Representation of Discrete Time Signals

Signals are represented in following way

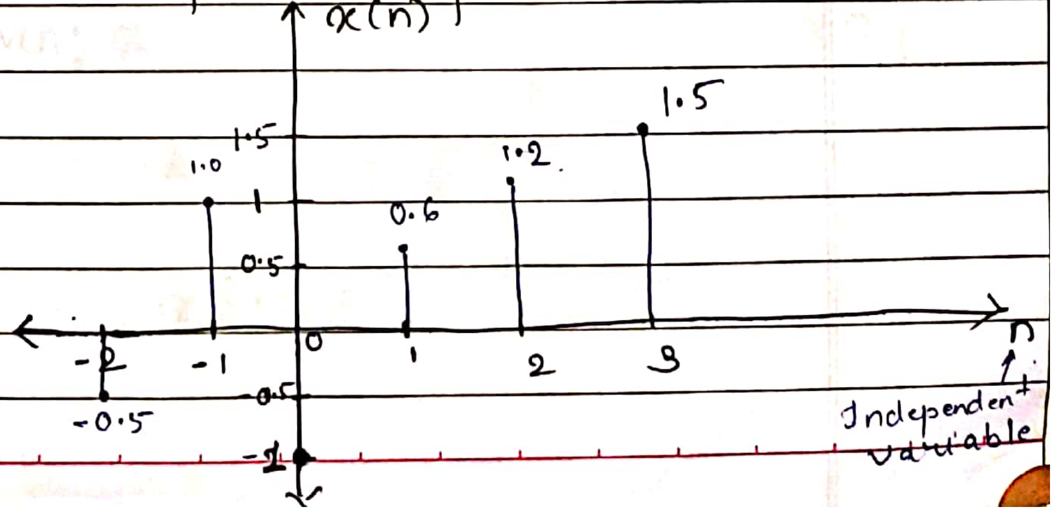
1. Functional Representation
2. Graphical Representation
3. Tabular Representation
4. Sequence Representation

1. Functional Representation

→ Represented in Mathematical Equation

Eg:
$$x(n) = \begin{cases} -0.5 & ; n = -2 \\ 1.0 & ; n = -1 \\ -1.0 & ; n = 0 \\ 0.6 & ; n = 1 \\ 1.2 & ; n = 2 \\ 1.5 & ; n = 3 \\ 0 & ; \text{other } n. \end{cases}$$

2. Graphical Representation :



3. Tabular Representation

n	.....	-2	-1	0	1	2	3	....
$x(n)$	.....	-0.5	1.0	-1.0	0.6	1.2	1.5	

→ Independent variable

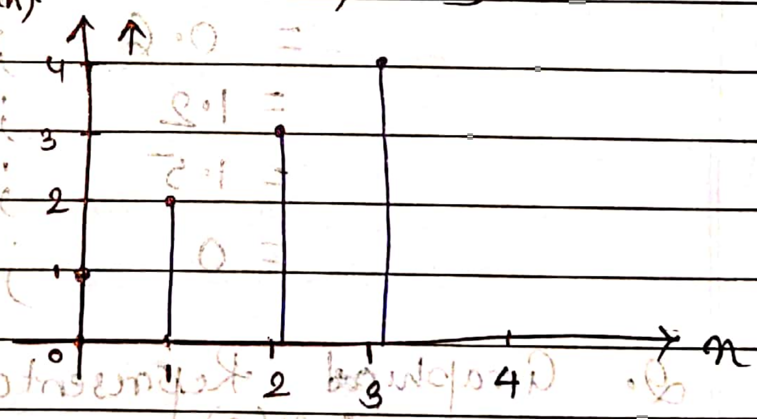
→ Signal value for each variable

4. Sequence representation

→ Represented in one dimensional array

$$x(n) = \{\ldots, -0.5, 1.0, \underset{\uparrow}{-1.0}, 0.6, 1.2, 1.5 \ldots$$

Eg: $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$



Mathematical operations on Discrete Time Signals.

1. Scaling
   - ↳ Amplitude scaling
   - ↳ Time scaling
2. Folding
3. Shifting
   - ↳ Right shift (advance)
   - ↳ Left shift (delay).
4. Addition
5. Multiplication

1. (a) Amplitude Scaling

↳ Multiply value of every signal sample by A (constant).

Eg: $y(n) = A \cdot x(n)$

amplitude scaled signal — constant

~~Eg~~ Given: $x(n) = \{10, 16, 20\}$

$\uparrow$ (under 10)

$A = 0.2$

~~x(n)~~

$$y(n) = A\,x(n)$$

$n = 0$; $y(0) = 0.2 \times 10 = 2.0$

$n = 1$; $y(1) = 0.2 \times 16 = 3.2$

$n = 2$; $y(2) = 0.2 \times 20 = 4.0$

$$y(n) = \{\underset{\uparrow}{2.0},\ 3.2,\ 4.0\}$$

Example 2

$$x(n) = \{\underset{\uparrow}{2},\ 1,\ 2,\ 1\}$$

$$A = 2$$

$\rightarrow$ $y(n) = A\,x(n)$

$$y(n) = \{\underset{\uparrow}{4},\ 1,\ 4,\ 1\}$$

1(b) Timescaling (Downsampling or Upscali

$\Rightarrow$ decreasing or increasing the signal strength

~~$x(2n) \Rightarrow$~~ (a) Downsampling

eg:

$$x(n) = \{1,\ \underset{\uparrow}{1},\ 1,\ 1,\ 2\}$$

Find $x(2n) = ?$

$n = -1$; $x(2n) = x(2\times -1) = x(-2) = 0$

$n = 0$; $x(2n) = x(0) = 1$

$n = 1$; $x(2n) = x(2\times 1) = x(2) = 1$

$n = 2$; $x(2n) = x(4) = 0$

$$x(2n) = \{0, \underset{\uparrow}{1}, 1, 0\}$$

(*) Upsampling $= x(n/I)$

~~$n = -1$; $x(-1/2) = 0$~~

Eg: $x(n) = \{1, \underset{\uparrow}{1}, 1, 2\}$

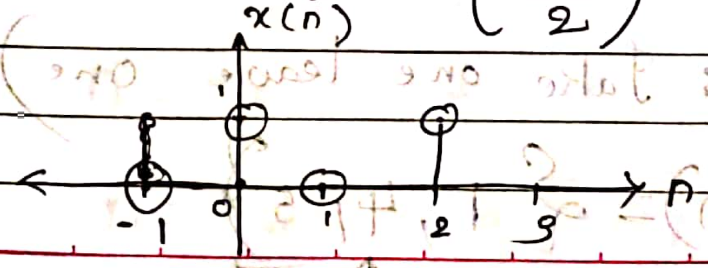
$n = -1$; $x\left(\frac{n}{2}\right) = x\left(\frac{-1}{2}\right) = 0$

$n = 0$; $x\left(\frac{n}{2}\right) = x(0) = 1$

$n = 1$; $x\left(\frac{n}{2}\right) = x\left(\frac{1}{2}\right) = 0$

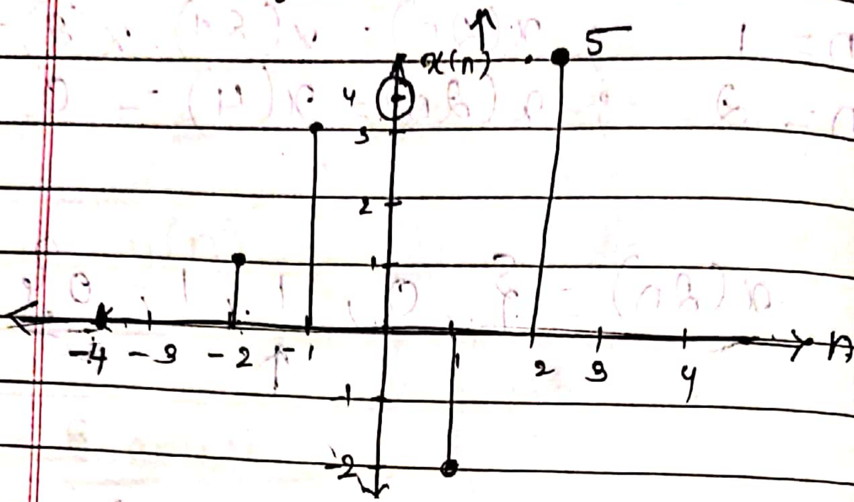
$n = 2$; $x\left(\frac{2}{2}\right) = x(1) = 1$

$n = 3$; $x\left(\frac{3}{2}\right) = 0$



Additional Examples

$$x(n) = \{1, 3, \underset{\uparrow}{4}, -2, 5\}$$



(a) Downscaling

$$y(n) = x(2n)$$

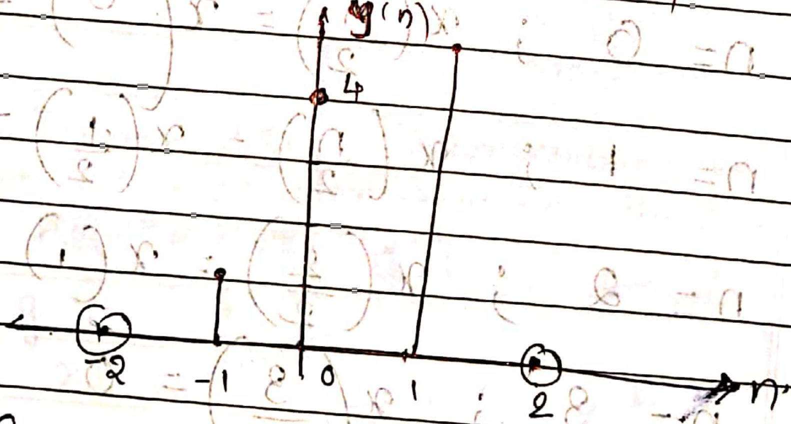
$n = -2$; $y(n) = x(-4) = 0$

$n = -1$; $y(n) = x(-2) = 1$

$n = 1$; $y(n) = x(2) = 5$

$n = 2$; $y(n) = x(4) = 0$

$n = 0$; $y(n) = x(0) = 4$



(Hint : Take one leave one)

$$y(n) = \{1, \underset{\uparrow}{4}, 5\}$$

$y(n) = x(-n)$

$n = -2 \quad y(-2) = x(2) = 1.6$

$n = -1 \quad y(-1) = x(1) = 0.8$

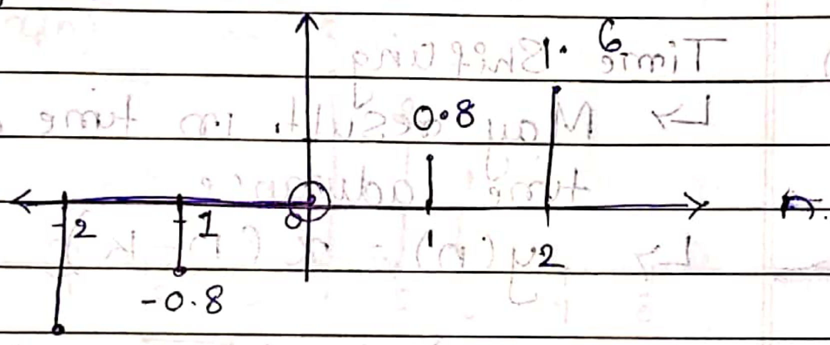
$n = 0 \quad y(0) = x(0) = 0$

$n = 1 \quad y(1) = x(-1) = -0.8$

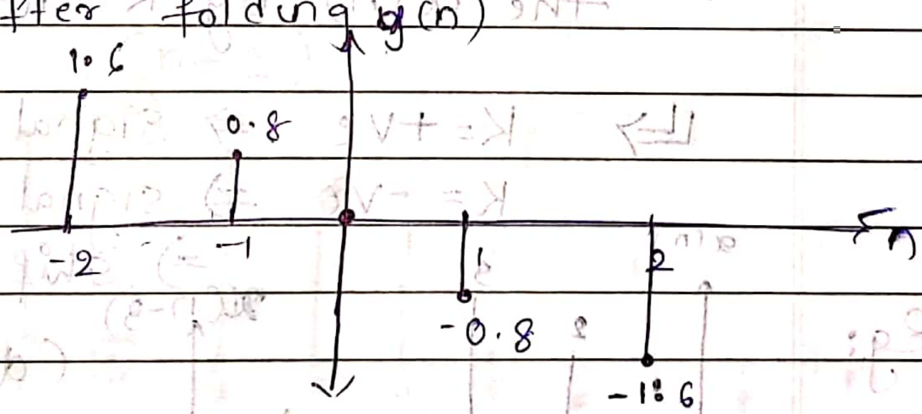
$n = 2 \quad y(2) = x(-2) = -1.6$

$$y(n) = \{1.6, 0.8, \underset{\uparrow}{0}, -0.8, -1.6\}$$

Before. $x(n)$



After folding $y(n)$



eg 2 :

$$x(n) = \{0.5, 1, 2, \underset{\uparrow}{3}, 1, 1, 1, 1, \infty\}$$

$$y(n) = \{\infty, 1, 1, 1, 1, \underset{\uparrow}{3}, 2, 1, 0.5\}$$

[Process of increasing the signal strength by inserting zeros between signal samples]

Downsampling

(b) $x(n) = \{1, 3, 4, -2, 5\}$
$\uparrow$

$y(n) = x\left(\frac{n}{3}\right)$

$n = -2 \quad y(-2) = x(-2/3) = 0$

$n = -1 \quad y(-1) = x(-1/3) = 0$

$\rightarrow n = 0 \quad y(0) = x(0) = 4$

$n = 1 \quad y(1) = x(1/3) = 0$

$n = 2 \quad y(2) = x(2/3) = 0$

$n = 3 \quad y(3) = x(3/3) = -2$

$n = 4 \quad y(4) = x(4/3) = 0$

$n = 5 \quad y(5) = x(5/3) = 0$

$n = 6 \quad y(6) = x(6/3) = 5$

$n = -3 \quad y(-3) = x(-3/3) = 3$

$n = -4 \quad y(-4) = x(-4/3) = 0$

$n = -5 \quad y(-5) = x(-5/3) = 0$

$n = -6 \quad y(-6) = x(-6/3) = 1$

Original Signal $x(n) = \{1, 3, \underset{\uparrow}{4}, -2, 5\}$

Upscaled Signal $y(n)$

$y(n) = \{1, 0, 0, 3, 0, 0, \underset{\uparrow}{4}, 0, 0, -2, 0, 0, 5\}$

2. Folding (Reflection or Transpose or Time reversal)

~~x(n) = x(2n)~~

$\Rightarrow$ Let $x(n)$ be original signal then $y(n) = x(-n)$ is the folded signal.

Eg:

$x(n) = 0.8\,n \quad -2 \leq n \leq 2$

$x(-2) = -1.6$

$x(-1) = -0.8$

$x(0) = 0$

$x(1) = 0.8$

$x(2) = 1.6$

$x(n) = \{-1.6, -0.8, \underset{\uparrow}{0}, 0.8, 1.6\}$

$y(n) = x(-n)$

$n = -2 \quad y(-2) = x(2) = 1.6$

$n = -1 \quad y(-1) = x(1) = 0.8$

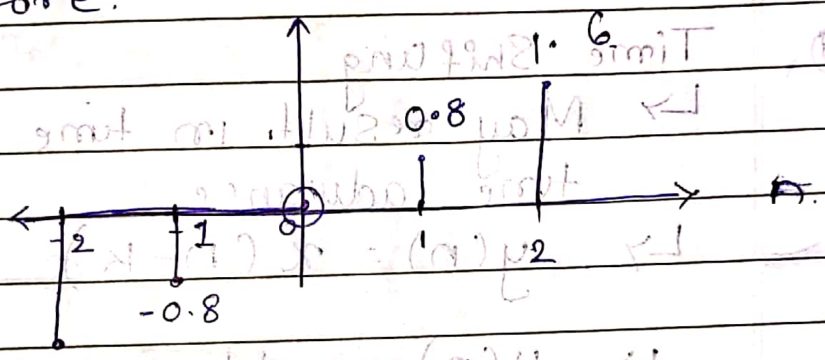
$n = 0 \quad y(0) = x(0) = 0$

$n = 1 \quad y(1) = x(-1) = -0.8$

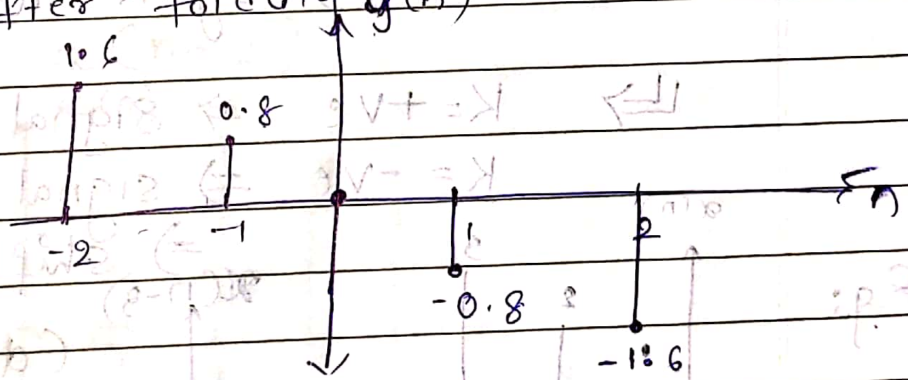
$n = 2 \quad y(2) = x(-2) = -1.6$

$$y(n) = \{1.6, 0.8, \underset{\uparrow}{0}, -0.8, -1.6\}$$

Before. $x(n)$



After folding $y(n)$



:

$$x(n) = \{0.5, 1, 2, \underset{\uparrow}{3}, 1, 1, 1, 1, \ldots\}$$

$$y(n) = \{\ldots 1, 1, 1, 1, \underset{\uparrow}{3}, 2, 1, 0.5\}$$

Example: Time scaling

$x(n) = \{ \underset{\uparrow}{1}, 2, 8, 4, 6 \}$

Downscaling

$x(2n) = \{ \underset{\uparrow}{1}, 8, 6 \}$

Upscaling

$x\left(\frac{n}{2}\right) = \{ \underset{\uparrow}{1}, 0, 2, 0, 8, 0, 4, 0, 6 \}$

---

(3) Time Shifting

↳ May result in time delay or time advance.

