Reinvestable Principal Guaranteed Note

HSBC Business School

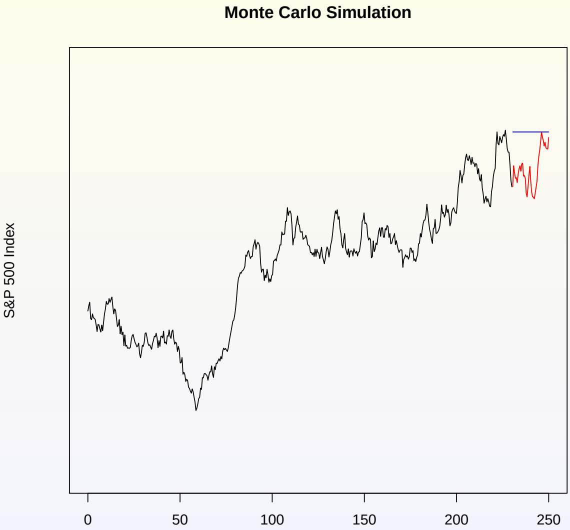
First Intuition

1. Economic Background
   - With low interest rate, we are seeking to develop an alternative investment opportunity.
2. Product Feature
   - Principal guaranteed;
   - Track the performance of the market portfolio;
   - Eliminate liquidity preference;
   - Double win (bank and investors);
   - ‘Bermudan’ style

Product Description

1. Initial investment: I;
2. Maturity time: $\{T,\ 2T,\ 3T\}$;
   Our product gives investors the option to continue or terminate the contract. Hence, the maturity is up to the investors' decision.
3. Decision time: $\{t^*,\ \ T+t^*\}$;
4. Underlying Asset: S&P 500 Index;
5. Participation rate: π;
6. Payoff at T: $I+\pi\times max\{\overline{S}_{t^*}^{T}-K,\ 0\}$.
   This is the payoff given that investor decided to continue the contract; otherwise, the payoff should be $I+\pi\times max\{S_T-K,\ 0\}$

Product Description



Structuring the Product

- The issuer could use a *call on call*, or *call*^{2} to structure this product;
- From the bank side, it needs a lookback call option which starts at t^*, and matures at T, with strike S_0, and most importantly, *priced as if* $S_{t^*} = S_0$;
- Define

$$K^{lookback} \equiv Call_{t^*}^{lookback}(S_{t^*} = S_0, K = S_0, \tau = T - t^*)$$

- The 'outer' would be an European call option, with strike $K^{lookback}$, maturity of t^*, and the underlying is the above mentioned lookback option.

Participation Rate

- Denote by π the participation rate;
- The money allocated to the options should be $B = I \times \{1 - e^{-rT}\}$;
- Considering the premium paid for $call^2$ at $t = 0$ and the strike price paid at $t = t^*$, the participation rate should satisfy

$$e^{rt^*}\left[B - \pi Call_0^2(K^{lookback})\right] = \pi K^{lookback}, \quad S_{t^*} \geq S_0.$$

$$\pi \equiv \frac{Be^{rt^*}}{K^{lookback} + e^{rt^*}Call_0^2(K^{lookback})}$$

Participation Rate

- If the realized $S_{t^*} > S_0$, i.e., the lookback option becomes more expensive than predicted, the bank could just exercise the *call*[2];

- On the other hand, if $S_{t^*} < S_0$, bank does not need to spend all the reserved money on buying the lookback options, which means bank could gain profit

$$\pi \times \left(K^{lookback} - Call_{t^*}^{lookback}(S_{t^*}, K = S_0, \tau = T - t^*) \right),$$

