006-11-06   Class :character 1st Qu.   :-0.0145011
##  Median :2011-04-25   Mode  :character Median    : 0.0002956
##  Mean   :2011-04-25                      Mean     : 0.0012686
##  3rd Qu.:2015-10-09                      3rd Qu.  : 0.0172373
##  Max.    :2020-03-27                      Max.     : 0.3522296
```

```
# Microsoft
summary(filter(df_returns, Stocks == "MSFT"))
```

```
##      date           Stocks      Return
##  Min.    :2000-12-04   Length:4858   Min.     :-0.1594534
##  1st Qu.:2005-10-05   Class :character 1st Qu.   :-0.0078078
##  Median :2010-08-03   Mode  :character Median    : 0.0003524
##  Mean   :2010-08-03                      Mean     : 0.0004342
##  3rd Qu.:2015-06-01                      3rd Qu.  : 0.0087174
##  Max.    :2020-03-27                      Max.     : 0.1706255
```

10 Plot Returns Using Facets

Instead of creating a separate plot for each stock, we can use the `facet_wrap()` function in `ggplot` to draw separate plot for each of the four assets. We will use data in `df_returns`. In Figure 4 shows that unlike stock

prices, returns do not appear to follow a trend. Also, there are times when the variation in returns is very high. This variation or volatility is usually measured by standard deviation. The high volatility during some period and not other is called **volatility clustering** (Ruppert and Matteson, 2015, p. 46).

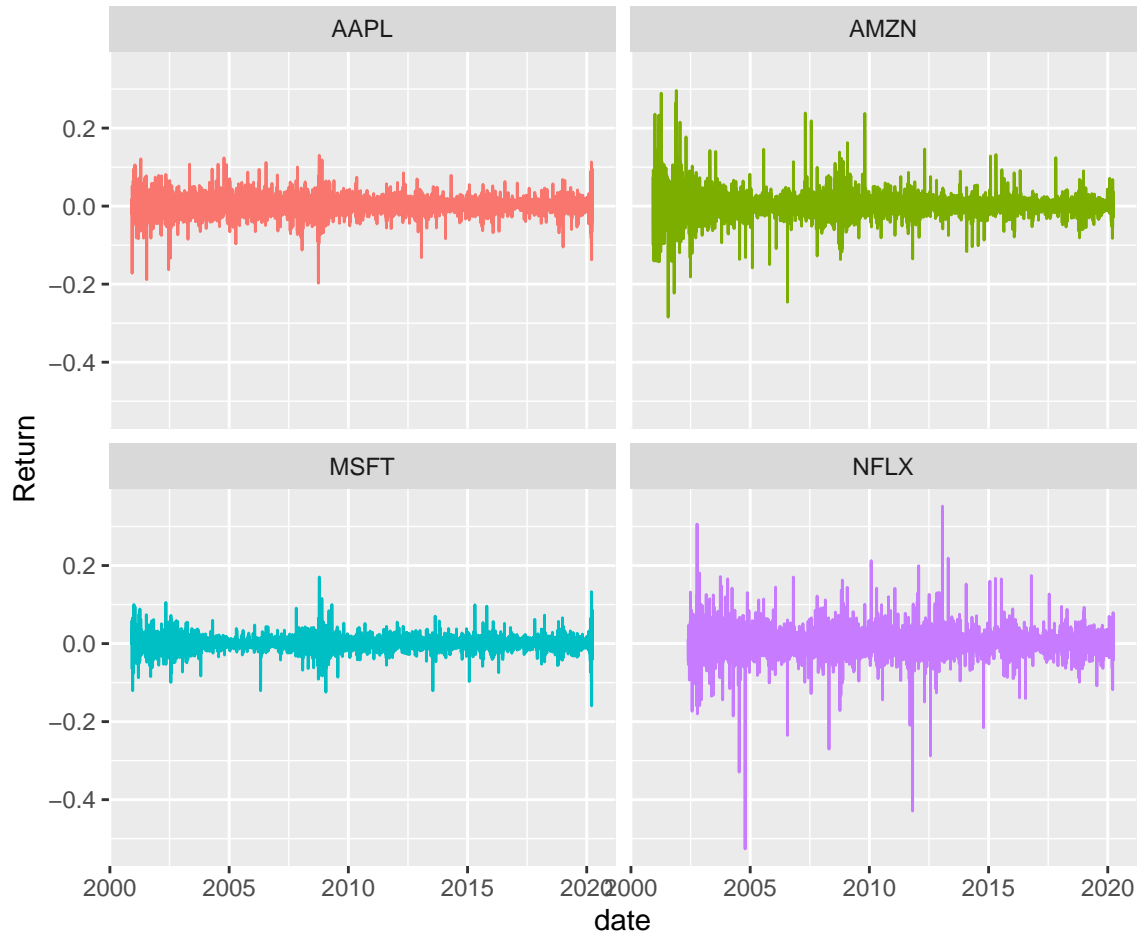


Figure 4: Returns for Four Assets

11 Density Plot of Returns

Earlier we used histograms to visualise the distribution of financial data. A better way to visualise distribution is to create a density plot. We do this for returns of four stocks below.

