ALGEBRA IN MY BROWSER AND RESPONSIBLE NUTRITION

Brian Beckman

November 2012

ALGEBRA IN MY BROWSER AND RESPONSIBLE ~~NUTRITION~~ DRINKING

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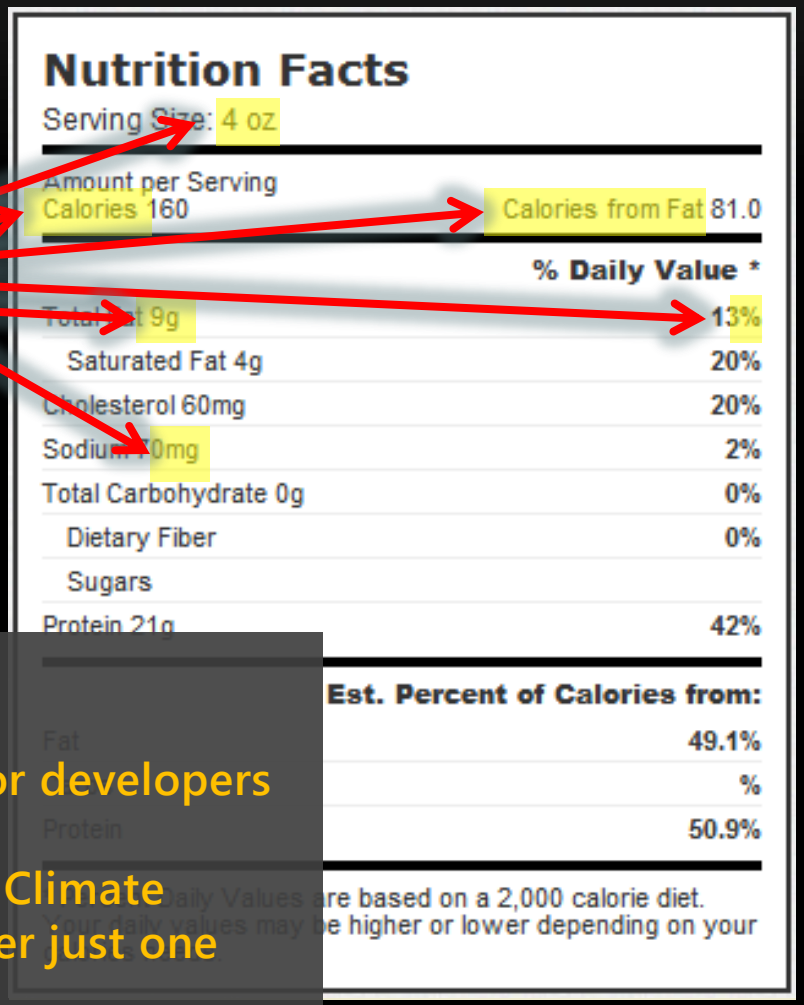
PICK A NUTRITION-FACTS LABEL AT RANDOM FROM THE WEB

There are at least eight different units of measure expressed or implied



Nutrition Facts

Serving Size: 4 oz

Amount per Serving
Calories 160 — Calories from Fat 81.0

	% Daily Value *
Total Fat 9g	13%
Saturated Fat 4g	20%
Cholesterol 60mg	20%
Sodium 70mg	2%
Total Carbohydrate 0g	0%
Dietary Fiber	0%
Sugars	
Protein 21g	42%

Est. Percent of Calories from:

Fat	49.1%
	%
Protein	50.9%

Daily Values are based on a 2,000 calorie diet.
Your daily values may be higher or lower depending on your

What are the chances that the label is consistent: that implied values match stated values?

UNITS OF MEASURE

Notoriously tricky for developers

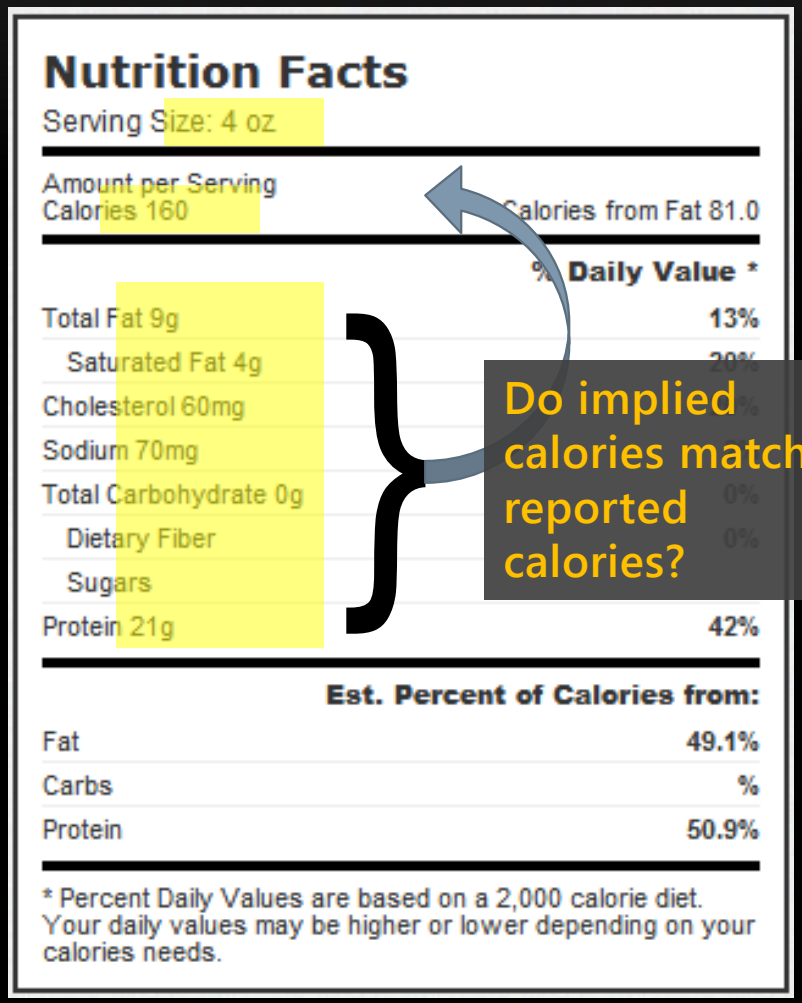
NASA crashed Mars Climate Observer in 1999 over just one units error

PICK A NUTRITION-FACTS LABEL AT RANDOM FROM THE WEB

Two questions we can ask immediately:

Do these weights add up to this serving size?

Do implied calories match reported calories?



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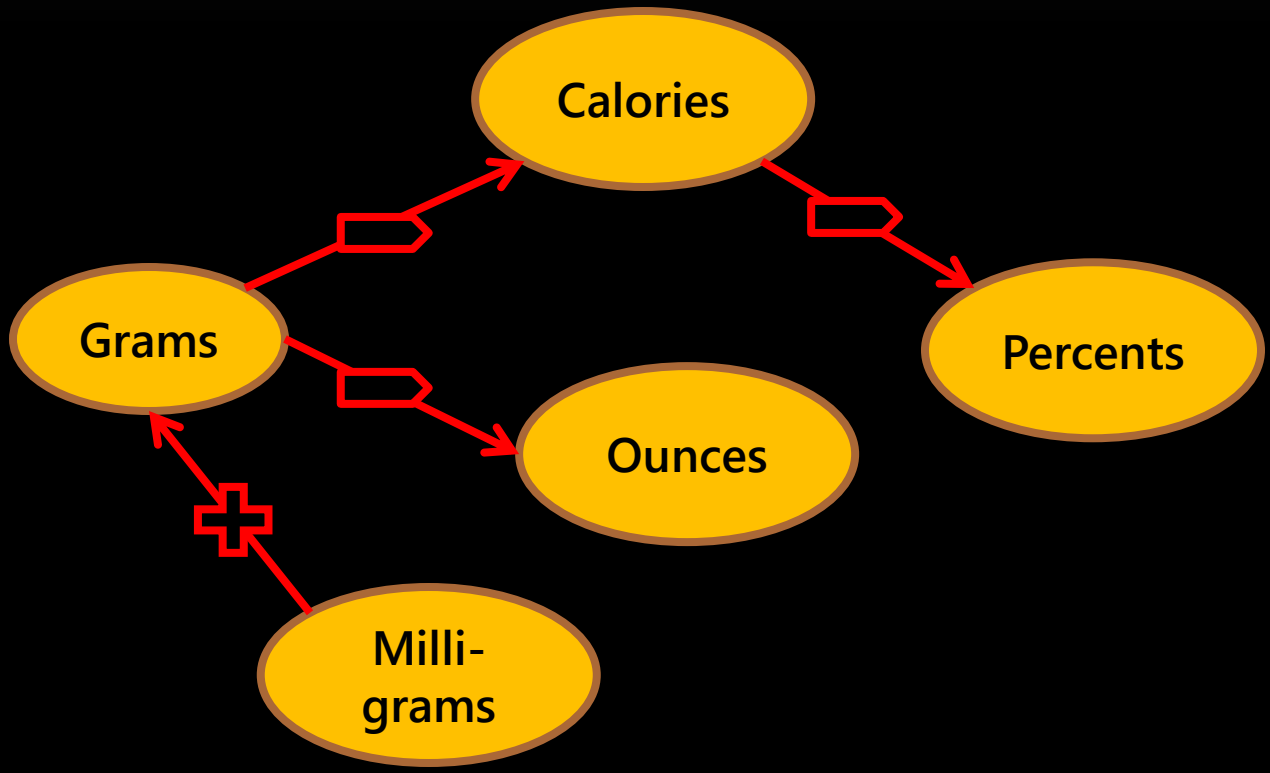
Est. Percent of Calories from:

Fat	49.1%
Carbs	%
Protein	50.9%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calories needs.

