HW1 *(due March 21,2021 Tuesday before recitation)*

1- *Sketch a composed TM* ***M*** *that performs the computation :*

$(s, \lozenge \# \omega) \vdash^*_M (h, \lozenge \# \omega^c)$

where ω^c *is the compressed version of* ω *where all the characters '\$' within* ω *are removed.*

2- *Problems from the main text book :*

4.1.4, 4.1.7,4.1.12, 4.2.2, 4.3.1, 4.3.2 (a);(b),4.3.6

An optional HW question

Question *– A Turing machine* M_n *operates on an abstract* ***n-dimensional quadrant*** *given by* $X = N\times \ldots \times N = N^n$ *where* $N = \{0,1,2, \ldots , \}$ *denotes the set of natural numbers. The head can move in positive and negative directions along each of the dimensions; hence it has precisely* ***2n*** *moves at each slot position. Each slot is an* ***n-dimensional cube*** *that carries data represented by a symbol in its alphabet set* Σ. *A standard TM* ***M*** *to simulate* M_n *must assign a unique slot on its tape corresponding to each n-dimensional cubic slot of* M_n.

This is accomplished by defining functions $f_k : N^k \to N$ *defined inductively via functions* $f_1,f_2 ,\ldots,f_n$ *, each with the domain* N^k *for* $k= 1,\ldots,n$ *as follows :*

$f_1(i_1) = i_1$ *for all* $i_1 \in N$

$f_k(i_1,i_2, \ldots, i_k) := (f_{k-1}(i_1,i_2, \ldots,i_{k-1}) +i_k)(f_{k-1}(i_1,i_2,\ldots,i_{k-1}) +i_k+1)/2 +i_k , k =2,3,\ldots,n \; \ldots\ldots (1)$

Hence each $f_k (i_1,i_2, \ldots,i_k)$ *can be computed sequentially using the inductive recipe above.*

For example $f_2(i_1,i_2) = (i_1+i_2) (i_1+i_2 +1)/2 + i_2$ *etc.*

Theorem

The function $f_n : N^n \to N$ *described above by* ***(1)*** is a ***bijection .***

i.e. :

(i) $f_n(i_1,i_2, \ldots, i_n) = f_n(j_1,j_2, \ldots, j_n) \Rightarrow i_p = j_p$ *for* $p=1,\ldots, n$ *(hence it is an* ***injection*** *(1-to-1))*

(ii) for every $u \in N$ *there exist* $(i_1 ,\ldots, i_n) \in N^n$ *such that* $f_n(i_1,i_2, \ldots, i_n) = u$ *(hence it is a* ***surjection*** *(onto))*

Using this theorem describe how M can simulate M_n*. In particular describe how* ***M*** *moves its head when* M_n *oves its head from slot* $(i_1, \ldots, i_p \ldots , i_n)$ *to* $(i_1 , \ldots , i_p \pm 1, \ldots . i_n)$.

(To get a feeling for the problem first solve it for ***n=2 ,*** *namely simulating a 2D-TM !)*

Meanwhile try also to prove the theorem above using induction on n