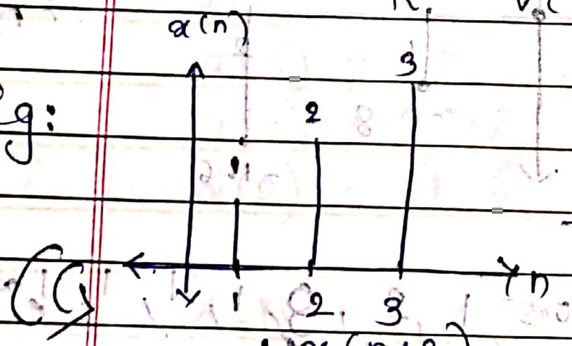
↳ $y(n) = x(n-k)$

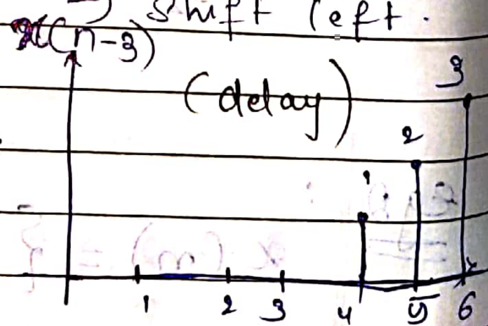
↳ $y(n)$ is obtained by shifting the $x(n)$ by 'k' units

↳ $k = +ve \Rightarrow$ signal delay $\Rightarrow$ Shift Right

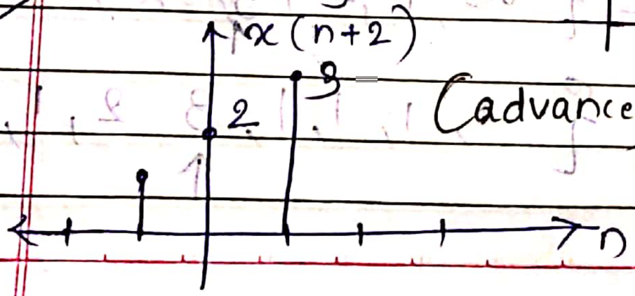
$k = -ve \Rightarrow$ signal Advance $\Rightarrow$ Shift left.

Eg:





(delay)

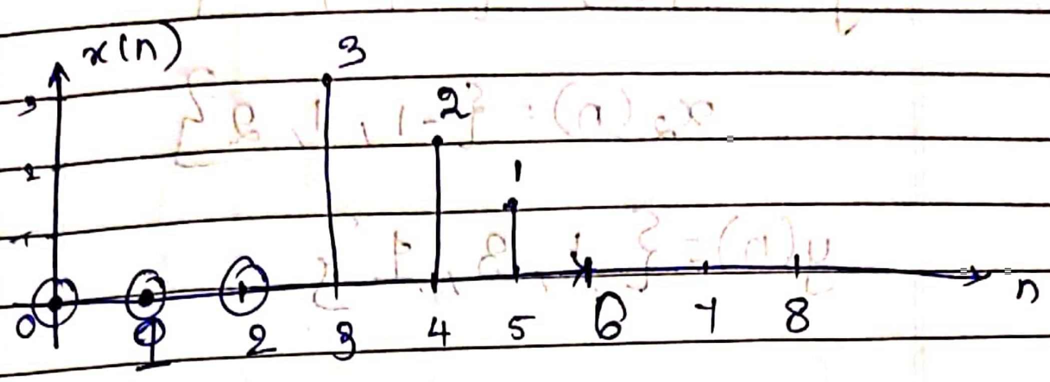


(advance)

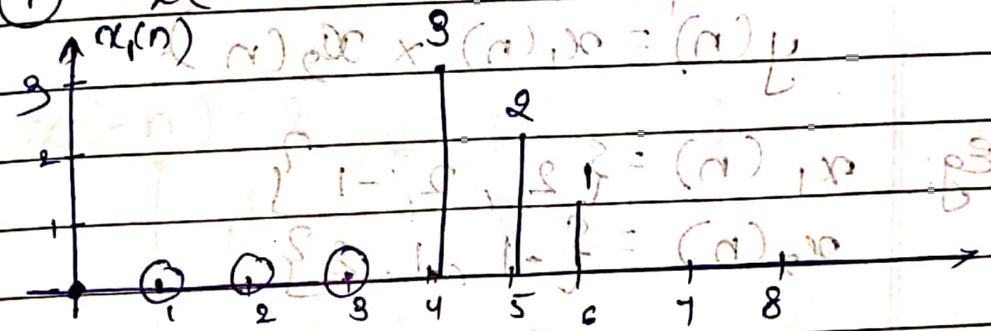
Eg 2

~~$x(n) = \{3, 2, 1\}$~~

$$x(n) = \{\underset{\uparrow}{0}, 0, 0, 3, 2, 1, 0, 0 \ldots\}$$

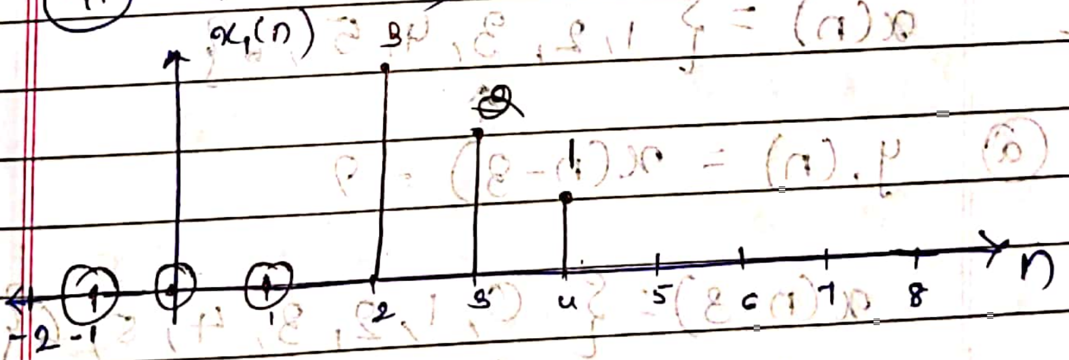


(i) $x(n-1)$



" Delay the Signal "

(ii) $x(n+2)$



4. Addition

$$y(n) = x_1(n) + x_2(n)$$

Eg $x_1(n) = \{2, 2, -1\}$

$x_2(n) = \{-1, 1, 2\}$

$y(n) = \{1, 3, 1\}$

5. Multiplication

$$y(n) = x_1(n) \times x_2(n)$$

Eg: $x_1(n) = \{2, 2, -1\}$

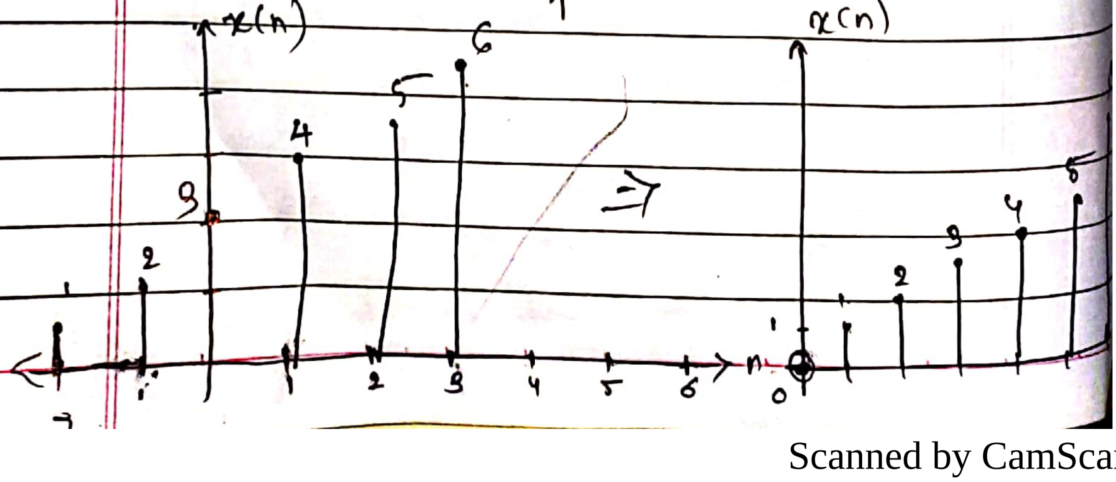
$x_2(n) = \{-1, 1, 2\}$

$y(n) = \{-2, 2, -2\}$

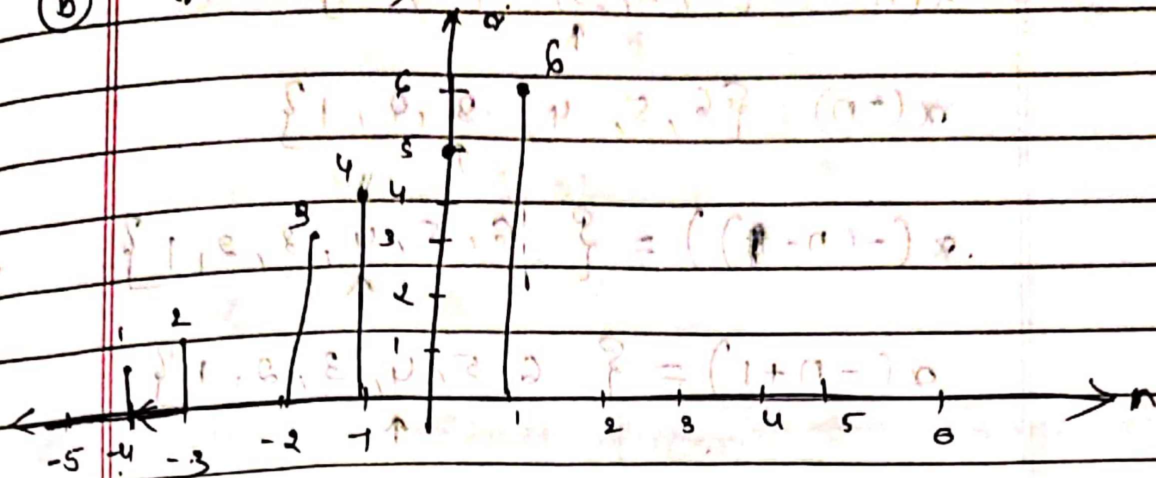
Eg. $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5, 6\}$

(a) $y_1(n) = x(n-3) = ?$

$x(n-3) = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 5, 6\}$

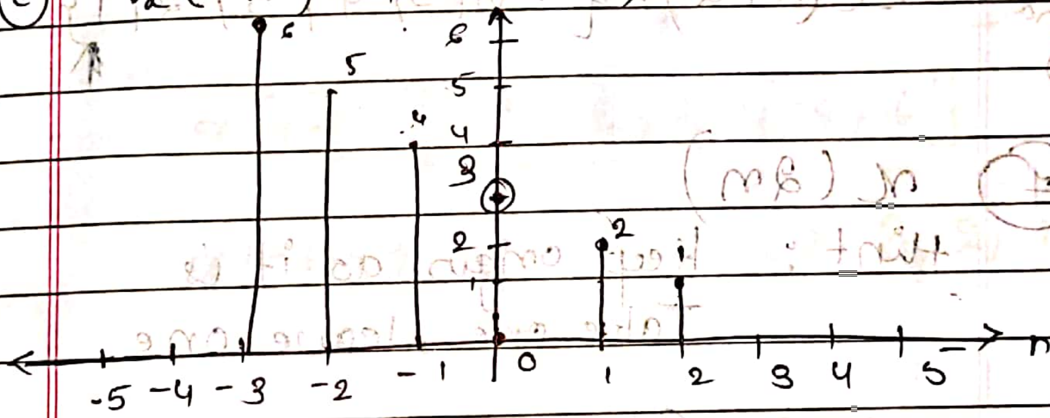


(b) $x(n+2) = ?$



$x(n+2) = \{1, 2, 3, 4, 5, 6\}$
$\uparrow$

(c) $x(-n) = ?$



$x(-n) = \{6, 5, 4, 3, 2, 1\}$
$\uparrow$

(d) $x(-n+1)$ (P.T.O for alternate solution)

$= x(-(n-1))$ ⇒ Fold & delay operation

$x(n) \xrightarrow{\text{Fold}} x(-n) \xrightarrow{\text{delay}} x(-(n-k))$
$\Downarrow$
$x(-n+k)$

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5, 6\}$

$x(-n) = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1\}$

$x(-(n-1)) = \{6, 5, \underset{\uparrow}{4}, 3, 2, 1\}$

$x(-n+1) = \{6, 5, \underset{\uparrow}{4}, 3, 2, 1\}$

*(6) $x(-n-2) = x(-(n+2))$

Fold & advance signal

$x(-n) = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1\}$

$x(-(n+2)) = \{6, 5, 4, 3, 2, \underset{\uparrow}{1}\}$

(7) $x(2n)$

Hint : Keep origin as it is
Take one, leave one

$x(2n) = \{1, \underset{\uparrow}{3}, 5\}$

*(8) $x(3n-1)$

Hint : Right to left after
1. delay by 1
2. Downscaling by 3

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5, 6\}$

$x(n-1) = \{1, \underset{\uparrow}{2}, 3, 4, 5, 6\}$

$x(3n-1) = \{\ldots 2, 5\}$

Alternate Solution

$x(-n-2) = ?$

Hint: Start from ← Right to Left

1. Delay by 2
2. Fold.

$x(n) = \{1, 2, 3, 4, 5, 6\}$

$x(n-2) = \{1, 2, 3, 4, 5, 6\}$

$x(-n-2) = \{6, 5, 4, 3, 2, 1\}$

Alternate Solution

$x(-n+1)$

1. Advance signal by 1
2. Fold.

$x(n+1) = \{1, 2, 3, 4, 5, 6\}$

$x(-n+1) = \{6, 5, 4, 3, 2, 1\}$

Eg

$x(n) = \{1, 2, 3\}$
$\uparrow$ (at 1)

(i) Amplitude scaling

$A = 2$

$y(n) = A\, x(n)$
$= 2 \cdot x(n)$

$y(n) = \{2, 4, 6\}$

(2). Time scaling

(a) Downscaling

$\Rightarrow$ Decreasing the signal strength

$x(n) = \{1, 2, 3\}$
$\uparrow$ (at 1)

$y(n) = x(2n)$

Hint: 1. Take origin 2. Take 1 leave 1

$\Rightarrow y(n) = \{1, 3\}$
$\uparrow$ (at 1)

Long Method:

$y(-1) = x(2 \times -1) = 0$
$y(0) = x(0) = 1$
$y(1) = x(2 \times 1) = 3$
$y(2) = x(2 \times 2) = 0$

$y(n) = \{1, 3\}$
$\uparrow$ (at 1)

(b) Upscaling:

⇒ Increase the signal strength by inserting zeros

~~eg:~~ ~~x(~~ $y(n) = x\left(\frac{n}{A}\right)$

Example:

$x(n) = \{1, 2, 3\}$
$\uparrow$

Short Method: Hint Insert 2 zeros between each sample.

$$\boxed{x\left(\frac{n}{2}\right) = \{\underset{\uparrow}{1}, 0, 2, 0, 3\}}$$

Long Method:

$y(-1) = x\left(\frac{-1}{2}\right) = 0$

$y(0) = x\left(\frac{0}{2}\right) = 1$

$y(1) = x\left(\frac{1}{2}\right) = 0$

$y(2) = x\left(\frac{2}{2}\right) = 2$

$y(3) = x\left(\frac{3}{2}\right) = 0$

$y(4) = x\left(\frac{4}{2}\right) = 3$

$y(n) = x\left(\frac{n}{2}\right) = \{\underset{\uparrow}{1}, 0, 2, 0, 3\}$

3. Folding

eg1:

$$x(n) = \{\underset{\uparrow}{1}, 2, 3\}$$

$$x(-n) = \{3, 2, \underset{\uparrow}{1}\}$$

eg2: $\{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$$x(-n) = \{5, 4, \underset{\uparrow}{3}, 2, 1\}$$

4. Time Shifting

⇒ Result in delay or advance in signal

$$\boxed{y(n) = x(n-k)}$$

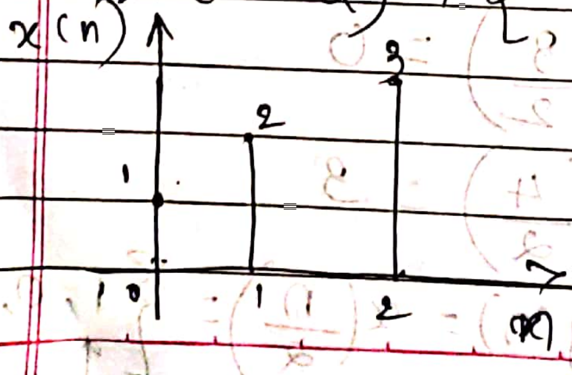
$(n-k)$ ⇒ Delay → Shift Right →

$(n+k)$ ⇒ Advance → Shift left ←

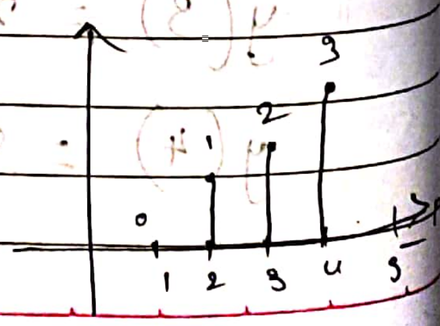
$$x(n) = \{\underset{\uparrow}{1}, 2, 3\}$$

$x(n-k)$ ⇒ delay

$$x(n-2) \Rightarrow \{\underset{\uparrow}{0}, 0, 1, 2, 3\}$$

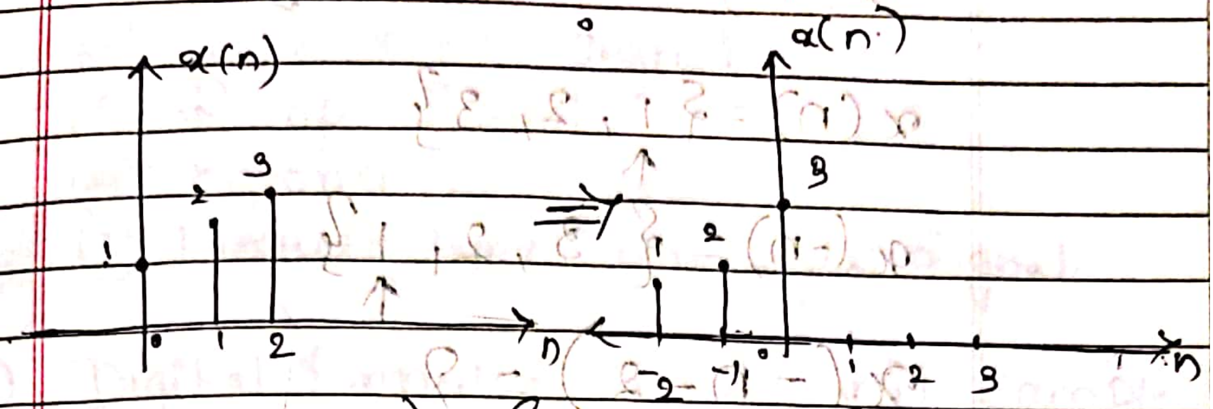


⇒



$x(n+k) \rightarrow$ Advance

$x(n+2) = ?$



$x(n+2) = \{1, 2, 3\}$ with 3 marked as the origin ($\uparrow$)

5. Fold and delay ↳ i.e. shift to → (Right)

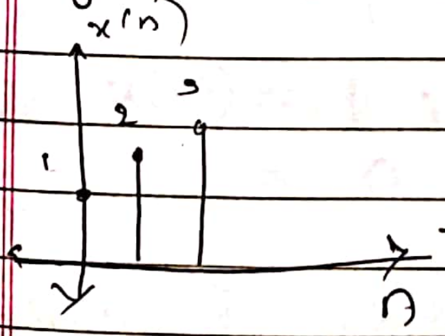
Fold ↓ replace n by $-n$ in $x(n)$

$x(n) \xrightarrow{\text{Fold}} x(-n)$

$x(n) \xrightarrow{\text{Delay}} x(n-k)$

$\therefore x(-n) \rightarrow x(-(n-k)) = x(-n+k)$

eg: $x(n) = \{1, 2, 3\}$ with 1 marked as the origin ($\uparrow$); $x(-n+2) = ?$



$x(-n) = \{3, 2, 1\}$ with 1 marked as the origin ($\uparrow$)

$x(-n+2) = x(-(n-2))$

$x(-n+2) = \{3, 2, 1\}$ with 3 marked as the origin ($\uparrow$)

7) Fold & advance

$\downarrow$ $x(-n)$ $\qquad$ $\downarrow$ $x(n+2)$

$x(n) = \{\underset{\uparrow}{1}, 2, 3\}$

$x(-n) = \{3, 2, \underset{\uparrow}{1}\}$

$x(-n-2) = ?$

$x(-(n+2))$

↳ Fold.