CENTRAL PROBLEM

How to write the validator app?



CENTRAL PROBLEM

JavaScript or # programmer st k with no innate w y to tr k units of meas re

```
var burgerNutritionFacts =
{ ServingSize       :   4  , // Ounce
  AmountPerServing  : 160  , // Calorie
  CaloriesFromFat   :  81.0, // Calorie
  SaturatedFat      :   4  , // Gram
  Cholesterol       :  60  , // Milligram
  Sodium            :  70  , // Milligram
  DietaryFiber      :   0  , // Gram
  Sugars            :   0  , // Gram
  TotalFat          :   9  , // Gram
  Protein           :  21  , // Gram
  TotalCarbohydrate :   0  , // Gram
};
```

```
var addWeights = function(nutritionFacts) {,
return nutritionFacts.TotalFat + ,
       nutritionFacts.DietaryFiber +,
       nutritionFacts.Protein +,
       nutritionFacts.Cholesterol +,
       nutritionFacts.Sodium +,
       nutritionFacts.TotalCarbohydrate;
};
document.writeln(addWeights(burgerNutritionFacts));
```

160

Can you spot the mistake?

CENT
PROI

JavaS
progra
with no
to tra

```
var addWe
return nu
        nu
        nu
        nu
        nu
        nu
};
document.
```

```
s =
 4   , // Ounce
60   , // Calorie
81.0, // Calorie
 4   , // Gram
60   , // Milligram
70   , // Milligram
 0   , // Gram
 0   , // Gram
 9   , // Gram
21   , // Gram
 0   , // Gram
```

Nutrition Facts

Serving Size: 4 oz

Amount per Serving
Calories 160

Calories

Calories from Fat 81.0

	% Daily Value *
Total Fat 9g	13%
Saturated Fat 4g	20%
Cholesterol 60mg	20%
Sodium 70mg	2%
Total Carbohydrate 0g	0%
Dietary Fiber	0%
Sugars	
Protein 21g	42%

Estimated Calories from:

Fat	49.1%
Carbs	%
Protein	50.9%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calories needs.

Legit Coincidence?
or
Programming Error?

To eliminate coincidence, we must cross-check the calories

Is this another mistake?

weight

160

Can you spot the mistake?

CENTRAL PROBLEM

TRY REFACTOR NG:
Inv t a
representatio for
quantities with its
and write co to
manage em

```
var burgerNutritionFacts =
{ ServingSize        : [  4  , "Ounce"     ],
  AmountPerServing   : [160  , "Calorie"   ],
  CaloriesFromFat    : [ 81.0, "Calorie"   ],
  SaturatedFat       : [  4  , "Gram"      ],
  Cholesterol        : [ 60  , "Milligram" ],
  Sodium             : [ 70  , "Milligram" ],
  DietaryFiber       : [  0  , "Gram"      ],
  Sugars             : [  0  , "Gram"      ],
  TotalFat           : [  9  , "Gram"      ],
  Protein            : [ 21  , "Gram"      ],
  TotalCarbohydrate  : [  0  , "Gram"      ]
};
```

```
label.forEach(function (k, v) {
  switch(v[1]) {
    case "Ounce":
      result.k = [
        v[0] * 28.34952,
        "Gram"
      ],   ... more cases ...
  }  ... more code ...
});
```

We may write all the conversions we need for this app this way,

but generalize just a little, and we've invented an

ad-hoc, buggy, fragmentary simulation of computer algebra

OK SO WHY NOT JUST *USE* COMPUTER ALGEBRA? MATHEMATICA == BEST OF BREED

In[62]:=
```
(beefedUpBurgerNutritionFacts = {ServingSize → 4 * Ounce,
    AmountPerServing → 160 * Calorie, CaloriesFromFat → 81.0 * Calorie,
    SaturatedFat → 4 * Gram * saturated fat,
    Cholesterol → 60 * Milli * Gram * choleste
    Sodium → 70 Milli * Gram * sodium, Dietary
    Sugars → 0 * Gram * sugar, TotalFat → 9 * Gi
    Protein → 21 * Gram * protein,
    TotalCarbohydrate → 0 * Gram * carbohydra
```

Out[62]=

ServingSize	Times 4 Ounce
AmountPerServing	Times 160 Calorie
CaloriesFromFat	Times 81. Calorie
SaturatedFat	Times 4 fat Gram sa
Cholesterol	Times 60 cholesterol
Sodium	Times 70 Gram Milli
DietaryFiber	0
Sugars	0
TotalFat	Times 9 fat Gram
Protein	Times 21 Gram prote
TotalCarbohydrate	0

In[63]:=
```
(calorieFacts = {
    Gram * saturated * fat → 9 * Calorie,
    Gram * fat → 9 * Calorie,
    Gram * sugar → 4 * Calorie,
    Gram * carbohydrate → 4 * Calorie,
    Gram * protein → 4 * Calorie,
    Gram * cholesterol → 0 * Calorie,
    Gram * fiber → 0 * Calorie,
    Gram * sodium → 0 * Calorie,
    Milli * Gram → Gram * 0.001}) // gridRules
```

Out[63]=

Times fat Gram saturated	Times 9 Calorie
Times fat Gram	Times 9 Calorie
Times Gram sugar	Times 4 Calorie
Times carbohydrate Gram	Times 4 Calorie
Times Gram protein	Times 4 Calorie
Times cholesterol Gram	0
Times fiber Gram	0
Times Gram sodium	0
Times Gram Milli	Times 0.001 Gram

NOW IMAGINE EVALUATING MATHEMATICA EXPRS IN JAVASCRIPT

Observe : syntactically, Mathematica is not horribly different from JavaScript

Observe : semantically, we
Mathematica's basic evalu

Observe : natural correspo
Mathematica expressions

- Author in Mathematica and ru
- Store expressions themselves
- Avoid the trap of innovating i

jump in, start swimming...

Language innovation is a risky, expensive socialogical process, not a technological process:

A CS intern can invent a new language, but getting adoption takes an

institution (e.g. ECMA),
an *authority* (e.g. Odersky),
or *luck* (e.g. CoffeeScript)

CHECKING WEIGHTS

Let arithmetic include symbolic constants

Represent objects and functions as lists of replacement rules

```
burgerNutritionFacts = {
  ServingSize        ->   4   * Ounce,
  AmountPerServing   -> 160   * Calorie,
  CaloriesFromFat    ->  81.0 * Calorie,
  SaturatedFat       ->   4   * Gram,
  Cholesterol        ->  60   * Milli * Gram,
  Sodium             ->  70   * Milli * Gram,
  DietaryFiber       ->   0   * Gram,
  Sugars             ->   0   * Gram,
  TotalFat           ->   9   * Gram,
  Protein            ->  21   * Gram,
  TotalCarbohydrate  ->   0   * Gram
}
```

OBJECTS ≅ FUNCTIONS ≅ RULES

```
var obj = {a: 1, b: 2};
```

```
obj.a ⇝ 1     obj["a"] ⇝ 1
```

Symbols in JavaScript are actually strings!
∃ only before colons and after dots

```
function obj (x) {
    return x === "a" ? 1 :
           (x === "b" ? 2 : undefined);
}
```

```
obj("a") ⇝ 1
```

```
obj = {"a" -> 1, "b" -> 2, _ -> undefined}
```

```
"a" /. obj ⇝ 1
```

The meaning of expr /. rule is "Apply the rule to the expression"

```
obj = {a -> 1, b -> 2, _ -> undefined}
```

```
a /. obj ⇝ 1
```

Real Computer Algebra has symbolic constants as a distinct type

CHECKING WEIGHTS

```
burgerNutritionFacts = {
  ServingSize       ->   4    * Ounce,
  AmountPerServing  -> 160    * Calorie,
  CaloriesFromFat   ->  81.0  * Calorie,
  SaturatedFat      ->   4    * Gram,
  Cholesterol       ->  60    * Milli * Gram,
  Sodium            ->  70    * Milli * Gram,
  DietaryFiber      ->   0    * Gram,
  Sugars            ->   0    * Gram,
  TotalFat          ->   9    * Gram,
  Protein           ->  21    * Gram,
  TotalCarbohydrate ->   0    * Gram
}
```

```
{ Milli -> 0.001,
  Gram  -> Ounce / 28.3495 }
```

