MODULE *Euclid*

EXTENDS *Integers*, *GCD*, *TLC*

CONSTANTS M, N

ASSUME $\{M, N\} \subseteq Nat \setminus \{0\}$ *subset* doesn't seem to work

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****************************************************************************
--fair algorithm Euclid
  {  variables x ∈ 1 .. N, y ∈ 1 .. N, x0 = x, y0 = y ;
    { while_step: while ( x ≠ y )
        The Greek word for the following process of repeated subtraction
        is antanaresis.
      { if ( x < y )
          { antanaresis_x: y := y − x ;  }
          else
          { antanaresis_y: x := x − y ;  }
       } ;
      assert (x = y) ∧ (x = GCD(x0, y0)) ;
   }  }
****************************************************************************
```

BEGIN TRANSLATION

VARIABLES x, y, $x0$, $y0$, pc

$vars \triangleq \langle x, y, x0, y0, pc\rangle$

$Init \triangleq$ Global variables
$\land x \in 1 \,..\, N$
$\land y \in 1 \,..\, N$
$\land x0 = x$
$\land y0 = y$
$\land pc =$ "while_step"

$while\_step \triangleq \land pc =$ "while_step"
$\land$ IF $x \neq y$
THEN $\land$ IF $x < y$
THEN $\land pc' =$ "antanaresis_x"
ELSE $\land pc' =$ "antanaresis_y"
ELSE $\land Assert((x = y) \land (x = GCD(x0, y0)),$
"Failure of assertion at line 19, column 7.")
$\land pc' =$ "Done"
$\land$ UNCHANGED $\langle x, y, x0, y0\rangle$

$antanaresis\_x \triangleq \land pc =$ "antanaresis_x"
$\land y' = y - x$
$\land pc' =$ "while_step"

$$\wedge \text{ UNCHANGED } \langle x,\ x0,\ y0\rangle$$

$$\begin{aligned} \mathit{antanaresis\_y} \triangleq\ & \wedge\ pc = \text{“antanaresis\_y”} \\ & \wedge\ x' = x - y \\ & \wedge\ pc' = \text{“while\_step”} \\ & \wedge \text{ UNCHANGED } \langle y,\ x0,\ y0\rangle \end{aligned}$$

$$\begin{aligned} \mathit{Next} \triangleq\ & \mathit{while\_step} \vee \mathit{antanaresis\_x} \vee \mathit{antanaresis\_y} \\ & \vee \text{ Disjunct to prevent deadlock on termination} \\ & \quad (pc = \text{“Done”} \wedge \text{ UNCHANGED } \mathit{vars}) \end{aligned}$$

$$\begin{aligned} \mathit{Spec} \triangleq\ & \wedge\ \mathit{Init} \wedge \Box[\mathit{Next}]_{\mathit{vars}} \\ & \wedge\ \mathrm{WF}_{\mathit{vars}}(\mathit{Next}) \end{aligned}$$

$$\mathit{Termination} \triangleq \Diamond(pc = \text{“Done”})$$

END TRANSLATION

$$\mathit{PartialCorrectness} \triangleq (pc = \text{“Done”}) \Rightarrow (x = y) \wedge (x = \mathit{GCD}(x0,\ y0))$$

* Modification History
* Last modified Sun *Feb* 16 10:12:36 *PST* 2014 by *bbeckman*
* Created *Fri Feb* 14 15:53:12 *PST* 2014 by *bbeckman*