↳ Advance

$x(-n) = \{\underset{\uparrow}{3}, 2, 1\}$

$x(-(n+2)) = \{3, 2, \underset{\uparrow}{1}\}$

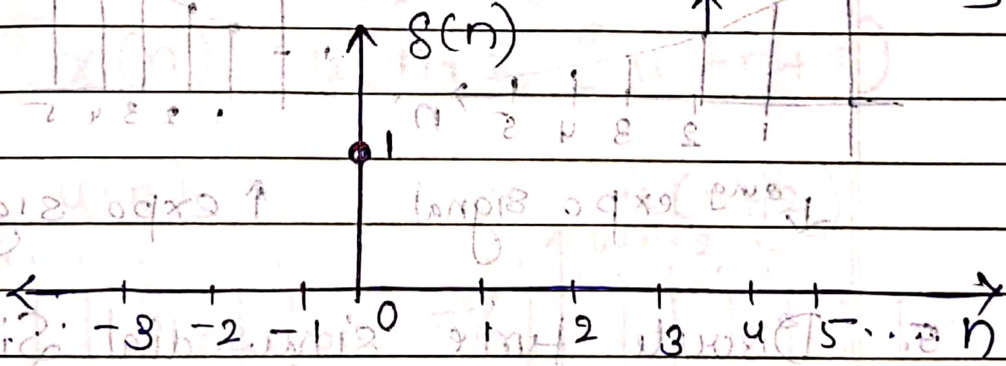
* Standard DT signals:

1. Digital Impulse signal / unit sample sequence.
2. unit step signal
3. Ramp Signal
4. Exponential Signal.
5. Discrete time sinusoidal Signal.

1. Digital Impulse signal / unit sample sequence

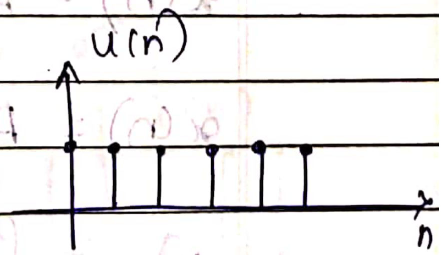
$$\delta(n) = 1 ; n = 0$$
$$= 0 ; n \neq 0$$

$$\delta(n) = \{ \cdots 0, 0, 0, \underset{\uparrow}{1}, 0\, 0\, 0, \cdots \}$$



2. Unit step signal

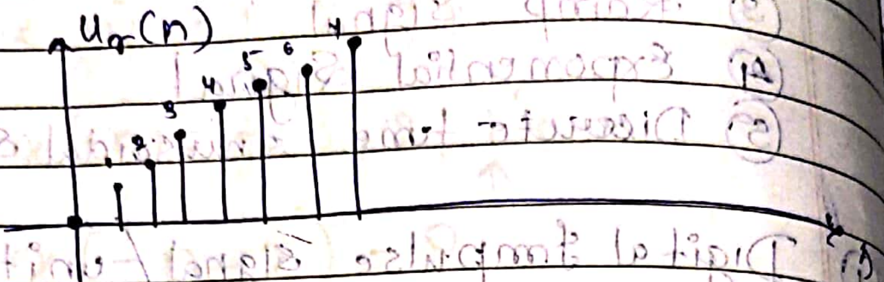
$$u(n) = 1 ; n \geq 0$$
$$= 0 ; n < 0$$



$$u(n) = \{ \cdots 0, 0, 0, \underset{\uparrow}{1}, 1, 1, 1 \cdots \}$$

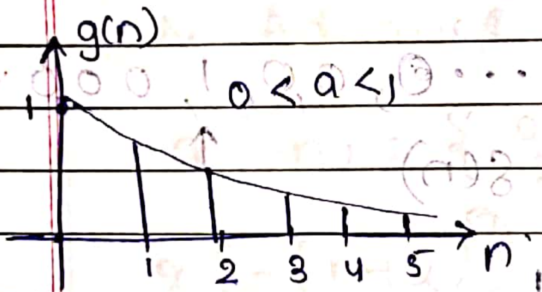
3. Ramp signal

$u_r(n) = n \; ; n \geq 0$
$\quad\quad = 0 \; ; n < 0$

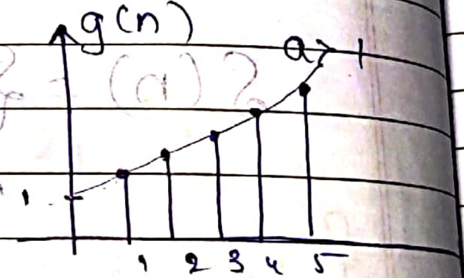


4. Exponential Signal

$g(n) = a^n \; ; n \geq 0$
$\quad\quad = 0 \; ; n < 0$



↓ sing expo signal



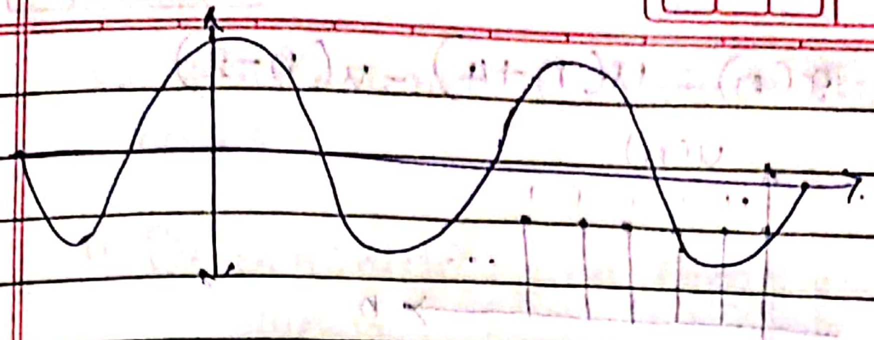
↑ expo signal

5. Discrete time sinusodial Signal

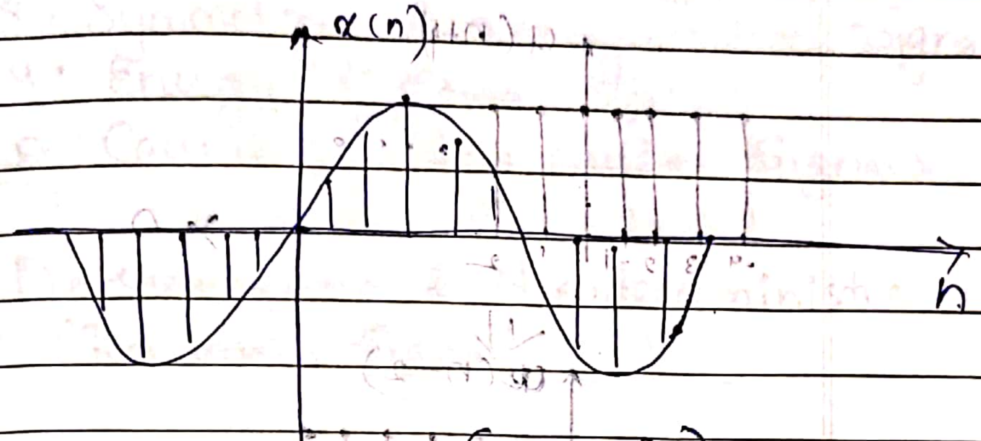
$x(n) = A\cos(\omega_0 n + \theta) \quad -\infty < n < +\infty$

$x(n) = A\sin(\omega_0 n + \theta) \quad -\infty < n < +\infty$

ω_0 = freq in radians/sample
θ = Phase in radians
$f = \dfrac{\omega_0}{2\pi}$ freq in cycles/sample



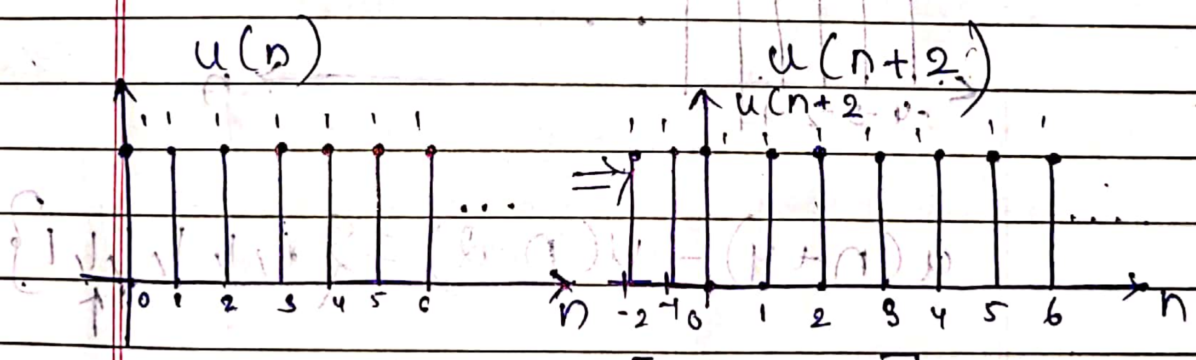
$x(n) = A\cos(\omega_0(n))$



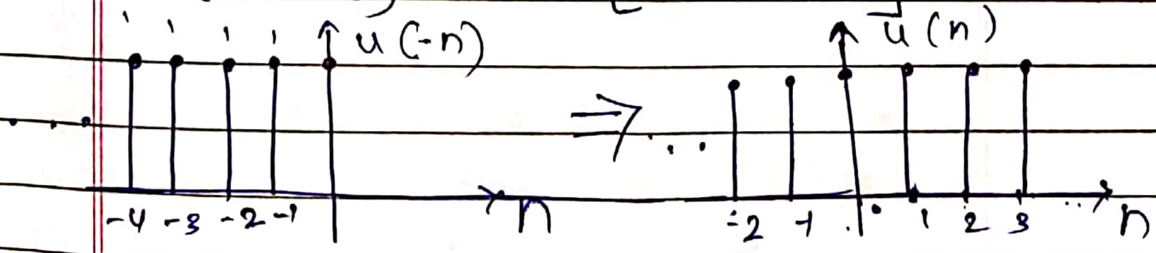
$x(n) = A\sin(\omega_0(n))$

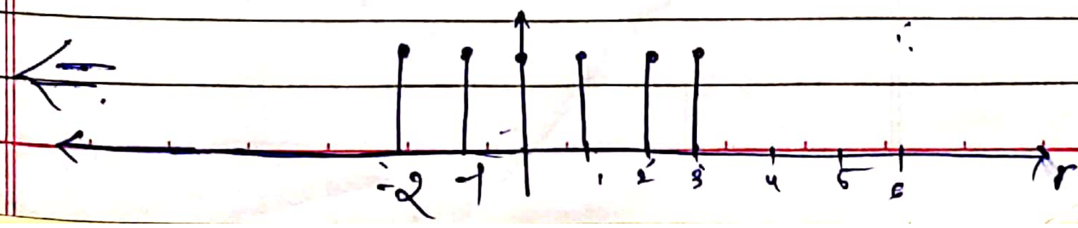
---x---0---x---

eg: $x(n) = u(n+2)\ \&\ u(-n+3)$

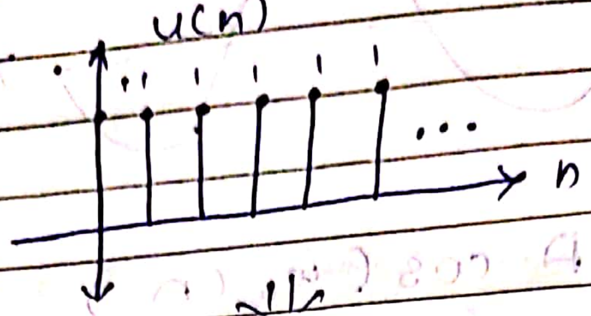


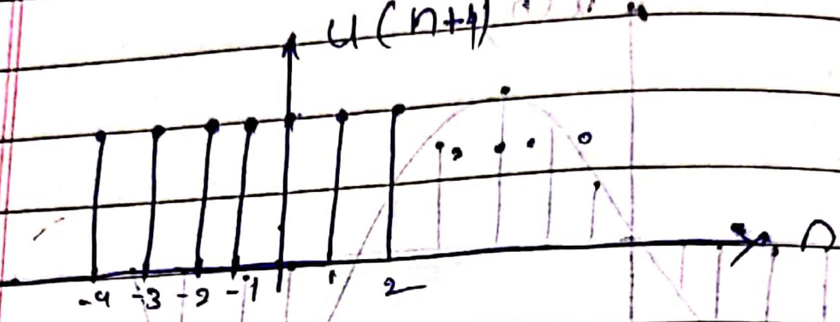
$u(-n+3) \Rightarrow u[-(n-3)]$

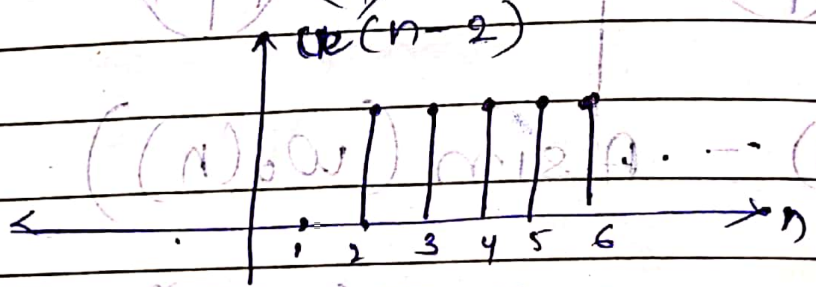


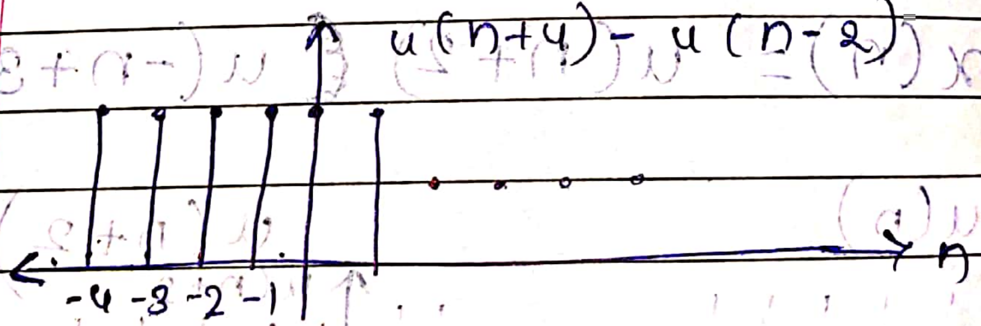


$x(n) = u(n+4) - u(n-2)$









$u(n+4) - u(n-2) = \{1, 1, 1, 1, 1, 1 \ldots$

Classification of Discrete Time Signals

1. Deterministic and Nondeterministic Signals
2. Periodic (Even) & Aperiodic (odd) Signals
3. Symmetric & antisymmetric Signals
4. Energy & power signals
5. Causal & Non causal Signals

1. Deterministic & Nondeterministic (Random) Signal:

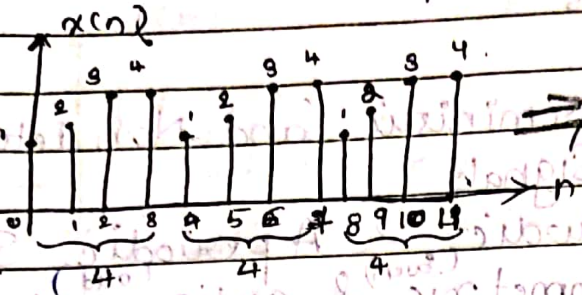
Deterministic Signal → Can be represented in mathematical Equation

Eg: ~~eg.~~ Sinusoidal $x(n) = \cos \omega n$
Expo. signal, ramp signal ..

Random Signal → Cannot be represented by any mathematical equation.

Eg: Noise signal.

2. Periodic & Aperiodic Signal



⟹ Periodic

The above signal is said to be periodic as the signal is getting repeated ~~for~~ after 4 intervals

$$\boxed{x(n) = x(n+N)}$$

↑ Period of Signal.

