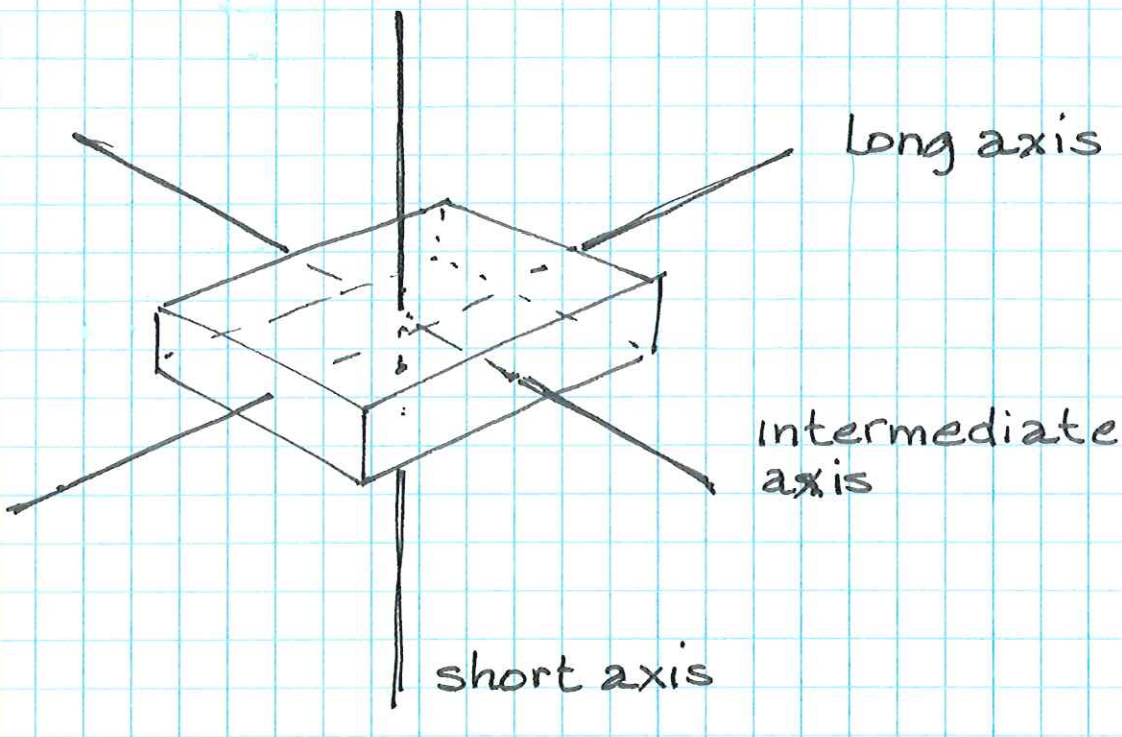
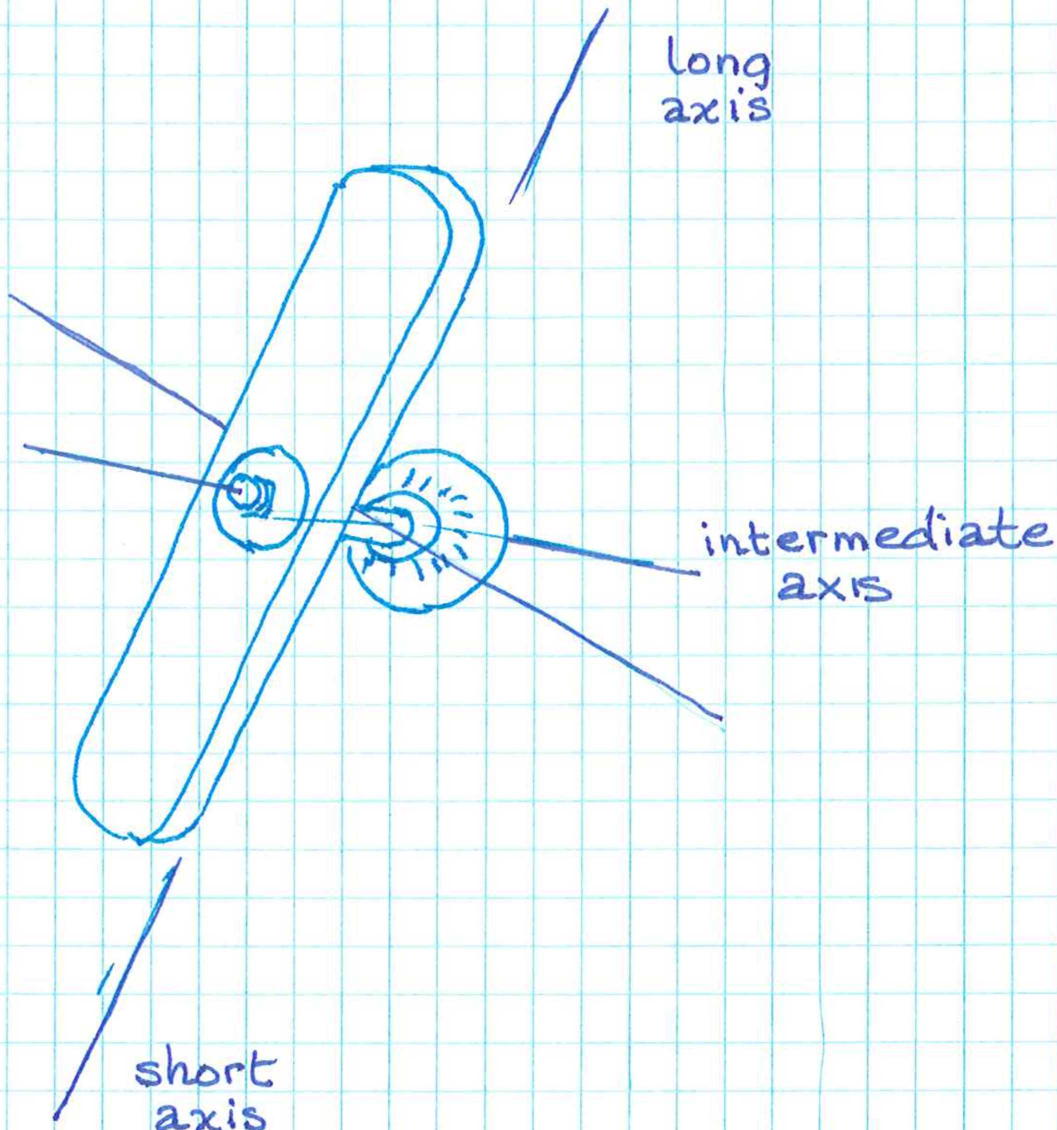
DZHANYBEKHOV EFFECT

