Digital Signal and Image Processing
CSC 701

Subject In-charge

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Module 1

Discrete-Time Signal and Discrete-Time System

Module 1

Module No.	Unit No.	Topic details	Hrs.
1.0		**Discrete-Time Signal and Discrete-Time System**	**14**
	1.1	Introduction to Digital Signal Processing, Sampling and Reconstruction, Standard DT Signals, Concept of Digital Frequency, Representation of DT signal using Standard DT Signals, Signal Manipulations(shifting, reversal, scaling, addition, multiplication).	
	1.2	Classification of Discrete-Time Signals, Classification of Discrete-Systems	
	1.3	Linear Convolution formulation for 1-D and 2-D signal (without mathematical proof), Circular Convolution (without mathematical proof), Linear convolution using Circular Convolution. Auto and Cross Correlation formula evaluation, LTI system, Concept of Impulse Response and Step Response, Output of DT system using Time Domain Linear Convolution.	

Introduction to Digital Signal Processing

What is a signal?

- A signal is defined as any physical quantity that varies with time, space, or any other independent variable or variables.
- Mathematically , we describe a signal as a function of one or more independent variables.
- E.g.

$$s_1(t) = 5t$$

$$s_2(t) = 20t^2$$

Other Examples

- A microphone is a device that measures these variations and generates an electrical signal that represents sound.
- A speaker is a device that takes an electrical signal and produces sound.
- Microphones and speakers are called transducers because they transduce, or convert, signals from one form to another.

Other dimensions

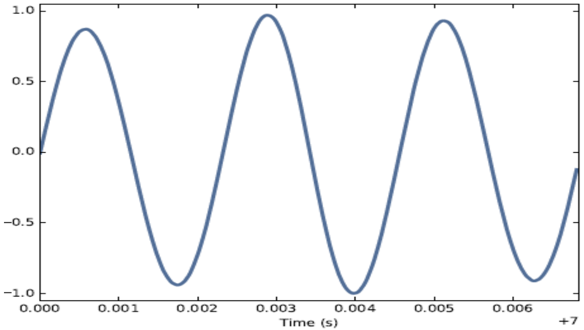
- Consider the function

$$s(x, y) = 3x + 2xy + 10y^2$$

- This function describes a signal of two independent variables x and y that could represent the two spatial coordinates in a plane.
- Examples :
  - an image is a signal that varies in two-dimensional space.
  - a movie is a signal that varies in two-dimensional space and time.
- we start with a simple one-dimensional signal.

Periodic Signals

- Periodic signals are signals that repeat themselves after some period of time.
- For example, if you strike a bell, it vibrates and generates sound and plot of the transduced signal looks like

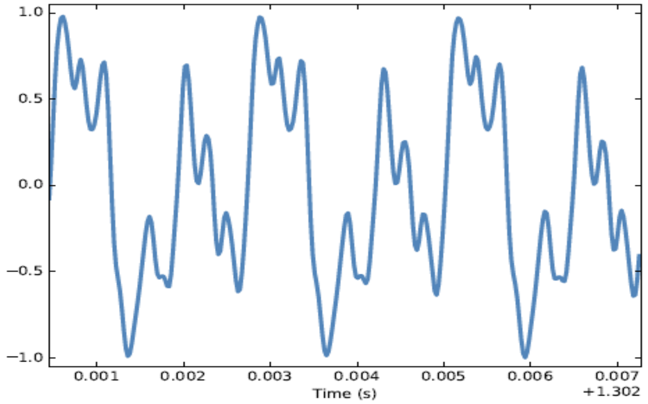


Periodic Signals

- This signal resembles a sinusoid, which means it has the same shape as the trigonometric sine function.
- This signal is periodic and repeats itself in full repetitions, also known as cycles.
- The duration of each cycle, called the period, is about 2.3 ms.
- The frequency of a signal is the number of cycles per second, which is the inverse of the period.

Periodic Signals

- Most musical instruments produce periodic signals, but the shape of these signals is not sinusoidal.
- A segment from a recording of a violin is given as follows:



Periodic Signals

- Again we can see that the signal is periodic, but the shape of the signal is more complex.
- The shape of a periodic signal is called the waveform.
- Most musical instruments produce waveforms more complex than a sinusoid.
- These are cases where a functional relationship is unknown or too highly complicated to be of any practical use.

Periodic Signals

- Example : Speech signal
- It cannot be described functionally.
- In general, a segment of speech may be represented to a high degree of accuracy as a sum of several sinusoids of different amplitudes and frequencies.

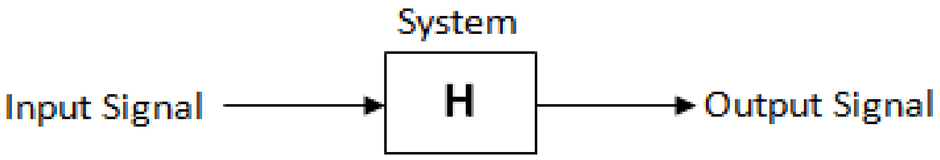
$$\sum_{i=1}^{N} A_i(t) \sin[2\pi F_i(t)t + \theta_i(t)]$$

Speech Signal: $\sum_{i=1}^{N} A_i(t) \sin[2\pi F_i(t)t + \theta_i(t)]$

- $A_i(t)$, $F_i(t)$ and $Ɵ_i(t)$ are the sets of possibly time varying amplitudes, frequencies and phases respectively of the sinusoids.
- One way to interpret the speech signal is to measure the amplitudes, frequencies and phases contained in the short time segment of the signal
- These are but few examples of the countless number of natural signals encountered in practice.

What is a System?

- A system may also be defined as a physical device that performs an operation on a signal.
- For example, a filter used to reduce the noise and interference corrupting a desired information-bearing signal
- The system is characterized by the type of operation that it performs on the signal.
- For example, if the operation is linear, the system is called linear.

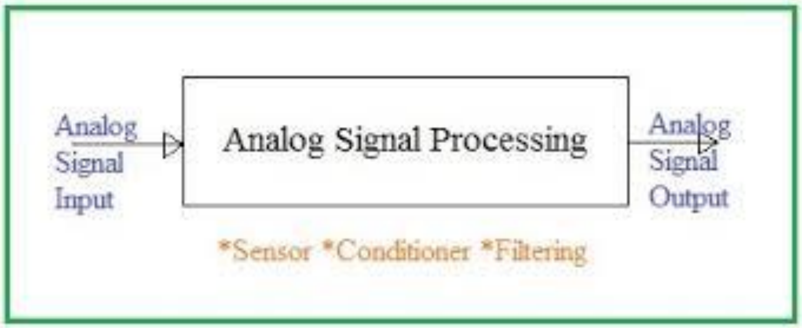


Signal Processing

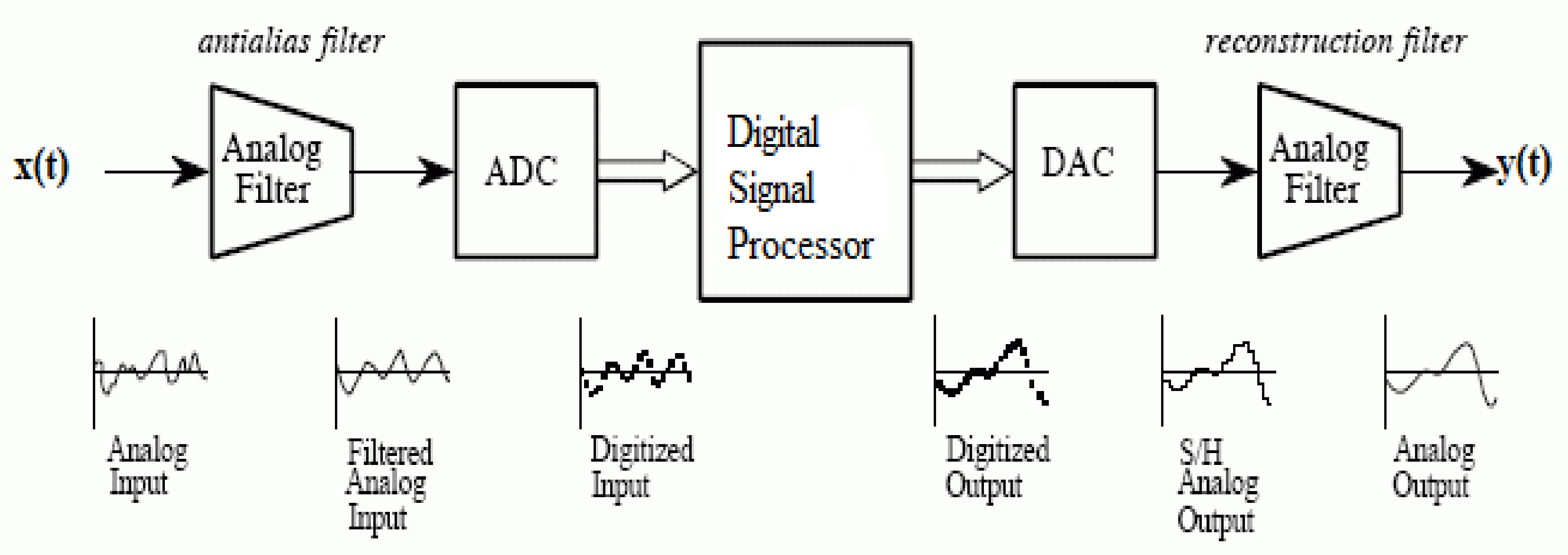
- It is any operation that changes the characteristics of a signal.
- The characteristics include amplitude, frequency and phase of a signal.
- They are broadly classified into
  - Analog Signal Processing
  - Digital Signal Processing

Analog Signal Processing

- Most signals encountered are analogous in nature which are functions of continuous variable (time or space).
- Such signals can be processed directly by the analog systems to perform any operation.
- Here both input and output are in analog form.



Digital Signal Processing



Digital Signal Processing

1. Antialiasing Filter:

It is a low pass filter used to remove the high frequency noise.

2. ADC:

It converts analog input signal to digital signal by sampling and quantization process.

