Characterizing the time dependence of source intensity using the JCMT

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August 29th, 2018

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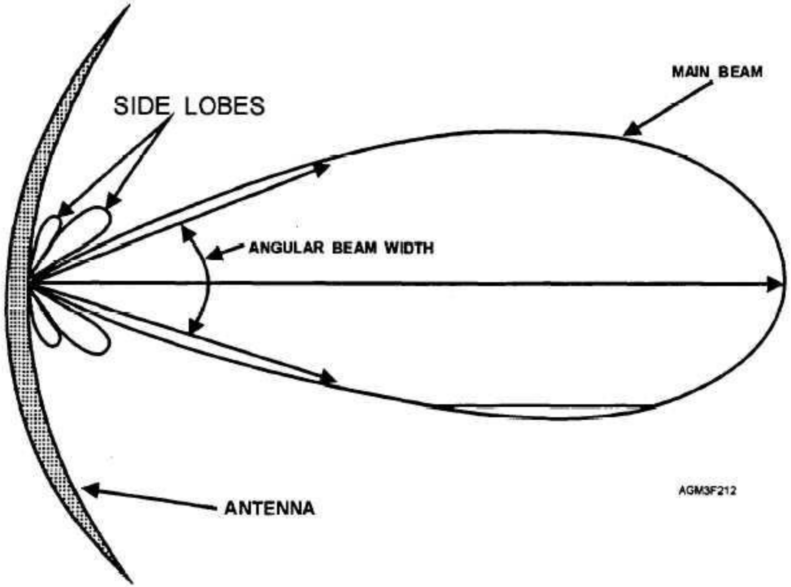
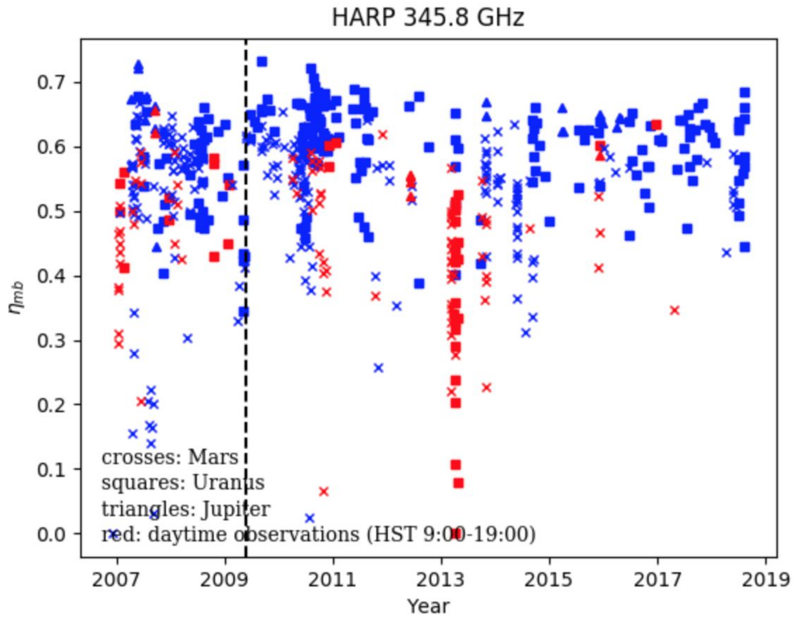
Outline

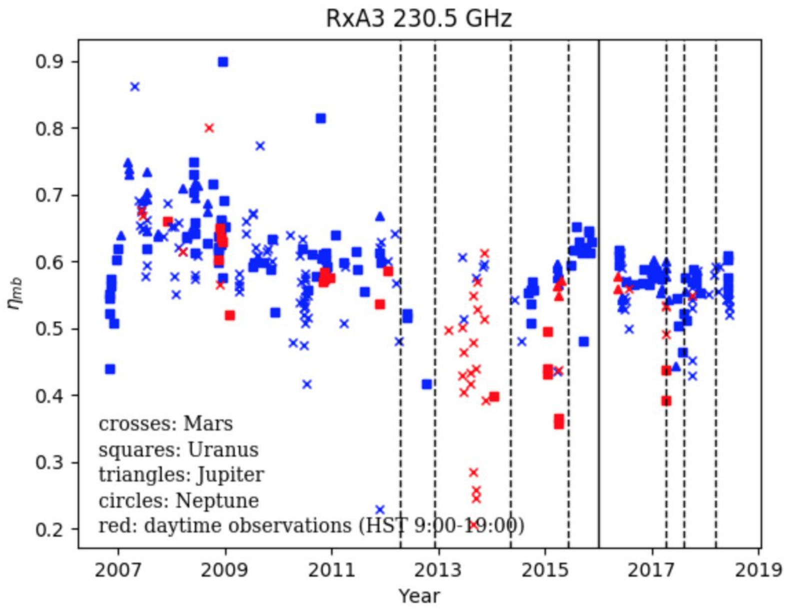
1. Motivation and Background
2. Project Goal
3. Bayesian Inference and MCMC
4. Constructing a Likelihood
5. Forward Modeling
6. Summary
7. Future Work