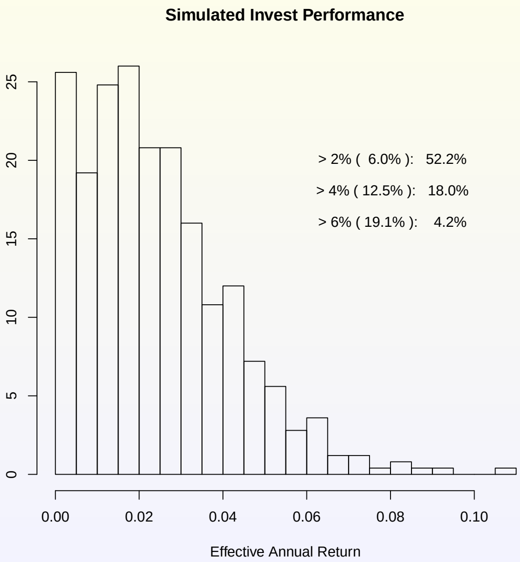
while still maintains the original participation rate for the investor.

Performance: Simulation

- We are seeking to evaluate the performance of this product;
- In simulation, we made several assumptions to simplify:
  1. Investor may always choose to continue the contract;
  2. Bank maintains no profit.
- The parameters used are

$$\begin{aligned} r &: \quad 2\% \\ \mu &: \quad 8\% \\ \sigma &: \quad 25\% \end{aligned}$$

Simulated Performance



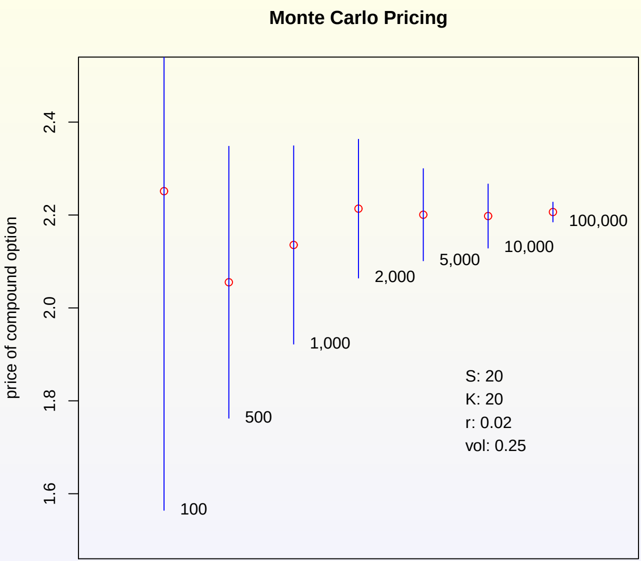
Option Pricing

- We need to price two options: *fixed strike lookback option* and *call on lookback*;
- Under the Black-Scholes framework, $dS_t = rS_t dt + \sigma S_t dW_t$, there is explicit formula for the lookback option. *Conze and Viswanathan (1991)* gives

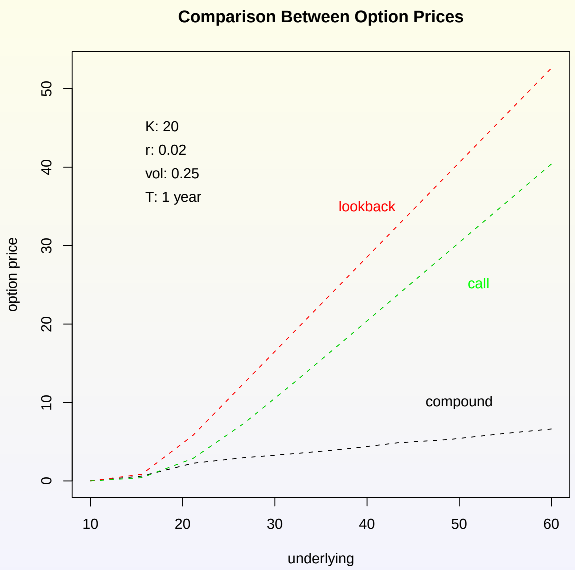
$$
\begin{aligned}
& (M-K)e^{-r(T-t)} + Se^{-q(T-t)}N(d) - Me^{-r(T-t)}N(d-\sigma\sqrt{T-t}) \\
+ \quad & Se^{-r(T-t)}\frac{\sigma^2}{2(r-q)}\Big\{ -\Big[\frac{S}{M}\Big]^{-\frac{2(r-q)}{\sigma^2}} N\Big[d - \frac{2(r-q)\sqrt{T-t}}{\sigma}\Big] + e^{(r-q)(T-t)}N\Big[d\Big]\Big\}
\end{aligned}
$$

$$
d = \frac{ln(S/M) + (r-q+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \quad M = S \vee K
$$

- For the *Call*^{2} option, we use Monte Carlo method to price.