Non Periodic Signal:

$$\boxed{x(n) \neq x(n+N)}$$

Eg: Determine whether given signal is periodic or Aperiodic. If periodic then find fundamental period

✓ (i) $\cos(0.01\pi n)$

$$x(n) = \cos(0.01\pi n)$$

$$x(n) = \cos \omega n$$

$$\therefore \omega = 0.01\pi$$

$$\therefore 2\pi f = 0.01\pi$$

$$f = \frac{0.01\cancel{\pi}}{2\cancel{\pi}} = \frac{1}{200} \frac{\text{cycles}}{\text{samples}}$$

$\therefore \boxed{N = 200}$

$\because \boxed{f = \frac{K}{N}} \rightarrow$ Condition for periodicity

(ii) $\cos(8\pi n)$

$x(n) = \cos(8\pi n)$

$x(n) = \cos(\omega n)$

$\therefore \omega = 8\pi$

$2\cancel{\pi} f = 8\cancel{\pi}$

$f = \frac{8}{2}$

$\because f$ is ratio of rational nos.

$\therefore \cos 8\pi n$ is periodic signal

By condition of periodicity

$f = \frac{K}{N}$

$\therefore \boxed{N = 2} \rightarrow$ 'fundamental Period'

(iii) $x(n) = \sin(3n)$

$x(n) = \sin \omega n$

$\omega = 3$

$2\pi f = 3$

$\therefore f = \frac{3}{2\pi}$

$\because f$ is ratio of irrational nos

$\therefore \sin(3n)$ is Aperiodic signal

(iv) $\cos\left(\frac{n}{8}\right) \cdot \cos\left(\frac{\pi n}{8}\right)$

$x(n) = \cos \omega_1 n \cdot \cos \omega_2 n$

$\omega_1 = \frac{1}{8}$ & $\omega_2 = \frac{\pi}{8}$

$2\pi f_1 = \frac{1}{8}$ & $2\pi f_2 = \frac{\pi}{8}$

$f_1 = \frac{1}{16\pi}$ & $f_2 = \frac{\cancel{\pi}}{16\cancel{\pi}}$

$= \frac{1}{16}$

f_1 is Aperiodic

f_2 is periodic

Multiplication of periodic & Aperiodic is Aperiodic

(v) $y(n) = \sin\frac{\pi}{5}n + 2\cos\frac{\pi}{\sqrt{2}}n$

$y(n) = y_1(n) + y_2(n)$

$y_1(n) = \sin\frac{\pi}{5}n$

$= \sin \omega n$

$\therefore \omega = \frac{\pi}{5}$

$2\pi f = \frac{\pi}{5}$

$f = \frac{\pi}{5 \times 2\pi} = \frac{1}{10}$

Periodic signal $\Rightarrow N_1 = 10$

$$y_2(n) = 2\cos\frac{\pi}{\sqrt{2}}n$$

$$\cancel{y_2(n) = A\cos\frac{\pi}{\sqrt{2}}\omega n}$$

$$\therefore y_2(n) = A\cos\omega n$$

$$\omega = \frac{\pi}{\sqrt{2}}$$

$$2\pi f = \frac{\pi}{\sqrt{2}}$$

$$f = \frac{\pi}{2\pi \times \sqrt{2}}$$

$$f = \frac{1}{2\sqrt{2}}$$

$$\because f = \frac{K}{N} \qquad N_2 = 2\sqrt{2}$$

$y(n)$ is Periodic if N_1/N_2 is ratio of 2 integers

$$\therefore \frac{N_1}{N_2} = \frac{10}{2\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$\therefore y(n)$ is non-Periodic in nature

(vi) $x_1(n) = e^{j\left(\frac{\pi}{4}n\right)}$

For discrete time, complex exponential equation is

$$x(n) = e^{j\omega n}$$

$$\omega = \frac{\pi}{4}$$

$$2\pi f = \frac{\pi}{4}$$

$$f = \frac{\cancel{\pi}}{4 \times 2 \times \cancel{\pi}}$$

$$= \frac{1}{8}$$

$$f = \frac{K}{N}$$

$$\boxed{\therefore N = 8}$$

(vii) $x(n) = 8 \sin\left(\frac{1}{8}n\right)$

$$x(n) = A \sin \omega n$$

$$\omega = \frac{1}{8}$$

$$2\pi f = \frac{1}{8}$$

$$f = \frac{1}{16\pi}$$

$\therefore$ It is ratio of irrational no

$\therefore$ Non-periodic

(viii) $x(n) = \cos\left(\frac{\pi}{2} + 0.3n\right)$

$x(n) = \cos(\theta + \omega n)$

$\therefore \omega = 0.3$ & $\theta = \frac{\pi}{2}$

$2\pi f = 0.3$ (phase)

$f = \frac{0.3}{2\pi}$

$f = \frac{3}{20\pi}$

which not rational

$\therefore x(n)$ is non-periodic.

Continuous Signal:

$x(t) = x(t + T_0)$

T_0: Period

Note: Discrete Time signal is represented in 'n'.

Continuous time signal is represented in 't'.

eg: (1) $x(t) = 2\cos 100\pi t + 5\sin 50t$

$x(t) = x_1(t) + x_2(t)$

$x_1(t) = 2\cos 100\pi t$

$x_1(t) = A\cos\omega_1 t$

$\omega = 100\pi$

$2\pi f_1 = 100\pi$

$f_1 = \frac{100}{2}$

But, $f_1 = \frac{1}{T_1}$ (freq. is inverse of time period)

$\therefore \frac{1}{T_1} = \frac{100}{2}$

$\therefore T_1 = \frac{2}{100} = \frac{1}{50}$

(2) $x_2(t) = 5\sin 50t$

$\therefore x_2(t) = A\sin\omega_2 t$

$\therefore \omega_2 = 50$

$2\pi f = 50$

$f = \frac{50}{2\pi}$

$\frac{1}{T_2} = \frac{50}{2\pi}$

$T_2 = \frac{2\pi}{50} = \cancel{\frac{\pi}{25}}$

$$\therefore \frac{T_1}{T_2} = \frac{1}{50} \div \frac{2\pi}{50}$$

$$= \frac{1}{\cancel{50}} \times \frac{\cancel{50}}{2\pi}$$

$$= \frac{1}{2\pi}$$

$\therefore x(t)$ is non periodic.

(ii) $x(t) = 3 \sin 4t$

$\omega = 4$

$2\pi f = 4$

$f = \frac{4}{2\pi}$

$\therefore$ Non-Periodic Signal

(iii) $x(t) = 10 \sin(12\pi t) + 4 \sin(18\pi t)$

$x_1(t) = x_1(t) + x_2(t)$

$x_1 = 10 \sin(12\pi t)$

$x_1 = A \sin \omega_1 t$

$\omega_1 = 12\pi$

$2\pi f_1 = 12\pi$

$f_1 = 6$

~~$\frac{1}{T_1} = \frac{1}{6}$~~

~~$T_1 = 6$~~

$\frac{1}{T_1} = 6$

$\boxed{\therefore T_1 = 1/6}$

$x_2 = 4 \sin(18\pi t)$

$= A \sin \omega_2 t$

$\omega_2 = 18\pi$

$2\pi f = 18\pi$

$f = 9$

$\frac{1}{T_2} = 9$

$\boxed{\therefore T_2 = 1/9}$

$$\frac{T_1}{T_2} = \frac{1}{6} \div \frac{1}{9}$$

$$= \frac{1}{6} \times 9$$

$$= \frac{3}{2}$$

Hence periodic signal

(iv) $x(t) = 3 + t^2$ --- (1)

$\therefore$ A continous signal is said to be periodic if

$$x(t) = x(t + T_0)$$

Replace t with $t+T_0$ in Eq (1)

$$\therefore x(t + T_0) = 3 + (t + T_0)^2$$

$$= 3 + t^2 + 2tT_0 + T_0^2 \quad \text{--- (ii)}$$

$$\therefore x(t) \neq x(t + T_0)$$

$\therefore$ Aperiodic signal

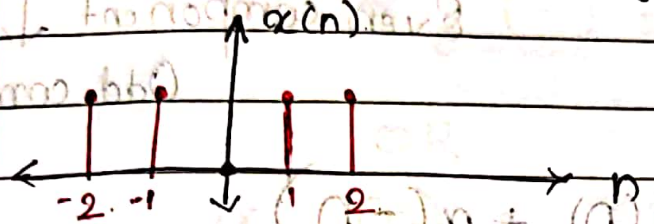
Even & Odd Signal :

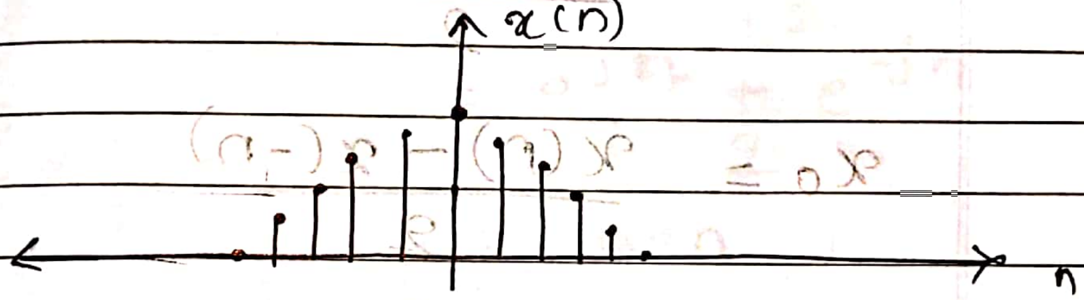
Even signal : (Symmetric Signal)

$$x(n) = x(-n) \quad \text{for all } n.$$

→ Symmetrical across vertical axis

→ Eg: cosine signal.





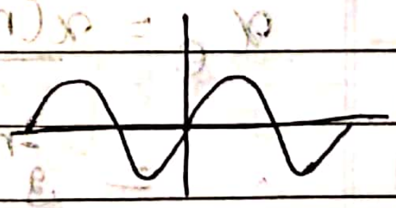
Odd Signal : (anti Symmetric Signal)

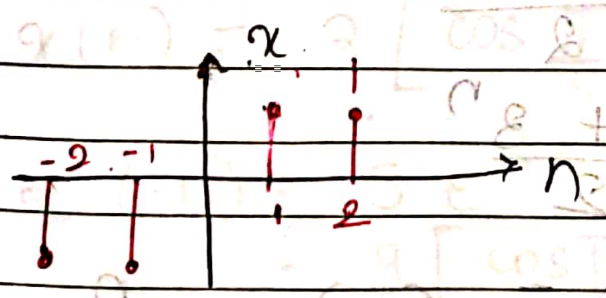
$\cancel{x(-n) = -x(n)}$

$$x(n) = -x(-n) \quad \text{for all } n.$$

→ Antisymmetric across vertical axis

→ Sine wave.





Any signal is sum of even + odd components

signal = even + odd

i.e: $\underbrace{x(n)}_{} = \underbrace{x_e(n)}_{\text{Even component}} + \underbrace{x_o(n)}_{\text{Odd component}}$

$$x_e = \frac{x(n) + x(-n)}{2}$$

$$x_o = \frac{x(n) - x(-n)}{2}$$

Eg: ① Find ~~whether~~ For given signal, even & odd component

$$x(n) = 3^{-n}$$

$$x(-n) = 3^{-(-n)}$$

$$x(-n) = 3^{n}$$

$$x_e = \frac{x(n) + x(-n)}{2}$$

$$= \frac{3^{-n} + 3^{n}}{2}$$

$$x_o = \frac{x(n) - x(-n)}{2} = \frac{3^{-n} - 3^{n}}{2}$$

eg (ii) $x(n) = e^{jn}$

$x(n) = \cos n + j\sin n$

$x(n) = x_e(n) + x_o(n)$

$e^{jn} = x_e(n) + x_o(n)$

$\therefore x_e(n) = \cos n$

$x_o(n) = j\sin n$

OR

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \frac{e^{jn} + e^{-jn}}{2}$$

$$= \cos n$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$= \frac{1}{2}(e^{jn} - e^{-jn})$$

$$= j\sin n$$

eg (iii) $x(n) = 3e^{j\pi/5 n}$

$$x(n) = 3\left[\cos\frac{\pi}{5}n + j\sin\frac{\pi}{5}n\right]$$

$$x(-n) = 3e^{-j\pi/5 n}$$

$$= 3\left[\cos\frac{\pi}{5}n - j\sin\frac{\pi}{5}n\right]$$

Even part $x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$

$= \frac{1}{2}\left[\left[3\cos\frac{\pi}{5}n + 3j\sin\frac{\pi}{5}n\right] + \left[3\cos\frac{\pi}{5}n - 3j\sin\frac{\pi}{5}n\right]\right]$

$= \frac{1}{2} \times 6\cos\frac{\pi}{5}n$

$= 3\cos\frac{\pi}{5}n.$

Odd part $x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$

$= \frac{1}{2}\left[\left(3\cos\frac{\pi}{5}n + 3j\sin\frac{\pi}{5}n\right) - \left(3\cos\frac{\pi}{5}n - 3j\sin\frac{\pi}{5}n\right)\right]$

$= \frac{1}{2}\left[6j\sin\frac{\pi}{5}n\right]$

$= 3j\sin\frac{\pi}{5}n$

Eg: $x(n) = \{\underset{\uparrow}{1}, 2, 1, 3\}$

$x(-n) = \{3, 1, 2, \underset{\uparrow}{1}\}$

$x_e(n) = \frac{x(n) + x(-n)}{2}$

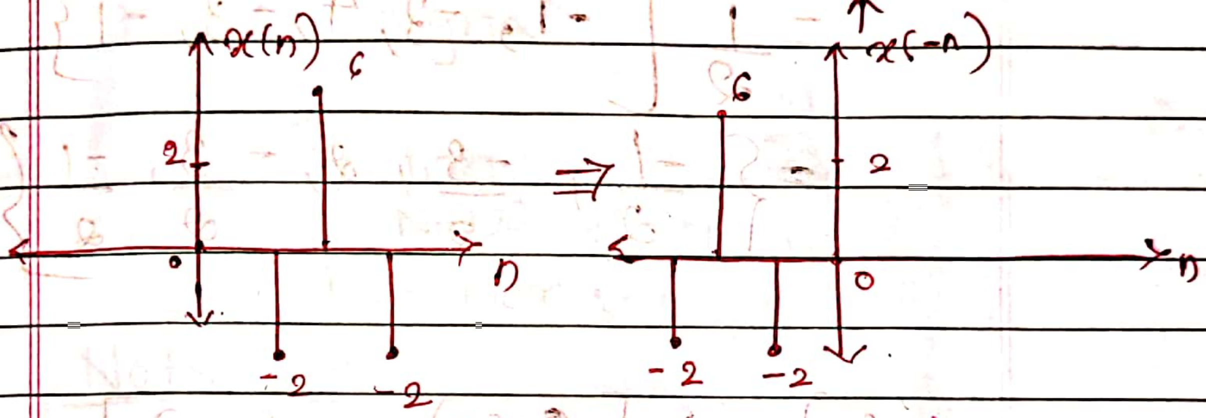
$= \frac{\{\underset{\uparrow}{1}, 2, 1, 3\} + \{3, 1, 2, \underset{\uparrow}{1}\}}{2}$

$$x_o(n) = \frac{\{3, 1, 2, \underset{\uparrow}{2}, 2, 1, 3\}}{2}$$

$$= \left\{\frac{3}{2}, \frac{1}{2}, 1, 1, 1, 1, 3\right\}$$

(*) $x(n) = \{\underset{\uparrow}{2}, -2, 6, -2\}$

$x(-n) = \{-2, 6, -2, \underset{\uparrow}{2}\}$



$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \frac{1}{2}\left[(-2+0), (6+0), (-2+0), \underset{\uparrow}{(2+2)}, (-2+0), (6+0), (-2+0)\right]$$

$$= \frac{1}{2}\left[-2, 6, -2, \underset{\uparrow}{4}, -2, 6, -2\right]$$

$$= \{-1, 3, -1, \underset{\uparrow}{2}, -1, 3, -1\}$$

$$x_o(n) = \frac{1}{2}\left(x(n) - x(-n)\right)$$

$\{1, -3, 1, 0, -1, 3, -1\}$ $= \frac{1}{2}\{2, -6, 2, \underset{\uparrow}{0}, -2, 6, -2\}$

(iii) $x(n) = \{-3, 1, 2, -4, 2\}$

$x(-n) = \{2, -4, 2, 1, -3\}$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$= \frac{1}{2}\left[(-3+2), (1-4), (2+2), (-4+1), (2-3)\right]$$

$$= \frac{1}{2}\left[-1, -3, 4, -3, -1\right]$$

$$= \left\{\frac{-1}{2}, \frac{-3}{2}, 2, -\frac{3}{2}, \frac{-1}{2}\right\}$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

$$= \frac{1}{2}\left[(-3-2), (1+4), (2-2), (-4-1), (-2+3)\right]$$

$$= \frac{1}{2}\left[-5, 5, 0, -5, 5\right]$$

$$= \left\{\frac{-5}{5}, \frac{5}{2}, 0, \frac{-5}{2}, \frac{5}{2}\right\}$$

* Energy & Power Signal

Energy signal:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

E = non-zero & finite.
then it is energy signal

Power signal

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

P = non zero & finite.

Note:

For Energy signal $\Rightarrow P = 0$

For Power signal $\Rightarrow E = \infty$

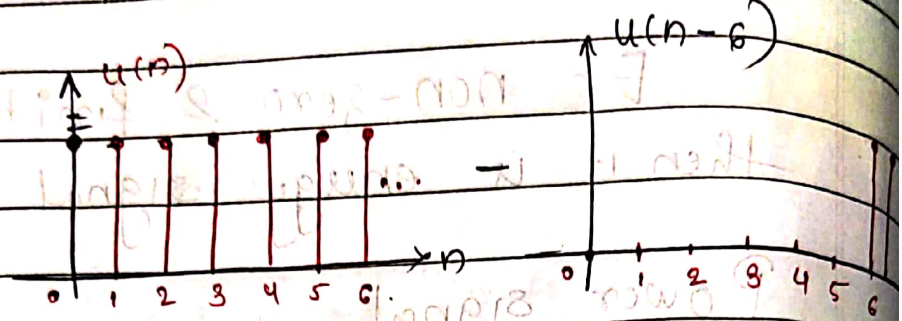
No signal is both Energy & power signal.