```
ggplot(df_returns) +  
  geom_density(aes(x = Return,  
                  group = Stocks,  
                  color = Stocks) ) +
```



```
facet_wrap(~Stocks, ncol = 2) +
theme(legend.position = "none")
```

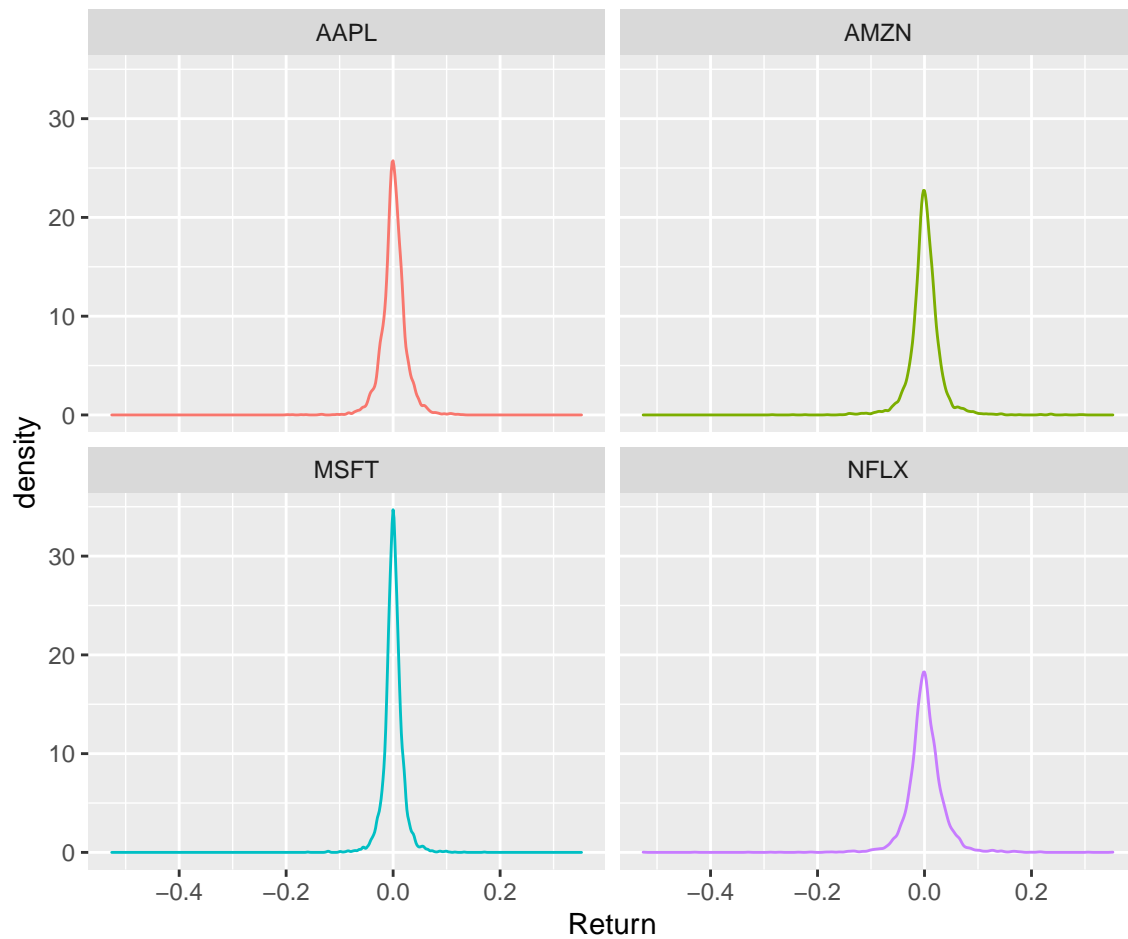


Figure 5: Density Plot for Returns

12 Scatterplot

12.1 Possible Relationship Between Two Variables

A **scatterplot** is a useful visualisation tool to explore possible relationship between two variables. We draw a scatterplot for returns for Apple and Amazon. You will draw scatterplot for Microsoft and Netflix. However, we first need to change our data from **long** to **wide** format. This is done below using the **spread()** function.