JCMT Observing

- James Clerk Maxwell Telescope
- NESS (Nearby Evolved Stars Survey)
  - Long term goals to investigate physical mechanisms such as AGB mass loss and dust formation
- Main beam efficiency
  - Fraction of power in main beam







Plots of main beam efficiency

Project Goal

- Fit a function to characterize the time dependence of main beam efficiency
  - Eventually integrate into STARLINK for NESS and JCMT community
- I.e. input a date, get out a value and the uncertainty to propagate

Bayesian Inference

- An approach to probabilistic problems
- Uses Bayes' Theorem to incorporate new or already known information into computing probabilities
- Instead of "given a model, how likely is it to observe my data" → "given my data (which I've already observed), how likely is this model?"

Bayes' Theorem

Probability of our collected data given the model is true (**Likelihood**)

Probability of model being true before collecting data (**Prior**)

(represented by parameters θ)

(y)

$$P(model \mid data) = \frac{P(data \mid model)\, P(model)}{P(data)}$$

Probability of the model given our collected data is true (**Posterior**)

Probability of collecting the data (Evidence)

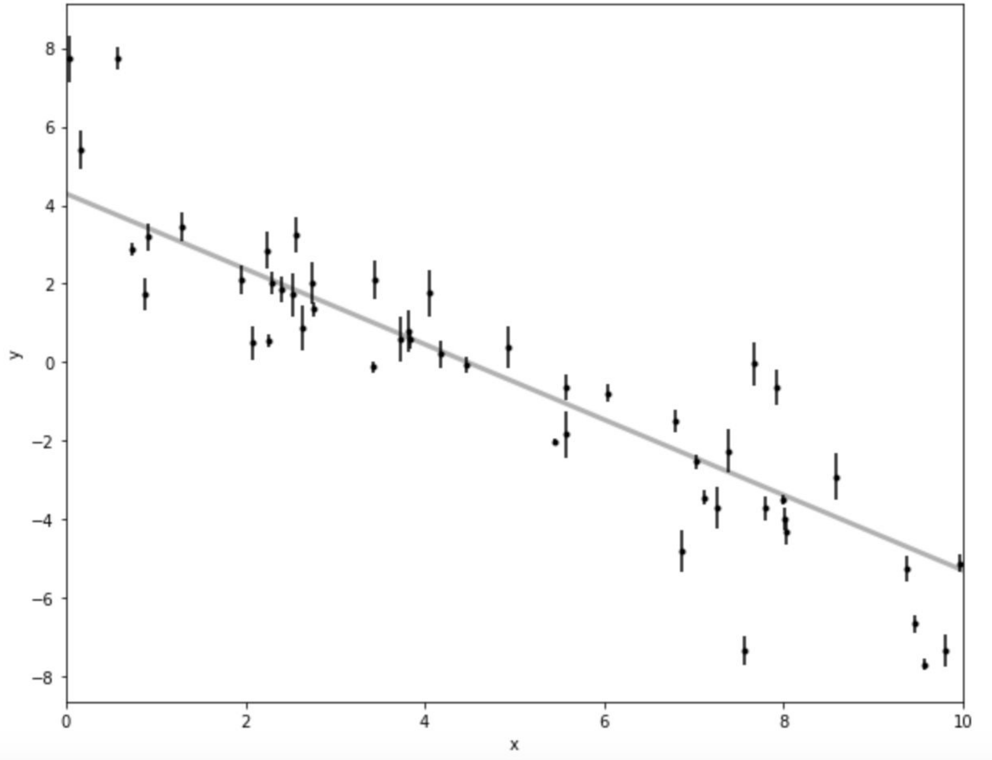
MCMC

- Markov Chain Monte Carlo
- *Emcee* : Python implementation of an MCMC Ensemble sampler (Foreman-Mackey et al. 2012)
- How it works:
    - Set of “walkers” step through the parameter space, accepting or rejecting samples
    - End up with a set of samples for each parameter