Digital Signal Processors

- A digital signal processor can be
- **Programmable Machine(Digital Computer/Microprocessor):**
  - They provide the flexibility to change signal processing operations through a change in the software.
- **Hardwired Digital Processors :**
  - They are difficult to reconfigure.
  - They are used when the signal processing operations are well defined.
  - They help in optimizing the speed and cost of the required hardware.

Digital Signal Processing

4. DAC: It converts the digital signal to analog by dequantization and filtering. When the application needs information in digital form, it is not required.

5. Reconstruction Filter: It is used to construct a smooth analog signal.

Advantages of Digital Signal Processing

1. They provide flexibility in reconfiguration
2. It provides better control of accuracy requirements (e.g Word-length : fixed or floating point)
3. It can be easily stored without any distortion
4. It allows implementation of sophisticated processing algorithms.
5. It is cheaper compared to its analog counterpart in some cases

Limitations of DSP

1. Speed of Operation : Signals with extreme high bandwidth requires fast ADCs
2. Reconstruction of analog signals from digital form results in distortion due to quantization

Discrete Time Signals

Discrete Time Signal

- For discrete time signal the independent variable is time n, and it is represented by x(n).
- A discrete time signal x(n) is a function of an independent variable that is an integer(n).
- Discrete-time signals are signals which are defined only at discrete instants of time.
- A discrete time signal is not defined at instants between two successive samples.
- It is incorrect to think that x(n) is equal to zero if n is not an integer.

Representing Discrete-Time signals

- There are following four ways of representing discrete-time signals:

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequence representation

Representing Discrete-Time signals

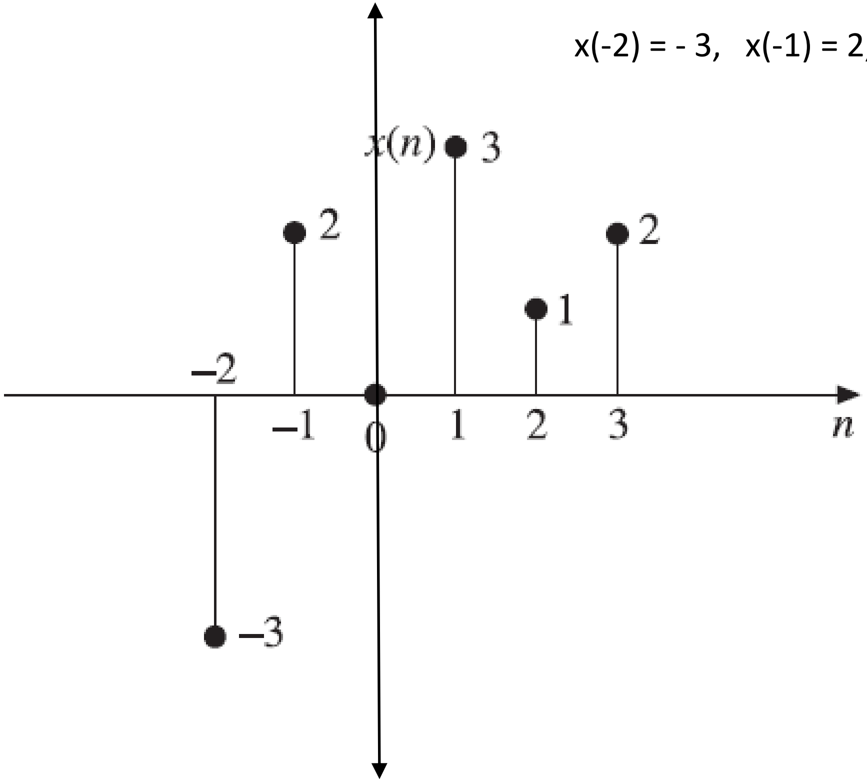
- Consider a signal x(n) with values

$x(-2) = -3$, $x(-1) = 2$, $x(0) = 0$, $x(1) = 3$, $x(2) = 1$ and $x(3) = 2$

- We will see the different representations of this signal

1. Graphical Representation

x(-2) = - 3,   x(-1) = 2,   x(0) =0,   x(1) = 3,   x(2) = 1 and x(3) = 2



2. Functional Representation

$$x(n) = \begin{cases} -3 & \text{for } n = -2 \\ 2 & \text{for } n = -1 \\ 0 & \text{for } n = 0 \\ 3 & \text{for } n = 1 \\ 1 & \text{for } n = 2 \\ 2 & \text{for } n = 3 \end{cases}$$

Another Example :

$$x(n) = 2^n u(n)$$

$$x(n) = \begin{cases} 2^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

3. Tabular Representation

- In this, the sampling instant n and the magnitude of the signal at the sampling instant are represented in the tabular form as follows:

n	-2	-1	0	1	2	3
$x(n)$	-3	2	0	3	1	2

4. Sequence Representation

- A finite duration sequence given can be represented as follows:

$$x(n) = \left\{ \begin{matrix} -3, & 2, & 0, & 3, & 1, & 2 \\ & & \uparrow & & & \end{matrix} \right\}$$

- Another example which is of infinite duration is given as follows:

$$x(n) = \left\{ \begin{matrix} \ldots 2, & 3, & 0, & 1, & -2, \ldots \\ & & \uparrow & & \end{matrix} \right\}$$

- The arrow mark $\uparrow$ denotes the n = 0 term. When no arrow is indicated, the first term corresponds to n = 0.

$$x(n) = \{3, 5, 2, 1, 4, 7\}$$

Exercise Problem 1:

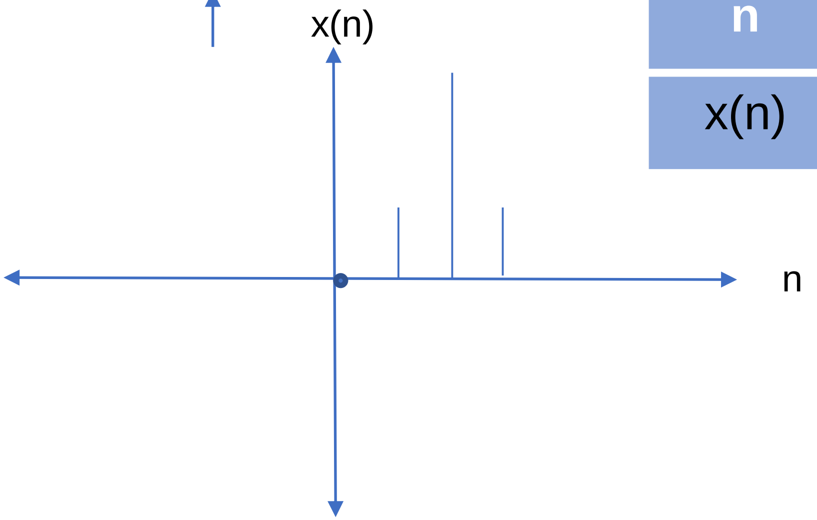
1. Represent the following signal in all forms:

$$x(n) = \begin{cases} 1, & \text{for } n = 1, 3 \\ 4, & \text{for } n = 2 \\ 0, & \text{elsewhere} \end{cases}$$

Solution

$$x(n) = \begin{cases} 1, & \text{for } n = 1, 3 \\ 4, & \text{for } n = 2 \\ 0, & \text{elsewhere} \end{cases}$$

- $x(n) = \{\underset{\uparrow}{0}, 1, 4, 1\}$



n	0	1	2	3
x(n)	0	1	4	1

Exercise Problem 2:

2. Represent the following signal in all forms:

$$
\begin{aligned}
x(n) &= n(0.25)^n \quad \text{for } 0 < n < 5 \\
&= 0 \qquad\qquad \text{Otherwise}
\end{aligned}
$$

Solution

$$x(n) = n(0.25)^n \quad \text{for } 0 < n < 5$$
$$= 0 \quad \text{Otherwise}$$

- $x(n) = \{\underset{\uparrow}{0}, 0.25, 0.125, 0.047, 0.015\}$

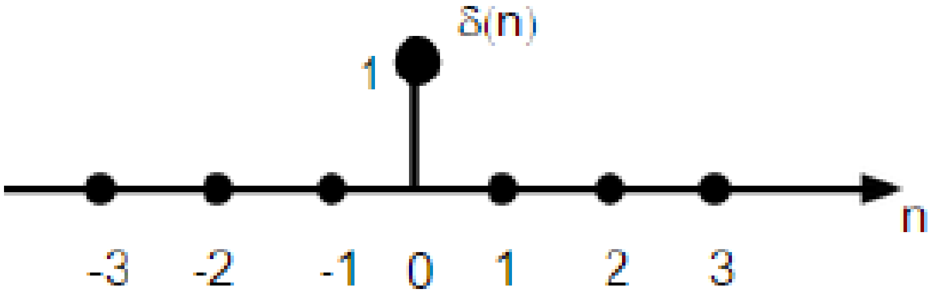
n	0	1	2	3	4
x(n)	0	0.25	0.125	0.047	0.015

Standard DT Signals

- Unit Impulse Sequence
- Unit Step Signal
- Unit Ramp Function
- Sinusoidal Signal

