HW5 (*due May 23 Tuesday before recitation)*

0- Suppose ***M'*** is a *TM* that *semidecides* a language ***L*** . Construct a *TM* ***M*** making use of ***M'*** that *semidecides* the language ***L****.

1- Prove the ***transitivity*** of the polynomial reduction operator $\boldsymbol{\alpha}$:

i.e. , $L_1 \,\alpha\, L_2$ and $L_2 \,\alpha\, L_3$ implies that $L_1 \,\alpha\, L_3$

2 - Given a ***SAT*** problem define a set of literals in ***SAT*** a ***consistent set*** if a literal x_j and its complement literal x^c_j are *NOT* both members of this set.

Prove that ***SAT*** has a solution **if and only if** there exists a **consistent set** of literals, whose members are selected, one from each clause C_j .

3 - Prove the following **:** ***IS*** α ***CLIQUE*** , ***IS*** α ***NC***, ***SAT*** α ***MAXSAT***, ***HC*** α ***UHC*** where ***IS*= *Independent Set* ,** ***NC* = *Node Cover*** and ***UHC* = *HC*** for undirected graphs.

4- ***(a)*** Formulate the ***2SAT*** problem where each vertex corresponds to a Boolean literal and there is a directed edge from vertex ***x*** to vertex ***y*** corresponding to ***x*** implies ***y*** ($x \Rightarrow y$ or $\neg x \vee y$ or $(\neg x, y)$ is a clause)

(b) Show that $2SAT \in \boldsymbol{P}$

5 - Given an ***EC*** problem with $U = \{ u_0, u_1, u_2, u_3, u_4 \}$;

$F = \{ \{u_0, u_3, u_4\}, \{u_2, u_4\}, \{u_0, u_1, u_2\}, \{u_0, u_2, u_4\}, \{u_1, u_2\} \}$

State the ***KS*** and the ***HC*** problems obtained from the above ***EC*** problem by the ***polynomial reduction*** methods discussed in class. State solution(s) of the three problems ***EC*** , ***KS*** and ***HC*** if one exists for each case.