Every bounded & periodic signal ⇒ Power Signal
Every bounded & non-periodic signal ⇒ Energy-"-

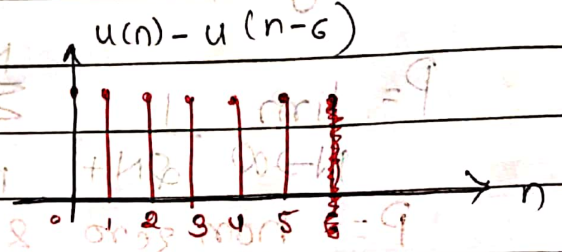
Eg: $x(n) = u(n) - u(n-6)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left| (u(n) - u(n-6)) \right|^2$$



=



$$E = \sum_{n=-\infty}^{-1} |0|^2 + \sum_{n=0}^{5} |1|^2$$

$$= 0 + 1 + 1 + 1 + 1 + 1 + 1$$

$$\boxed{E = 6 \text{ joules}}$$

Eg: $x(n) = n\,u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{-\infty}^{\infty} |n\,u(n)|^2$$

$$= \sum_{n=-\infty}^{-1} n^2 (u(n))^2 + \sum_{n=0}^{\infty} n^2 [u(n)]^2$$

$$= 0 + \sum_{n=0}^{\infty} n^2 \cdot (1)^2$$

$$= \sum_{n=0}^{\infty} n^2.$$

$$\boxed{E = \infty}$$

Power signal

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |n \cdot u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2 \cdot 1$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[0^2 + 1^2 + 2^2 + 3^2 + N^2 \right]$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[0 + 1 + 4 + \cdots + \infty \right]$$

$$= 0$$

Energy is finite & $P = 0$

$\therefore$ Energy signal.

(2) * $x(n) = (0.5)^n u(n)$

Sol// $$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |0.5^n u(n)|^2$$

$$\because \sum_{n=-\infty}^{-\infty} u(n) = 0$$

$$= \sum_{n=0}^{\infty} |0.5^n u(n)|^2$$

$$= \sum_{n=0}^{\infty} (0.25)^n \cdot 1$$

Geometric Series

$$\sum_{n=0}^{\infty} A^n = \frac{1}{1-A}$$

$$= \frac{1}{1-0.25}$$

$$= \frac{1}{0.75}$$

$$= 4/3 \text{ J}$$

$\Rightarrow$ Energy signal

Scanned by CamScanner

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2.$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| 0.5^n \, u(n) \right|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (0.25)^n \cdot 1$$

Geometric Series

$$\sum_{n=0}^{N} A^n = \frac{A^{N+1} - 1}{A - 1}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[\frac{0.025^{n+1} - 1}{0.25 - 1} \right]$$

$\lim_{N \to \infty} a^n = 0$ if $a < 1$

$\therefore 0.25^{n+1} = 0$

$$= \frac{1}{2 \cdot \infty + 1} \cdot \frac{0.25^{\infty+1} - 1}{-0.75} \to 0$$

$$= \frac{1}{\infty} \cdot 0$$

$$\boxed{\lim_{N \to \infty} \frac{1}{n} = 0}$$

$$= 0$$

$$= \lim_{N \to \infty} \frac{1/N}{2 + 1/N} = \frac{0}{2+0} = 0$$

$P = 0$

(x) $x(n) = n\,u(n)$

$$E = \sum_{-\infty}^{\infty} |x(n)|^2$$

$=$

$$E = \sum_{n=-\infty}^{\infty} |n\,u(n)|^2$$

$$= \sum_{n=0}^{\infty} n^2$$

$$= 1^2 + 2^2 + 3^2 + 4^2 + \dots \infty$$

$$= \infty$$

Not an Energy signal

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} n^2 u(n)$$

$$= \boxed{\lim_{N\to\infty} \frac{1}{2N+1}} \sum_{n=0}^{N} n^2 = \lim_{n\to\infty} \frac{1}{2N+1} \frac{n(n+1)(2n+1)}{6} = \lim_{n\to\infty} \frac{n(n+1)}{6}$$

$$= \infty$$

∴ Power is ∞.

∴ Not a Power signal

3. $$x(n) = \begin{cases} n^2 & 0 \le n \le 3 \\ 10-n & 4 \le n \le 6 \\ n & 7 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{-1} 0 + \sum_{n=0}^{3} (n^2)^2 + \sum_{n=4}^{6} (10-n)^2 + \sum_{n=7}^{9} n^2$$

$$= \cancel{0^2 + 1^2 + 2^2 + 3^2 + (10-4) + (10-5) + (10-6)}$$

$$\cancel{= 1 + 4 + 9 +}$$

$$= 0^4 + 1^4 + 2^4 + 3^4 + \sum_{n=4}^{6} (100 - 20n + n^2)$$

$$+ \sum 7^2 + 8^2 + 9^2$$

$$= 0 + 1 + 16 + 81 + 100 - 20 \times 4 + 4^2 +$$

$$100 - 20 \times 5 + 5^2 + 100 - 20 \times 6 + 6^2.$$

$$+ 49 + 64 + 81$$

$$= 369 \text{ J.}$$

So, Energy signal

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[\sum_{n=-1}^{-\infty} 0 + \sum_{n=0}^{3} n^4 + \sum_{n=4}^{6} (10-4)^2 + \sum_{n=7}^{9} n^2 \right]$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[369 \right]$$

$$= \frac{1}{2\infty + 1} \left[369 \right]$$

$$\frac{1}{\infty} 369 = 0.369$$

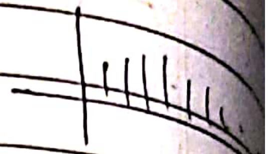
$$= 0$$

Power is zero. Energy sig

(*) Causal & Non Causal Signal.

Causal $x(n) = 0 \; ; n < 0$

$\Rightarrow$ Positive signal



Eg $u(n)$

~~Non-Causal~~

Anti-Causal $x(n) = 0 \; ; n > 0$

$\Rightarrow$ Negative signal

Eg $u(-n)$

Non-causal

$\Rightarrow$ A signal which exist for +ve & -ve time.

Convolution

$\Rightarrow$ Mathematical operation, just like mult[illegible] addition, to combine 2 signals to form a third signal.

$\Rightarrow$ denoted by $*$

$\Rightarrow$ $y(n) = x(n) * h(n)$

$\Rightarrow$ eqs

Types of Convolution

1. Linear Convolution
2. Circular Convolution
3. Linear using Circular Convolution

① Linear Convolution

→ Calculates o/p for any LTI (Linear Time Invariant System) given its input & its impulse response

Method 1: Matrix Method

(*) $x(n) = \{4, 2, \underset{\uparrow}{1}, 3\}$

$h(n) = \{1, \underset{\uparrow}{2}, 2, 1\}$

$y(n) = x(n) * h(n)$

	1	2	2	1
4	4	8	8	4
2	2	4	4	2
1	1	2	2	1
3	3	6	6	3

$y(n) = \{4, 10, 13, \underset{\uparrow}{13}, 10, 7, 3\}$

-3, -2, -1, 0

How to decide origin of y(n)

- lower index of x(n) + lower index of h(n)

$-2 - 1 = -3$

(*) Method 2: Graphical Method

Procedure:

1. Replace n with m.
   ∴ x(n) & h(n) is replaced to $x_1(m)$ & $h_1(m)$
2. Sketch $x_1(m)$ & h(m)
3. Fold h(m) to h(-m)

4. Shift $h(-m)$ to left. & Multiply $x(m)$ & $h(-m)$ & sum up all samples of o/p sequence

5. To get next sample of o/p sequence shift $h(-m)$ of previous step to one position right & multiply & sum.

6. Repeat Steps.

Eg:

$x(n) = \{\underset{\uparrow}{1}, 2, 0.5, 1\}$

$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$

$$y(n) = x(n) * h(n)$$
$$= \sum_{m=-\infty}^{\infty} x(m) * h(n-m)$$

lower index $[x(n)] = 0$

lower index $[h(n)] = -1$

$0 - 1 = -1$

∴ o/p sequence starts at -1

len $[x(n)] = 4$

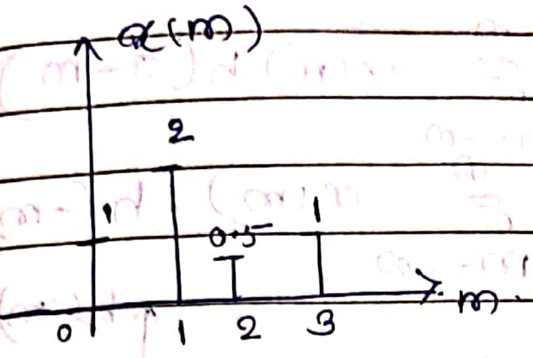
len $[h(n)] = 4$

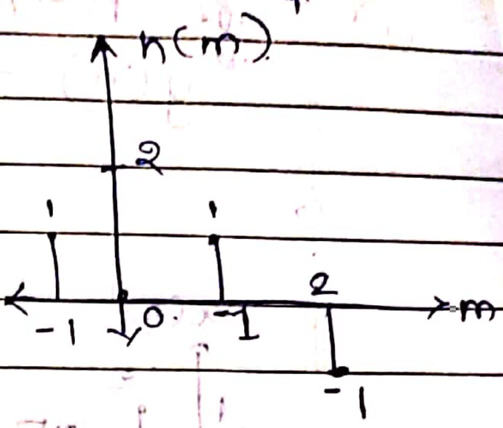
$4 + 4 - 1 = 7$

o/p sequence consists of 7 samples

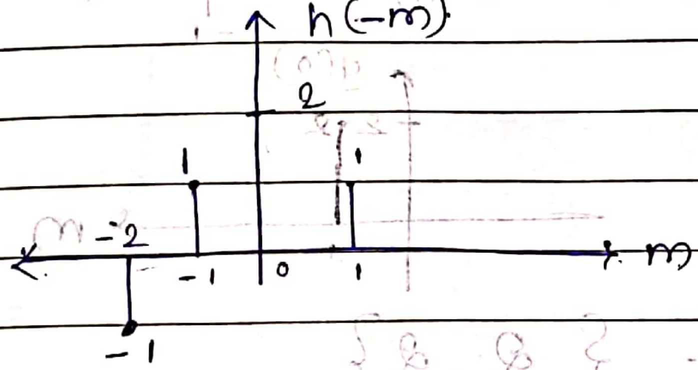
1. Replace $x(n) \to x(m) = \{1, 2, 0.5, 1\}$
$h(n) \to h(m) = \{1, 2, 1, -1\}$

2.





3. Fold $h(m)$ to $h(-m)$

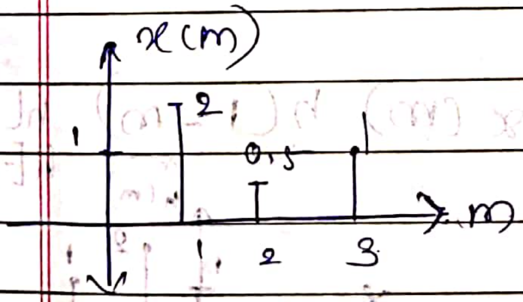


4.

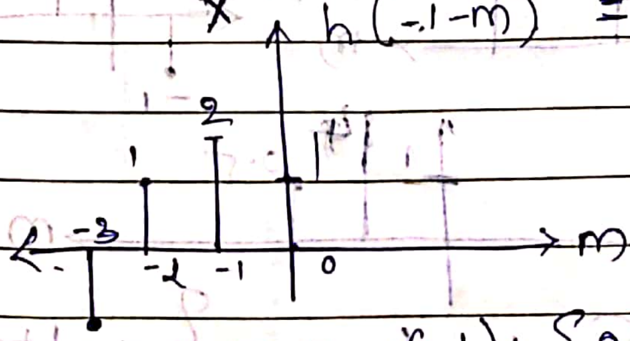
(i) $n = -1$

$$y(-1) = \sum_{m=-\infty}^{\infty} x(m) \underbrace{h(-1-m)}_{h[-1(1+m)]}$$

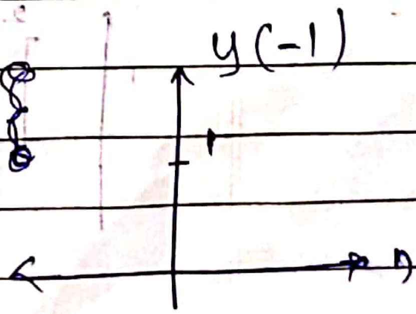
Fold   Advance



$\times$



$=$



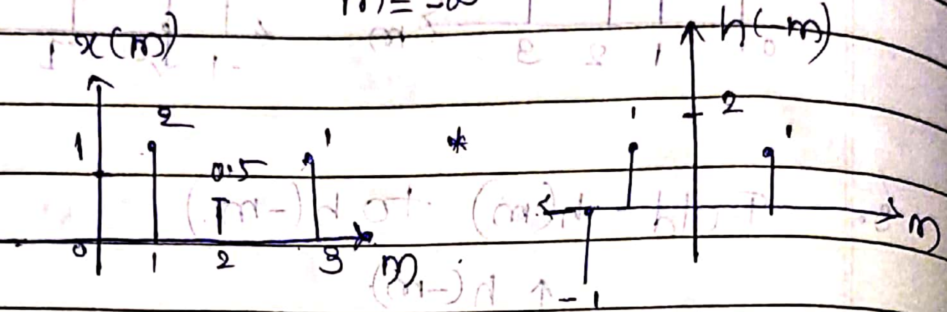
$y(-1) = \{0, 0, 0, 1, 0, 0, 0\}$

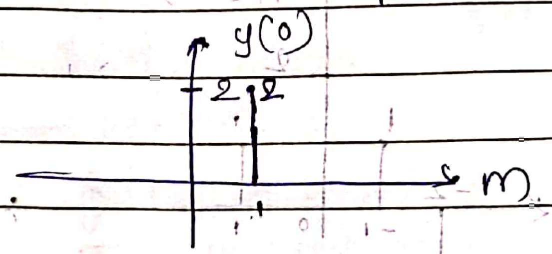
$y(-1) = 1$

(ii) for $n = 0$

$$y(0) = \sum_{m=-\infty}^{\infty} x(m)\, h(0-m)$$

$$= \sum_{m=-\infty}^{\infty} x(m) \cdot h(-m)$$



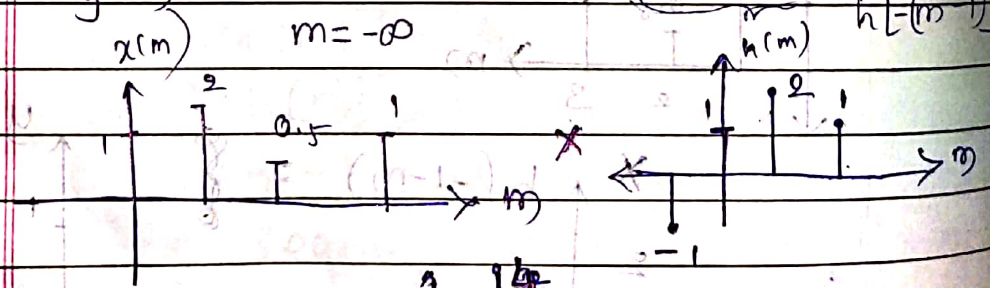


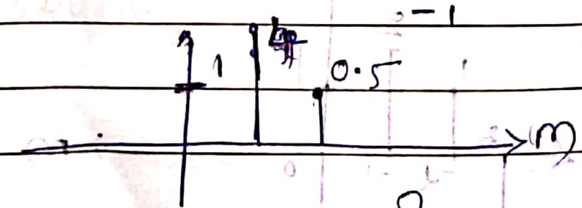
$y(0) = \{2, 2\}$

$y(0) = 2 + 2 = 4$

(iii) for $n = 1$

$$y(1) = \sum_{m=-\infty}^{\infty} x(m)\, h(1-m), \quad h[-m+1] \quad h[-(m-1)]$$





$y(1) = \{1, \cancel{4}, 0.5\} = 1 + \cancel{4} + 0.5$

$= 5.5$

(iv) $n = 2$

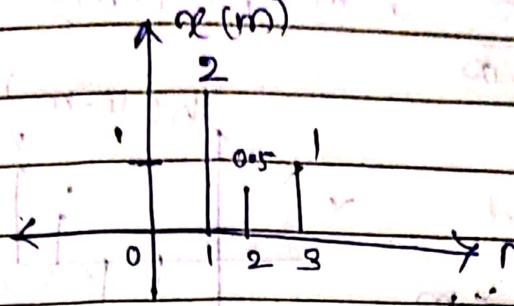
$$y(2) = \sum_{m=-\infty}^{\infty} x(m)\cdot h(2-m)$$