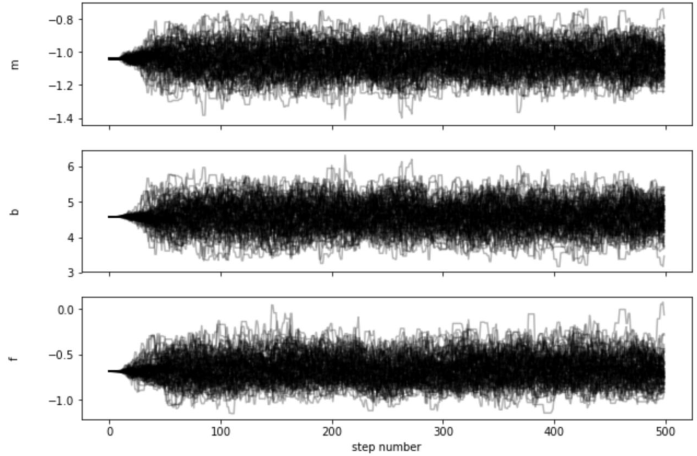
Quick Review and Example

1. Write up our likelihood
2. Write up our priors
3. Obtain our posterior
4. Sample from the posterior using MCMC

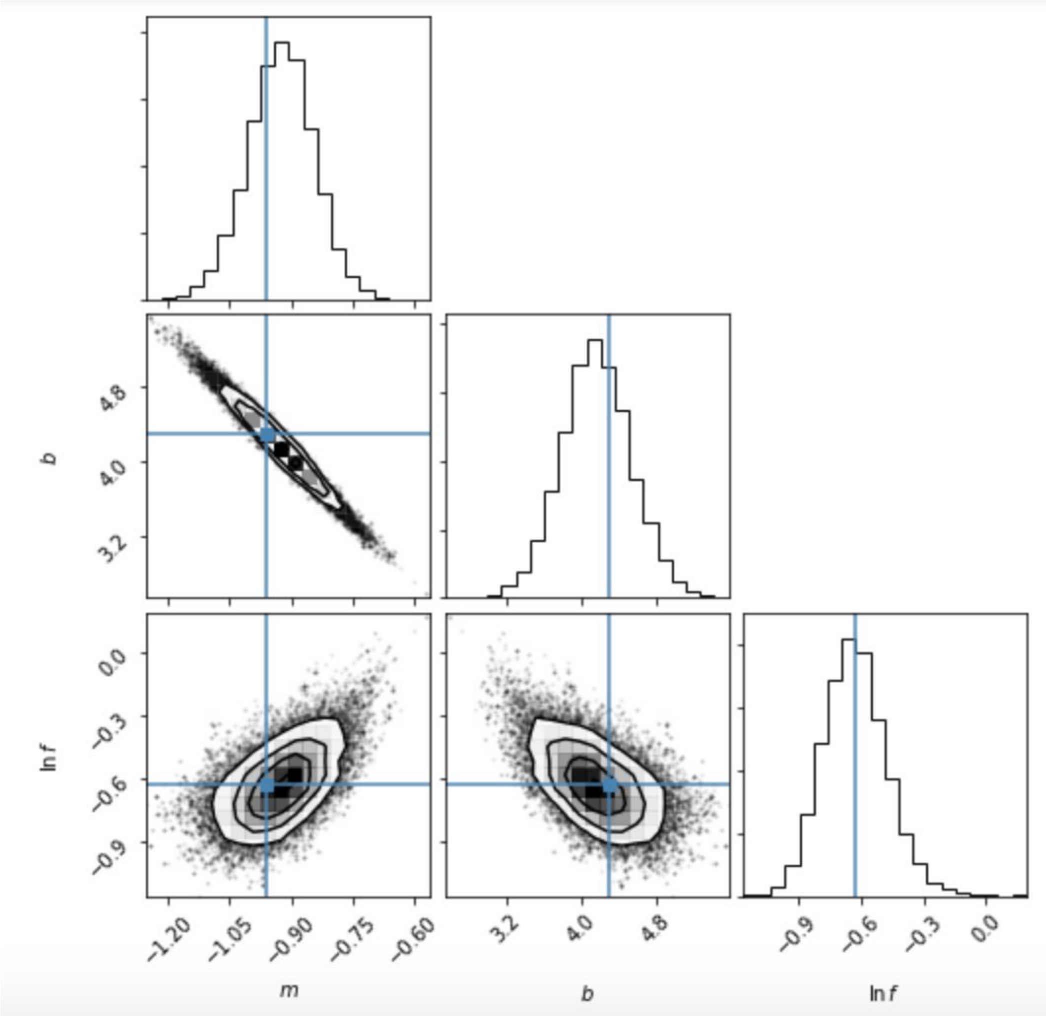
$$Posterior = \frac{Likelihood \times Prior}{Evidence}$$