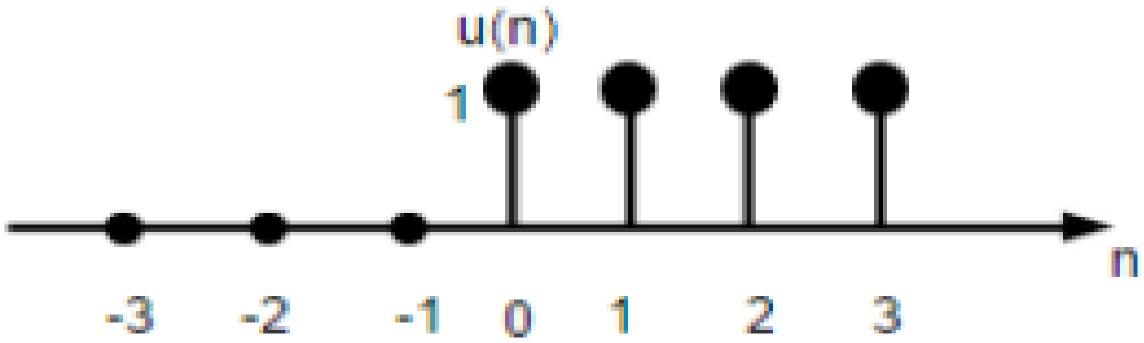
Unit Impulse Sequence

$$\delta(n) = \begin{cases} 1, & for \quad n = 0 \\ 0, & Otherwise \end{cases}$$



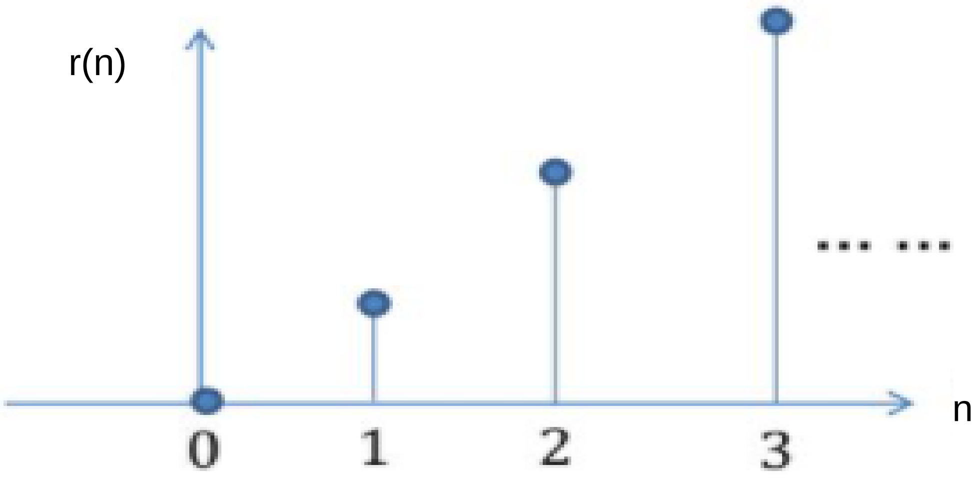
Unit Step Signal

$$U(n) = \begin{cases} 1, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$



Unit Ramp Function

$$r(n) = \begin{cases} n, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$



Basic Operations on Signal (Signal Manipulations)

Basic Operations on Signal

- The discrete time sequence may undergo several manipulations involving the independent variable or the amplitude of the signal.
- The basic operations on sequences are as follows:
  1. Time Shifting
  2. Time Reversal
  3. Time Scaling
  4. Amplitude Scaling
  5. Signal Addition
  6. Signal Multiplication
- The first 3 operations transform in independent variable n of a signal.
- The last 3 operations transform amplitude of a signal.

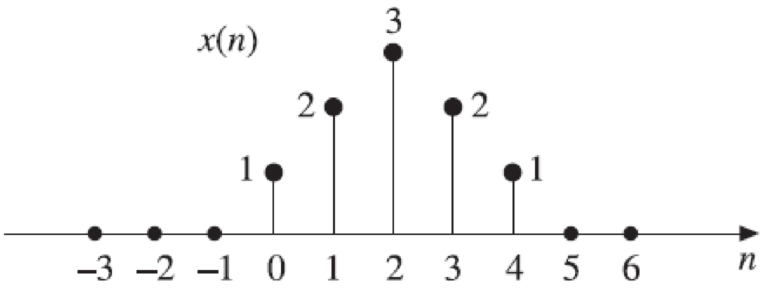
Time Shifting

- The time shifting of a signal may result in time delay or time advance.
- The time shifting operation of a discrete-time signal x(n) can be represented by

$$y(n) = x(n - k)$$

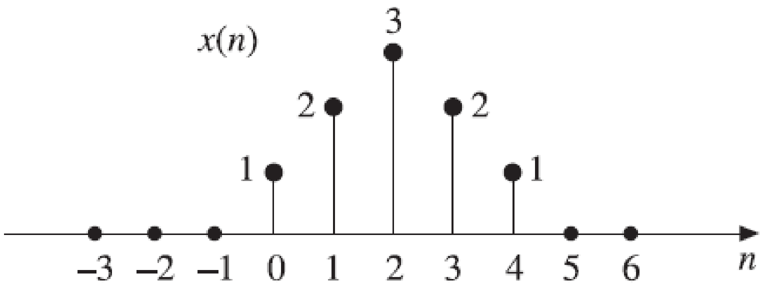
- This shows that the signal y(n) can be obtained by time shifting the signal x(n) by k units.
- If k is positive, it is delay and the shift is to the right
- If k is negative, it is advance and the shift is to the left.

Example:



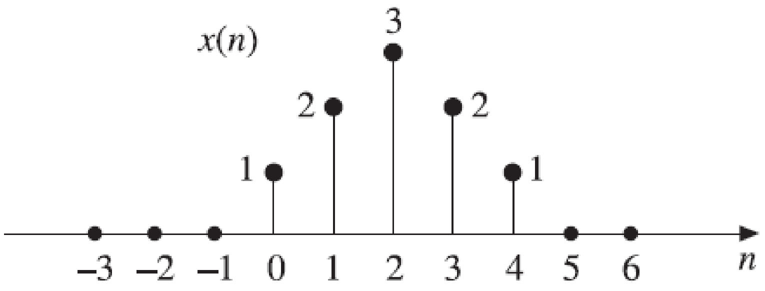
Find x(n-3) and x(n+2)

Example:

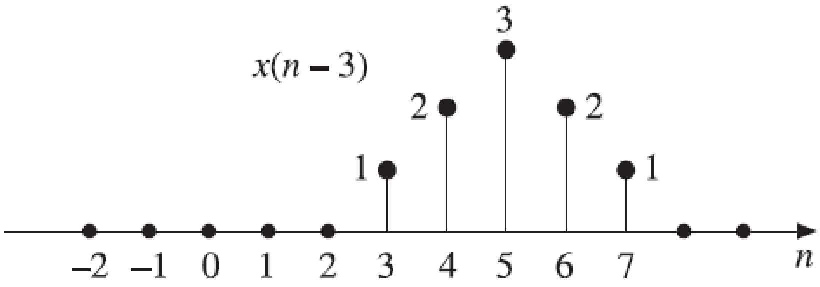


Delay : x(n-3)

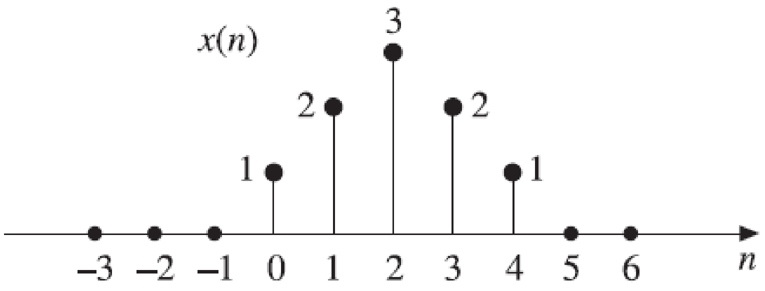
Example:



Delay

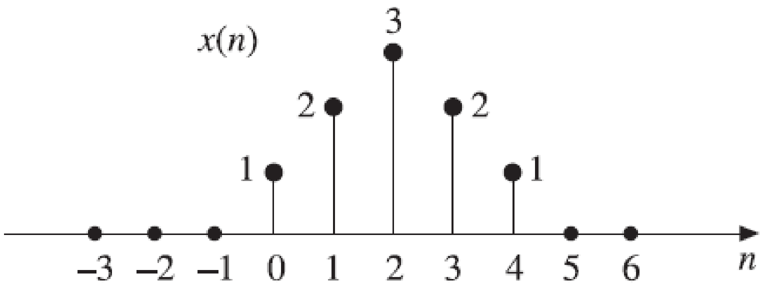


Example:

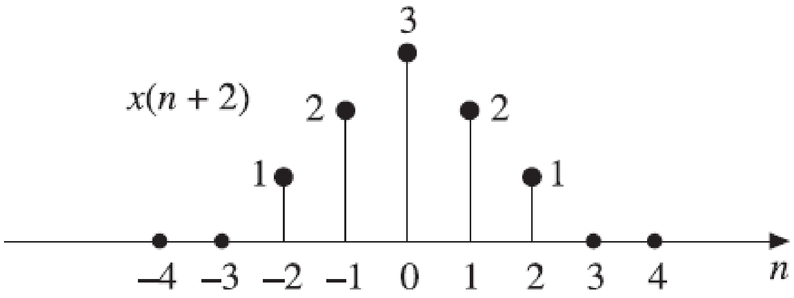


Advance : x(n+2)

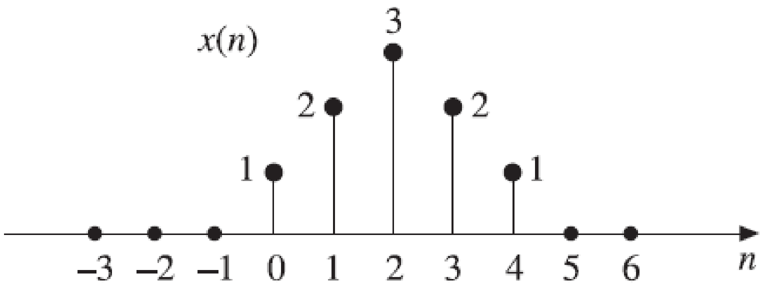
Example:



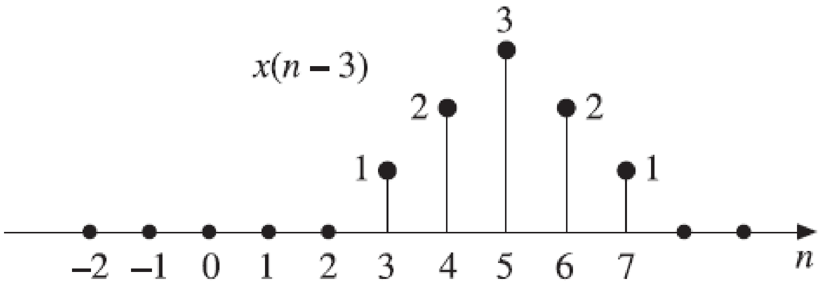
Advance



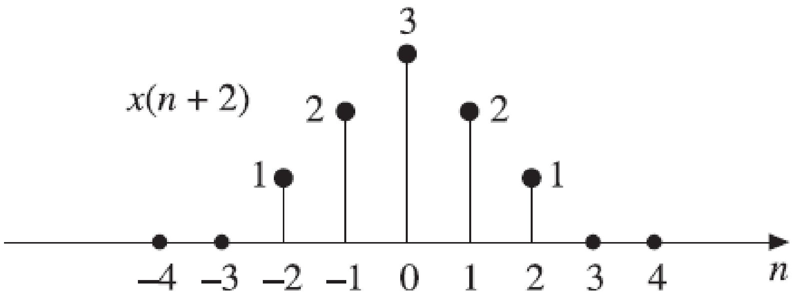
Example:



Delay



Advance

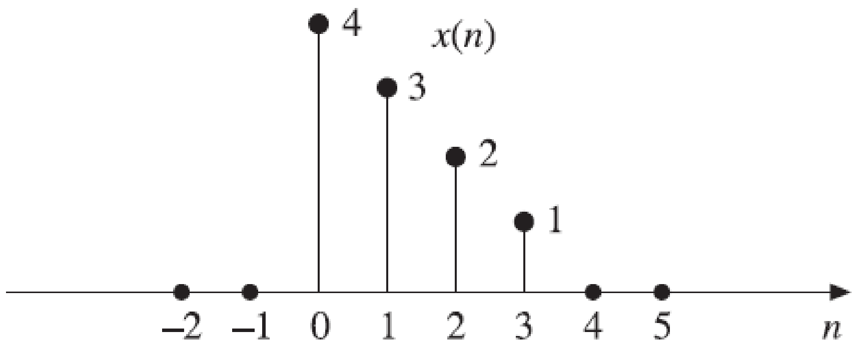


Time Reversal

- It is also called time folding of the signal
- A discrete-time signal x(n) can be obtained by folding the sequence about n = 0.
- The time reversed signal is the reflection of the original signal.
- It is obtained by replacing the independent variable n by –n.

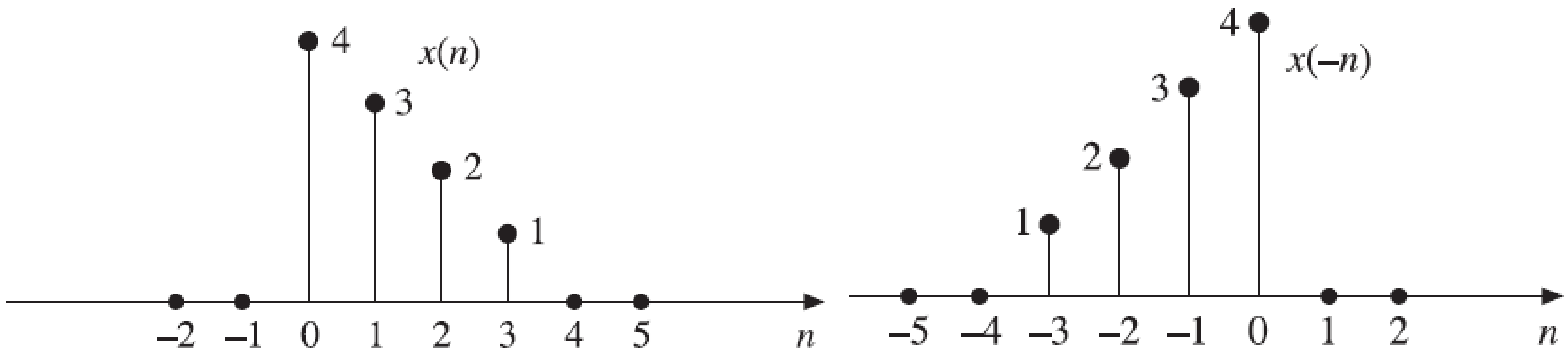
Time Reversal : Problem

- For an arbitrary discrete-time signal x(n) shown below, find its time reversed version x(–n), x(-n+3)



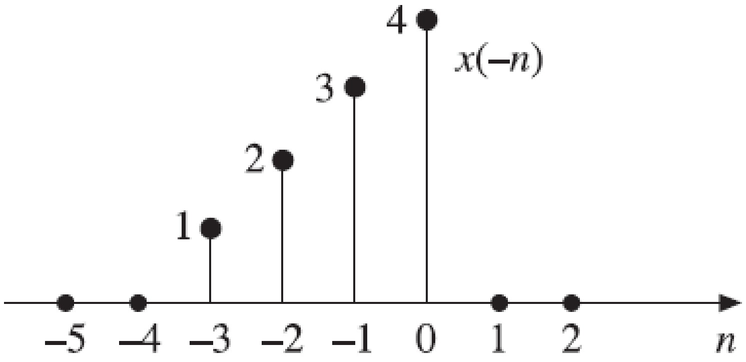
Time Reversal : Problem

- x(–n)



Time Reversal

- The signal x(–n+3) is obtained by delaying (shifting to the right) the time reversed signal x(–n) by 3



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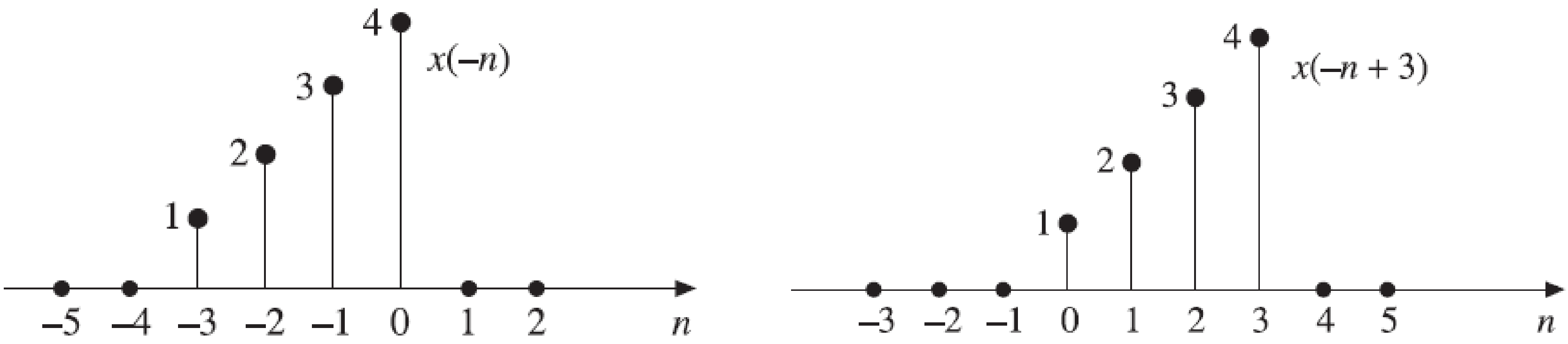
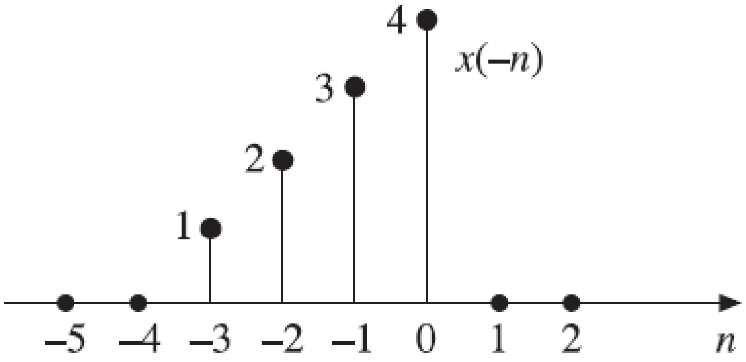


Time Reversal

- The signal x(–n–3) is obtained by advancing (shifting to the left) the time reversed signal x(–n) by 3 units



Time Reversal

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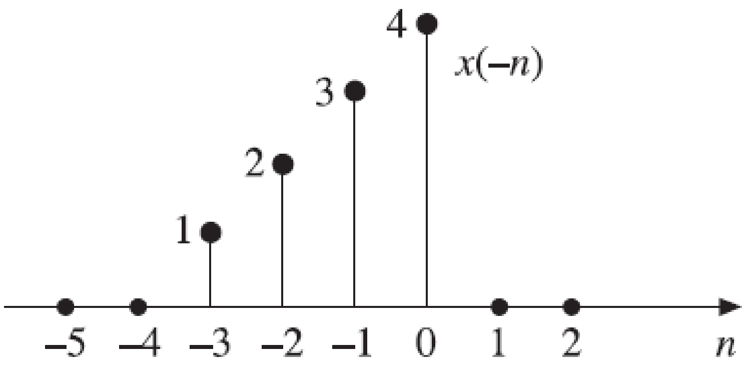
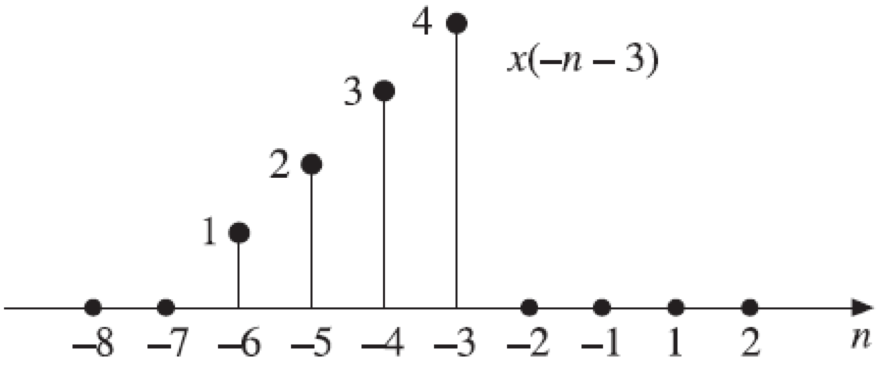