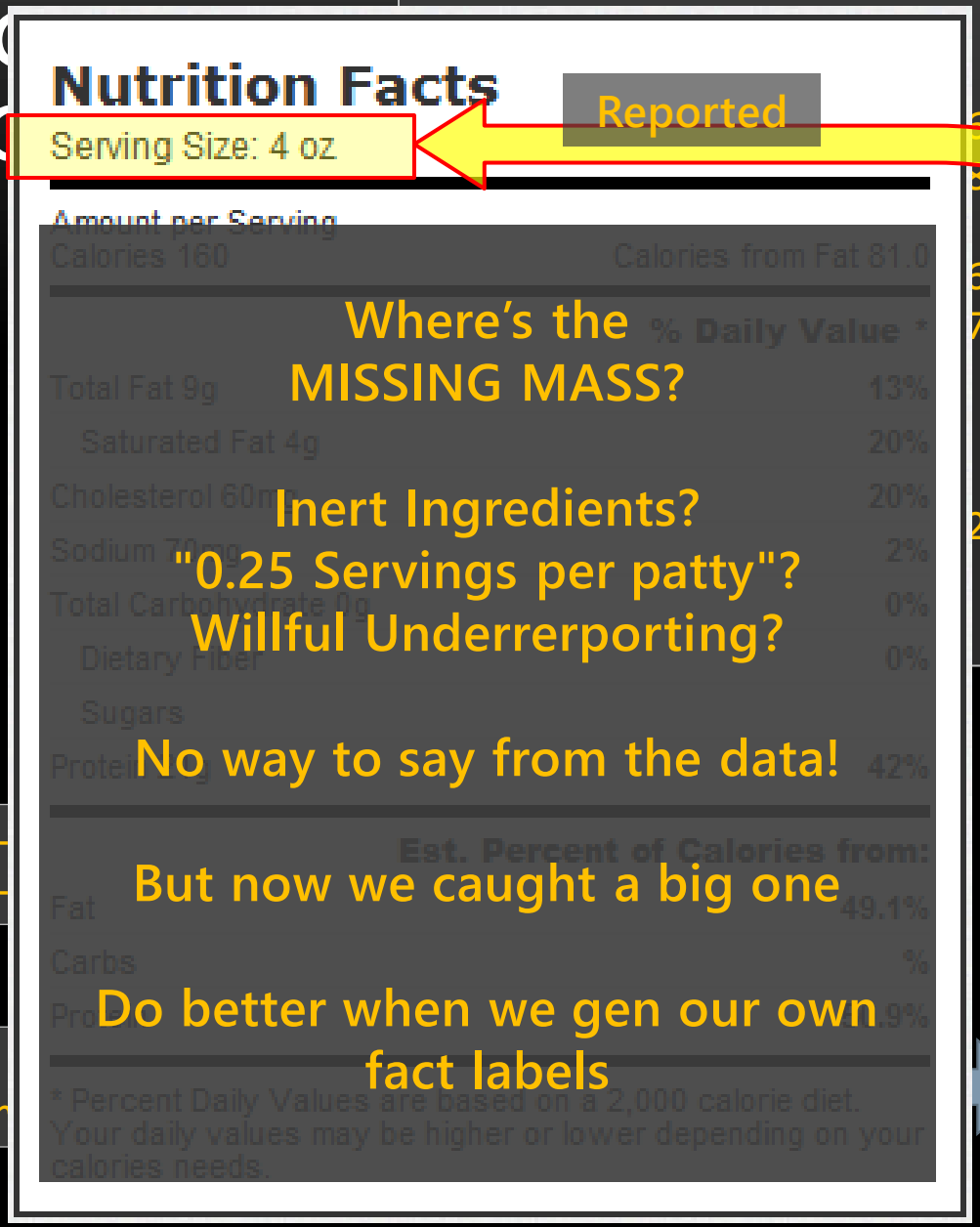
Basic computation strategy is to "repeatedly apply rules to expressions"

i.e., to rewrite expressions

```
TotalFat + DietaryFiber + Protein +
Cholesterol + Sodium + TotalCarbohydrate
```

```
1.0628 * Ounce
```

CHE
WEI



Reported

Where's the MISSING MASS?

Inert Ingredients?
"0.25 Servings per patty"?
Willful Underrerporting?

No way to say from the data!

But now we caught a big one

Do better when we gen our own fact labels

```
4    * Ounce,
50   * Calorie,
81.0 * Calorie,
4    * Gram,
60   * Milli * Gram,
70   * Milli * Gram,
0    * Gram,
0    * Gram,
9    * Gram,
21   * Gram,
0    * Gram
```

Inferred

```
{ Milli -
  Gram  -
```

```
TotalFat
Cholester
```

```
1.0628 * Ounce
```

TERM REWRITING

Term Rewriting is a general computational strategy

- Can simulate lambda calculus and vice-versa
- Commonplace in Computer Algebra and Theorem Proving
- Enables very concise statements of sophisticated algorithms

Basic idea: "Replace patterns with expressions after variable substitution; iterate until nothing changes"

```
fib[n_ /; n <= 2] = 1
fib[n_] := fib[n - 1] + fib[n - 2]
fib[3] ~~> fib[2] + fib[1] ~~> 1 + 1 ~~> 2 ~~> 2 DONE!
```

- Not a function call, but can simulate one
- Can do much more, and much less, however

Patterns are like Regular Expressions, only they work on expressions themselves

- Expressions are "auto-iconic" – they are their own representation language

CHECKING CALORIES

Not all grams are created equal –

grams fat ≠ grams carbs

```
beefedUpBurgerNutritionFacts = {
  ServingSize       ->   4 * Ounce,
  AmountPerServing  -> 160 * Calorie,
  CaloriesFromFat   ->  81 * Calorie,
  SaturatedFat      ->   4 * Gram * saturatedFat,
  Cholesterol       ->  60 * Milli * Gram * cholesterol,
  Sodium            ->  70 * Milli * Gram * sodium,
  DietaryFiber      ->   0 * Gram * fiber,
  Sugars            ->   0 * Gram * sugar,
  TotalFat          ->   9 * Gram * fat,
  Protein           ->  21 * Gram * protein,
  TotalCarbohydrate ->   0 * Gram * carbohydrate}
```

Now mine some more facts from the web

```
calorieFacts = {
  Gram * saturatedFat -> 9 * Calorie,
  Gram * fat          -> 9 * Calorie,
  Gram * sugar        -> 4 * Calorie,
  Gram * carbohydrate -> 4 * Calorie,
  Gram * protein      -> 4 * Calorie,
  Gram * cholesterol  -> 0 * Calorie,
  Gram * fiber        -> 0 * Calorie,
  Gram * sodium       -> 0 * Calorie,
  Milli * Gram        -> Gram * 0.001}}
```

We could dump all these zeros, but keep them around as reminders and for future generalization

CHECKING CALORIES

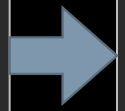
Rules are expressions too:

we can rewrite

rules-as-objects using rules-as-functions

```
beefedUpBurgerNutritionFacts = {
  ServingSize       ->   4 * Ounce,
  AmountPerServing  -> 160 * Calorie,
  CaloriesFromFat   ->  81 * Calorie,
  SaturatedFat      ->   4 * Gram * saturatedFat,
  Cholesterol       ->  60 * Milli * Gram * cholesterol,
  Sodium            ->  70 * Milli * Gram * sodium,
  DietaryFiber      ->   0 * Gram * fiber,
  Sugars            ->   0 * Gram * sugar,
  TotalFat          ->   9 * Gram * fat,
  Protein           ->  21 * Gram * protein,
  TotalCarbohydrate ->   0 * Gram * carbohydrate}
```

```
calorieFacts = {
  Gram * saturatedFat -> 9 * Calorie,
  Gram * fat          -> 9 * Calorie,
  Gram * sugar        -> 4 * Calorie,
  Gram * carbohydrate -> 4 * Calorie,
  Gram * protein      -> 4 * Calorie,
  Gram * cholesterol  -> 0 * Calorie,
  Gram * fiber        -> 0 * Calorie,
  Gram * sodium       -> 0 * Calorie,
  Milli * Gram        -> Gram * 0.001}}
```



```
{ ServingSize       ->   4 * Ounce,
  AmountPerServing  -> 160 * Calorie,
  CaloriesFromFat   ->  81 * Calorie,
  SaturatedFat      ->  36 * Calorie,
  Cholesterol       ->   0 * Calorie,
  Sodium            ->   0 * Calorie,
  DietaryFiber      ->   0 * Calorie,
  Sugars            ->   0 * Calorie,
  TotalFat          ->  81 * Calorie,
  Protein           ->  84 * Calorie,
  TotalCarbohydrate ->   0 * Calorie}
```