Monte Carlo Pricing
price of compound option
2.4
2.2
2.0
1.8
1.6
100
500
1,000
2,000
5,000
10,000
100,000
S: 20
K: 20
r: 0.02
vol: 0.25


Comparison Between Option Prices
K: 20
r: 0.02
vol: 0.25
T: 1 year
lookback
call
compound
option price
underlying
0
10
20
30
40
50
60

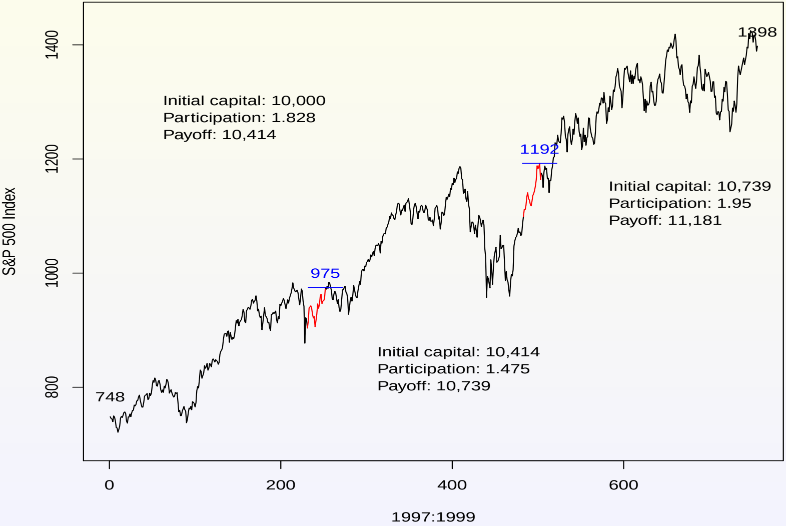
Historical Data Application

- We examine the S&P 500 Index data;
- For the investor, the payoff should be
  - If continues to invest in the next period

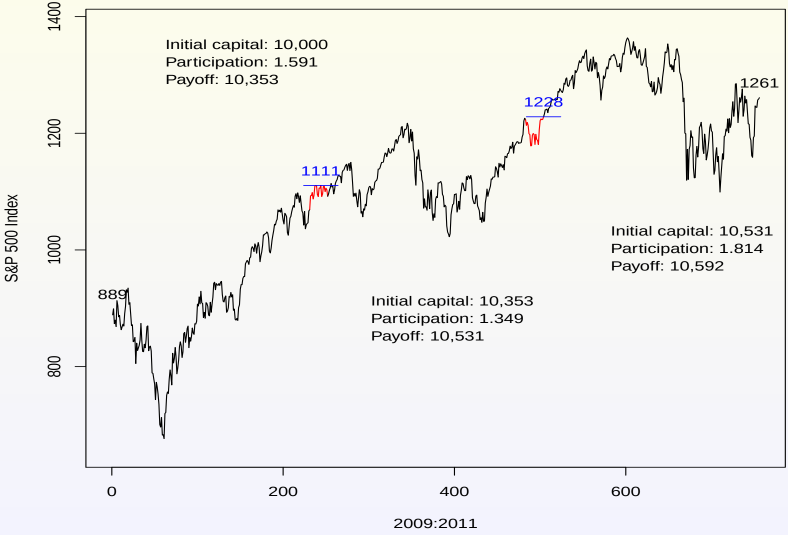
$$I + \pi \times max\{\overline{S}_{t^*}^{T} - S_0,\ 0\};$$