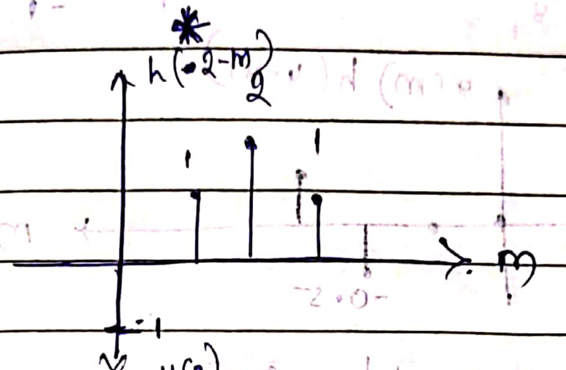
$h(2-m)$

$= h(-m+2)$

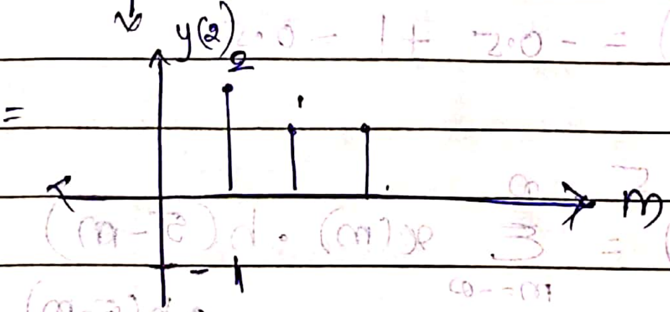
$= h[-(m-2)]$

↑ Fold ↑ delay





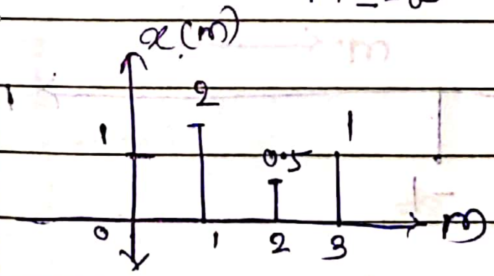
$=$



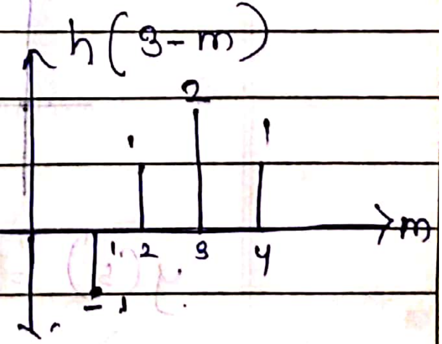
$y(2) = -1 + 2 + 1 + 1 = 3$

(v) When $n = 3$

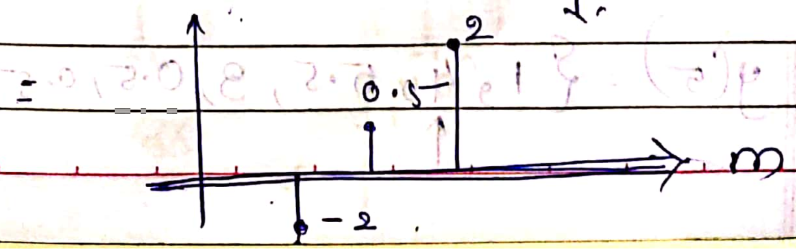
$$y(3) = \sum_{m=-\infty}^{\infty} x(m)\cdot h(3-m)$$



$\times$



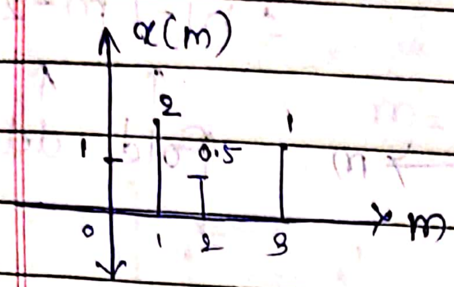
$=$



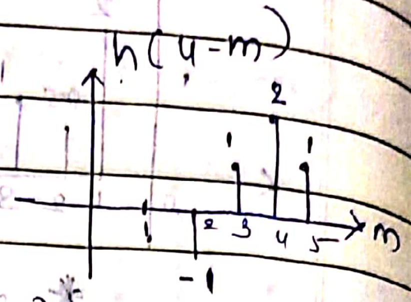
$y(3) = -2 + 0.5 + 2 = 0.5$

(vi) When $n = 4$

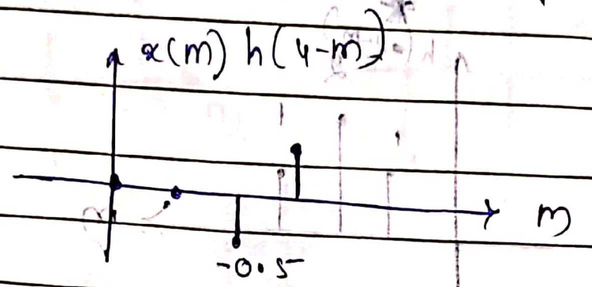
$$y(4) = \sum_{m=-\infty}^{\infty} x(m)\cdot h(4-m)$$



$\times$



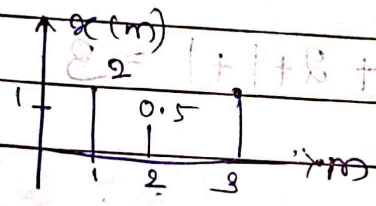
=



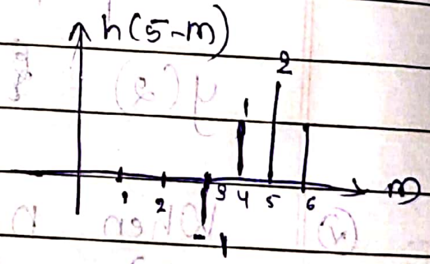
$y(4) = -0.5 + 1 = 0.5$

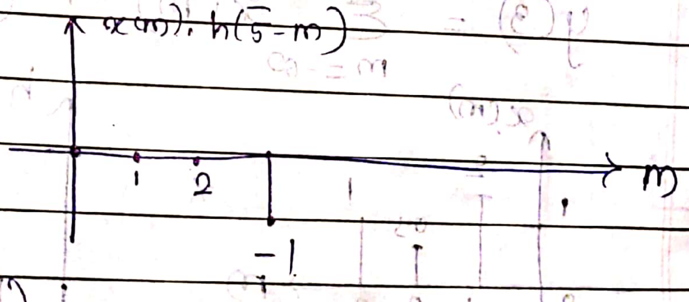
(vii) $n = 5$

$$y(5) = \sum_{m=-\infty}^{\infty} x(m)\cdot h(5-m)$$



$\times$





$y(5) = -1$

$$y(5) = \{1, \underset{\uparrow}{4}, 5.5, 3, 0.5, 0.5, -1\}$$

(2) $x(n) = \{1, 2, 3, 1\}$
$\uparrow$ (under the first 1)

$h(n) = \{1, 2, 1, -1\}$
$\uparrow$ (under the 2)

Replace $x(n) \rightarrow x(m) = \{1, 2, 3, 1\}$
$\uparrow$ (under the first 1)

$h(n) \rightarrow h(m) = \{1, 2, 1, -1\}$
$\uparrow$ (under the 2)

(i) lower index $[x(n)] = 0$
— " — $[h(n)] = -1$

$0 - 1 = -1$

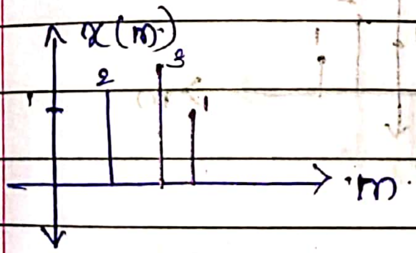
O/P sequence starts at -1

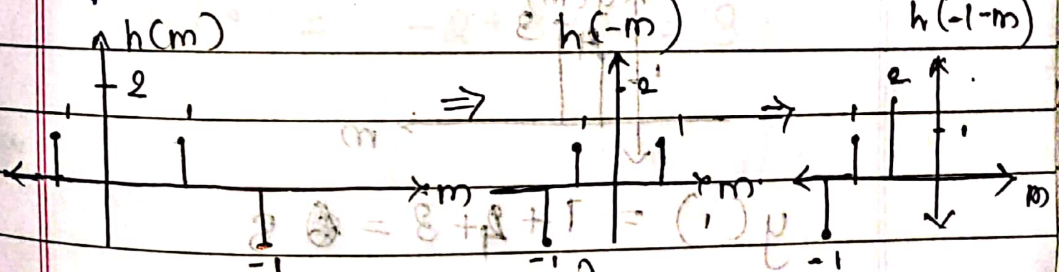
Len $[x(n)] = 4$
Len $[h(n)] = 4$
$4 + 4 - 1 = 7$

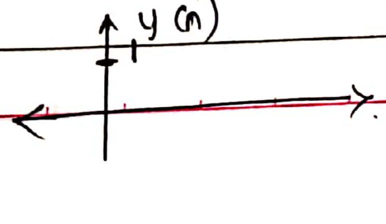
O/P sequence = 7 samples

(i) When $n = -1$

$$y(-1) = \sum_{m=-\infty}^{\infty} x(m) \cdot h(-1-m)$$





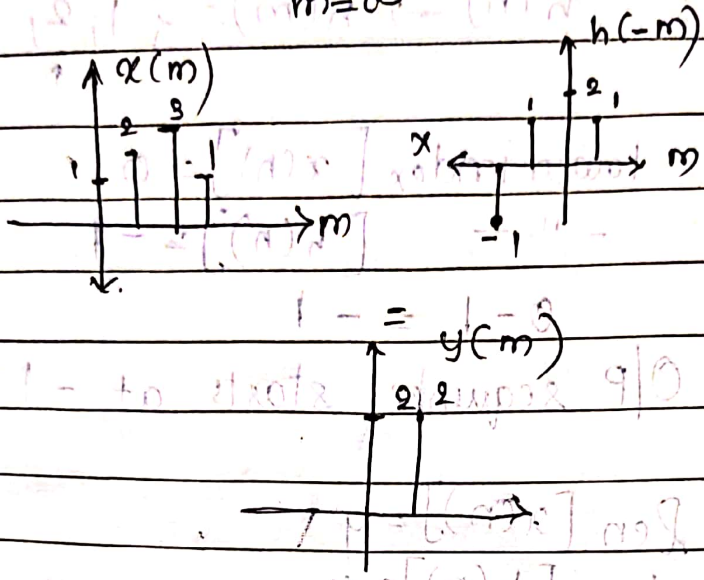


$$\boxed{y(-9) = 1}$$

(ii) $n = 0$

$$y(0) = \sum_{m=-\infty}^{\infty} x(m) \cdot h(0-m)$$

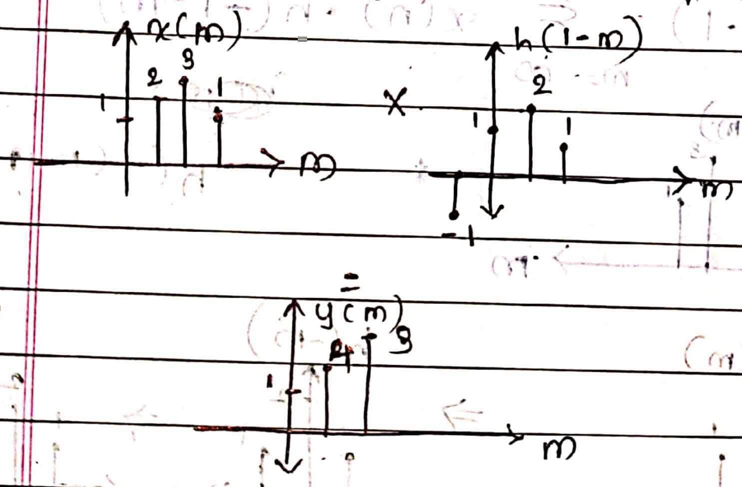
$$= \sum_{m=\infty}^{\infty} x(m) \cdot h(-m)$$



$$y(0) = 2 + 2 = 4$$

(iii) $n = 1$

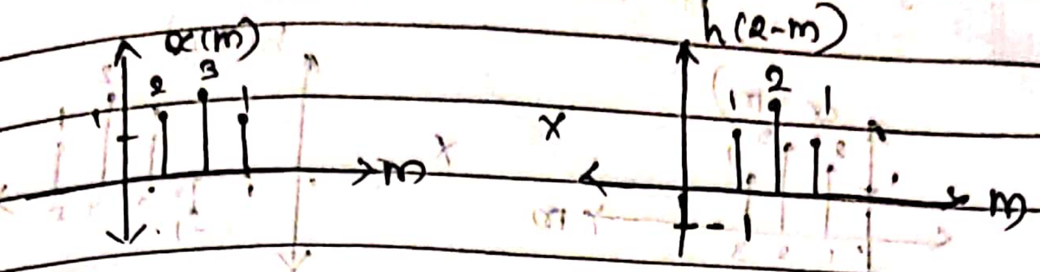
$$y(1) = \sum_{m=-\infty}^{\infty} x(m) \cdot \underbrace{h(1-m)}_{h(-m+1)} = h[-(m-1)]$$



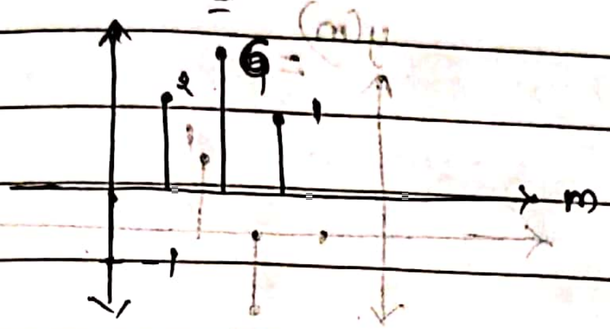
$$y(1) = 1 + 4 + 3 = 8$$

(iv) $n = 2$

$$y(2) = \sum_{m=-\infty}^{\infty} x(m)\, h(2-m)$$



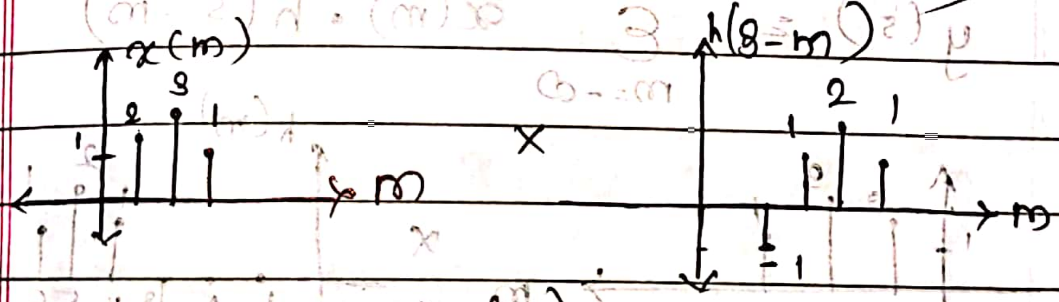
=



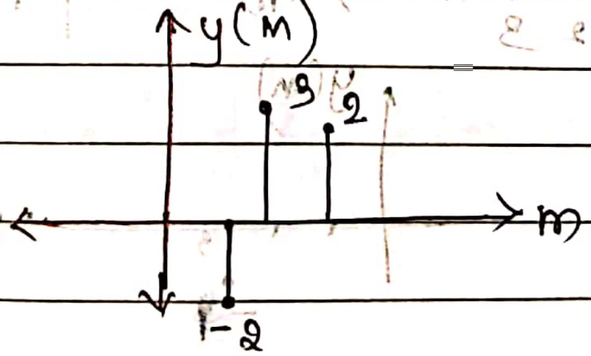
$$y(2) = -1+2+6+1 = 8$$

(v) $n = 3$

$$y(3) = \sum_{m=-\infty}^{\infty} x(m) \cdot h(3-m)$$



=

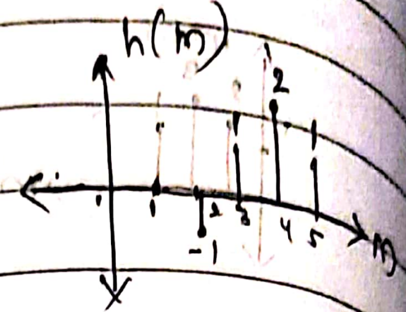


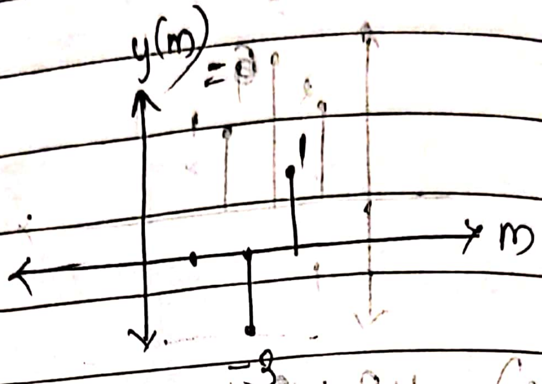
$$y(3) = -2+3+2 = 3$$

(vi) $n = 4$

$$y(4) = \sum_{m=-\infty}^{\infty} x(m)\cdot h(4-m)$$

×

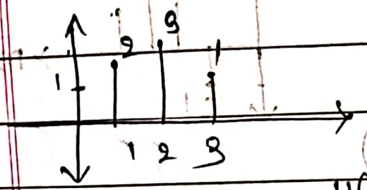




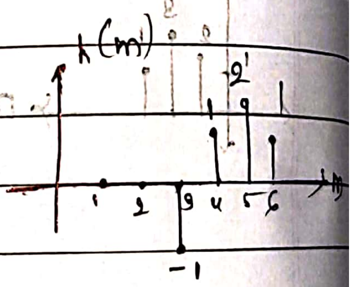
$$y(4) = -3 + 1 = -2$$

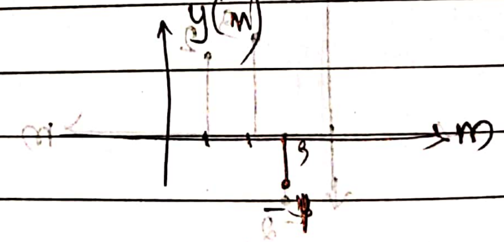
(vii) $n = 5$

$$y(5) = \sum_{m=-\infty}^{\infty} x(m)\cdot h(5-m)$$



×





$y(5)$

$$y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$$

Circular Convolution

→ Circular Convolution is performed on periodic sequences

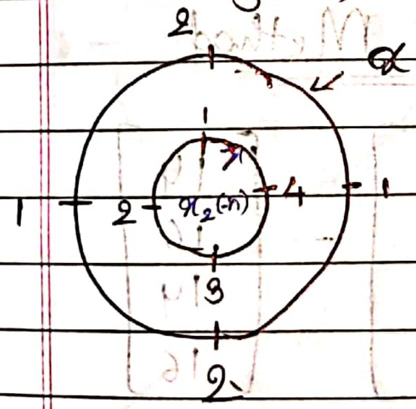
Formula:

$$\boxed{\begin{aligned} x_3(n) &= x_1(n) \circledast x_2(n) \\ x_3(n) &= \sum_{m=0}^{N-1} x_1(m) \cdot x_2(n-m) \end{aligned}}$$

