

Ref: ~~Barfi~~

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Q3]

A]

• [ii]

① Covariance is a measure of how changes in one variable are associated with changes in a second variable

② Specifically, covariance measures the degree to which two variables are linearly associated. However, it is also often used informally as a general measure of how monotonically related the two variables are.

Sol<sup>n</sup>:-

$$\bar{x}_m = \frac{\sum x}{N} = 1.82$$

$$\bar{y}_m = \frac{\sum y}{N} = 1.91$$

$$\text{Covariance matrix } C = \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix}$$

$$C_{xx} = \frac{\sum (x - \bar{x}_m)^2}{N-1} = 0.6165$$

$$C_{xy} = C_{yx} = \frac{\sum (x - \bar{x}_m)(y - \bar{y}_m)}{N-1} = 0.61544$$

$$C_{yy} = \frac{\sum (y - \bar{y}_m)^2}{N-1} = 0.7165$$

$$C = \begin{bmatrix} 0.6165 & 0.61544 \\ 0.61544 & 0.7165 \end{bmatrix}$$



Rekha

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$$C_{xx} = \frac{\sum (x - x_m)^2}{N-1}$$

$$= 0.6012$$

$$C_{xy} = \frac{\sum (x - x_m)(y - y_m)}{N-1} \therefore \{C_{xy} = C_{yx}\}$$

	$x_i - x_m$	$y - y_m$	$(x - x_m)x$ $(y - y_m)y$
$= 5.438$	0.68	0.49	0.333
$9$	-1.82	-1.21	0.1597
$= 0.6042$	0.38	0.99	0.3762
	0.08	0.29	0.023
	1.28	1.09	1.395
$C_{yy} = \frac{\sum (y - y_m)^2}{N-1}$	0.48	0.79	0.379
$= 0.7032$	0.18	-0.31	-0.055
	-0.82	-0.81	0.664
	-0.32	-0.31	0.0992
	-0.62	-1.01	0.626
$\Sigma$			$\Sigma = 5.438$

$$\therefore C = \begin{bmatrix} 0.6012 & 0.6042 \\ 0.6042 & 0.7032 \end{bmatrix}$$



Rebecca

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Q3]

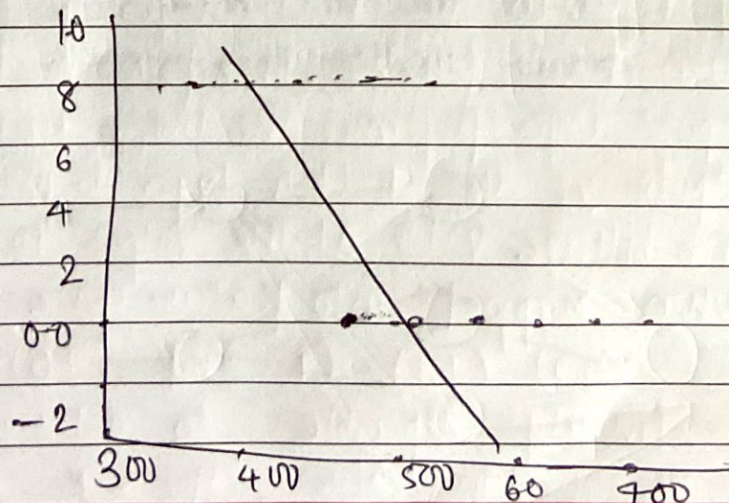
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i]

- ① The logit function is the natural log of the odds that  $Y$  equals one of the categories.
- ② The log of the link function is what the logit function is.
- ③

$$\ln \left( \frac{p}{1-p} \right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_K X_K$$

- ④ The logit function is particularly popular because its results are relatively easy to interpret. But many of the others work just as well.
- ⑤ Once we fit the model, we can then back-transform the estimated regression coefficients off of a log scale so that we can interpret the conditional effects of each  $X$ .
- ⑥ If we use  $Y$  as the outcome variable and tried to fit a line, it wouldn't be very good representation of the relationship.





Refer

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⑦ The graph shows an attempt to fit a line between  $x$  variable and a binary outcome  $y$ . You can see a relationship. There, higher values of  $x$  are associated with more 0s and lower values of  $x$  have more 1's.

But it's not a linear relationship.

⑧ Spam detection is an example where we need to classify it as spam or no spam.