CHEC
CALO

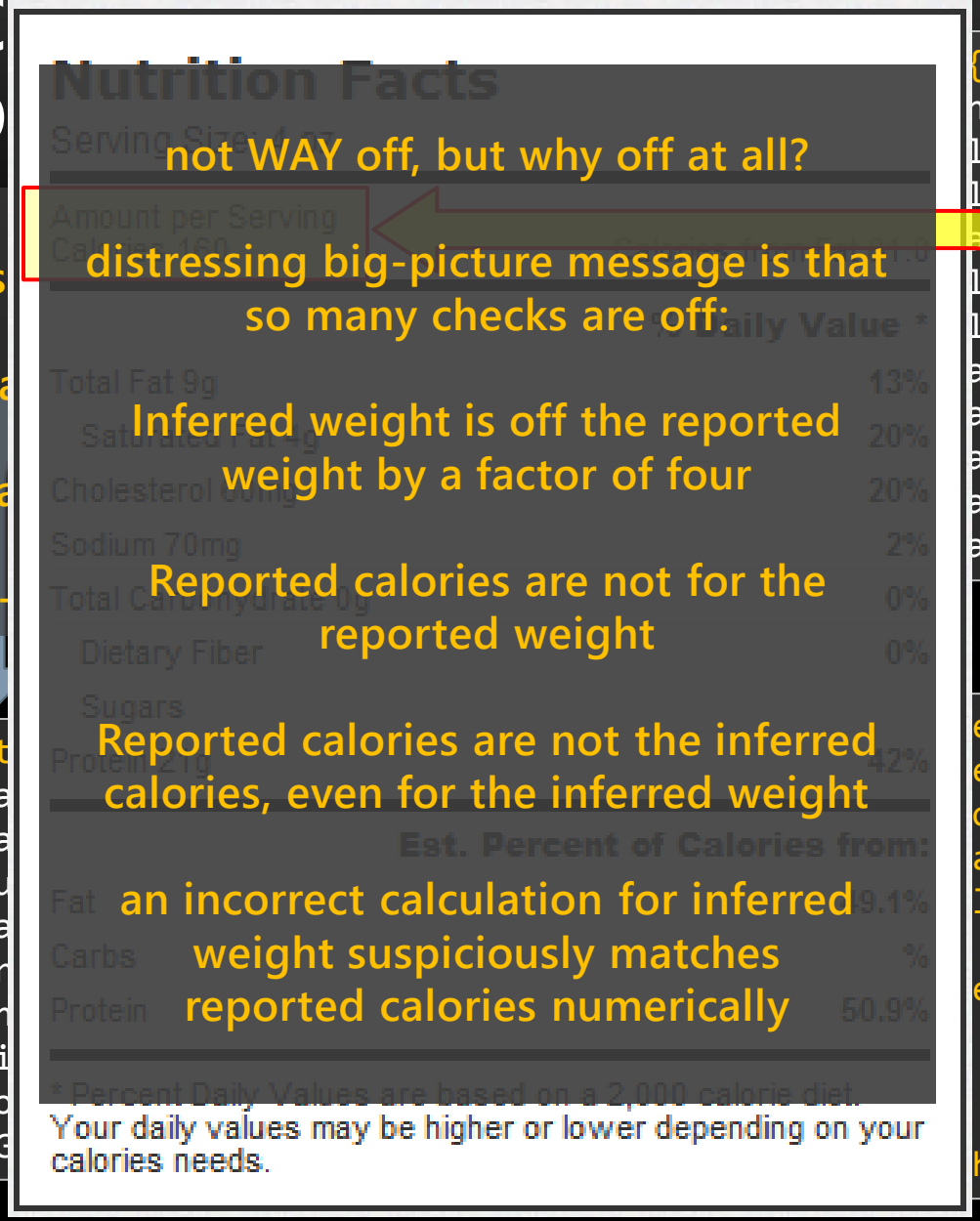
expres

we ca

rules-a

rules-as-

```
calorieFact
  Gram * sa
  Gram * fa
  Gram * su
  Gram * ca
  Gram * pr
  Gram * ch
  Gram * fi
  Gram * so
  Milli * G
```



not WAY off, but why off at all?

distressing big-picture message is that so many checks are off:

Inferred weight is off the reported weight by a factor of four

Reported calories are not for the reported weight

Reported calories are not the inferred calories, even for the inferred weight

an incorrect calculation for inferred weight suspiciously matches reported calories numerically

```
{
nce,
lorie,
lorie,
am * saturatedFat,
lli * Gram * cholesterol,
lli * Gram * sodium,
am * fiber,
am * sugar,
am * fat,
am * protein,
am * carbohydrate}
```

```
e          ->   4 * Ounce,
erving     -> 160 * Calorie,
omFat      ->  81 * Calorie,
at         ->  36 * Calorie,
l          ->   0 * Calorie,
           ->   0 * Calorie,
er         ->   0 * Calorie,
           ->   0 * Calorie,
           ->  81 * Calorie,
           ->  84 * Calorie,
hydrate    -> 165 * Calorie
```

SYMBOLIC ARITHMETIC JUSTIFIED

Robustly track and convert units of measure

"Easy on the Eyes": Minimal syntactic disruption to familiar JavaScript programming patterns

"Easy to Implement": Basic pattern-matching and rewriting is a small JavaScript library

Patterns-and-Rules strategy will extend our reach: next is robust creation of new labels from recipes...

HOW ABOUT NEW LABELS ON-THE-FLY?

Nutrition Facts

Serving Size: 4 oz

Amount per Serving	
Calories 160	Calories from Fat 81.0
	% Daily Value *
Total Fat 9g	13%
Saturated Fat 4g	20%
Cholesterol 60mg	20%
Sodium 70mg	2%
Total Carbohydrate 0g	0%
Dietary Fiber	0%
Sugars	
Protein 21g	42%
	Est. Percent of Calories from:
Fat	49.1%
Carbs	%
Protein	50.9%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calories needs.

What would the label be for this meal?

STEPS IN THE SOLUTION

Ok, we are NOT doing image reco!

Assume we start with recipe data

Ingredients

sparagus $

1 tablespoon olive oil $

1/2 cup chopped onion $

2 garlic cloves, minced

t 10 ounces

rmesan cheese,

1/4 cup chopped fresh parsley

1/2 teaspoon salt

1/2 teaspoon pepper

STEPS IN THE SOLUTION

Ingredients

2 cups (1-inch) diagonally sliced asparagus $

2 cups sliced carrot $

1 1/2 cups snow peas

1 tablespoon olive oil $

1/2 cup chopped onion $

2 garlic cloves, minced

6 cups hot cooked fettuccine (about 10 ounces uncooked pasta) $

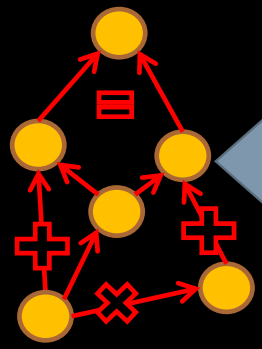
1/2 cup (2 ounces) grated fresh Parmesan cheese, divided $

1/4 cup chopped fresh parsley

2 tablespoons dry white wine $

1/2 teaspoon salt

1/2 teaspoon pepper



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Dietary Fiber	0%
Sugars	
Protein 21g	42%

Est. Percent of Calories from:	
Fat	49.1%
Carbs	%
Protein	50.9%

* Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calories needs.

THIS IS A VECTOR-SPACE PROBLEM

Each ingredient is a basis vector in Nutrition-Fact-Label (NFL) Vector Space

- ***Normalize*** **them into unit vectors**

The recipe is a sum of scaled ingredients

- **Must put all ingredients into commensurable units**
- **i.e., *Canonicalize* the units**

COMPUTATIONAL PLAN

Rewrite the Recipe in grams

- Convert volumes to masses via data-mined densities
- Convert whole-item ingredients to masses via data-mined averages

Data-mine ingredients

- Normalize each into a 1-gram "serving size"
- Proportionately scale all dependent quantities (calories, percents)

Lookup, scale and sum to write new NFL

- Optional postprocess: scale and convert result into user-specified units

REWRITING A RECIPE

write the recipe as a list of quantified ingredients

some volumes
some masses
some whole-items

It's ok to multiply numbers by symbols by strings – such has no other meaning than itself until there is a rule to transform it

```
myRecipe = {
  1.0  * Tablespoon * "olive oil",
 16.0  * Ounce      * "zucchini",
  3.5  * Teaspoon   * "salt",
  1.5  * Pound      * "eggplant",
  1.0               * "onion",
  2.0               * "bell pepper",
 14.5  * Ounce      * "stewed tomato",
  0.5  * Teaspoon   * "black pepper",
  0.5  * Teaspoon   * "dried basil",
  0.5  * Teaspoon   * "sugar",
 12.0  * Ounce      * "pasta",
  0.25 * Cup        * "parmesan cheese"}
```

Now mine density facts from the web

```
density["olive oil"]       = Mean[{6.68,7.67}] * Pound / Gallon;
density["salt"]            = 5.69 Gram / Teaspoon;
density["black pepper"]    = 2.1 Gram / Teaspoon;
density["dried basil"]     = 1.0 Gram / Teaspoon;
density["sugar"]           = 4.2 Gram / Teaspoon;
density["parmesan cheese"] = 88 Gram / Cup;
```

REWRITING A RECIPE

Bracket notation is just another way of writing a rule…
a rule that is always applied whenever seen…
a rule stored in session state – on purpose

