Reinvestable Principal Guaranteed Note

1 Product Description

We are seeking to develop an alternative investment opportunity for those who want to track performance of the market portfolio. Besides, since Chinese financial market is not a completely opened market, another target is to offer investor an opportunity to invest in foreign market without too many restrictions.

1.1 Product description

We introduce the **RPG Note** from several perspectives.

1. Currency and price of a unit

2. Time to maturity
   We sell the product once upon a year. The longest maturity of the product is set to be 3 years. Our product gives investors the option to continue or terminate the contract. Hence, the maturity is up to the investors' decision. The decision point is set to be 1 month before the termination time, which is 11 months and 23 months $(t^*, t^* + T)$ after the initial investment time point.

3. Underlying asset
   In our analysis, we use S&P 500 Index as an example.

4. Payoff
   We denote participation rate by π. At time T, if the investor decides to continue the contract, he or she will get payoff $I + \pi \times max\{\overline{S}_{t^*}^T - K,\ 0\}$. $\overline{S}_{t^*}^T$ is the highest stock price in the observation period. Otherwise the payoff should be $I + \pi \times max\{S_T - K,\ 0\}$, paid at time T[1].

[1] We can also create an product which pays 'opposite' to our original product, $I + \pi \times max\{K - \underline{S}_{t^*}^T,\ 0\}$, the structure and hedging are similar.

5. Management fee

1.2 Product features

Here is a brief summary of our product, without any reference to technical terms.

1. Principal guaranteed

2. Track the performance of the market portfolio

3. Compensate for liquidity preference
   Since the payoff of the product will be higher if the investor decides to continue the contract, the investor has incentive to invest for a longer time.

4. 'Bermudan' style
   Our product has predetermined time points upon which you can withdraw money and get payoff contingent on the stock index. This is somewhat like Bermudan option.

2 Structure the Product

In this section, we introduce the structuring of this product, as well as how the issuer could determine the participation rate, which is crucial for a capital guaranteed product.

2.1 Capital Allocation: Fixed Income Security

We invest $I \times e^{-rT}$ in Chinese government bond.

2.2 Capital Allocation: Options

We suppose a investor who will always choose to continue the contract. But in real world, investors have their own demand and expectation, so they may have different choices. To structure a continued contract, we need call on call option.

From the bank side, at decision time it needs a fixed strike lookback call option which matures at T, with strike S_0, and most importantly, priced as if $S_{t^*} = S_0$. A fixed strike lookback call pays $max\{\overline{S}_{t^*}^T - K,\ 0\}$, $\overline{S}_{t^*}^T$ is the maximum stock price during time period $t^*\ to\ T$. We define

$$K^{lookback} \equiv LB(S_{t^*} = S_0, K = S_0, \tau = T - t^*),$$

which is the call price of the lookback call option with staring sock price S_0.

Then we construct a call on the lookback call. The 'outer' of call on call would be an European call option, with strike $K^{lookback}$, maturity of t^*, and the underlying is the lookback option.

If we know in advance that a certain proportion of the investors will terminate the contract in the following year. Then we just need to invest in the Chinese government bond and a call option on S&P 500 Index. Since the participation rate is calculated by using lookback option and call on call, which will be more expensive than European call, the payoff can totally be covered by the bonds and options we buy.

2.3 Participation Rate and Issuer's Profit

Denote by π the participation rate, which should satisfy

$$e^{rt^*}\left[B - \pi Call_0^2(K^{lookback})\right] = \pi K^{lookback}, \quad S_{t^*} \geq S_0,$$

and hence is given by

$$\pi \equiv \frac{Be^{rt^*}}{K^{lookback} + e^{rt^*}Call_0^2(K^{lookback})}.$$

3 Risk

For the issuer, as long as the portfolio is managed dynamically, even leveraging the position may not introduce too much risk. The only possible risk for the issuer could be the exchange rate risk. In our product, once the contract has been initialized, the cash flow pattern is quite stable, which implies the issuer could use a typical currency swap to hedge away all the exchange rate risk.

4 Option Pricing in Other Models

Though we used Black-Scholes-Merton framework in our analysis, there are other well-established models for the underlying asset evolution.

The first candidate is the jump model proposed by *Kou (2002)*. In this model, the underlying moves as

$$dS_t = rS_{t-}dt + \sigma S_{t-}dW_t + d\Big(\prod_{i=1}^{N(t)}(V_i - 1)\Big),$$

where $N(t)$ is a Poisson process, and $ln(V_i)$ follows exponential distributions (up or down jump). One advantage of this jump model is that there is explicit formula for the underlying movement, hence no need to approximate the SDE by numerical method.

Another widely used model for the underlying evolution is proposed by *Heston (1993)*. In this model,

$$dS_t = rS_t dt + \sigma_t S_t dW_t^1,$$

and

$$d\sigma_t = k(\lambda - \sigma_t)dt + \beta\sqrt{\sigma_t}dW_t^2$$

where λ is the long-run mean value of volatility, k is the speed of convergence, and $W_t^i,\ i = 1, 2$ are Wiener Processes with correlation $-1 \leq \rho \leq 1$.

5 Hedging and Further Issues

A typical compound involves three time points and two contracts. t_0 is the initiation of the compound option; t_1 is the expiration of the compound option, and $T > t_1$ is the maturity of the underling option.

Existing literatures suggest several methods to hedge a compound option, one of them is the delta hedge. *Davis, et al. (2000)* has examined the performance of delta hedge in several settings, including transaction costs or stochastic volatility. Generally speaking, the performance of delta hedge is satisfactory.

Another hedging strategy is static hedge. Assuming the underlying of the compound option is an European call with strike K and predetermined p_1 paid at t_1, is the compound is exercised. *Thomas, (1999)* has proposed static hedge. The issuer of the compound option could buy a European call option with maturity T and strike $K + e^{r(T-t_1)}p_1$. Such a hedging strategy is supported by Davis, et al. (2000), in which the authors proved

$$C(t, T, K + e^{r(T-t_1)}p_1) > C(t, T, K) - e^{-r(t_1-t)}p_1 + P(t, t_1, p_1)$$

where t_1 and T are the expiration times of the compound call and European call options; and $P(\cdot)$ is the price of a European put option.

If the compound option expires at t_1 in-the-money, we have

$$C(t_1, T, K + e^{r(T-t_1)}p_1) > C(t_1, T, K) - p_1$$

which means the issuer could sell the European call with strike $K + e^{r(T-t_1)}p_1$ and the proceedings are enough to cover the money paid to the counter party, $C(t_1, T, K) - p_1$.

There are two remaining issues, though important, beyond the scope of this document. One is the investor behavior, and the other is the issuer's revenue.

Since the **RPG Note** has provided the investors the option to continue/terminate the contract, a key issue for the manager (portfolio management) is to estimate the investor's behavior. With different expectation of the investor's behavior, the portfolio management might be quite different.

Another issue is the issuer's revenue. In this stage, however, we think it is too early to consider this issue, hence ignored in the body of this document.

6 Reference

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