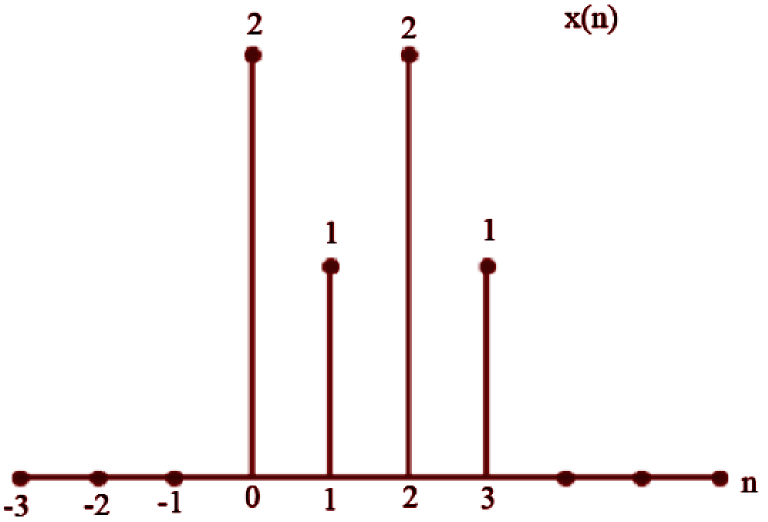


Exercise Problem 2:

From the x(n) given below, generate the following signals

(i) x(n-1) (ii) x(n+2) (iii)x(-n) (iv) x(-n-2)

(v) x(-n+2)



Time Scaling

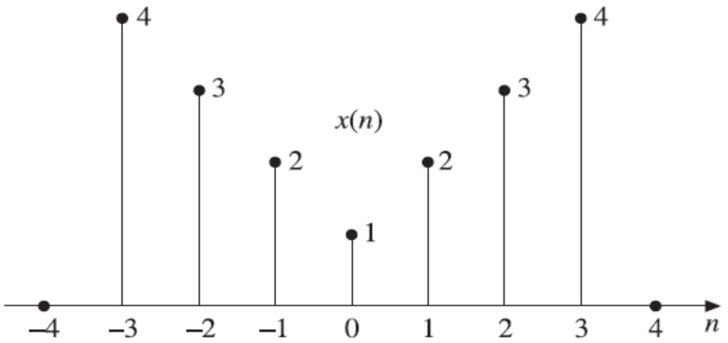
- Time scaling may be time expansion or time compression.
- The time scaling of a discrete time signal x(n) can be accomplished by replacing n by an in it.
- Mathematically, it can be expressed as:

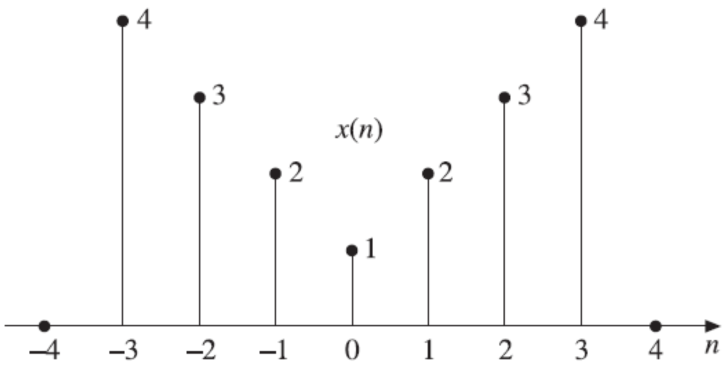
$$y(n) = x(an)$$

- When a > 1, it is time compression and when a < 1, it is time expansion.
- Time scaling is very useful when data is to be fed at some rate and is to be taken out at a different rate.

Example

- Consider an arbitrary discrete-time signal x(n), generate the following time scaled version of the signal x(n).
- (i) y(n) = x(2n) (ii) y(n) = x(n/2)

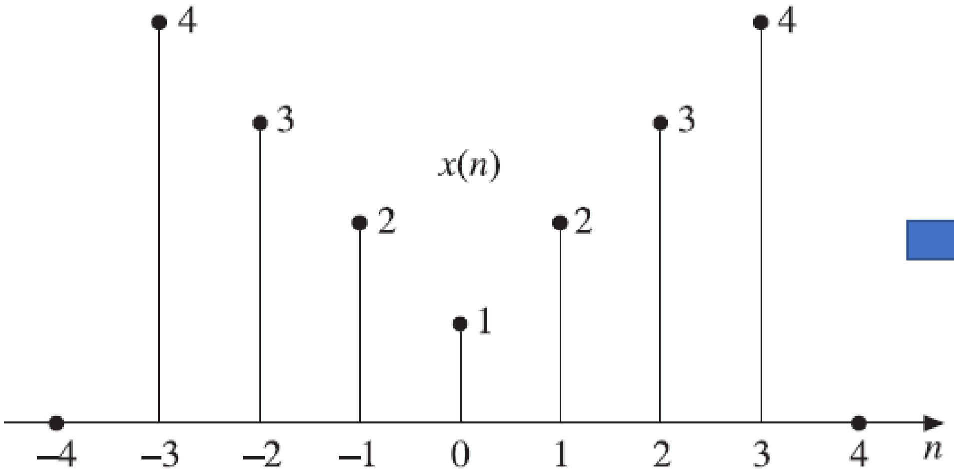


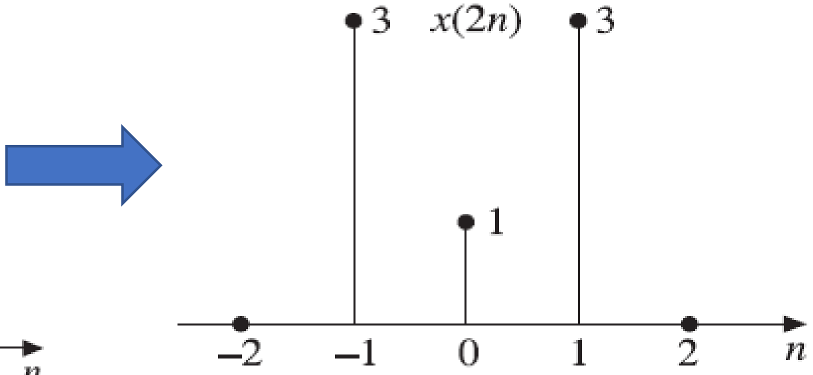


$y(n) = x(2n)$

Example

- So to plot x(2n) we have to skip odd numbered samples in x(n).





Example

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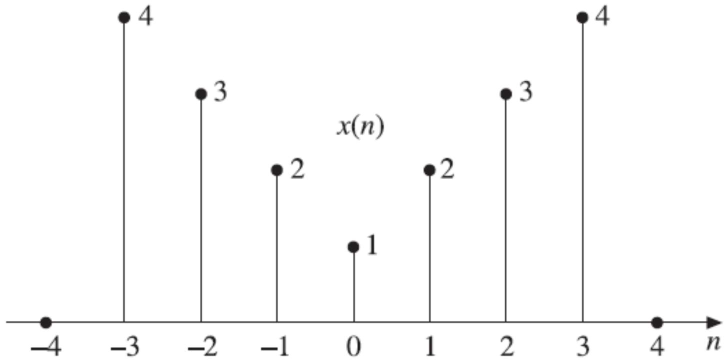
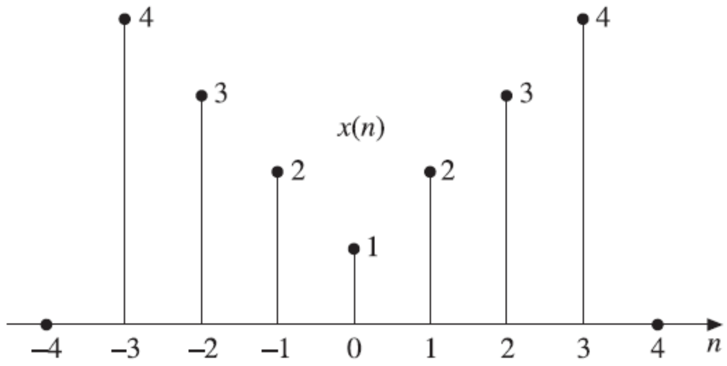
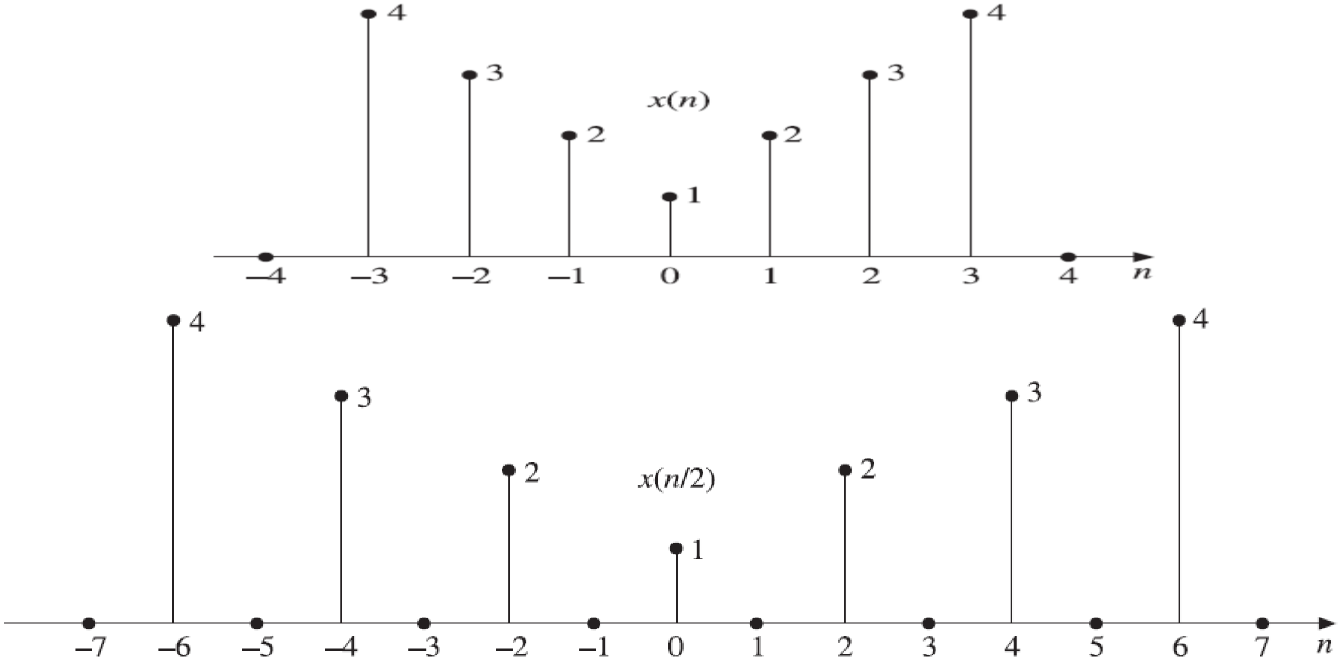




y(n) = x(n/2)

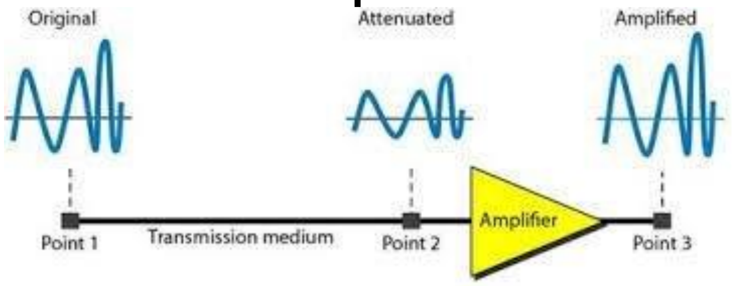
Example

- Thus, here the signal is expanded by 2.