Resembles function definition on purpose

left-hand side = pattern (like a regular expression)
right-hand side = replacement

```
density["olive oil"]       = Mean[{6.68,7.67}] * Pound / Gallon;
density["salt"]            = 5.69 Gram / Teaspoon;
density["black pepper"]    = 2.1 Gram / Teaspoon;
density["dried basil"]     = 1.0 Gram / Teaspoon;
density["sugar"]           = 4.2 Gram / Teaspoon;
density["parmesan cheese"] = 88 Gram / Cup;
```

REWRITING A RECIPE

Must create rules for converting quantified ingredients into weights in grams

```
weightRuleFromQuantifiedIngredientVolume[
  quantity_?NumberQ   *
  ingredient_String   *
  volume : (Teaspoon | Tablespoon | Cup | FluidOunce | Pint | Gallon)] :=
```

The pattern has *variables_* distinguished by underscores

They're bound to actual values during the match

This pattern will match any line in the recipe that is a numerical *quantity_* times an *ingredient_* of type *String* times one of the given symbolic constants; the matched one is given the name *volume*

The ?NumberQ test could be any other boolean-valued expression

REWRITING A RECIPE

```
weightRuleFromQuantifiedIngredientVolume[
  quantity_?NumberQ   *
  ingredient_String   *
  volume : (Teaspoon | Tablespoon | Cup | FluidOunce | Pint | Gallon)] :=

ingredient * volume ->
  ingredient * gramPerTargetVolumeFromDensity[
    volume,
    density[ingredient] ] * volume;
```

If we match such a line, rewrite it as a rule that, when applied, will convert the ingredient times the volume into the ingredient times the gram-density: grams per unit of the matched volume

REWRITING A RECIPE

```
weightRuleFromQuantifiedIngredientVolume[
  quantity_?NumberQ   *
  ingredient_String   *
  volume : (Teaspoon | Tablespoon | Cup | FluidOunce | Pint | Gallon)] :=

ingredient * volume ->
  ingredient * gramPerTargetVolumeFromDensity[
    volume,
    density[ingredient] ] * volume;
```

```
gramPerTargetVolumeFromDensity[
  targetVolume_,
  d_?NumberQ * weight_ / volume_] :=

 (d * Convert[weight, Gram]) / Convert[volume, targetVolume]
```

rewrite data-mined densities in terms of recipe volumes

REWRITING A RECIPE

```
weightRuleFromQuantifiedIngredientVolume[
  quantity_?NumberQ   *
  ingredient_String   *
  volume : (Teaspoon | Tablespoon | Cup | FluidOunce | Pint | Gallon)] :=

ingredient * volume ->
  ingredient * gramPerTargetVolumeFromDensity[
    volume,
    density[ingredient] ] * volume;
```

```
gramPerTargetVolumeFromDensity[
  targetVolume_,
  d_?NumberQ * weight_ / volume_] :=

 (d * Convert[weight, Gram]) / Convert[volume, targetVolume]
```

```
weightRuleFromQuantifiedIngredientVolume[___] := {}
```

```
volumeRules =
 SelectMany[myRecipe, weightRuleFromQuantifiedIngredientVolume]
```

Need one more rule to cover cases that don't match...
Then the magical monadic bind: SelectMany...
the Swiss army knife of programming with collections

REWRITING A RECIPE

Times olive oil Tablespoon	Times 12.713 olive oil Gram
Times salt Teaspoon	Times 5.69 salt Gram
Times black pepper Teaspoon	Times 2.1 black pepper Gram
Times dried basil Teaspoon	Times 1. dried basil Gram
Times sugar Teaspoon	Times 4.2 sugar Gram
Times parmesan cheese Cup	Times 88 parmesan cheese Gram

```
volumeRules =
 SelectMany[myRecipe, weightRuleFromQuantifiedIngredientVolume]
```

REWRITING A RECIPE

likewise for whole items:

```
wholeItemWeight["onion"] = (1.0/3) Pound;
wholeItemWeight["bell pepper"] = 0.5 Pound/4;
```

```
weightRuleFromQuantifiedWholeItemIngredient[_ * _String * _Symbol] = {};

weightRuleFromQuantifiedWholeItemIngredient[_?NumberQ * ingredient_] :=
  ingredient -> ingredient * wholeItemWeight[ingredient];

weightRuleFromQuantifiedWholeItemIngredient[___] = {};

wholeItemRules =
  SelectMany[myRecipe,
    weightRuleFromQuantifiedWholeItemIngredient]
```

onion	Times | 0.33333 | onion | Pound
bell pepper	Times | 0.125 | bell pepper | Pound

REWRITING A RECIPE

```
myRecipe /.
volumeRules /.
wholeItemRules
```

```
myRecipe = {
  1.0  * Tablespoon * "olive oil",
 16.0  * Ounce      * "zucchini",
  3.5  * Teaspoon   * "salt",
  1.5  * Pound      * "eggplant",
  1.0               * "onion",
  2.0               * "bell pepper",
 14.5  * Ounce      * "stewed tomato",
  0.5  * Teaspoon   * "black pepper",
  0.5  * Teaspoon   * "dried basil",
  0.5  * Teaspoon   * "sugar",
 12.0  * Ounce      * "pasta",
  0.25 * Cup        * "parmesan cheese"}
```

Times	12.713	olive oil	Gram
Times	16.	zucchini	Ounce
Times	19.915	salt	Gram
Times	1.5	eggplant	Pound
Times	0.33333	onion	Pound
Times	0.25	bell pepper	Pound
Times	14.5	stewed tomato	Ounce
Times	1.05	black pepper	Gram
Times	0.5	dried basil	Gram
Times	2.1	sugar	Gram
Times	12.	pasta	Ounce
Times	22.	parmesan cheese	Gram

REWRITING A RECIPE

```
myRecipe = {
  1.0  * Tablespoon * "olive oil",
 16.0  * Ounce      * "zucchini",
  3.5  * Teaspoon   * "salt",
  1.5  * Pound      * "eggplant",
  1.0               * "onion",
  2.0               * "bell pepper",
 14.5  * Ounce      * "stewed tomato",
  0.5  * Teaspoon   * "black pepper",
  0.5  * Teaspoon   * "dried basil",
  0.5  * Teaspoon   * "sugar",
 12.0  * Ounce      * "pasta",
  0.25 * Cup        * "parmesan cheese"}
```

```
recipeInGrams =
 Map[
  Function[ingredient,
   Convert[ingredient, Gram]],
  myRecipe /.
    volumeRules /.
      wholeItemRules
```

Times	12.713	olive oil	Gram
Times	453.59	zucchini	Gram
Times	19.915	salt	Gram
Times	680.39	eggplant	Gram
Times	151.2	onion	Gram
Times	113.4	bell pepper	Gram
Times	411.07	stewed tomato	Gram
Times	1.05	black pepper	Gram
Times	0.5	dried basil	Gram
Times	2.1	sugar	Gram
Times	340.19	pasta	Gram
Times	22.	parmesan cheese	Gram

REWRITING A RECIPE

Now a way to create new basis vectors

add the name to a list

add the basis vector object to a lookup rule by name

```
createNutritionFactsLabel[ name_,
 servingSize_, totalCalories_, fatCalories_,
 totalFat_, totalFatPercent_,
 saturatedFat_, saturatedFatPercent_, transFat_,
 cholesterol_, cholesterolPercent_,
 sodium_, sodiumPercent_,
 totalCarbohydrates_, totalCarbohydratesPercent_,
 dietaryFiber_, dietaryFiberPercent_,
 sugars_, protein_, proteinPercent_,
 vitaminAPercent_, vitaminCPercent_, calciumPercent_, ironPercent_ ]:=
(AppendTo[nflNames, name]; nfls[name] = {
  "name" -> name, "serving size" -> servingSize, "total calories" -> totalCalories,
  "fat calories" -> fatCalories, "total fat" -> totalFat,
  "% daily total fat" -> totalFatPercent, "saturated fat" -> saturatedFat,
  "% daily saturated fat" -> saturatedFatPercent, "trans fat" -> transFat,
  "cholesterol" -> cholesterol, "% daily cholesterol" -> cholesterolPercent,
  "sodium" -> sodium, "% daily sodium" -> sodiumPercent,
  "total carbohydrates" -> totalCarbohydrates,
  "% daily carbohydrates" -> totalCarbohydratesPercent,
  "dietary fiber" -> dietaryFiber, "%daily dietary fiber" -> dietaryFiberPercent,
  "sugars" -> sugars, "protein" -> protein, "% daily protein" -> proteinPercent,
  "vitamin A" -> vitaminAPercent, "vitamin C" -> vitaminCPercent,
  "calcium" -> calciumPercent, "iron" -> ironPercent});
```

REWRITING A RECIPE

```
createNutritionFactsLabel[
"black pepper", 1. Tablespoon,
16 Calorie, 2 Calorie, 0 Gram,
0 Percent, 0 Gram, 0 Percent, 0 Gram,
0 Gram, 0 Percent, 3 Milli Gram,
0 Percent, 4 Gram, 1. Percent,
2 Gram, 7 Percent, 0 Gram, 1. Gram,
0 Percent, 0 Percent, 2 Percent,
3 Percent, 10 Percent]
```

