A deterministic model for heat pump operation and control

Abstract

A deterministic optimization model which can be used for the optimal operation and control of residential heat pumps is presented in this document. The model minimizes the operation cost of a heat pump considering both the electricity consumption cost and the grid fees while it keeps the room temperature between some predefined limits. PV generation and battery storage are also considered in the model.

1 Mathematical formulation

$$min \sum_{t \in T} \left(C_t^{energy} + C_t^{grid,energy} \right) + C^{grid,power} \tag{1}$$

$$C_t^{energy} = \lambda_t^{buy} \cdot P_t^{buy} \cdot \delta_t - \lambda_t^{sell} \cdot P_t^{sell} \cdot \delta_t \ \forall t \tag{2}$$

$$C_t^{grid,energy} = \lambda_t^{grid,energy} \cdot P_t^{buy} \cdot \delta_t \ \forall t \tag{3}$$

$$COST^{grid,power} = \lambda^{grid,power} \cdot p^{contract} + \lambda^{grid,penalty} \cdot P^{over} \tag{4}$$

$$P^{over} \geq P_t^{buy} - p^{contract} \ \forall t \tag{5}$$

$$P_t^{sell} - P_t^{buy} = pvgen_t + B_t^- - load_t - P_t^{heat} - B_t^+ \ \forall t \tag{6}$$

$$Q_t = cop_t^{heat} \cdot P_t^{heat}, \ \forall t \tag{7}$$

$$0 \leq Q_t \leq q_t^{max}, \ \forall t \tag{8}$$

$$-q^{\downarrow} \cdot q_t^{max} \leq Q_t - Q_{t-1} \leq q^{\uparrow} \cdot q_t^{max}, \ \forall t \tag{9}$$

$$T_{t+1}^{in} - T_t^{in} = \frac{\delta_t \cdot 3600}{c} \cdot \left[Q_t - u \cdot \left(T_t^{in} - t_t^{out} \right) \right] \ \forall t \tag{10}$$

$$t^{min} \leq T_t^{in} \leq t^{max} \ \forall t \tag{11}$$

$$SOC_t - SOC_{t-1} = \eta^+ \cdot B_t^+ \cdot \delta_t - \frac{B_t^-}{\eta^-} \cdot \delta_t \ \forall t \tag{12}$$

$$soc^{min} \cdot soc^{max} \leq SOC_t \leq soc^{max} \; \forall t \tag{13}$$

$$0 \leq B_t^{+} \leq b^{+,max} \; \forall t \tag{14}$$

$$0 \leq B_t^{-} \leq b^{-,max} \; \forall t \tag{15}$$

The objective function (1) of the model stands for the minimization of the aggregated electricity costs of the building. These include: a) the cost (or revenue) of energy C_t^{energy} exchanged with the grid at period t , b) the grid cost $C_t^{grid,energy}$ for the energy imported at period t and c) the grid cost $C^{grid,power}$ for the power level. Analytically, the cost of energy (2) is the cost of energy bought from the grid at price λ_t^{buy} minus the revenue from energy sold to the grid at price λ_t^{sell}. P_t^{buy} is the power bought while P_t^{sell} is the power sold. δ_t is the period length that is used to translate power into energy.

The grid energy cost (3) is the cost for the imported energy where $\lambda_t^{grid,energy}$ is the grid fee. No cost for the exported energy is considered although it can be easily added if needed.

The grid power cost (4) includes the cost for the power level contract and the penalty cost when the imports exceed this level. The parameter $p^{contract}$ is the level of the power contract. However $p^{contract}$ can be replaced with the variable $P^{contract}$ and in this case the optimal level of the power contact is calculated. P^{over} is the maximum amount of power that exceeds the power contract level for the horizon that is considered in the model. This is calculated by (5).

The energy balance of the microgrid is given by (6). This defines that the difference between the power sold and bought in a period t is the total power produced by the PV $pvgen_t$ and the battery when discharging B_t^{-}, minus the total power consumed by the load (except the heat pump) $load_t$, the heat pump P_t^{heat} and the battery when charging B_t^{+}.

The heat pump heat generation Q_t is given by (7) and is proportional to the power consumption P_t^{heat}. cop_t^{heat} is the coefficient of performance of the heat pump. It depends on the weather conditions that's why it has been made time dependent in the model. The heating power is limited above by the heat pump's capacity q_t^{max} (8) which also depends on the weather conditions. The heating ramp limits are set in (9) where $q^{\uparrow}$ and $q^{\downarrow}$ are the ramp up and down rates respectively given as percentage of the heat pump's capacity.

The room temperature change between two consecutive periods is modeled in (10). This depends on the heat generation Q_t and the difference between the room and the outdoor temperature $T_t^{in} - t_t^{out}$. c is the heat capacity of the building and u is the heat loss coefficient. This formulation is the simple version of the temperature model presented in [1] [1] and is based on Newton's cooling law. The room temperature must be kept within some minimum t^{min} and maximum t^{max} limits (11) for a pleasing living environment [2].