- Example from emcee documentation
- Starting point for project
- Generate data from mx+b with underestimated uncertainty f

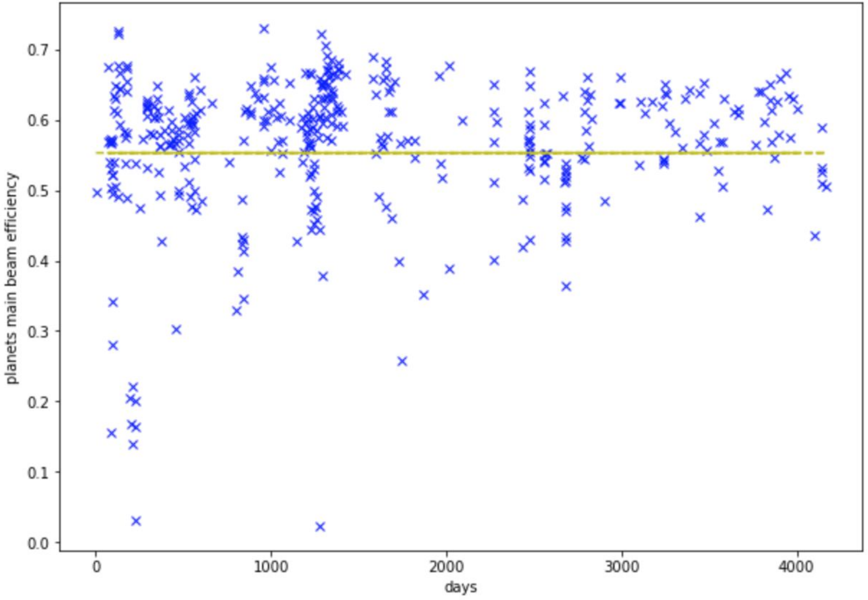


- Contour plot
- Visual display of sampling results



Constructing the Likelihood

- Want to account for uncertainty, scatter, and bias/systematic error
- Started with a simple linear model, *mx* + *b*, with a 10% uncertainty
  - Resulting b is similar to the least squares fit



2D Uncertainty and Scatter

- Hogg, Bovy, and Lang (2010)
- Begin with points lying on a narrow linear relation, with a Gaussian offset
- Introduce intrinsic Gaussian variance V, which is orthogonal to the line
  - Likelihood becomes convolution of 2D uncertainty Gaussian with V

$$\ln \mathscr{L} = K - \sum_{i=1}^{N} \frac{1}{2} \ln(\Sigma_i^2 + V) - \sum_{i=1}^{N} \frac{\Delta_i^2}{2\,[\Sigma_i^2 + V]}$$

$$\Delta_i = \hat{\boldsymbol{v}}^\top \boldsymbol{Z}_i - b\,\cos\theta$$

$$\Sigma_i^2 = \hat{\boldsymbol{v}}^\top \boldsymbol{S}_i\,\hat{\boldsymbol{v}}$$

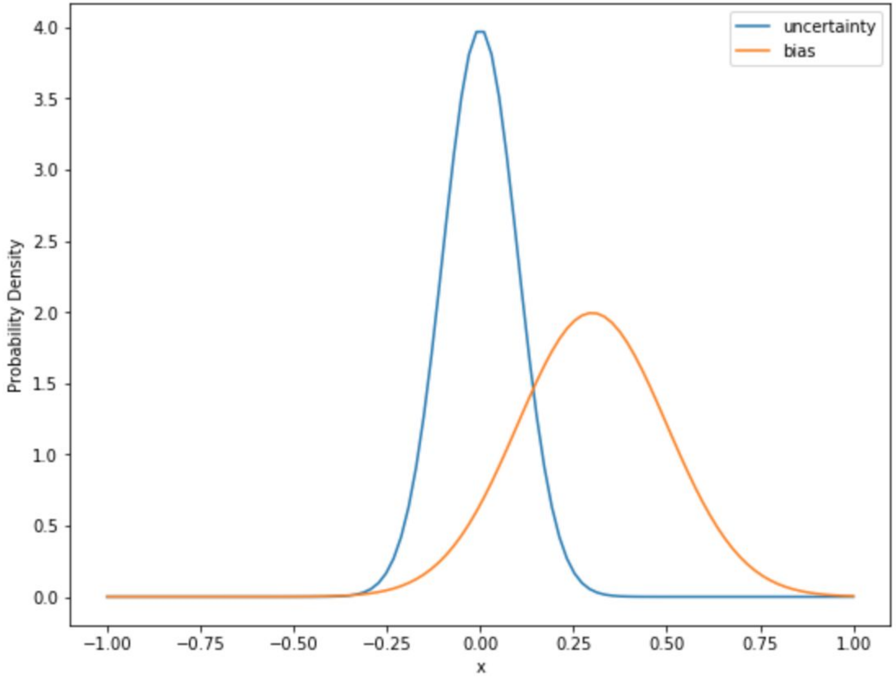
$$\hat{\boldsymbol{v}} = \frac{1}{\sqrt{1+m^2}} \begin{bmatrix} -m \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