Terse and convenient for database

There are lots of ways we could make it prettier and more robust

name	black pepper
serving size	Times | 1. | Tablespoon
total calories	Times | 16 | Calorie
fat calories	Times | 2 | Calorie
total fat	0
% daily total fat	0
saturated fat	0
% daily saturated fat	0
trans fat	0
cholesterol	0
% daily cholesterol	0
sodium	Times | 3 | Gram | Milli
% daily sodium	0
total carbohydrates	Times | 4 | Gram
% daily carbohydrates	Times | 1. | Percent
dietary fiber	Times | 2 | Gram
%daily dietary fiber	Times | 7 | Percent
sugars	0
protein	Times | 1. | Gram
% daily protein	0
vitamin A	0
vitamin C	Times | 2 | Percent
calcium	Times | 3 | Percent
iron	Times | 10 | Percent

REWRITING A RECIPE

```
canonicalizeUnits[nfl_] := (* for each rule
  Map[ Function[ rule, rule[[1]] -> Convert
    (nfl /. {  (* pattern to match against
      (keyWithVolume_ ->  (* this arrow is
         amount_?NumberQ  *
         volume : (Teaspoon | Tablespoon |

      :>  (* this arrow is part of the rule

      (* this is the resulting new rule *)
      keyWithVolume -> amount  *  volume  *
        gramPerTargetVolumeFromDensity[volu
```

Convert all weights and volumes to grams

honor "hippocratic principle" – don't damage lines that don't match

name	black pepper
serving size	Times 6.3 Gram
total calories	Times 16. Calorie
fat calories	Times 2. Calorie
total fat	0.
% daily total fat	0.
saturated fat	0.
% daily saturated fat	0.
trans fat	0.
cholesterol	0.
% daily cholesterol	0.
sodium	Times 0.003 Gram
% daily sodium	0.
total carbohydrates	Times 4. Gram
% daily carbohydrates	Times 1. Percent
dietary fiber	Times 2. Gram
%daily dietary fiber	Times 7. Percent
sugars	0.
protein	Times 1. Gram
% daily protein	0.
vitamin A	0.
vitamin C	Times 2. Percent
calcium	Times 3. Percent
iron	Times 10. Percent

REWRITING A RECIPE

A list of norms, a list of basis vectors

A function to scale any vector

➔ normalized basis vectors

```
nflList = Map[ Function[name, nfls[name]], nflNames ];
(* make a list of nfls from the lookup rules, for mapping *)
canonicalizedNfls = Map[ canonicalizeUnits, nflList ];
```

```
norms = Map[ Function[nfl, ("serving size" / Gram /. nfl)], canonicalizedNfls ]

{216., 273.12, 82., 160., 186., 101., 6.3, 1., 2., 128., 100., 100.}
```

```
scaleNfl[nfl_, scalar_] :=
 Map[
  Function[line, If[line[[1]] === "name",
    line, (* skip the name line (hippocratically) *)
    line[[1]] -> line[[2]] * scalar]],
  nfl]
```

```
normalizedNfls = Zip[ scaleNfl, {canonicalizedNfls, 1 / norms} ];
```

```
normalizedNflsObj =
(* convert the list into an object (a list of rules) *)
  Map[ Function[nfl, ("name" /. nfl) -> nfl ], normalizedNfls];
```

REWRITING A RECIPE

A function to scale an NFL from an ingredient in grams

A function to add two NFLs

A Fold and an ad-hoc scale

```
scaledNflFromIngredient[qtty_?NumberQ * name_String * Gram] :=
 If[(name /. normalizedNflsObj) === name,
 (* ingredient wasn't in DB *) {}, (* for SelectMany to flatten *)
  {scaleNfl[name /. normalizedNflsObj, qtty]}]
```

```
scaledNfls = SelectMany[recipeInGrams, scaledNflFromIngredient]
```

```
sumNfls[nfl1_, nfl2_] :=
 Zip[
  Function[{line1, line2},
   If[line1[[1]] === line2[[1]],
    line1[[1]] -> (line1[[2]] + line2[[2]]),
    Throw["dimensions didn't match"]]],
  {nfl1, nfl2}]
```

```
scaleNfl[Fold[sumNfls, First[scaledNfls], Rest[scaledNfls]], 1/6]
```

REWRITING A RECIPE

cale an NFL from an ingredient in grams

ion to add two NFLs

and an ad-hoc scale

```
scaledNflFromIngredien
 If[(name /. normalize
 (* ingredient wasn't
  {scaleNfl[name /. no
```

```
scaledNfls = SelectMan
```

```
sumNfls[nfl1_, nfl2_]
 Zip[
  Function[{line1, lin
   If[line1[[1]] === l
    line1[[1]] -> (lin
    Throw["dimensions
  {nfl1, nfl2}]
```

```
scaleNfl[Fold[sumNfls,
```

serving size	Times	368.02	Gram
total calories	Times	309.72	Calorie
fat calories	Times	58.413	Calorie
total fat	Times	6.546	Gram
% daily total fat	Times	9.8998	Percent
saturated fat	Times	1.5959	Gram
% daily saturated fat	Times	7.5358	Percent
trans fat	0		
cholesterol	Times	0.044422	Gram
% daily cholesterol	Times	14.795	Percent
sodium	Times	1.7696	Gram
% daily sodium	Times	73.6	Percent
total carbohydrates	Times	53.543	Gram
% daily carbohydrates	Times	17.71	Percent
dietary fiber	Times	6.8463	Gram
%daily dietary fiber	Times	25.73	Percent
sugars	Times	5.8525	Gram
protein	Times	12.1	Gram
% daily protein	Times	2.7658	Percent
vitamin A	Times	13.434	Percent
vitamin C	Times	107.62	Percent
calcium	Times	11.903	Percent
iron	Times	19.675	Percent

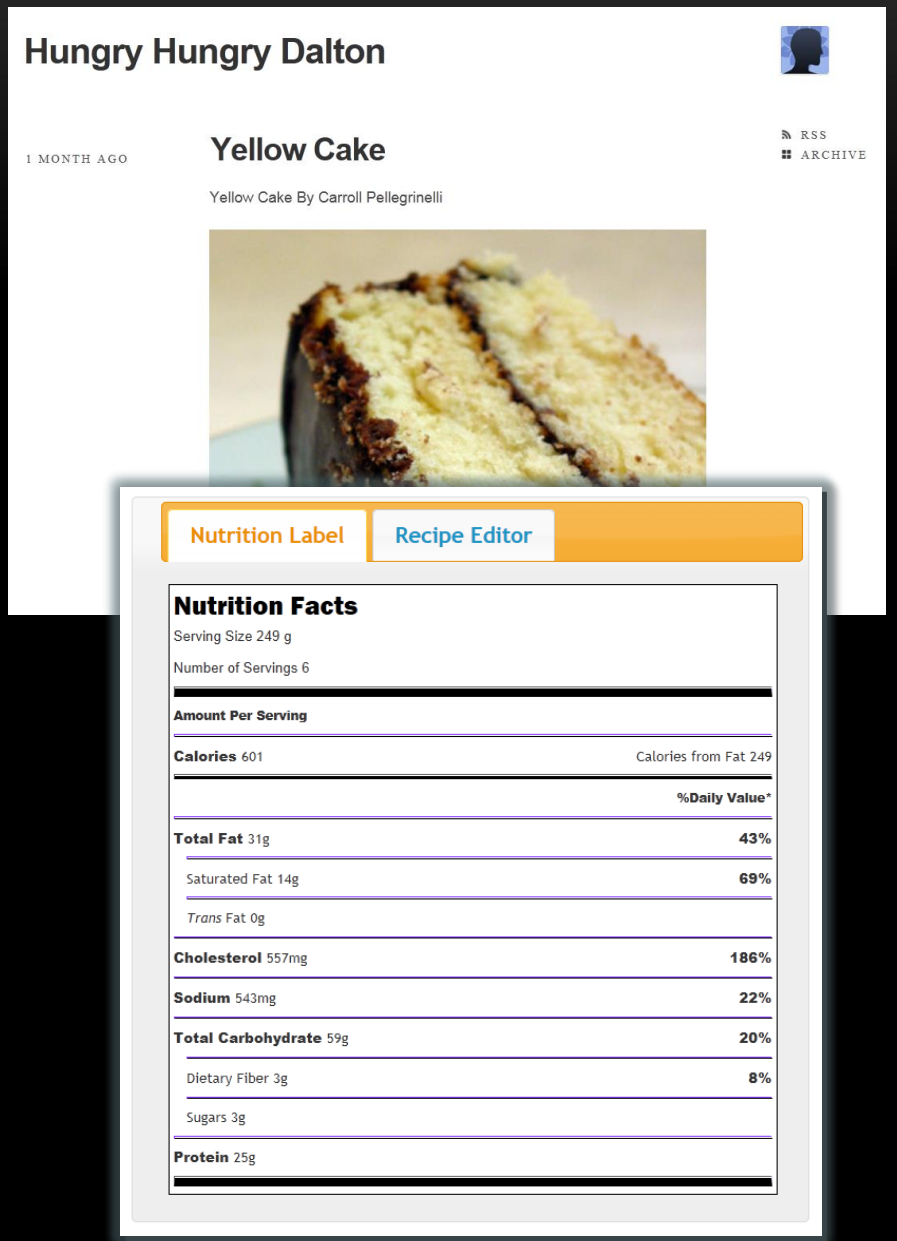
http://dghubble.tumblr.com/

REFINEMENTS

CSS for formatting results as a label

UI for editing recipes

Remoting expressions



UNITS OF MEASURE ARE EVERYWHERE

Finance

```
{ Euro -> 1.24 * Dollar,
  Yuan -> 0.15715 * Dollar, ... }
```

```
TradeTrigger[priceSpread_] := ...
```