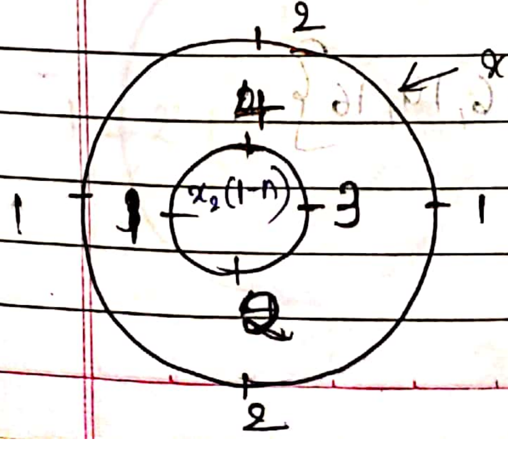
Method 1 : Graphical Method
(Concentric Circle Method)

eg: $x_1(n) = \{1, 2, 1, 2\}$
$x_2(n) = \{4, 3, 2, 1\}$

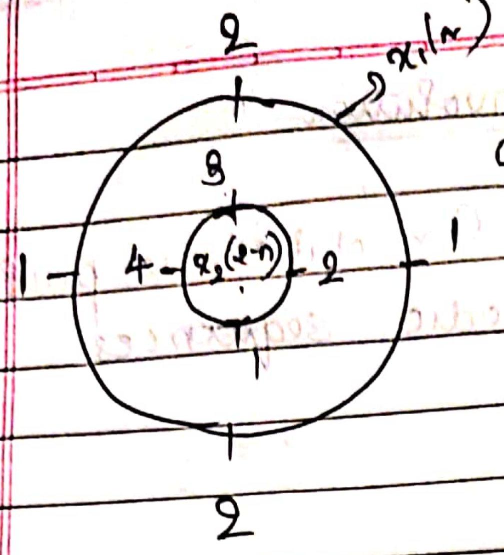
$$x_3(n) = x_1(n) \circledast x_2(n)$$



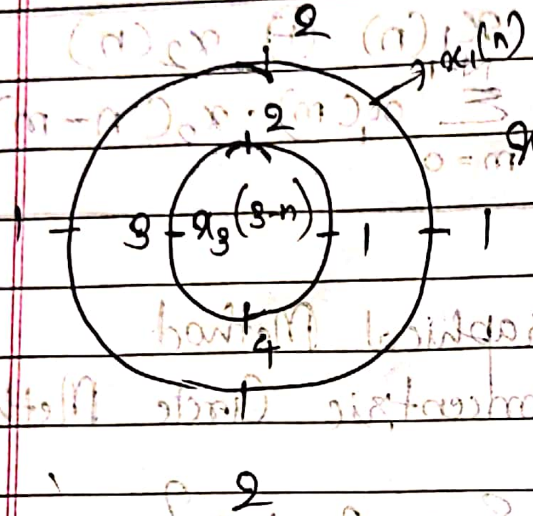
$x_3(0) = 1\times4 + 2\times1 + 2\times1 + 3\times2 = 14$



$x_3(1) = 1\times3 + 4\times2 + 1\times1 + 2\times2$
$= 16$



$$x_3(2) = 2\times1 + 3\times2 + 1\times4 + 1$$
$$= 14$$



$$x_4(2) = 1\times1 + 2\times2 + 3\times1 + 4\times2$$
$$= 16$$

$$x_3(n) = \{14, 16, 14, 16\}$$

Method 2: Matrix Method

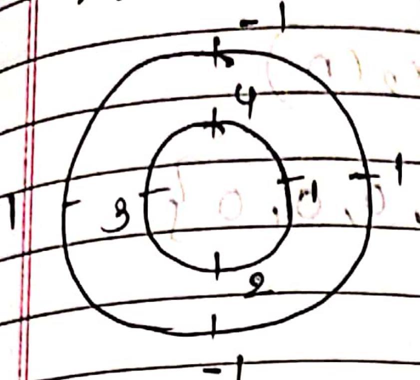
$$\begin{bmatrix} 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

$$x_3(n) = \{14, 16, 14, 16\}$$

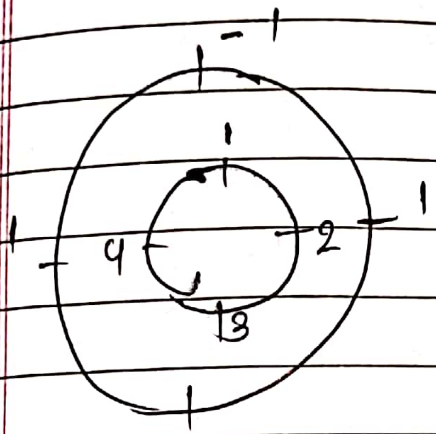
eg2 $x_1(n) = \{1, -1, 1, -1\}$

$x_2(n) = \{1, 2, 3, 4\}$

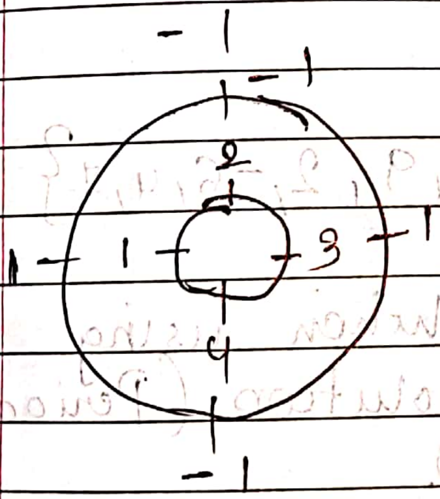
$\boxed{x_3(n) = x_1(n) \circledast x_2(n)}$



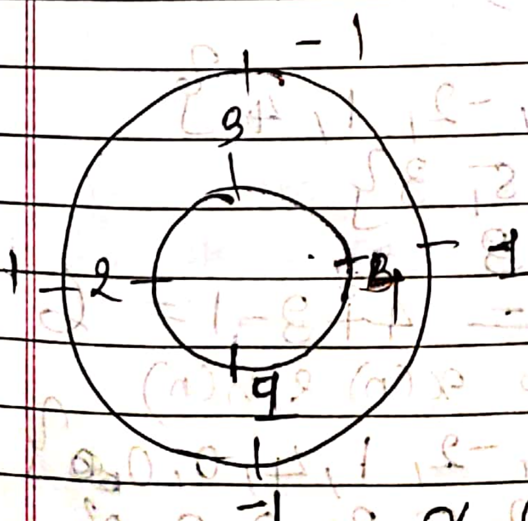
$x_3(0) = 1 - 4 + 3 - 2$
$= -2$



$x_3(1) = 2 - 1 + 4 - 3$
$= 2$



$x_3(2) = -2 + 3 + (-4) + 1$
$= -2$



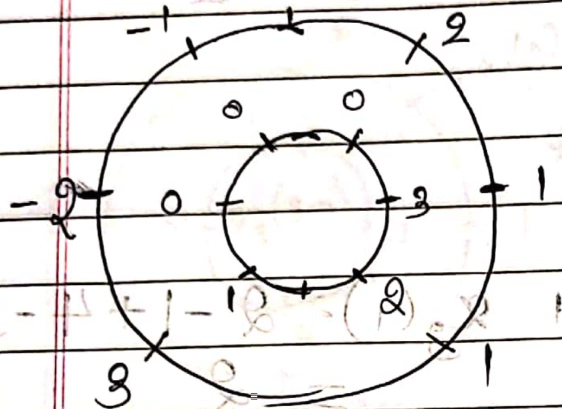
$x_3(3) = 4 - 3 + 2 - 1$
$= 2$

$x_3(n) = \{-2, 2, -2, 2\}$

Eg. $x_1(n) = \{1, 2, -1, -2, 3, 1\}$
$x_2(n) = \{3, 2, 1\}$

Pad zeros to $x_2(n)$:

$x_2(n) = \{3, 2, 1, 0, 0, 0\}$



⋮

Ans: $x_3(n) = \{8, 9, 2, -6, 4, 7\}$

(*) Linear Convolution using Circular Convolution (Periodic Convolution).

Eg: $x(n) = \{3, -2, 1, 4\}$
$h(n) = \{2, 5, 3\}$
$M = 4 \quad N = 3$
$\therefore M + N - 1 = 4 + 3 - 1 = 6$
Add zeros to $x(n)$ & $h(n)$
$\therefore x(n) = \{3, -2, 1, 4, 0, 0\}$
$h(n) = \{2, 5, 3, 0, 0, 0\}$

$$\begin{bmatrix} 3 & 0 & 0 & 4 & 1 & -2 \\ -2 & 3 & 0 & 0 & 4 & 1 \\ 1 & -2 & 3 & 0 & 0 & 4 \\ 4 & 1 & -2 & 3 & 0 & 0 \\ 0 & 4 & 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 1 \\ 7 \\ 23 \\ 12 \end{bmatrix}$$

$y(n) = \{3, -2, 1, 4\} * \{2, 5, 3\}$

$= \{3, -2, 1, 4, 0, 0, 0\} \circledast \{2, 5, 3, 0, 0, 0\}$

$= \{6, 11, 1, 7, 23, 12\}$

Correlation

- Mathematical operation used to compare 2 signals.
- Used in radar & sonar systems to find location of target by comparing the transmitted & reflected signals.
- Other applications are:
  - Image processing
  - Control Engineering
  - Seperate Noise from signals

2 Types of Correlation

(1) Cross-Correlation (2) Auto-Correlation

$R_{xy}(n) = x(n) * h(-n)$ $R_{xx}(n) = x(n) * x(-n)$

$$R_{xy}(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot y(k-n)$$

$$R_{xx}(n) = \sum_{k=-\infty}^{\infty} x(k)\, x(k-n)$$

(i) Cross Correlation

$x(n) = \{2, 3, 1, 4\}$

$y(n) = \{1, 3, 2, 1\}$

$y(-n) = \{1, 2, 3, 1\}$

$$\boxed{R_{xy}(n) = x(n) * y(-n)}$$

$x(n)$ \ $y(-n)$	1	2	3	1
2	2	4	6	2
3	3	6	9	3
1	1	2	3	1
4	4	8	12	4

$R_{xy}(n) = \{2, 3+4, 1+6+6, 4+2+9+2, 8+3+3, 12+1, 4\}$

$= \{2, 7, 13, 17, 14, 13, 4\}$

(ii) Auto correlation

$x(n) = \{2, 3, 1, 4\}$

$x(n) = \{4, 1, 3, 2\}$

$R_{xx}(n) = x(n) * x(-n)$

$x(n)$ \ $x(-n)$	4	1	3	2
2	8	2	6	4
3	12	3	9	6
1	4	1	3	2
4	16	4	12	8

$R_{xx}(n) = \{8, 14, 13, 30, 13, 14, 8\}$

eg: $x(n) = \{3, \underset{\uparrow}{5}, 1, 2\}$

$h(n) = \{\underset{\uparrow}{1}, 4, 3\}$

$h(-n) = \{3, 4, \underset{\uparrow}{1}\}$

	3	4	1
3	9	12	3
5	15	20	5
1	3	4	1
2	6	8	2

$R_{xy}(n) = x(n) * h(n)$

$= \{9, \underset{\uparrow}{27}, 26, 15, 9, 2\}$

$-1 - 0 = -1$

eg: How $x(n) = \{\underset{\uparrow}{2}, 5, -4\}$

Ans: $R_{xx}(n) = \{-8, -10, 45, -10, -8\}$

Scanned by CamScanner

⊛ Discrete Time Systems

System acts on input Signal & transforms it into an o/p signal

$$x(n) \xrightarrow{i/p} \boxed{\text{DT System}} \xrightarrow{o/p} y(n)$$

Classification of DT systems :

1. Static / Dynamic systems
2. Causal / Non-causal Systems
3. Linear / non-Linear - "
4. Time invariant / time invariant
5. Stable / Unstable

1. Static (Memoryless) & Dynamic (Memory) Systems

Static (Memoryless) → if o/p depends only on present i/p

Dynamic (Memory) → if o/p depends ~~only~~ on past & future i/ps.

Eg: Static systems

(i) $y(n) = x(n)$

(ii) $y(n) = 2x^2(n)$

Eg: Dynamic systems

(i) $y(n) = x(2n)$

(ii) $y(n) = x(n) + x(n-2)$

(iii) $y(n) = x(n) + 4\{y(n-1) + 4y(n-2)$

Eg: Find whether following systems are static or dynamic.

(i) $y(n) = x(n+2)$

o/p depends on future i/ps

$\therefore$ Dynamic system

(ii) $y(n) = x^2(n)$

o/p depends on present i/p

$\therefore$ Static System

(iii) $y(n) = x(n-2) + x(n)$

o/p depends on past i/p

$\therefore$ Dynamic System

Causal & Non Causal Systems

Causal { → o/p of system depends on present & past i/p
→ Causal system are real time systems
→ They are physically realizable

Eg: $y(n) = n\,x(n)$
$y(n) = x(n-2) + x(n-1) + x(n)$

Non causal → o/p depends on future i/p.
→ O/p is produced even before i/p is given
→ Non-exist in real time
→ Not physically realizable

Eg: $y(n) = x(n) + x(2n)$
$y(n) = x^2(n) + 2x(n+2)$

Eg: (i) $y(n) = x(n) + x(n-2)$

$n = -2$ $y(-2) = x(-2) + x(-4)$
$n = 0$ $y(0) = x(0) + x(-2)$
$n = +2$ $y(2) = x(2) + x(0)$

∴ o/p depends on present & past
⇒ Causal Systems

(ii) $y(n) = x(2n)$

$n = 0 \quad y(0) = x(0) \rightarrow$ Present

$n = -2 \quad y(-2) = x(-4) \rightarrow$ past

$n = 2 \quad y(2) = x(4) \rightarrow$ future

for positive values of n, the O/P depends on future values of I/P

$\Rightarrow$ Non - causal

(iii) $y(n) = \sin[x(n)]$

$\Rightarrow$ Causal.

(iv) $y(n) = x(-n)$

$n = -2 \quad y(-2) = x(2) \Rightarrow$ future

$n = 0 \quad y(0) = x(0) \rightarrow$ present

$n = 2 \quad y(2) = x(-2) \rightarrow$ past

For -ve 'n' O/P depends on future values of I/P

System is noncausal

* Time invariant & Time variant system:

→ A system is time invariant if its i/p / o/p characteristics donot change with time.

$y(n) = x(n) - x(n-1)$.

$y(n) = T[x(n)]$

$= x(n) - x(n-1)$

Delay i/p by k units & apply to system

$\therefore y(n,k) = x(n-k) - x(n-k-1)$ — (I) i.e o/p

Now delay y(n) by k units in time

$$y(n-k) = \overset{x(n-k)}{-} x(n-k-1)$$ — (II)

From (I) & (II)

$y(n,k) = y(n-k)$

⇒ Time invariant

2) $y(n) = n\,x(n)$

$y(n) = T[x(n)]$

$= n\,x(n)$

Delay i/p $x(n)$ by k units in time

$y(n,k) = n\,x(n-k)$ — (1)

Now, delay o/p y(n) by k units

$y(n-k) = (n-k)x(n-k)$

$= n\,x(n-k) - k\,x(n-k)$ —— II

From (I) & II

$y(n,k) \neq y(n-k)$

$\therefore$ System is time-variant system

(3) $y(n) = x(-n)$

$y(n) = T[x(n)] = x(-n)$

Delay i/p by k units

$y(n,k) = x(-n-k)$ —— I

Delay o/p y(n) by k units.

$y(n-k) = x(-(n-k))$

$= x(-n+k)$ —— II

$y(n,k) \neq y(n-k)$

$\therefore$ System is time variant

(4) $y(n) = x(n) \cos \omega_0 n$

$y(n) = T[x(n)]$

$= x(n) \cdot \cos \omega_0 n$

Delay i/p by k units

$\therefore y(n,k) = x(n-k) \cdot \cos \omega_0 n$ —— I

Delay o/p by k units

$y(n-k) = x(n-k) \cdot \cos \omega_0 (n-k)$ —— II

From I & II $y(n,k) \neq y(n-k)$

System is time variant

~~From I & II~~

~~$y(n,k) \neq y(n-k)$~~

(5) $y(t) = t\,x(t)$

$y(t) = T[x(t)]$

$= t\,x(t)$

Delay i/p by k units

$y(t,k) = t\,x(t-k)$ — I

Delay o/p by k units

$y(t-k) = (t-k)\,x(t-k)$ — II

$\therefore$ System time variant.

(6) $y(t) = e^{x(t)}$

$y(t) = T[x(t)]$

$y(t,k) = e^{x(t-k)}$ — I

$y(t-k) = e^{x(t-k)}$ — II

From I & II

$y(t,k) = y(t-k)$

$\therefore$ System is time invariant

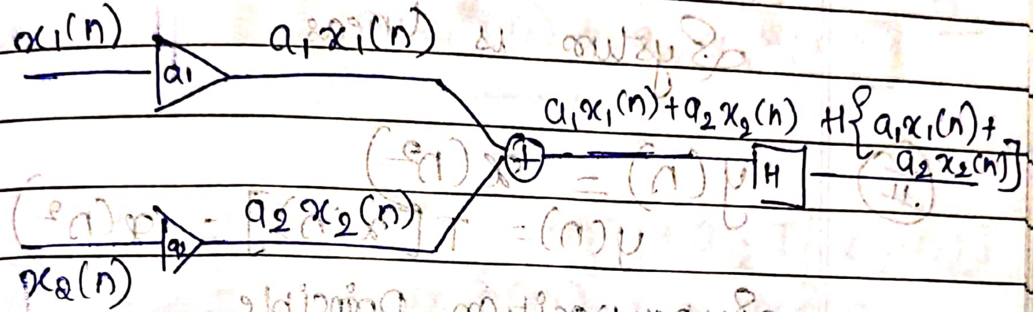
(4) Linear & Non-Linear Systems

Linear $\rightarrow$ satisfies superposition principle

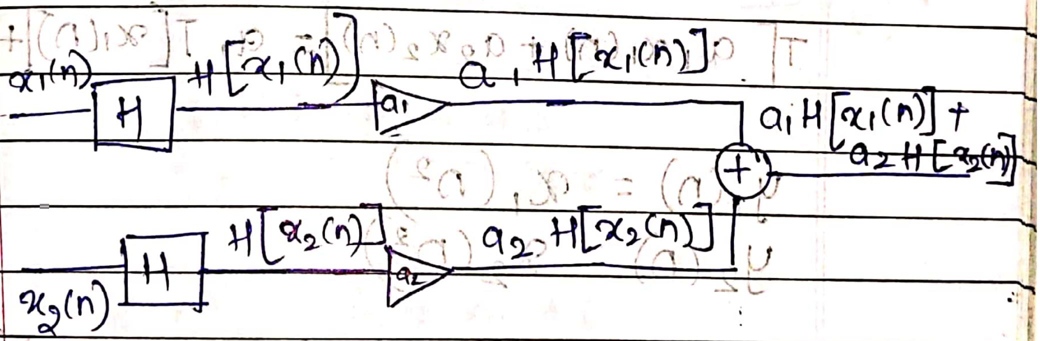
① ~~$y(n) = n\,x(n)$~~

Superposition Principle:

$$H\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 H\{x_1(n)\} + a_2 H\{x_2(n)\}$$



=



Eg: $y(n) = n\,x(n)$

$x(n) \rightarrow \boxed{H} \rightarrow y(n) = H\{x(n)\} = n\,x(n)$

Consider 2 signals $x_1(n)$ & $x_2(n)$

$x_1(n) \rightarrow \boxed{H} \rightarrow y_1(n) = H\{x_1(n)\} = n\,x_1(n)$

$x_2(n) \rightarrow \boxed{H} \rightarrow y_2(n) = H\{x_2(n)\} = n\,x_2(n)$

$$a_1 H\{x_1(n)\} + a_2 H\{x_2(n)\} = a_1 n\,x_1(n) + a_2 n\,x_2(n) \quad \text{—①}$$

Now $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$.

$$x_3 \to \boxed{H} \to H[x_3(n)] =$$

$$H[x_3(n)] = H[a_1 x_1(n) + a_2 x_2(n)]$$
$$= n\,[a_1 x_1(n) + a_2 x_2(n)]$$
$$= a_1 n\, x_1(n) + a_2 n\, x_2(n) \quad \text{(II)}$$

From I & (II)

System is linear.