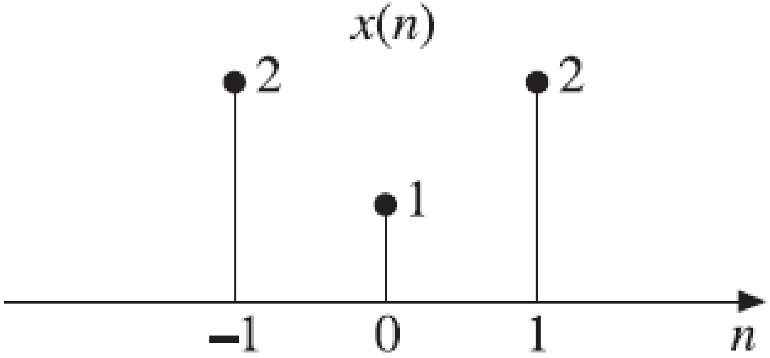


Amplitude Scaling (Scalar Multiplication)

- Amplitude scaling of a signal by a constant A is obtained by multiplying the value of every signal sample by A.
- y(n) = A x(n)
- The amplitude of y(n) at any instant is equal to a times the amplitude of x(n) a
- If a > 1, it is amplification
- If a < 1, it is attenuation

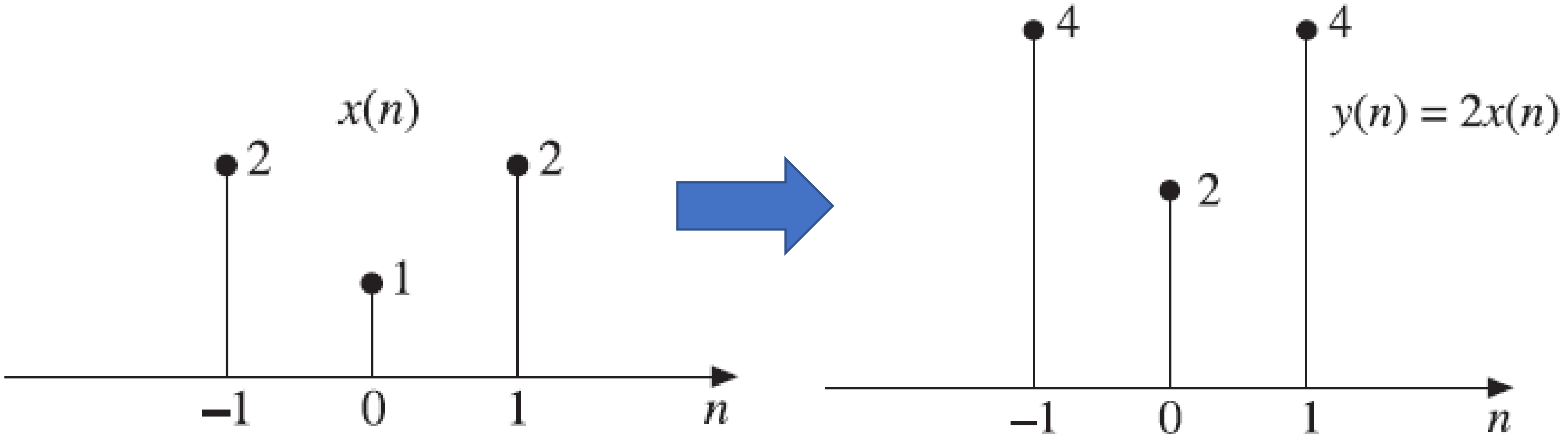


Amplitude Scaling Example:



Find y(n) = 2x(n)

Amplitude Scaling Example:



Signal Addition

- In discrete-time domain,

- the sum of two signals $x_1(n)$ and $x_2(n)$ can be obtained by adding the corresponding sample values and

- the subtraction of $x_2(n)$ from $x_1(n)$ can be obtained by subtracting each sample of $x_2(n)$ from the corresponding sample of $x_1(n)$

Example

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then

$x_1(n) + x_2(n)$

Example

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then

$x_1(n) + x_2(n)$

$= \{1 + 2, 2 + 3, 3 + 4, 1 + 1, 5 - 2\}$

Example

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then

$x_1(n) + x_2(n)$

$= \{1 + 2, 2 + 3, 3 + 4, 1 + 1, 5 - 2\}$

$= \{3, 5, 7, 2, 3\}$

Example

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then

$x_1(n) + x_2(n)$

$= \{1 + 2, 2 + 3, 3 + 4, 1 + 1, 5 - 2\}$

$= \{3, 5, 7, 2, 3\}$

$x_1(n) - x_2(n)$

Example

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then

$x_1(n) + x_2(n)$

$= \{1 + 2, 2 + 3, 3 + 4, 1 + 1, 5 - 2\}$

$= \{3, 5, 7, 2, 3\}$

$x_1(n) - x_2(n)$

$= \{1 - 2, 2 - 3, 3 - 4, 1 - 1, 5 + 2\}$

Example

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then

$x_1(n) + x_2(n)$

$= \{1 + 2, 2 + 3, 3 + 4, 1 + 1, 5 - 2\}$

$= \{3, 5, 7, 2, 3\}$

$x_1(n) - x_2(n)$

$= \{1 - 2, 2 - 3, 3 - 4, 1 - 1, 5 + 2\}$

$= \{-1, -1, -1, 0, 7\}$

Example Problem:

- Add two signals given below:

$$x_1(n) = \{\ 1,\ \ \underset{\uparrow}{2},\ \ 3\ \}$$

$$x_2(n) = \{\ \underset{\uparrow}{1},\ \ 2,\ \ 1\ \}$$

Solution

$$x_1(n) = \{\ 1,\ \ \underset{\uparrow}{2},\ \ 3\ \}$$

$$x_2(n) = \{\ \underset{\uparrow}{1},\ \ 2,\ \ 1\ \}$$

$$y(n) = x_1(n) + x_2(n)$$

$$y(n) = \{\ 1,\ \ \underset{\uparrow}{3},\ \ 5,\ \ 1\ \}$$

Signal Multiplication

- In a discrete-time domain, the product of two signals $x_1(n)$ and $x_2(n)$ can be obtained by multiplying the corre s.

$$y(n) = x_1(n) \times x_2(n)$$

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then find
- $x_1(n) \times x_2(n)$

Signal Multiplication

- In a discrete-time domain, the product of two signals $x_1(n)$ and $x_2(n)$ can be obtained by multiplying the corresponding sample values

$$y(n) = x_1(n) \times x_2(n)$$

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then
- $x_1(n) \times x_2(n)$

  $= \{1 \times 2, 2 \times 3, 3 \times 4, 1 \times 1, 5 \times -2\}$

Signal Multiplication

- In a discrete-time domain, the product of two signals $x_1(n)$ and $x_2(n)$ can be obtained by multiplying the corresponding sample values.

$$y(n) = x_1(n) \times x_2(n)$$

- If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$
- Then
- $x_1(n) \times x_2(n)$
- $= \{1 \times 2, 2 \times 3, 3 \times 4, 1 \times 1, 5 \times -2\}$
- $= \{2, 6, 12, 1, -10\}$

Example:

- Multiply two signals given below:

$$x_1(n) = \{\ 1,\ \underset{\uparrow}{2},\ 3\ \}$$

$$x_2(n) = \{\ \underset{\uparrow}{1},\ 2,\ 1\ \}$$

$$y(n) = x_1(n) \times x_2(n)$$

Example:

- Multiply two signals given below:

$$x_1(n) = \{\ 1,\ \ \underset{\uparrow}{2},\ \ 3\ \}$$

$$x_2(n) = \{\ \underset{\uparrow}{1},\ \ 2,\ \ 1\ \}$$

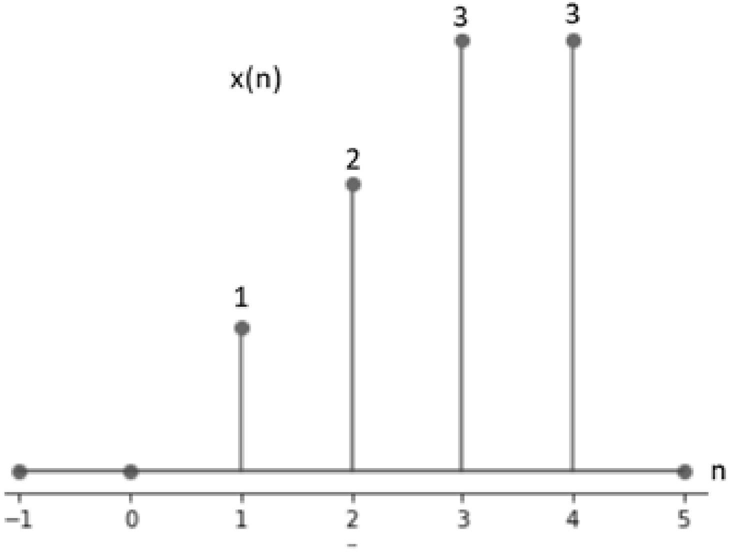
$$y(n) = x_1(n)\ \times\ x_2(n)$$

$$y(n) = \{\underset{\uparrow}{2}\quad 6\ \}$$

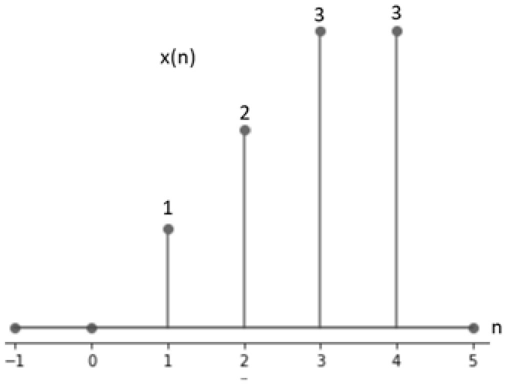
Exercise Problem 2:

- A discrete time signal x(n) is shown below. Sketch and label each of the following signals.