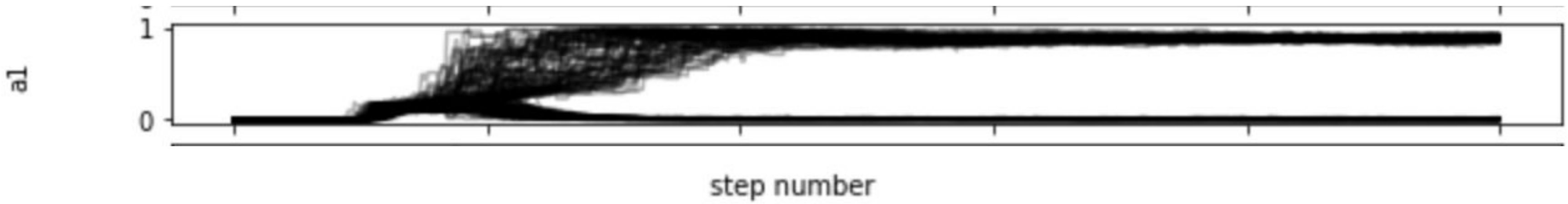
Bias

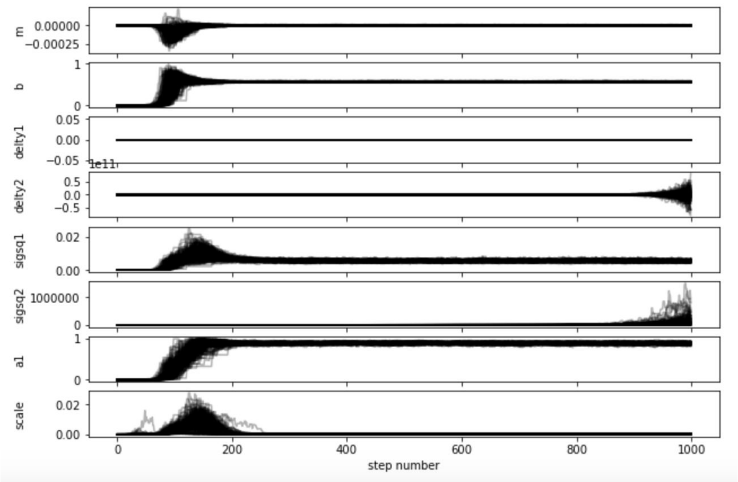
- Tried to implement bias using a scatter-like term
  - Unsuccessful, scatter is symmetric while bias is asymmetric (can only lower values)
- Back to Hogg -> non-Gaussian uncertainties
- Model noise using a mixture of *k* Gaussians

$$p(y_i|x_i, \sigma_{yi}, m, b) = \sum_{j=1}^{k} \frac{a_{ij}}{\sqrt{2\,\pi\,\sigma_{yij}^2}} \exp\left(-\frac{[y_i + \Delta y_{ij} - m\,x_i - b]^2}{2\,\sigma_{yij}^2}\right)$$