(II) $y(n) = x(n^2)$

$$y(n) = T[x(n)] = x(n^2)$$

Superposition principle,

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$y_1(n) = x_1(n^2)$$
$$y_2(n) = x_2(n^2)$$

$$\text{L.H.S} = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= a_1 x_1(n^2) + a_2 x_2(n^2) \quad \text{(I)}$$

$$\text{R.H.S} = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$= a_1 x_1(n^2) + a_2 x_2(n^2) \quad \text{(I)}$$

$\text{L.H.S} = \text{R.H.S}$

$\therefore$ System is Linear

(c) $y(n) = x^2(n)$

$y(n) = T[x(n)] = x^2(n)$

Superposition Principle,

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$L.H.S = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= [a_1 x_1(n) + a_2 x_2(n)]^2 \quad \text{--- (I)}$$

$$R.H.S = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$= a_1 x_1^2(n) + a_2 x_2^2(n) \quad \text{--- (II)}$$

$L.H.S \neq R.H.S$

$\therefore$ Non Linear

(d) $y(n) = Bx(n) + C$

$y(n) = T[x(n)] = Bx(n) + C$

Super position principle,

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$L.H.S = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= B[a_1 x_1(n) + a_2 x_2(n)] + C$$

$$R.H.S = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$= a_1[Bx_1(n) + C] + a_2[Bx_2(n) + C]$$
$$= a_1 B x_1(n) + a_1 C + a_2 B x_2(n) + a_2 C$$

$R.H.S \neq L.H.S$

$\therefore$ System is non-linear.

** Stable & Unstable System

$$y(n) = x(n)$$

Now, $n \Rightarrow$ can have infite value $\Rightarrow \infty$

But, $x(n)$ will produce finite value.

$\therefore$ Stable system

Eg: Case 1:
$$y(n) = n\,x(n)$$
$$= \infty \; x(\infty)$$
$$= \infty \times \text{finite}$$
$$= \infty$$

$\therefore$ Unstable system

Case 2: $y(n) = n \cdot x(n)$
$$= 0 \cdot x(\infty)$$
$$= 0$$

So stable

(*) $y(n) = \cos[x(n)]$

$\cos\theta$ lies between -1 to $+1$ for any value of θ

$\therefore$ ~~o/p y(n) is bounded~~

$\therefore$ Stable system

(ii) $y(n) = x(-n-3)$

$\Downarrow$

~~for~~ finite values

$\therefore$ $y(n)$ = Stable Systems

(i) $y(n) = \cos[x(n)]$

Check whether these are

1) Static or Dynamic

2) Linear or non linear

3) Shift invariant or shift variant

4) Causal or non-causal

5) Stable or unstable

(i) $y(n) = \cos[x(n)]$

(i) Static system $\Rightarrow$ O/P depends on present i/p

(ii) Linear & Non Linear

$y(n) = T[x(n)] = \cos[x(n)]$

Superposition principle

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T_1[x_1(n)] + a_2 T[x_2(n)]$

$$L.H.S = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= \cos[a x_1(n) + a_2 x_2(n)]$$

$$R.H.S = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$= a_1 . \cos[x_1(n)] + a_2 . \cos[x_2(n)]$$

$$L.H.S \neq R.H.S$$

$\therefore$ Nonlinear System

3) Shift Invariant vs Shift variant

$y(n) = \cos[x(n)]$

$= T[x(n)] = \cos x(n)$

$$y(n,k) = \cos[x(n-k)] \quad \text{--- (1)}$$
$$y(n-k) = \cos[x(n-k)] \quad \text{--- (2)}$$

(1) & (2)

$$y(n,k) = y(n-k)$$

$\Rightarrow$ ~~Linear~~ Time variant System

4) Causal & Non Causal

$y(n) = \cos[x(n)]$

Causal System as o/p depends on present i/p

5) Stable & Unstable

$y(n) = \cos[x(n)]$

For bounded value of $x(n)$ the cosine function has bounded o/p. $\Rightarrow$ Stable system

Thus the system is : Static
Non linear
Time variant
Causal
Stable

(x) $y(n) = \sum_{K=-\infty}^{n+1} x(K)$

(i) Static or Dynamic
System's o/p for n^{th} Sample is equal to sum of past i/p, present & one next sample
$\Rightarrow$ ~~Causal~~ Dynamic

(ii) Linear & Non-linear System
$$y(n) = \sum_{K=-\infty}^{n+1} x(K)$$
$$= x(-\infty) + \cdots x(-2) + x(-1) + x(0) + \cdots + x(n) + x(n+1)$$
$\Rightarrow$ System is Linear since it is summation of individual i/ps. Summation operations is a linear operation Hence Linear system

(iii) Shift variant & Invariant
y(n) is linear sum of i/ps.
If we shift the i/ps, then there will be corres. shift in o/p
Hence shift invariant.

iv) Causal & Non Causal

o/p $y(n)$ can be expressed as $y(n) = x(-\infty) + \cdots + x(0) + \cdots x(n) + \cdots$

o/p $y(n)$ denpends on past i/p like $x(-\infty)$, present $x(n)$ & future $x(n+1)$

$\therefore$ Non Causal

5) Stable / Unstable

Consider the system has $x(k)$ as unit step seq $u(k)$.

$$u(k) = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$\therefore$ System equation is

$$y(n) = \sum_{k=0}^{n+1} u(k)$$

$$= 1+1+1+ - (n+1)$$

$\therefore$ if $n \to \infty$

$y(n) = \infty$

Thus $u(k)$ is unit sample seq & it is bounded. But $y(n)$ is unbounded as $n \to \infty$

hence unstable system.

(iii) $y(n) = x(n) \cos(\omega_0 n)$

(i) Static / Dynamic.

$\Rightarrow$ o/p depends on present i/p

$\therefore$ Static system

(ii) Linear / NonLinear

Superposition principle.

$L.H.S = T[a_1 x_1(n) + a_2 x_2(n)]$

$= [a_1 x_1(n) + a_2 x_2(n)] \cos(\omega_0 n)$

$= a_1 x_1(n) \cos\omega_0 n + a_2 x_2(n) \cdot \cos(\omega_0 n)$

$R.H.S = a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a_1 x_1(n) \cos(\omega_0 n) + a_2 x_2(n) \cos(\omega_0 n)$

$L.H.S = R.H.S$

Linear System

(iii) Time variant & Time invariant

$y(n) = x(n) \cos(\omega_0 n)$

$y(n,k) = x(n-k) \cos\omega_0 n$ — (1)

$y(n-k) = x(n-k) \cos\omega_0 (n-k)$ — (2)

$(1) \neq (2)$

$\Rightarrow$ ~~Non Linear~~ Time invariant

(iv) Causal & Non Causal

$y(n) = x(n) \cos(\omega_0 n)$

$\Rightarrow$ o/p depends on present i/p

$\therefore$ Causal.

(v) Stable / Unstable

$\cos(\omega_0 n) \Rightarrow$ Bounded & $x(n) \Rightarrow$ Bounded

$\Rightarrow$ Stable

Module 2:

Discrete Fourier Transform

$\cos \pi = -1 \qquad \sin \pi = 0$

$\cos \pi/2 = 0 \qquad \sin \pi/2 = 1$

$\cos 3\pi/2 = 0 \qquad \sin 3\pi/2 = -1$

3 Types of DFT

→ 2 point

→ 4 point

→ 8 point

$x(n)$ = I/P sequence

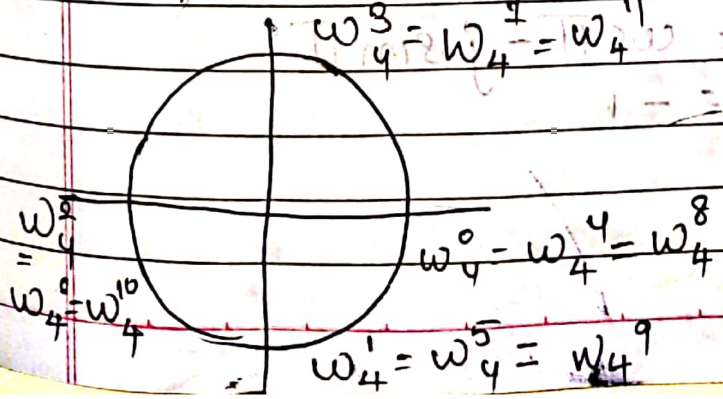
$X(K)$ = O/P sequence in DFT

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi nk}{N}}$$

$$X(K) = \sum x(n) \cdot W_N^{Kn}$$

$$= \begin{bmatrix} W_N^{Kn} \end{bmatrix} \begin{bmatrix} x(n) \end{bmatrix},$$

How to construct this twiddle factor Matrix?



$N = 4$

i.e. $n = 0$ to 3

$k = 0$ to 3

$$W_4 = \begin{array}{c|cccc|} & n=0 & n=1 & n=2 & n=3 \\ \hline k=0 & W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ k=1 & W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ k=2 & W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ k=3 & W_4^0 & W_4^3 & W_4^6 & W_4^9 \\ \hline \end{array}$$

$$W_4^0 = e^{-j2\pi nk/N}$$
$$= e^{-j\frac{2\pi(0)}{N}}$$
$$= e^0$$
$$= 1$$

$$W_4^1 = e^{-j2\pi \cdot \frac{nk}{N}}$$
$$= e^{-j2\pi(1)/4}$$
$$= e^{-j\pi/2}$$
$$= \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$
$$= 0 - j = -j$$

$$W_4^2 = e^{-j2\pi\frac{nk}{N}}$$
$$= e^{-j2\pi\frac{2}{4}}$$
$$= e^{-j\pi}$$
$$= \cos\pi - j\sin\pi$$
$$= -1$$

$$W_4^3 = e^{-j2\pi \frac{nk}{N}}$$

$$= e^{-j2\pi \frac{3}{4}}$$

$$= e^{-j3\pi/2}$$

$$= \cos\frac{3\pi}{2} - j\sin 3\pi/2$$

$$= j$$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3j \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

IDFT → Inverse Discrete Fourier Transform.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(K)\cdot W_N^{-nk}$$

$$x(n) = \frac{1}{N}\left[W_N^{*}\right]\cdot X_N$$

X_N = DFT Matrix

$*$ = complex conjugate

= change sign of j

$$[W_4^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$\text{IDF} = x_n = \frac{1}{N} [W_N^*] X_{N}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 6-2+2j-2-2-2j \\ 6-2j+2j^2+2+2j+2j^2 \\ 6+2-2j-2+2+2j \\ 6+2j-2j^2+2-2j-2j^2 \end{bmatrix}$$

$j^2=-1$

$$= \frac{1}{4} \begin{bmatrix} 0 \\ 4 \\ 8 \\ 12 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

Eg: 2 Find DFT of the Sequence $x(n) = \{1, 2, 1, 0\}$

$$\begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+1+0 \\ 1-2j-1+0 \\ 1-2+1-0 \\ 1+2j-1+0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2j \\ 0 \\ 2j \end{bmatrix}$$

$$DFT\{x(n)\} = X(k) = \{4, -2j, 0, 2j\}$$

$$IDF(x(n)) = \frac{1}{N}\left[W_N^{*nk}\right] X(k)$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 4 \\ -2j \\ 0 \\ 2j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4-2j+0+2j \\ 4-2j^2-0-2j^2 \\ 4+2j+0-2j \\ 4+2j^2+0+2j^2 \end{bmatrix}$$

$$\frac{1}{4} \begin{bmatrix} 4 \\ 8 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Eg 3 $x(n) = \{1, -1, 2, -2\}$

Ans: $X(K) = \{0, -1-j, 6, -1+j\}$

— x — o — x —

2-Point DFT

$N = 2$

$$W_N^{kn} = \begin{array}{c} \\ k=0 \\ k=1 \end{array} \begin{array}{cc} n=0 & n=1 \\ \hline W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{array}$$

$$W_2^0 = e^{-j2\pi \frac{kn}{N}} = e^{-j2\pi \cdot \frac{0}{2}}$$
$$= e^0 = 1$$

$$W_2^1 = e^{-j2\pi \frac{kn}{N}} = e^{-j2\pi \cdot \frac{1}{2}}$$
$$= e^{-j\pi}$$
$$= \cos\pi - j\sin\pi$$
$$= -1$$

$$W_2^{kn} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Eg: $x(n) = \{1, 1\}$ Find DFT & IDFT

$$X(k) = [W_N^{nk}]\, x(n)$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+1 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\therefore$ 2 point DFT is

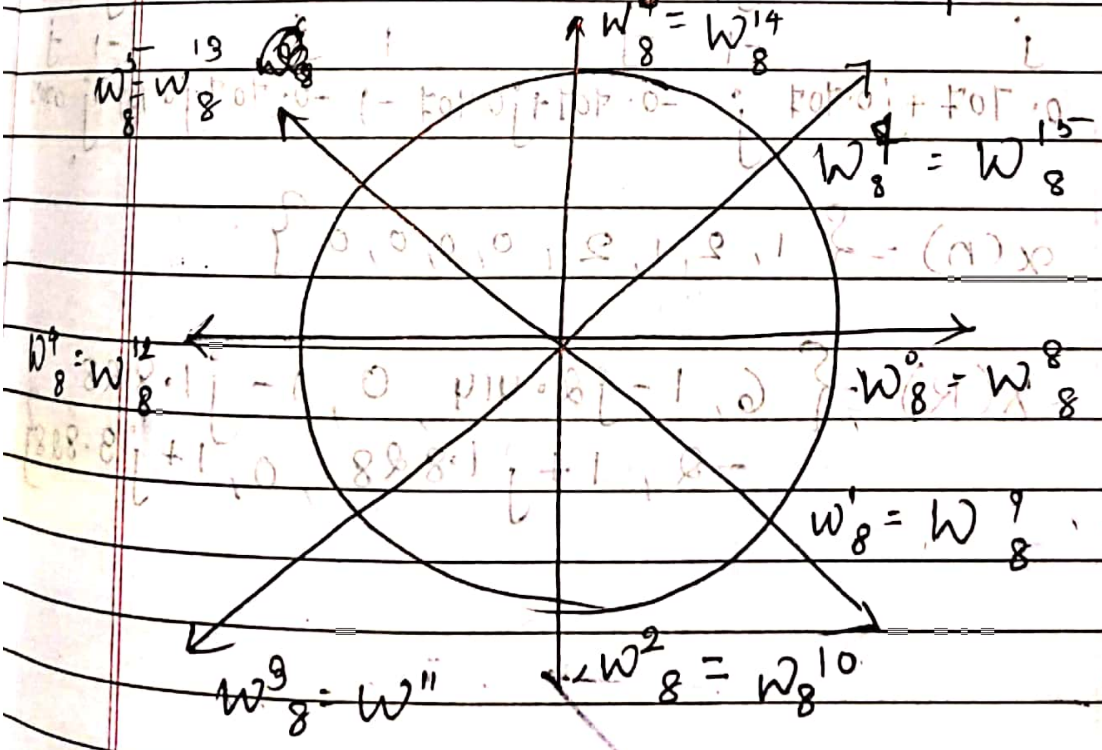
$X(k) = \{2, 0\}$

IDFT: $x(n) = \frac{1}{N}\left[W_N^{-kn}\right] \cdot X(k)$

$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2+0 \\ 2-0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

x	π	$\pi/2$	$\pi/3$	$\pi/4$	$\pi/6$
$\sin x$	0	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$
$\cos x$	-1	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$



$$W_N^{kn} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0.707-j0.707 & -j & -0.707-j0.707 \\ 1 & -j & -1 & j \\ 1 & -0.707-j0.707 & j & 0.707-j0.707 \\ 1 & -1 & 1 & -1 \\ 1 & -0.707+j0.707 & -j & 0.707+j0.707 \\ 1 & j & -1 & -j \\ 1 & 0.707+j0.707 & j & -0.707+j0.707 \end{bmatrix}$$

$$W_8^{kn} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707-j0.707 & -j & -0.707-j0.707 & -1 & -0.707+j0.707 & j & 0.\ldots j0.707 \\ 1 & -j & -1 & j & 1 & -j & -1 & \\ 1 & -0.707-j0.707 & j & 0.707-j0.707 & -1 & 0.707+j0.707 & -j & -0.\ldots +j0.707 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & \\ 1 & -0.707+j0.707 & -j & 0.707+j0.707 & -1 & 0.707-j0.707 & j & -0.\ldots -j0.707 \\ 1 & j & -1 & -j & 1 & j & -1 & \\ 1 & 0.707+j0.707 & j & -0.707+j0.707 & -1 & -0.707-j0.707 & -j & 0.\ldots -j0.707 \end{bmatrix}$$

Eg. $x(n) = \{1, 2, 1, 2, 0, 0, 0, 0\}$

$X(k) = \{6,\ 1-j2.414,\ 0,\ 1-j1.828,\ -2,\ 1+j1.828,\ 0,\ 1+j3.828\}$