  - Else, the payoff should be

$$I + \pi \times max\{S_T - S_0,\ 0\}.$$



S&P 500 Index
1997:1999
800
1000
1200
1400
0
200
400
600
748
975
1192
1398
Initial capital: 10,000
Participation: 1.828
Payoff: 10,414
Initial capital: 10,414
Participation: 1.475
Payoff: 10,739
Initial capital: 10,739
Participation: 1.95
Payoff: 11,181



Initial capital: 10,000
Participation: 1.591
Payoff: 10,353
Initial capital: 10,353
Participation: 1.349
Payoff: 10,531
Initial capital: 10,531
Participation: 1.814
Payoff: 10,592
889
1111
1228
1261
S&P 500 Index
1400
1200
1000
800
0
200
400
600
2009:2011

Hedging

- Existing literatures suggest several methods to *hedge a call on European call*, which we will mention here;

- Dynamic hedging:
  Theoretically, traditional *Delta Hedge* could be used to hedge a compound option. *Davis, et al. (2000)* has examined the performance of delta hedge, which is satisfactory.

- Static hedging:
  Thomas, (1999) has proposed static hedge. The issuer of the compound option could buy a European call option with maturity T and strike $K + e^{r(T-t_1)}p_1$, where K is the strike price of the 'inner' option, p_1 and t_1 are the strike price and maturity of the 'outer' option.

Limitations

- Difficult to hedge
  It might be difficult to buy 'call on call' in the market. Though we believe the hedging of a call on lookback should be somehow similar to that of a call on European, the implementation might not be trivial;

- Investor behaviour
  In reality, bank faces large amount of investors, which means bank needs to estimate the percentage of investors who might be willing to reinvest;

- Bank revenue
  Bank's revenue relies on the stock market. In worst case, the bank may not earn any profit.

Thanks!

Reference

- Conze, Viswanathan. 1991. Path Dependent Options: The Case of Lookback Options. *Journal of Finance*, 46, 1893-1907.
- B. Thomas. 1999. Exotic Options II. In Carol Alexander (editor): *The Handbook of Risk Management and Analysis*. John Wiley & Sons, Chichester, England.
- Davis, Schachermayer, Tompkins. 2000. Pricing, No-arbitrage Bounds and Robust Hedging of Installment Options. Working paper.
- S. G. Kou. 2002. A Jump-Diffusion Model for Option Pricing. *Management Science*, 48, 1086-1101.

Jump Model

- *Kou (2002)* proposed a model in which underlying follows a Brownian motion plus a compound Poisson process with jump sizes double exponentially distributed.
- Such a model could capture *the dividend effect* and *close-open price jump*;
- Also, this model may introduce fat tail, which is realistic.

$$dS_t = rS_{t-}dt + \sigma S_{t-}dW_t + d\Big(\prod_{i=1}^{N(t)}(V_i - 1)\Big),$$

or

$$S_t = S_0 exp\Big((r - \frac{1}{2}\sigma^2)t + \sigma W_t\Big)\prod_{i=1}^{N(t)} V_i$$

where $N(t)$ is a Poisson process, and $ln(V_i)$ follows exponential distributions (up or down jump).

Static Hedging of Compound Option

For a call on European call, *Davis, et al. (2000)* proved

$$C(t, T, K + e^{r(T-t_1)}p_1) > C(t, T, K) - e^{-r(t_1-t)}p_1 + P(t, t_1, p_1)$$

where t_1 and T are the expiration times of the compound call and European call options; p_1 and K are the strike prices of the compound call and European call options; and $P(\cdot)$ is the price of a European put option.

If the compound option expires at t_1 in-the-money, we have

$$C(t_1, T, K + e^{r(T-t_1)}p_1) > C(t_1, T, K) - p_1$$

which means the issuer could sell the European call with strike $K + e^{r(T-t_1)}p_1$ and the proceedings are enough to cover the money paid to the counter party, $C(t_1, T, K) - p_1$.