Rebecca Dias Roll no-19
TE CMPN A2

x	y	MAx	MAy
4	11	-4	2.5
8	4	0	-4.5
13	5	5	-3.5
7	14	-1	5.5

$\bar{x} = 8 \qquad \bar{y} = 8.5$

$$A = \begin{bmatrix} MAx & MAy \end{bmatrix} = \begin{bmatrix} -4 & 2.5 \\ 0 & -4.5 \\ 5 & -3.5 \\ -1 & 5.5 \end{bmatrix}$$

$$C = A^T A = \begin{bmatrix} -4 & 0 & 5 & -1 \\ 2.5 & -4.5 & -3.5 & 5.5 \end{bmatrix} \begin{bmatrix} -4 & 2.5 \\ 0 & -4.5 \\ 5 & -3.5 \\ -1 & 5.5 \end{bmatrix} = \begin{bmatrix} 42 & -33 \\ -33 & 69 \end{bmatrix}$$

Let $(C - \lambda I) = 0$

$$\therefore \begin{bmatrix} 42-\lambda & -33 \\ -33 & 69-\lambda \end{bmatrix} = 0$$

$$(42-\lambda)(69-\lambda) - (-33)(-33) = 0$$
$$2898 - 42\lambda - 69\lambda + \lambda^2 - 1089 = 0$$
$$\therefore \lambda^2 - 111\lambda + 1809 = 0$$
$$\therefore \lambda = 91.154, \quad \lambda = 19.845$$

when $\lambda = 91.154$

$$\begin{bmatrix} 42-91.154 & -33 \\ -33 & 89-91.154 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -49.154 & -33 \\ -33 & -22.154 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore -49.154x_1 - 33x_2 = 0$

$\therefore -33x - 22.154x_2 = 0$

$-33x_1 = 22.154x_2$

$\therefore x_1 = \dfrac{22.154}{-33} x_2$

when $x_2 = 1$

$x_1 = -0.671$

$$x_1 = \frac{-0.671}{\sqrt{(-0.671)^2 + (1)^2}} = -0.5571$$

$$x_2 = \frac{1}{\sqrt{(-0.671)^2 + (1)^2}} = 0.8303$$

$\therefore$ For $\lambda = 91.154$

Eigen vector $[-0.5571 \quad 0.8303]$

When $\lambda = 19.845$

$$\begin{bmatrix} 42-19.845 & -33 \\ -33 & 69-19.845 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 22.155 & -33 \\ -33 & 49.155 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$22.155x_1 - 33x_2 = 0$$
$$-33x_1 + 49.155x_2 = 0$$

$$33x_1 = 49.155x_2$$
$$\therefore x_1 = \frac{49.155}{33} x_2$$

When $x_2 = 1$

$$x_1 = 1.4895$$

$$x_1 = \frac{1.4895}{\sqrt{(1.4895)^2 + (1)^2}} = 0.8302$$

$$x_2 = \frac{1}{\sqrt{(1.4895)^2 + (1)^2}} = 0.5573$$

∴ For $\lambda = 19.845$

Eigen vector $\begin{bmatrix} 0.8302 & 0.5573 \end{bmatrix}$

Total of variance = $91.154 + 19.845$

$= 110.999$

$TM = A * EV$

$$= \begin{bmatrix} -4 & 2.5 \\ 0 & -4.5 \\ 5 & -3.5 \\ 1 & 5.5 \end{bmatrix} \begin{bmatrix} -0.5571 & 0.8302 \\ 0.8303 & 0.5573 \end{bmatrix} = \begin{bmatrix} 4.304 & -1.927 \\ -3.736 & -2.507 \\ -5.691 & 2.2004 \\ 5.1237 & 2.234 \end{bmatrix}$$