- Started with 2 Gaussians (uncertainty and bias), ignoring scatter temporarily
    - Too bimodal, caused walkers to branch, prioritizing only one Gaussian or the other
- Added another Gaussian to smooth the distribution






m
0.00000
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b
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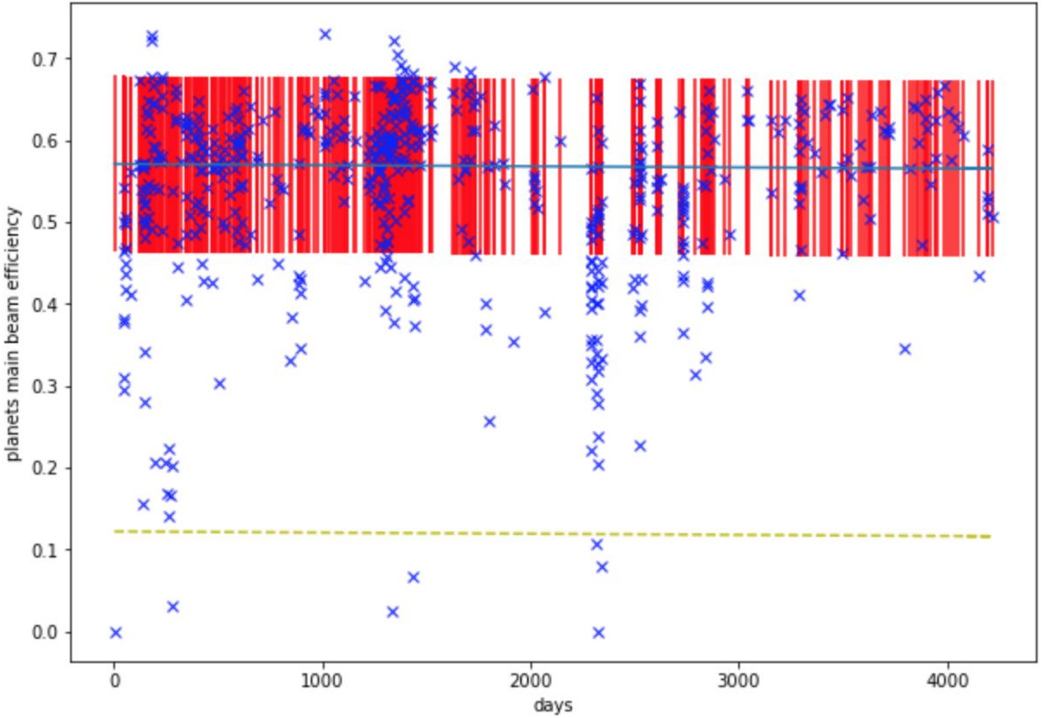
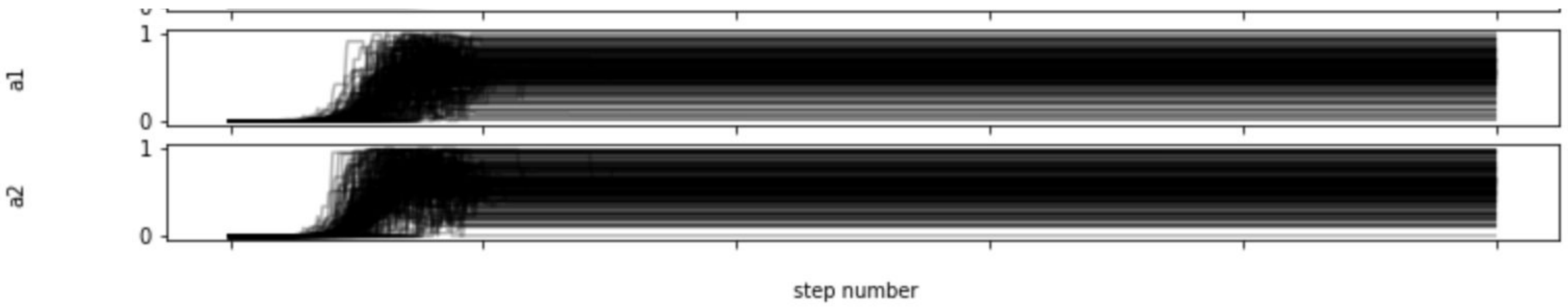
1000000
a1
scale
step number


planets main beam efficiency
days

Matrix Version

- Coded a matrix version (derived by Sundar)
- Constrained optimization
    - Lagrange multiplier to force non-zero weights on the Gaussians
    - Fixed the weighting problem
    - Reasonable parameters still seemed arbitrary or random



Results

- Very complicated to use MCMC for non-Gaussian likelihoods, asymmetric uncertainties
- Method should in principle work but inconclusive, inconsistent
- Tested the code on fake generated datasets
  - Similar problems
- Possible reasons for results:
  - Model fails to account for all the physics
  - Poor method for this dataset