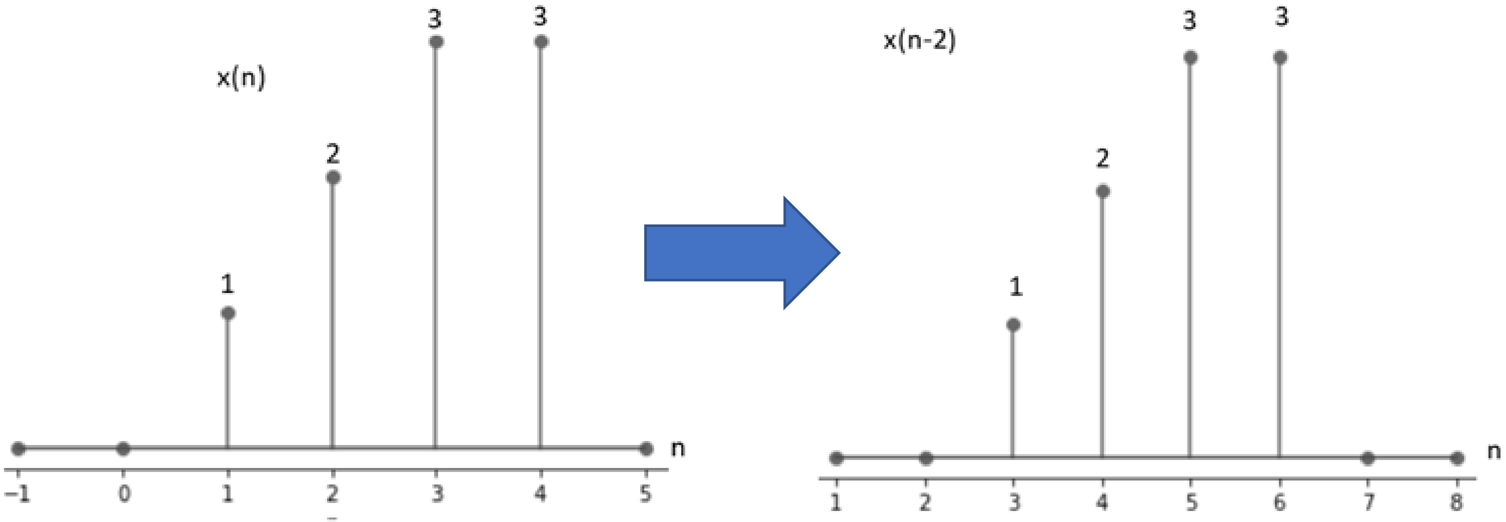
a. x(n-2)
b. x(2n)
c. x(-n)
d. x(-n+2)
e. x(-n-2)



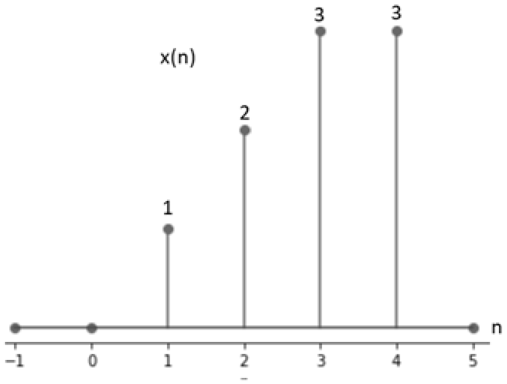
a. x(n-2)



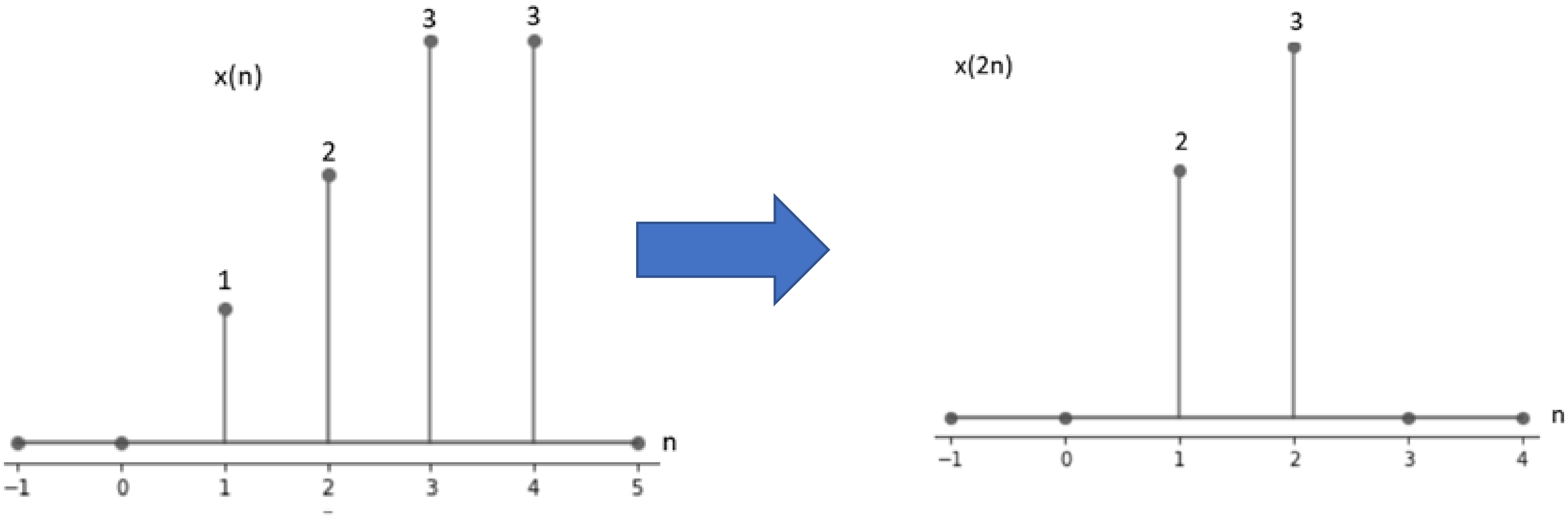
a. x(n-2)



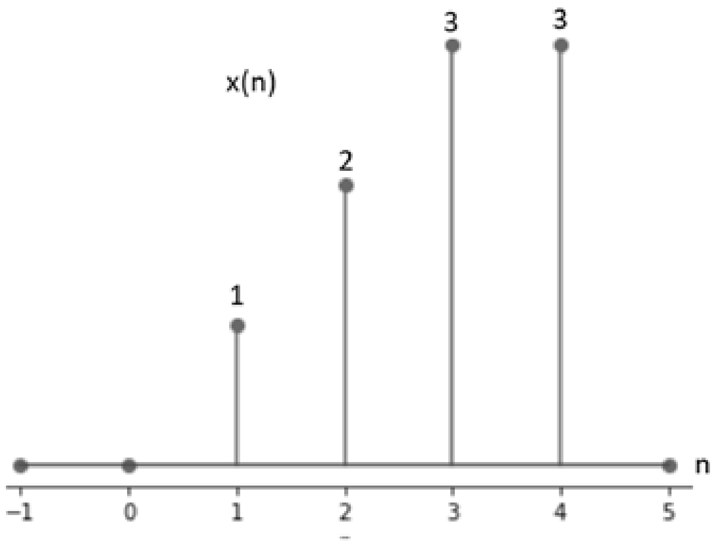
b. x(2n)



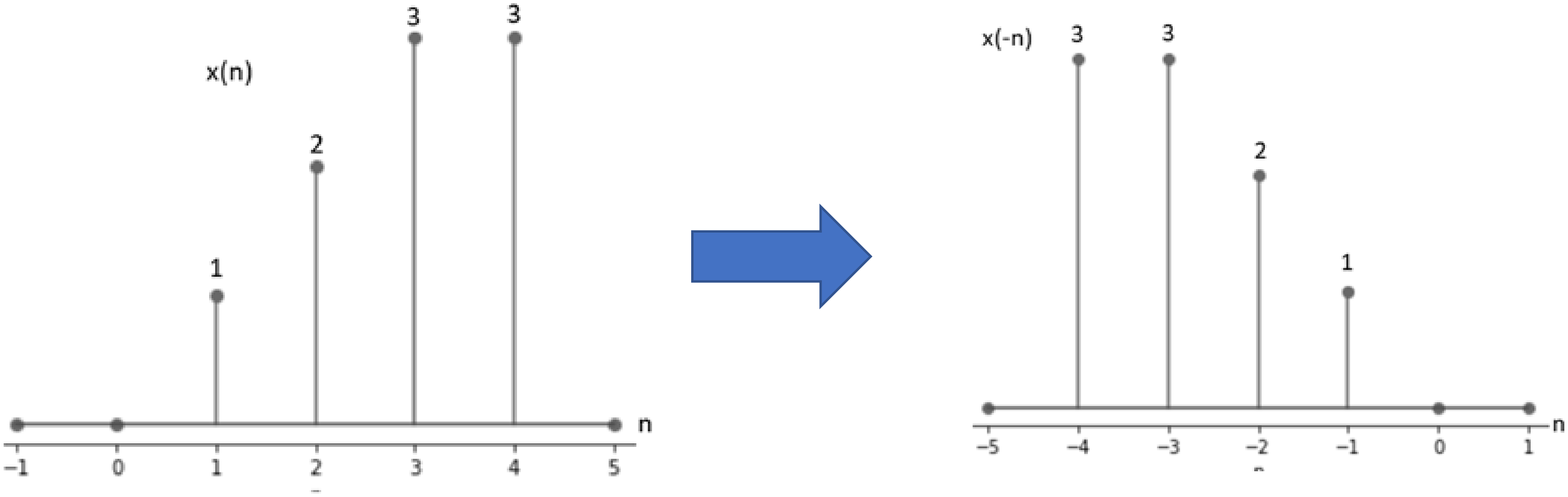
b. x(2n)