[1]

$$T_t^{in} = \alpha \left[T_{t-1}^{in} + \frac{\delta_t \cdot 3600}{c} \left[x_1 \cdot Q_{t-1} + x_2 \cdot Q_{t-2} + x_3 \cdot Q_{t-3} + \ldots - u \left(T_{t-1}^{in} - t_{t-1}^{out} \right) \right] \right] + b \tag{16}$$

The battery energy balance that calculates the battery state-of-charge (SOC) SOC_t is given by (12), where η^+ and η^- are the charging and discharging efficiencies respectively. The SOC (13) is limited above by the battery capacity soc^{max}, and below by a minimum level soc^{min} which is applied in order to extend the battery life by avoiding deep discharges. The charging and discharging powers of the battery are also limited above (14,15)

The previous formulation may result in an infeasible problem due to temperature limits when these are not possible to be satisfied. In order to avoid this, the model can be reformulated in the following way. Constraint 11 is replaced by the following two constraints:

$$T_t^{excess,+} \geq T_t^{in} - t^{max} \; \forall t \tag{17}$$

$$T_t^{excess,-} \geq t^{min} - T_t^{in} \; \forall t \tag{18}$$

These calculate the temperature excess of the room temperature compared to the maximum or minimum limit. When the room temperature is greater than the upper limit ($T_t^{in} > t^{max}$) then $T_t^{excess,+}$ becomes positive while $T_t^{excess,-}$ is zero. The opposite happens when the room temperature is lower than the lower limit ($T_t^{in} < t^{min}$). To avoid the temperature excess as much as possible, this is penalized in the objective function which becomes:

$$\begin{aligned} min \;\; & \sum_{t \in T} \left(C_t^{energy} + C_t^{grid,energy} \right) + C^{grid,power} \\ & + \sum_{t \in T} \left(\lambda^{t,penalty,+} \cdot T_t^{excess,+} + \lambda^{t,penalty,-} \cdot T_t^{excess,-} \right) \end{aligned} \tag{19}$$

where $\lambda^{t,penalty,+}$ and $\lambda^{t,penalty,-}$ are artificial positive price penalties for the temperature excess.

2 Nomenclature

Indices and Sets:

t	Index of time periods, $t \in T$

Parameters:

λ_t^{buy}	Electricity price (buying), [€/kWh]
λ_t^{sell}	Electricity price (selling), [€/kWh]
$\lambda_t^{grid,energy}$	Grid fee for importing electricity, [€/kWh]
$\lambda^{grid,power}$	Grid power fee, [€/kW]
$\lambda^{grid,penalty}$	Grid overcharge penalty, [€/kW]
$\lambda^{t,penalty,+/-}$	Temperature excess penalty, [€/K]
$p^{contract}$	Level of the power contract, [kW]
$pvgen_t$	PV power generation, [kW]
$load_t$	Load (excluding heat pump), [kW]
cop_t^{heat}	Heat pump coefficient of performance, [$\mathrm{kW_{heat}}$/kW]
q_t^{max}	Heat pump capacity, [$\mathrm{kW_{heat}}$]
$q^{\uparrow/\downarrow}$	Heat pump ramp up/down rate, [% q_t^{max}]
soc^{max}	Battery capacity, [kWh]
soc^{min}	Battery minimum state of charge, [% soc^{max}]
$b^{+,max}$	Battery maximum charging rate, [kW]
$b^{-,max}$	Battery maximum discharging rate, [kW]
η^{+}	Battery charging efficiency
η^{-}	Battery discharging efficiency
t_t^{out}	Outdoor temperature, [K]
t^{min}	Minimum allowed room temperature, [K]
t^{max}	Maximum allowed room temperature, [K]
u	Building heat loss coefficient, [kW/K]
c	Building heat capacity, [kJ/K]
δ_t	Length of a period t, [h]

Positive variables:

P_t^{buy}	Power bought, [kW]
P_t^{sell}	Power sold, [kW]
P^{over}	Grid power overcharge, [kW]
Q_t	Heat pump heat generation, [$\mathrm{kW_{heat}}$]
P_t^{heat}	Heat pump power consumption, [kW]
T_t^{in}	Room temperature, [K]
SOC_t	Battery state of charge, [kWh]
B_t^{+}	Battery charging rate, [kW]
B_t^{-}	Battery discharging rate, [kW]
T_t^{excess}	Temperature excess, [K]

Variables:

C_t^{energy}	Cost of energy trading, [€]
$C_t^{grid,energy}$	Grid energy import cost, [€]
$C^{grid,power}$	Grid power cost, [€]

References

[1] Hietaharju, P., Ruusunen, M., Leiviskä, K., 2018. A Dynamic Model for Indoor Temperature Prediction in Buildings. Energies 11, 1477. URL: https://www.mdpi.com/1996-1073/11/6/1477, doi:10.3390/en11061477. number: 6 Publisher: Multidisciplinary Digital Publishing Institute.

[2] Kem, O., Ksontini, F., 2020. A Multi-Agent Approach to Energy Optimisation for Demand-Response Ready Buildings, in: Sayed-Mouchaweh, M. (Ed.), Artificial Intelligence Techniques for a Scalable Energy Transition. Springer International Publishing, Cham, pp. 77–107. URL: http://link.springer.com/10.1007/978-3-030-42726-9_4, doi:10.1007/978-3-030-42726-9_4.