Homework 1 – Rebah Özkoç - 29207

1- Sketch a composed TM M that performs the computation: (s, ◊ <u>#</u> ω) --|* M (h, ◊ <u>#</u> ω c)

	Condition	**TM**	**Comments**
M>	--------	R A	
A	σ = #	$L_{\#}$ h	
	σ ≠ # and σ ≠ $	R A	
	σ = $	R B	
B	σ = $	x R B	X ∉ Σ
	σ ≠ $	x $L_{\$}$ σ R C	
C	σ = x	$ R C	
	σ ≠ x	$L_{\#}$ R A	

Verbal description:

Head goes to the first symbol of w and enters state A.

State A: If w is e (empty) or it does not contain any $ symbol, it goes back until the first blank symbol which is next to the blip, and it holds.

State A: If head sees a $ symbol while it is going to the left it goes one right and enters to state B.

State B: If the symbol at the right of the symbol $ is also $ head writes x which does not belong to the alphabet and it stays at the state B.

State B: While going to the right if head sees a symbol which is not $ it removes it by putting an x it goes to the left until it sees a $ and writes what is sees and goes one step to the right and enters state C.

State C: In this state the machine deletes the temporary x symbols which does not belong the current alphabet. If head sees an x it writes $ instead of it and goes one step to the right and stays at state C.

State C: If the head sees a symbol which is not x it goes to the left until the first blank symbol which is at the left of the w and it goes one step right and enters state A.

2.

4.1.4. Let M be the Turing machine $(K, \Sigma, \delta, s, \{h\})$, where

$$K = \{q_0, q_1, q_2, h\},$$
$$\Sigma = \{a, \sqcup, \triangleright\},$$
$$s = q_0,$$

and δ is given by the following table.

Let $n \geq 0$. Describe carefully what M does when started in the configuration $(q_0, \triangleright \sqcup a^n \underline{a})$.

q,	σ	$\delta(q, \sigma)$
q_0	a	$(q_1, \leftarrow)$
q_0	$\sqcup$	$(q_0, \sqcup)$
q_0	$\triangleright$	$(q_0, \rightarrow)$
q_1	a	$(q_2, \sqcup)$
q_1	$\sqcup$	$(h, \sqcup)$
q_1	$\triangleright$	$(q_1, \rightarrow)$
q_2	a	(q_2, a)
q_2	$\sqcup$	$(q_0, \leftarrow)$
q_2	$\triangleright$	$(q_2, \rightarrow)$

Solution:

$(q_0, \Diamond\ \#\ a^n\ \underline{a}) \vdash_M (q_1, \Diamond\ \#\ a^{n-1}\ \underline{a}\ a) \vdash_M (q_2, \Diamond\ \#\ a^{n-1}\ \underline{\#}\ a) \vdash_M (q_0, \Diamond\ \#\ a^{n-2}\ \underline{a}\ \#\ a) \vdash_M (q_1, \Diamond\ \#\ a^{n-3}\ \underline{a}\ a\ \#\ a) \vdash_M$

$(q_2, \Diamond\ \#\ a^{n-3}\ \underline{\#}\ a\ \#\ a) \vdash_M (q_0, \Diamond\ \#\ a^{n-4}\ \underline{a}\ \#\ a\ \#\ a) \vdash^*_M (q_0, \Diamond\ \#\ \underline{a}\ (\#a)^{n/2}) \vdash_M (q_1, \Diamond\ \underline{\#}\ a\ (\#a)^{n/2})$

$\vdash_M (h, \Diamond\ \underline{\#}\ (\#a)^{n/2})$

If n is even M halts on the blank but if n is odd M loops forever between the blank next to the blip to right and back.

4.1.7. Design and write out in full a Turing machine that scans to the right until it finds two consecutive a's and then halts. The alphabet of the Turing machine should be $\{a, b, \sqcup, \triangleright\}$.

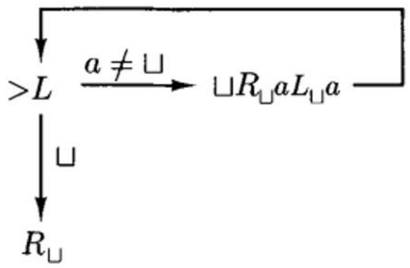
Solution:

This Turing machine semi-decides a language. If there is no two consecutive a's on the tape it goes right and reads the tape indefinitely.

	Condition	**TM**
>M	---------	R A
A	σ = a	R B
	σ ≠ a	R A
B	σ = a	h
	σ ≠ a	R A

4.1.12. Trace the operation of the Turing machine of Example 4.1.9 on $\triangleright \sqcup aabb\underline{\sqcup}$.

Example 4.1.9: The right-shifting machine $S_{\rightarrow}$, transforms $\sqcup w \underline{\sqcup}$, where w contains no blanks, into $\sqcup\sqcup\ w\underline{\sqcup}$. It is illustrated in Figure 4-9.◊



Solution:

L **U** **RU** **a** **LU** **a**

(◊ # a a b b $\underline{\#}$) |-$_{s\rightarrow}$

(◊ # a a b $\underline{b}$ #) |-$_{s\rightarrow}$ (◊ # a a b $\underline{\#}$ #) |-$_{s\rightarrow}$ (◊ # a a b # $\underline{\#}$) |-$_{s\rightarrow}$ (◊ # a a b # $\underline{b}$) |-$_{s\rightarrow}$ (◊ # a a b $\underline{\#}$ b) |-$_{s\rightarrow}$ (◊ # a a b $\underline{b}$ b) |-$_{s\rightarrow}$