Transportation

```
{ Mile   -> 1.609344 * Kilo * Meter,
  Gallon -> 3.785412 * Liter, ... }
```

```
Convert[22 * Mile / Gallon, ...
~~>    0.10692 Liter / Kilo*Meter
```

Textiles / Apparel

```
{ American -> 2 * British / 6,
  British  -> European / 6, ... }
```

Calendars **Engineering** **Geospatial**

Agriculture **Government** **Education**

UNITS CONVERSION IN GENERAL

```
165 SiRules = {
189   (* MASS *)
190
191  (* Gram         -> Kilogram / 1000, *)
192     Grain        -> 64.79891 * Milli * Gram,
193     Carat        -> (3 + 1/16) * Grain,
194     MetricCarat  -> 200 * Milli * Gram,
195     PoundMass    -> 7000 * Grain,
196     OunceMass    -> PoundMass / 16,
197     USTon        -> 2000 * PoundMass,
198     UKTon        -> 2240 * PoundMass,
199     Scruple      -> 20 * Grain,
200
201     AvdpPound    -> PoundMass,
202     AvdpDram     -> (27 + 11/32) * Grain,
203
204     TroyPound    -> 5740 * Grain,
205     TroyOunce    -> TroyPound / 12,
206
207     Shekel       -> Kilogram / 87.719298246,
208
209     ApothecaryOunce -> TroyOunce,
210     ApothecaryDram  -> 60 * Grain,
211
212   (* WEIGHT / FORCE *)
213
214     Newton         -> Kilogram * Meter / (Second^2),
215     PoundForce     -> 4.4482216152605 * Newton,
216     KilogramForce -> 9.80665
217     OunceForce     -> PoundFor
```

```
78  PluralsRules = {
79    (* LENGTH *)
80      Meters -> Meter,
81      Miles  -> Mile,
82      Yards  -> Yard,
83      Feet   -> Foot,
84      Inches -> Inch,
85
86    (* TIME *)
87      Fortnights -> Fortnight,
88      Weeks      -> Week,
89      Days       -> Day,
```

```
153 CommonParlanceRules = {
154   Quart          -> LiquidQuart,
155   Pint           -> LiquidPint,
156   Ounce          -> OunceMass,
157   USGallon       -> Gallon,
158   Litre          -> Liter,
159   ImperialGallon -> UKGallon,
160   Pound          -> PoundForce,
161   PoundWeight    -> PoundForce,
162   OunceWeight    -> OunceForce
```

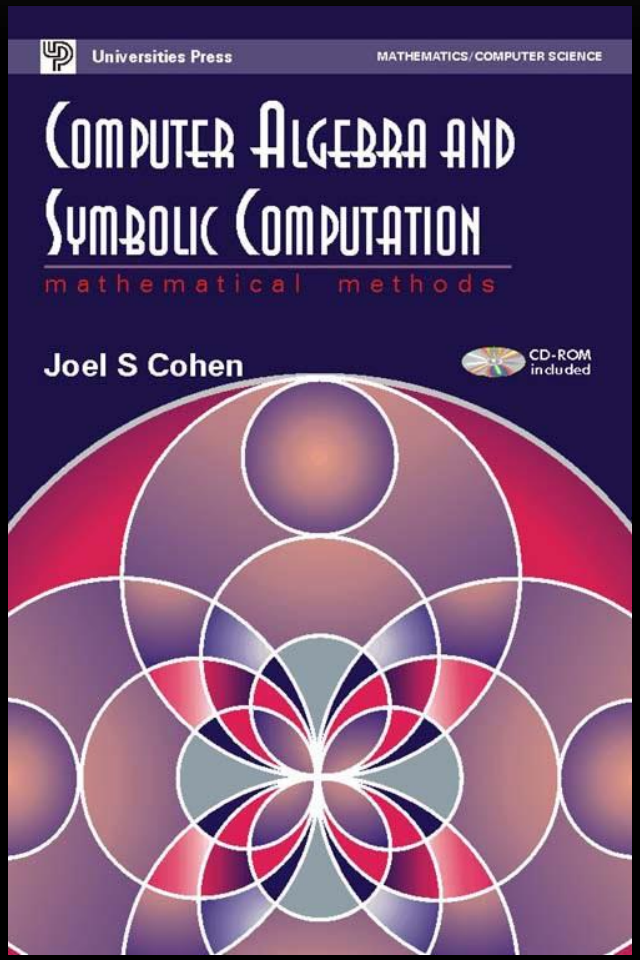
```
45  SiPrefixRules = {
46      Deci   -> 1 / 10,
47      Centi  -> 1 / 100,
48      Milli  -> 1 / 1000,
49      Micro  -> 1 / 1000000,
```

```
250 Convert[old_, new_] :=
251 (* Convert[old, new] = *)
252 (* Memoization trick: under investigation *)
253     (( (old / new)
254       //. PluralsRules
255       //. CommonParlanceRules
256       //. SiRules
257       //. SiPrefixRules ) * ( new //. PluralsRules //. CommonParlanceRules ))
```

http://bit.ly/SGwWuh

How to implement such an expression evaluator?

A bit of background reading, and...



Consider the following, *back-of-an-envelope*, calculation:

```
(9 - 5)^2 * (7 + 4) =
4^2  * (7 + 4) =
16 * (7 + 4) =
16 * 11 =
176
```

This is a perfect example of *term rewriting*: the initial expression `(9 - 5)^2 * (7 + 4)` is simplified

Many forms of simplification or symbolic manipulation can be expressed in this way. Recall the sim

```
(a + b)^2 = a^2 + 2ab + b^2
```

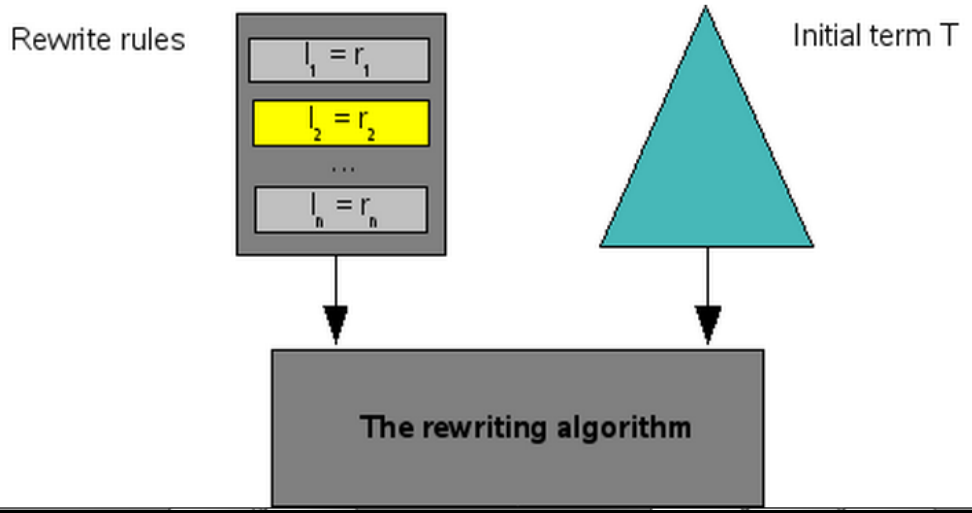
from high school algebra or the rule to calculate the derivative of the sum of two functions `u` and `v`:

```
d(u + v)/dx = du/dx + dv/dx
```

In both cases, there is a complex *left-hand side* that can be simplified into the expression appearing may differ and is explained below. Observe that some of the items on the left-hand side re-appear at

A simple view on term rewriting is shown in Figure 1.1, "The rewriting process". Given a set of re called the *normal form* of T. Later (in the section called "The term rewriting algorithm") we will fu

Figure 1.1. The rewriting process



MATHEMATICA GUIDE

The Mathematica specs are on line

Expressions

At the core of *Mathematica* is the foundational idea that every
formulas, graphics, documents—can be represented as symbol
unifying concept that underlies *Mathematica*'s symbolic progra
possible much of the unique power of the *Mathematica* langua

Expression Structure »
FullForm — the full form of an expression, without shortened
TreeForm ▪ **Head** ▪ **Length** ▪ **Depth** ▪ **Symbol** ▪ **...**

Transforming Expressions »
expr/.*rules* — make replacements for any occurrence of a patte

Applying Functions »
Map, **Apply** — map, apply a function at any level in any expre

Expression Testing »
SameQ (===) ▪ **FreeQ** ▪ **MemberQ** ▪ **NumberQ** ▪ **Orde**

Parts of Expressions »
Part (..[[..]]) — numbered parts of an expression, reset usin
Position ▪ **ReplacePart** ▪ **MapAt** ▪ **Delete** ▪ **...**
Cases — find occurrences of a pattern in an expression

Structural Operations »
Flatten ▪ **Thread** ▪ **Distribute** ▪ **FlattenAt** ▪ **Append** ▪

Controlling Expression Evaluation »
Hold ▪ **Evaluate** ▪ **HoldFirst** ▪ **HoldAll** ▪ **...**

Evaluation

The Standard Evaluation Sequence

The following is the sequence of steps that *Mathematica* follows in evaluating an expression like $h[e_1, e_2 \ldots]$. Every time the expression changes, *Mathematica* effectively starts the evaluation sequence over again.

