

# Model description and mathematical formulation

## 1 Mathematical formulation

$$\min \sum_{t \in T} \left( COST_t^{energy} + COST_t^{grid,energy} \right) + COST^{grid,power} \quad (1)$$

$$COST_t^{energy} = \lambda_t^{buy} \cdot P_t^{buy} \cdot \delta_t - \lambda_t^{sell} \cdot P_t^{sell} \cdot \delta_t \quad \forall t \quad (2)$$

$$COST_t^{grid,energy} = \lambda^{grid,in} \cdot P_t^{buy} \cdot \delta_t + \lambda^{grid,out} \cdot P_t^{sell} \cdot \delta_t \quad \forall t \quad (3)$$

$$COST^{grid,power} = \lambda^{grid,power} \cdot p^{contract} + \lambda^{grid,penalty} \cdot P^{over} \quad (4)$$

$$P^{over} \geq P_t^{buy} - p^{contract} \quad \forall t \quad (5)$$

$$P^{over} \geq P_t^{sell} - p^{contract} \quad \forall t \quad (6)$$

$$P_t^{sell} - P_t^{buy} = generation_t - load_t + B_t^- - B_t^+ \quad \forall t \quad (7)$$

$$SOC_t - SOC_{t-1} = \eta^+ \cdot B_t^+ \cdot \delta_t + \frac{B_t^-}{\eta^-} \cdot \delta_t \quad \forall t \quad (8)$$

$$soc^{min} \leq SOC_t \leq soc^{max} \quad \forall t \quad (9)$$

$$0 \leq B_t^+ \leq b^{+,max} \quad \forall t \quad (10)$$

$$0 \leq B_t^- \leq b^{-,max} \quad \forall t \quad (11)$$

## 2 Model description

The objective function (1) of the model stands for the minimization of sum of all costs related to the operation of the microgrid. These include: a) the cost of energy  $COST_t^{energy}$  exchanged with the grid at period  $t$ , b) the grid cost  $COST_t^{grid,energy}$  for the energy transferred at period  $t$  and c) the grid cost  $COST^{grid,power}$  for the power level. Analytically, the cost of energy (2) is the cost of energy bought from the grid at price  $\lambda_t^{buy}$  minus the revenue from energy sold to the grid at price  $\lambda_t^{sell}$ .  $P_t^{buy}$  is the power bought while  $P_t^{sell}$  is the power

sold.  $\delta_t$  is the period length that is used to translate power into energy. The grid energy cost (3) is equal to the cost for the imported energy plus the cost for the exported period.  $\lambda^{grid,in}$  and  $\lambda^{grid,out}$  are the grid fees for importing and exporting energy respectively. The grid power cost (4) includes the cost for the power level contract and the penalty cost when the imports or exports exceed this level.  $p^{contract}$  is the level of the power contract. Typically a microgrid has a contract for the power level and therefore this is a parameter with constant value. However  $p^{contract}$  can be replaced with the variable  $P^{contract}$  and in this case the optimal level of the power contract is calculated.  $P^{over}$  is the maximum amount of power that exceeds the power contract level for the horizon that is considered in the model. This is calculated by (5) and (6). The energy balance of the microgrid is given by (7). This defines that the difference between the power sold and bought in a period  $t$  is equal to the aggregated power generation  $generation_t$  minus the power consumption  $load_t$  in the microgrid, increased or decreased by the discharging  $B_t^-$  or charging  $B_t^+$  power of the battery. The energy balance that calculates the state-of-charge (SOC)  $SOC_t$  of the battery is given by (8), where  $\eta^+$  and  $\eta^-$  are the charging and discharging efficiencies respectively. The SOC (9) is limited above by the battery capacity  $soc^{max}$ , and below by a minimum level  $soc^{min}$  which is applied in order to extend the battery life by avoiding deep discharges. The charging and discharging powers of the battery are also limited above (10,11)