(◊ # a a $\underline{b}$ b b) |-$_{s\rightarrow}$ (◊ # a a $\underline{\#}$ b b) |-$_{s\rightarrow}$ (◊ # a a # b b $\underline{\#}$) |-$_{s\rightarrow}$ (◊ # a a # b b $\underline{b}$) |-$_{s\rightarrow}$ (◊ # a a $\underline{\#}$ b b b) |-$_{s\rightarrow}$ (◊ # a a $\underline{b}$ b b b) |-$_{s\rightarrow}$

(◊ # a $\underline{a}$ b b b b) |-$_{s\rightarrow}$ (◊ # a $\underline{\#}$ b b b b) |-$_{s\rightarrow}$ (◊ # a # b b b b $\underline{\#}$) |-$_{s\rightarrow}$ (◊ # a # b b b b $\underline{a}$) |-$_{s\rightarrow}$ (◊ # a $\underline{\#}$ b b b b a) |-$_{s\rightarrow}$ (◊ # a $\underline{a}$ b b b b a) |-$_{s\rightarrow}$

(◊ # $\underline{a}$ a b b b b a) |-$_{s\rightarrow}$ (◊ # $\underline{\#}$ a b b b b a) |-$_{s\rightarrow}$ (◊ # # a b b b b a $\underline{\#}$) |-$_{s\rightarrow}$ (◊ # # a b b b b a $\underline{a}$) |-$_{s\rightarrow}$ (◊ # $\underline{\#}$ a b b b b a a) |-$_{s\rightarrow}$

(◊ $\underline{\#}$ # a b b b b a a) |-$_{s\rightarrow}$ (◊ # $\underline{\#}$ a b b b b a a)

4.2.2. Present Turing machines that decide the following languages over $\{a, b\}$:

(a) $\emptyset$
(b) $\{e\}$
(c) $\{a\}$
(d) $\{a\}^*$

Solution:

a) It will hold with no directly.

	Condition	TM
>M	-----	h_{no}

b)

	Condition	TM
>M	----	R A
A	σ = #	h_{yes}
	σ ≠ #	h_{no}

c)

	Condition	TM
>M	-----	R A
A	σ = a	R B
	σ ≠ a	h_{no}
B	σ = #	h_{yes}
	σ ≠ #	h_{no}

d)

	Condition	TM
>M	-----	R A
A	σ = #	h_{yes}
	σ = b	h_{no}
	σ = a	R A

4.3.1. Formally define:
(a) M semidecides L, where M is a two-way infinite tape Turing machine;
(b) M computes f, where M is a k-tape Turing machine and f is a function from strings to strings.

a) A two-way infinite tape turing machine is (Q, Σ, t, s, H) where t is the transition function.
A configuration (q, u <u>a</u> v) belongs to Q x $((\Sigma - \{\#\})\ \Sigma^* \cup \{\#\}) \times (\Sigma^*\ (\Sigma - \{\#\}) \cup \{e\})$

M semidices L means if the input w belongs to L then M will hold. If the w does not belong to L it may continue to work indefinitely.
w belongs to L : (s, #, w) $|\text{-}^*_M$ (h, x, y) for some h belongs to H

b) M a k-tape Turing machine computes f if $\Sigma^*_0 \to \Sigma^*_0$ for all w belongs to Σ^*_0
$(q_0$, ◊ <u>#</u> w, ◊ <u>#</u> , … , ◊ <u>#</u>) $|\text{-}_M^*$ (h, ◊ <u>#</u> f(w), w_2 , … w_k) where h belongs to H.

4.3.2. Formally define:
(a) a k-head Turing machine (with a single one-way infinite tape);
(b) a configuration of such a machine;
(c) the yields in one step relation between configurations of such a machine. (There is more than one correct set of definitions.)

a) A k-head Turing machine with a single one-way infinite tape is defined as M = (Q, Σ, t, s, H), where Q, Σ, s and H are same with the standard Turing machine and t is the transition function goes from (Q-H) x Σ^k to Q x $(\Sigma \cup \{\leftarrow, \rightarrow\})^k$.
b) The configuration of the machine is an element of Q x $◊\Sigma^*$ x N^k

(q, u^k $\underline{a}^k$ v^k) where q is the current state of the machine which belongs to Q.

u^k is the string that is at the left of the k^{th} head (this can be the blip symbol.)

v^k is the string that is at the right of the kth head. (this can be blank or empty.)

a^k is the symbol under the hth head.

4.3.6. Formally define a Turing machine with a 2-dimensional tape, its configurations, and its computation. Define what it means for such a machine to decide a language L. Show that t steps of this machine, starting on an input of length n, can be simulated by a standard Turing machine in time that is *polynomial* in t and n.

A Turing machine with a 2 dimensional infinite tape is defined as M = (Q, Σ, t, s, H), where Q, Σ, s and H are same with the standard Turing machine and t is the transition function goes from (Q-H) x Σ to Q x (Σ U {<- , ->, up, down} .