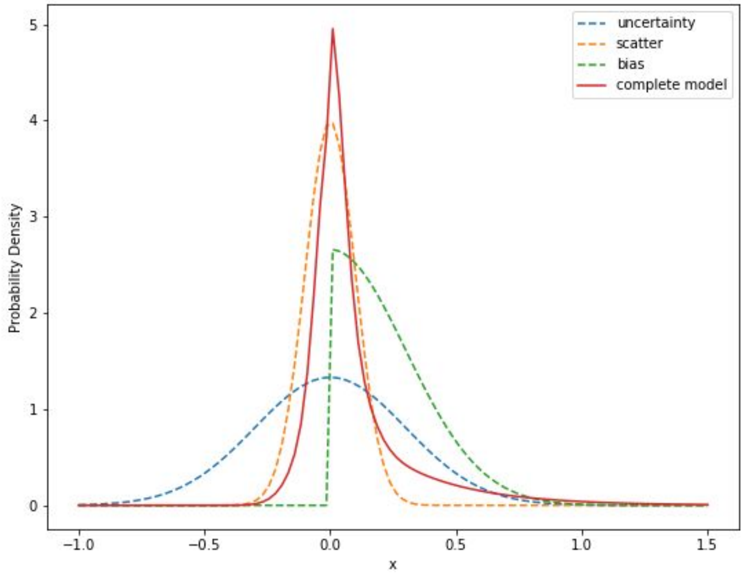
Forward Modeling

Write a model that generates datasets based on input parameters ⇒ Create a grid that represents the parameter space

⇒ Generate 1000 samples for each set of parameters ⇒ Compare datasets to real data ⇒ Obtain best fit with uncertainties

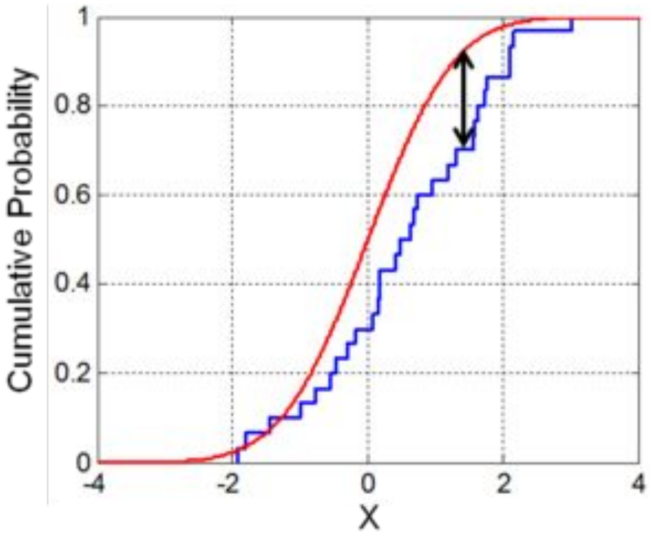
The Model

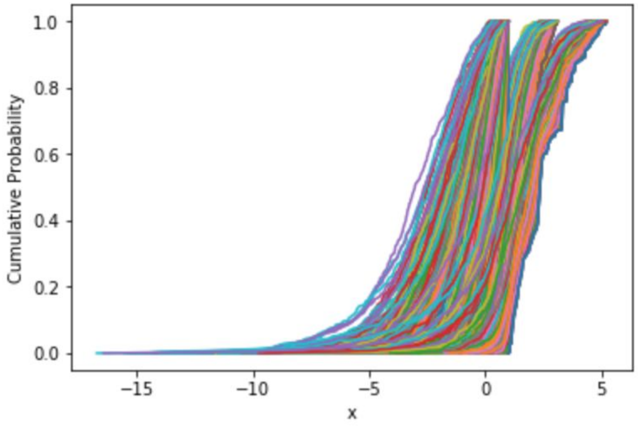
- Simple generative model
  - Line *mx+b*
  - *y* error and scatter drawn from normal distributions
  - Bias drawn from half-normal distribution
- Removed scatter term, as *y* error was large enough to account for it
- Changed bias to exponential distribution
- Added multiplicative factor on bias term



KS Test

- Kolmogorov-Smirnov test
- Compares Cumulative Distribution Functions (CDF)
- Looks at maximum distance between two CDF’s
- Minimized KS test statistic