Why do we care?

- Unit-testing dynamics S.W.
- Hypothetical unstable flight modes (Anecdotally, satellites have been lost while "coning" & looking for signals from Earth).


long axis
intermediate axis
short axis


long
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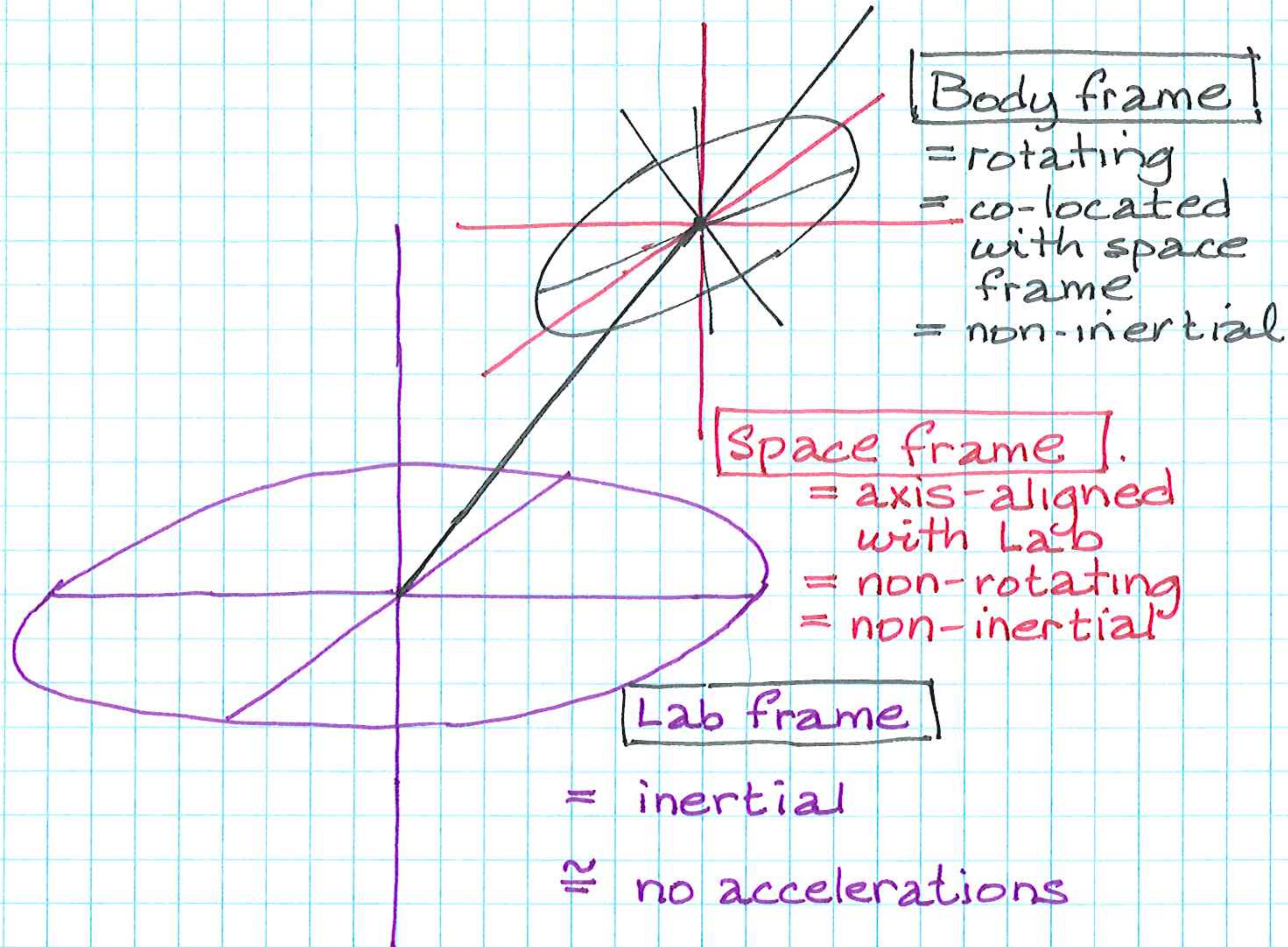
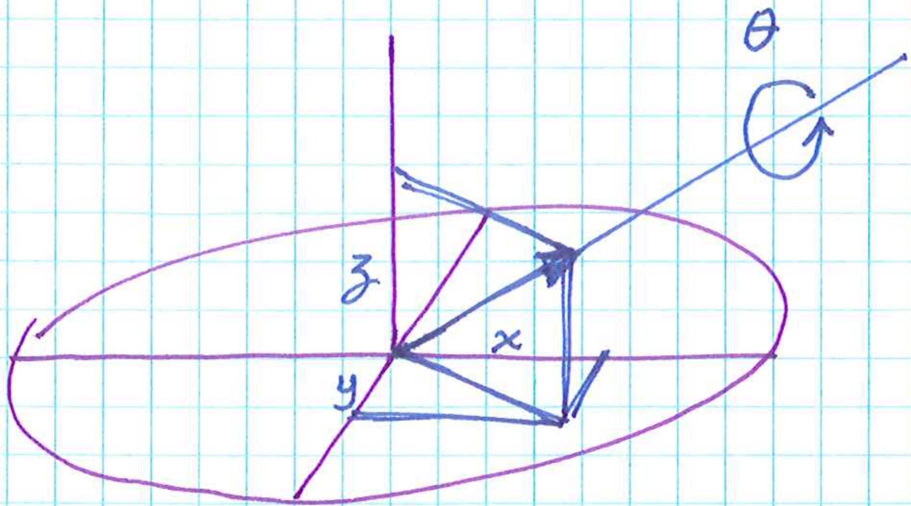