```
df_returns <- df_returns %>%
  spread(Stocks, Return) %>%
  drop_na()
```

```
ggplot(df_returns) +
  geom_point(aes(x = AAPL, y = AMZN),
             color = "steelblue", alpha = 0.4) +
  labs(title = "Scatterplot for Apple & Amazon",
       x = "Return on Apple Stock",
       y = "Return on Amazon Stock")
```

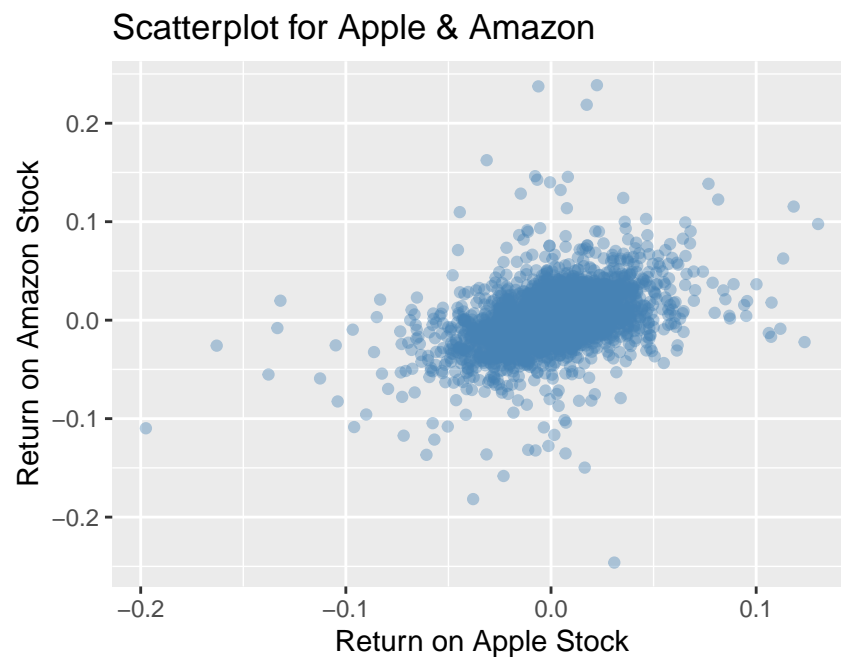


Figure 6: Scatterplot for Apple and Amazon

12.2 Exercise

Use `df_returns` to draw a scatterplot for Netflix and Microsoft.

```
# Your code
```

13 Next Steps

This session provided a brief overview of some of the visualisation tools available in the `ggplot2` package. The plot considered in this sessions can be used for preliminary exploration of data before conducting financial analysis. The next session provides an overview of essential concepts from probability and mathematical statistics. These are frequently used not only in finance but in many other disciplines.

References

- Chihara, L. and Hesterberg, T. (2018). *Mathematical Statistics with Resampling and R*. Wiley Online Library.
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- Wickham, H. and Grolemund, G. (2016). *R for Data Science: Import, Tidy, Transform, Visualize, and Model Data*. O'Reilly Media, Inc.