Results

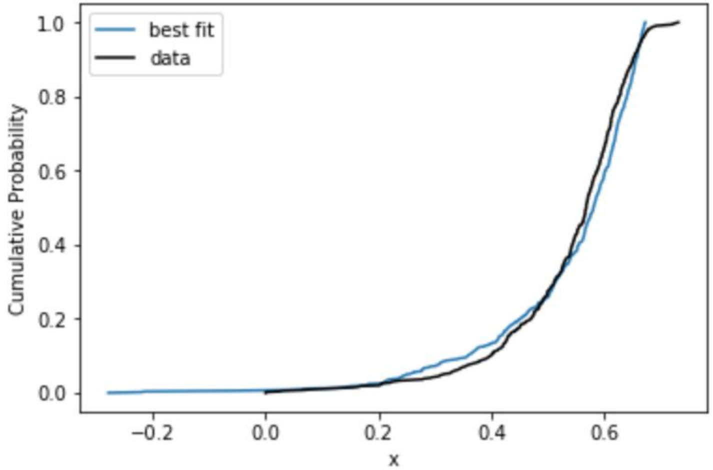
- After running 5 coarse grids
- Optimal parameters

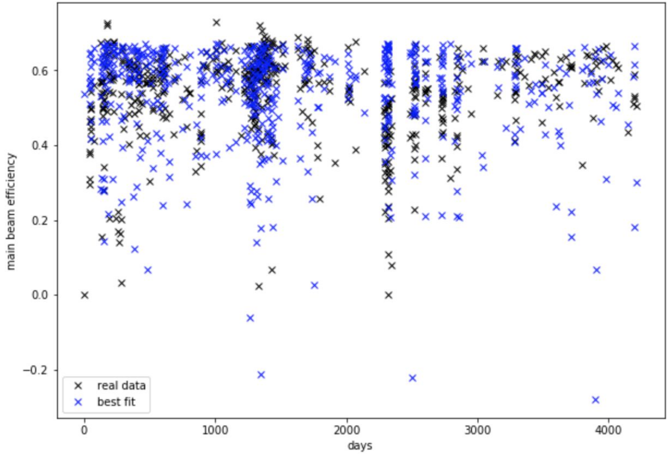
$$m : 3.3 \times 10^{-20} \pm 0.001$$

$$b : 0.57 \pm 0.014$$

$$bias\_sig : 0.35 \pm 0.014$$

$$rel : 0.45 \pm 0.049$$

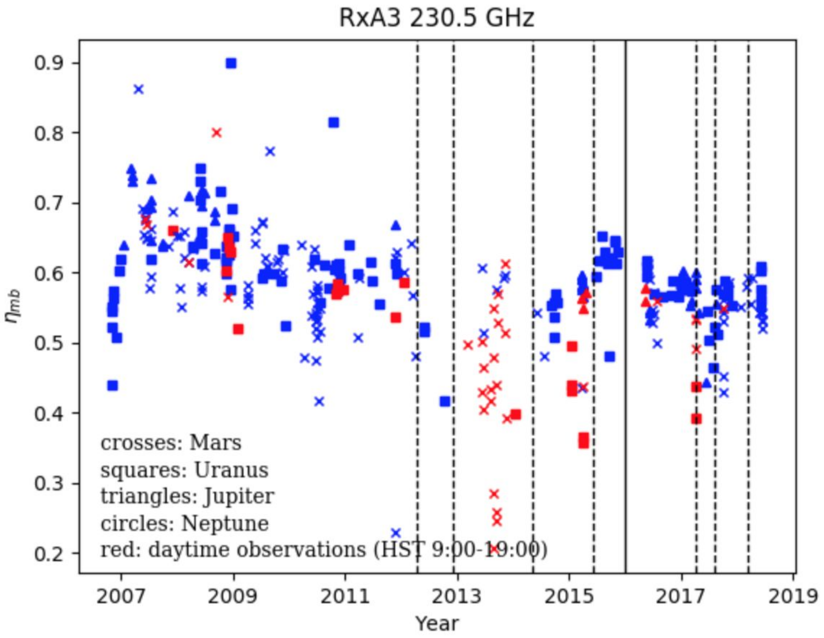




Summary

- Model main beam efficiency
- Wanted to account for uncertainty, scatter, and bias
- Bayesian Inference and MCMC
    - Gets very difficult when you're not working strictly with Gaussians or non symmetric functions
    - Method not effective or consistent for the HARP dataset -- not very clear why
- Forward Modeling
    - Required more pure computational power
    - Somewhat less statistically robust
    - Produced reasonable best fit results and uncertainties

Future Work



- Finer grid for forward modeling
- Still need to look at RxA3
  - Requires fitting a function for each section of the plot
- Could also try maximizing entropy for MCMC method

Acknowledgements

Thank you to my supervisors Sofia, Jonty, and Peter, as well as unofficial supervisor Sundar Srinivasan.

Questions?

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