Translational Motion decouples from Rotational Motion



$$\text{Quaternion}\left[\cos\frac{\theta}{2},\ \sin\frac{\theta}{2}\,\frac{\{x,y,z\}}{\sqrt{x^2+y^2+z^2}}\right]$$

All orientation information is contained in a quaternion

$$\underbrace{\left(\frac{d\vec{A}(t)}{dt}\right)_{space}}_{\text{do likewise in the space frame}} = \underbrace{\left(\frac{d\vec{A}}{dt}\right)_{body}}_{\text{decompose the vector into components in the body frame; compute derivatives; reassemble into a vector}} + \vec{\omega} \times \vec{A}(t)$$

$$\underbrace{\left(\frac{d\vec{L}}{dt}\right)_{space}}_{\text{zero in free rotation (no torques)}} = \left(\frac{d\vec{L}}{dt}\right)_{body} + \vec{\omega} \times \vec{L}(t)$$

LEMMA

$$\dot{q} = \frac{1}{2} q \vec{\omega}$$

$$\vec{r}_{space} = q \, \vec{r}_{body} \, \tilde{q}$$

$$\dot{\vec{r}}_{space} = \dot{q} \, \vec{r}_{body} \, \tilde{q} + q \, \vec{r}_{body} \, \dot{\tilde{q}}$$ because $\dot{r}_{body} = 0$

$$= \vec{\omega} \times \vec{r} \; ?$$

$$= \frac{1}{2}(q\vec{\omega})\vec{r}_{body}\tilde{q} + q\vec{r}_{body}\frac{1}{2}(\vec{\omega}\tilde{q})$$

because $\tilde{v} = v$ for any vector

$$= q\Big(\underbrace{\frac{1}{2}(\vec{\omega}\vec{r}_{body} + \vec{r}_{body}\vec{\omega})}\Big)\tilde{q}$$

$$= q(\vec{\omega} \times \vec{r}_{body})\tilde{q}$$ quaternion fact

$$= \vec{\omega} \times \vec{r}_{space}$$

$$\vec{L} = \overset{\rightrightarrows}{I}\,\vec{\omega}$$

$$\dot{\vec{L}} = -\vec{\omega} \times \vec{L} = \vec{L} \times \vec{\omega}$$

$$= \overset{\rightrightarrows}{I}\,\vec{\omega} \times \vec{\omega}$$

const. in body frame

$$= \overset{\rightrightarrows}{I}\,\dot{\vec{\omega}} = \overset{\rightrightarrows}{I}\,\vec{\omega} \times \vec{\omega}$$

$$\dot{\vec{\omega}} = \left(\overset{\rightrightarrows}{I}\right)^{-1}\left(\left(\overset{\rightrightarrows}{I}\,\vec{\omega}\right) \times \vec{\omega}\right)$$