0 = Not spam

1 = Spam.

In order to apply logistic regression to the spam detection problem, sender of email, occurrence of words/phrases like "offer", "prize", "gift" are checked.



Rebecca

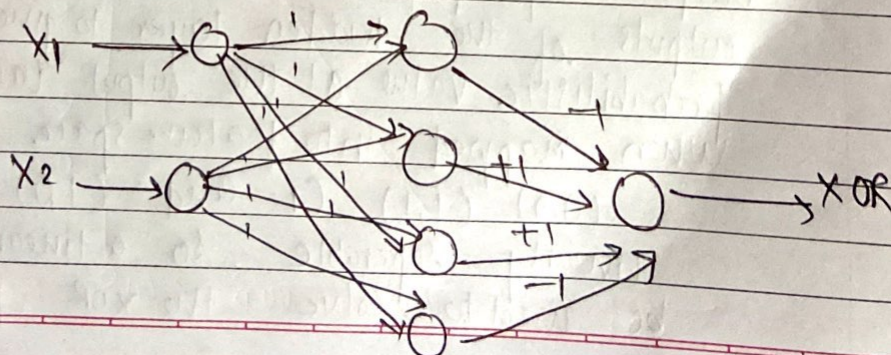
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Q3]

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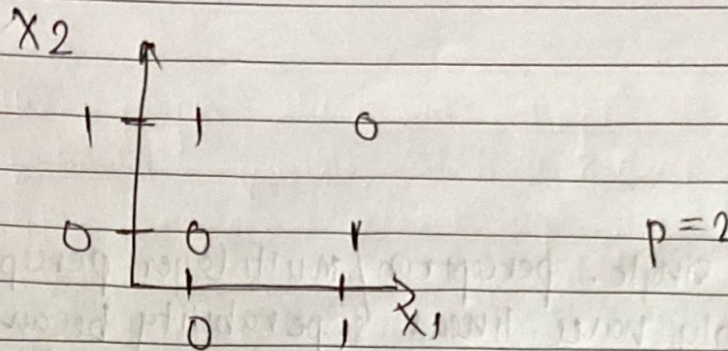
- ① In single perceptron / multi layer perceptron we only have linear separability because they are composed of input & output layers.
- ② Radial basis function, we define a receptor =  $t$ , we draw a confrontal map around the receptor and gaussian function are generally used for Radial Basis function. we define the radial distance  $r = ||x - t||$
- ③ Radial Basis Neural Network composes of input, hidden and output layers. RBNN is strictly limited to have exactly one hidden layer called as feature vector.
- ④ EOX function  
we have 4 inputs and will not increase dimension at the feature vector here. we select  $\phi(x)$ , we will have each receptor  $t$
- ⑤  $p = \#$  of input features  
 $M = \#$  of transformed vector dimensions  
each node in the hidden layer performs a set of non-linear radial basis function.

⑥





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$$\begin{aligned} \phi_1 &\rightarrow t_1 (0,0); \sigma_1 = 1 & \phi_1(x) &= e^{-\frac{\|x-t_1\|^2}{2}} \\ \phi_2 &\rightarrow t_2 (0,1); \sigma_2 = 1 & \phi_2(x) &= e^{-\frac{\|x-t_2\|^2}{2}} \\ \phi_3 &\rightarrow t_3 (1,0); \sigma_3 = 1 & \phi_3(x) &= e^{-\frac{\|x-t_3\|^2}{2}} \\ \phi_4 &\rightarrow t_4 (1,1); \sigma_4 = 1 & \phi_4(x) &= e^{-\frac{\|x-t_4\|^2}{2}} \end{aligned}$$

Input		$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$	$\sum W \cdot \phi$	Output
0	0	1.0	0.6	0.6	0.4	-0.2	0
0	1	0.6	1.0	0.4	0.6	0.2	1
1	0	0.6	0.4	1.0	0.6	0.2	1
1	1	0.4	0.6	0.6	1.0	-0.2	0
		-1	+1	+1	-1		

- Only nodes in the hidden layer perform the radial basis function transformation function
- Output layer performs the linear combination of the outputs of the hidden layer to give a final probabilistic value at the output layer.
- When mapped into feature space.

(0,0) (1,1) C1 and (1,0) (0,1) become linearly separable. So a linear classifier can be used to solve the XOR problem.