- If the expression is a raw object (e.g., `Integer`, `String`, etc.), leave it unchanged.
- Evaluate the head h of the expression.
- Evaluate each element e_i of the expression in turn. If h is a symbol with attributes `HoldFirst`, `HoldRest`, `HoldAll`, or `HoldAllComplete`, then skip evaluation of certain elements.
- Unless h has attribute `HoldAllComplete`, strip the outermost of any `Unevaluated` wrappers that appear in the e_i.
- Unless h has attribute `SequenceHold`, flatten out all `Sequence` objects that appear among the e_i.
- If h has attribute `Flat`, then flatten out all nested expressions with head h.
- If h has attribute `Listable`, then thread through any e_i that are lists.
- If h has attribute `Orderless`, then sort the e_i into order.
- Unless h has attribute `HoldAllComplete`, use any applicable transformation rules associated with f that you have defined for objects of the form $h[f[e_1, \ldots], \ldots]$.
- Use any built-in transformation rules associated with f for objects of the form $h[f[e_1, \ldots], \ldots]$.
- Use any applicable transformation rules that you have defined for $h[f[e_1, e_2, \ldots], \ldots]$ or for $h[\ldots][\ldots]$.
- Use any built-in transformation rules for $h[e_1, e_2, \ldots]$ or for $h[\ldots][\ldots]$.

Nonstandard Argument Evaluation

There are a number of built-in *Mathematica* functions that evaluate their arguments in special ways. The control structure `While` is an example. The symbol `While` has the attribute `HoldAll`. As a result, the arguments of `While` are not evaluated as part of the standard evaluation process. Instead, the internal code for `While` evaluates the arguments in a special way. In the case of `While`, the code evaluates the

REPRESENTATION FIRST

A JSON representation for every kind of expression

There are only two kinds: Atoms and non-Atoms, a.k.a., Normal Forms
e.g., *headExpr* [*partExpr1*, *partExpr2*, ...]

```
engine.CreateAtom(42).toJSON()
{number: {subtype: "Integer", value: 42}}
```

```
engine.CreateAtom(3.14159).toJSON()
{number: {subtype: "Real", value: 3.14159}}
```

```
engine.CreateAtom('"myString"').toJSON()
{string: "\"myString\""}
```

```
engine.CreateAtom("mySymbol").toJSON()
{symbol: "mySymbol"}
```

```
engine.Evaluate("{\"a\", b, c -> 3}")
{head: "List",
 parts: [
   {string: "\"a\""},
   {symbol: "b"},
   {head: "Rule",
    parts: [
      {symbol: "c"},
      {number: {subtype: "Integer", value: 3}}
   ]}
]}
```

Three kinds of Atoms: Numbers, Strings, & Symbols. Numbers are Integers, Real, Complex, Rational (N.Y.I.)

Normal Forms are just recursive structures: explicit abstract syntax trees (ASTs)

Patterns

One of the unique strengths of *Mathematica*'s core language is its powerful and succinct—yet highly readable—symbolic pattern language. Convenient both for immediate use in individual functions, and f
like r

Basic
_ (B
x_ —
__ (
___
Comp
p|p|p
p.. (R
x:p (
Exce

FullForm

`FullForm[`*expr*`]`
prints as the full form of *expr*, with no special syntax.

MORE INFORMATION

EXAMPLES

Basic Examples (2)

`FullForm` of a typeset expression:

In[1]:=
$$\texttt{FullForm}\left[\frac{x}{\sqrt{5}} + y^2 + 1 / z\right]$$

Out[1]//FullForm=

```
Plus[Times[Power[5, Rational[-1, 2]], x], Power[y, 2], Power[z, -1]]
```

`FullForm` of a graphic:

ENGINEERING THE EVAL ENGINE

Bootstrap gradually

- "Never more than a few minutes away from something that works"
- Focus on scenario-prioritized functions
- Implementation flows from the representation shown

Don't be distracted by syntax

- Bootstrap with FullForm ASTs & JSON reps
- Use Jison to implement parser incrementally

Start with patterns and replacement

- Patterns, MatchQ, ReplaceList, ReplaceAll, Set, SetDelayed

Remoting is almost free

- Run exactly the same engine in Node.JS and in the browser
- Most of the challenge is on the Mathematica side
  - Quoting expressions is sometimes non-trivial

MatchQ

MatchQ[*expr*, *form*]
returns True

EXAMPLES

Basic Exampl

Test if an e

```
In[1]:= MatchQ[
Out[1]= True
```

.....

Test if an e

```
In[1]:= MatchQ[
Out[1]= False
```

```
In[2]:= MatchQ[
Out[2]= True
```

ReplaceList

ReplaceList[*expr*, *rules*]
attempts to transform the entire expression *expr* by applying a rule or list of rules in all possible ways, and returns a list of the results obtained.

ReplaceList[*expr*, *rules*, *n*]
gives a list of at most *n* results.

MORE INFORMATION

EXAMPLES

Basic Examples (1)

Give the results of all possible replacements:

```
In[1]:= ReplaceList[{a, b, c, d, e, f}, {x__, y__} → {{x}, {y}}]
Out[1]= {{{a}, {b, c, d, e, f}}, {{a, b}, {c, d, e, f}},
  {{a, b, c}, {d, e, f}}, {{a, b, c, d}, {e, f}}, {{a, b, c, d, e}, {f}}}
```

Give only the first replacement that applies:

```
In[2]:= Replace[{a, b, c, d, e, f}, {x__, y__} → {{x}, {y}}]
Out[2]= {{a}, {b, c, d, e, f}}
```

REMOTING

Move Expressions to the Data

- Flexible expressions means "elastic platform" – scenarios need not be pre-canned
- Evaluator is a sandbox – not necessary to host all of JavaScript
- AFFINITY and PRIVACY are the drivers for remoting
- e.g., Evaluate the NFL vector computation on a server with the ingredients
- e.g., Evaluate business offers on the client device without moving private data

CONCLUSIONS

Term-Rewriting is an underutilized but generally useful and powerful computing strategy

- Robust Units-of-Measure is a huge risk-reducer
- Concise patterns-and-rules specs is a cost-reducer
- Remotable Expressions are a new kind of data resource

It is surprisingly easy to implement in standard web-computing settings

Established tools & practice (e.g., Mathematica) substantially increase attractiveness of the approach

ACKNOWLEDGEMENTS

ESSENTIAL CONTRIBUTORS

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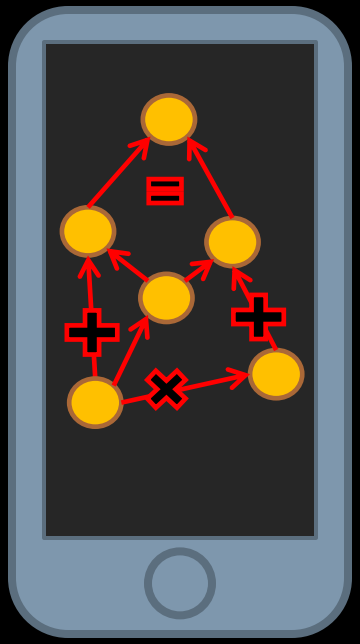
SPONSORS

IPE Core Platform

Bimal Mehta, James Whittaker, Gurdeep Singh Pall, Qi Lu

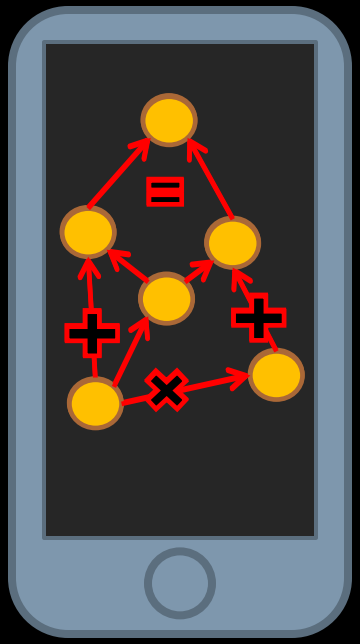
OVERFLOW SLIDES

REMOTING PROBLEM



Moving data to the computation is not the best use of $data plan$

REMOTING SOLUTION



Move the computation to the data

JaqSON

Remoting format for Jacquard expressions

MORE ONLINE SCENARIOS

- *Get me to the airport on time*
  - Reactively monitor traffic, flight info, current location
- *Average age of singers? Maximum salary of CEOs?*
  - Map-reduce style; statistics on-the-fly
- *Help me by a car*
  - Distributed workflow with privacy concerns