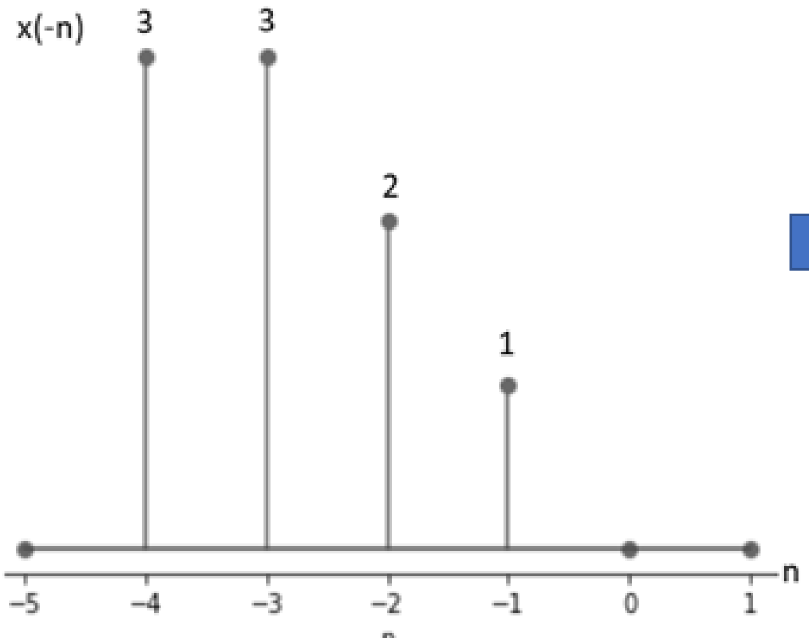
c. x(-n)



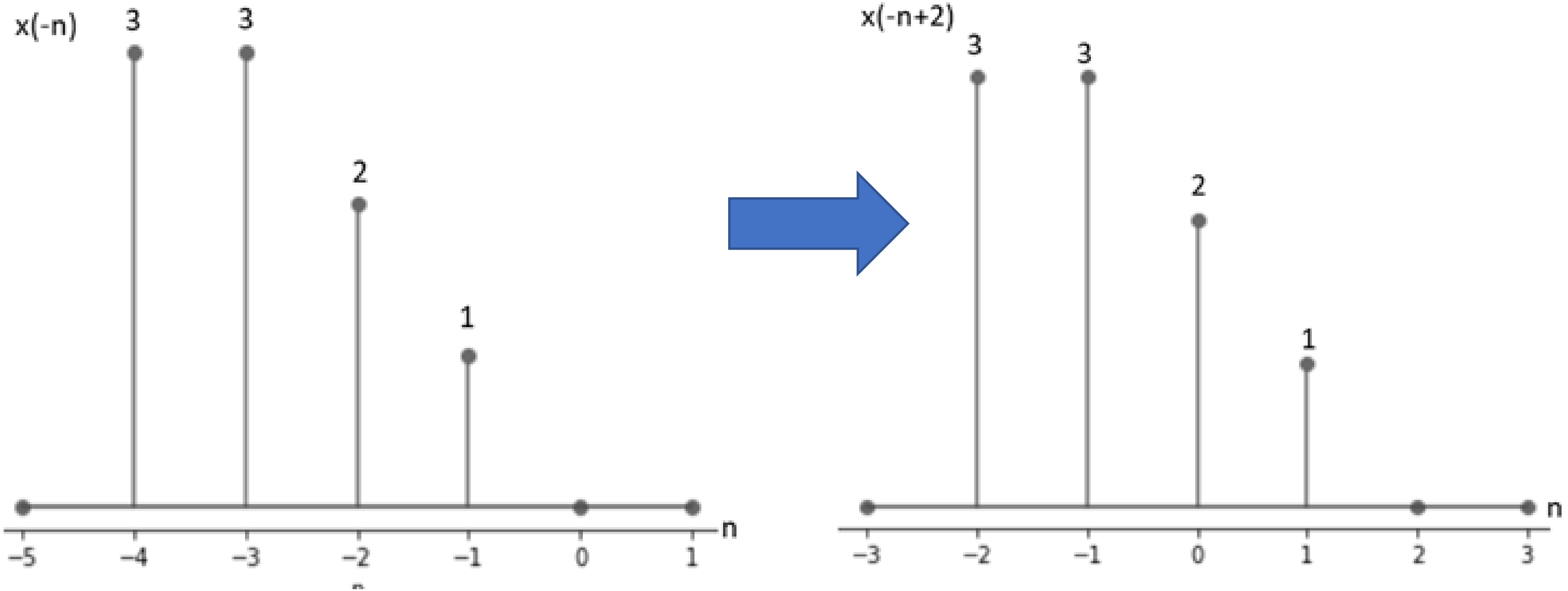
c. x(-n)



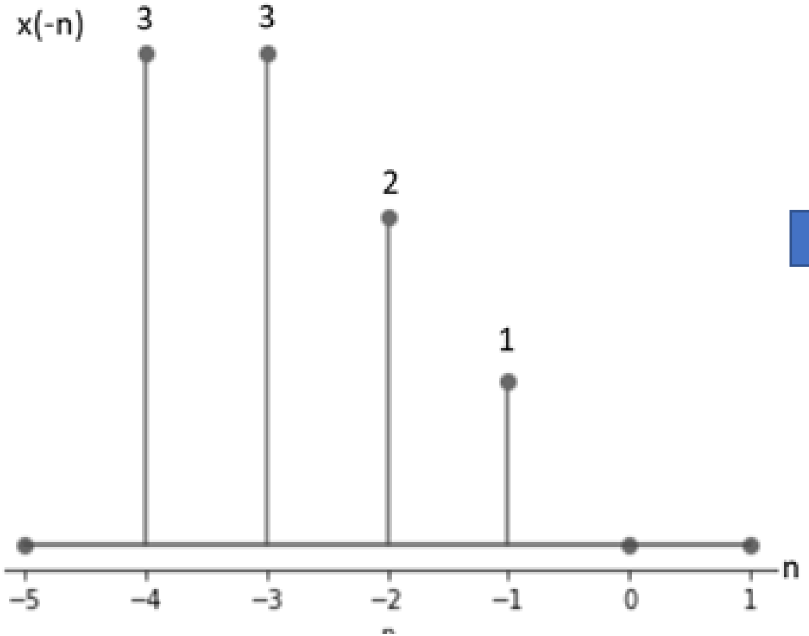
d. x (-n+2)



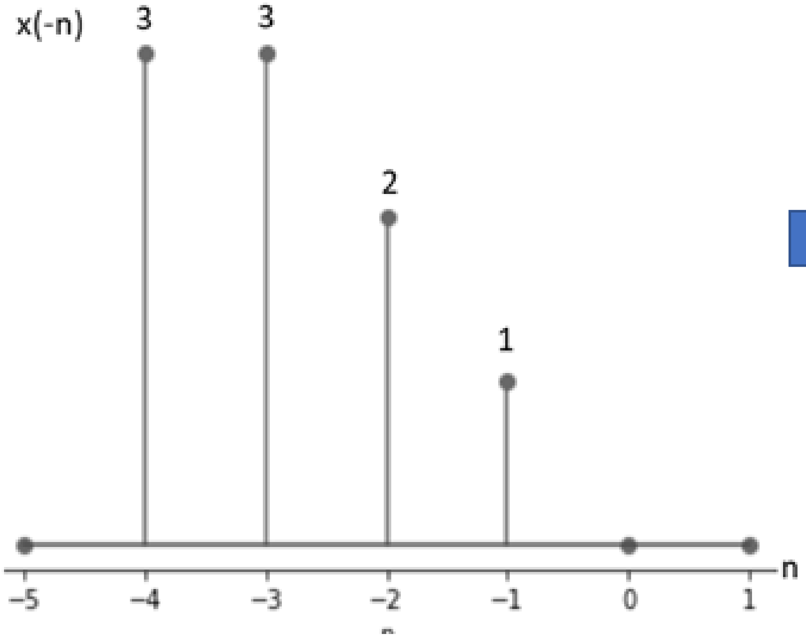
d. x (-n+2)

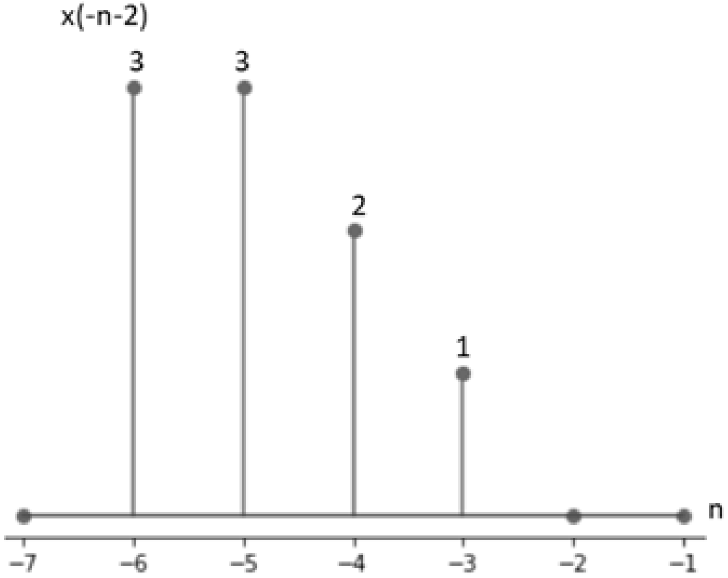


e. x(-n-2)



e. x(-n-2)





UQ 1:

For $x(n) = \{1,3,-1,2,0,4\}$, plot the following discrete time signals

(i) $x(n+2)$

(ii) $x(-n-1)$

(iii) $2x(n)$

(iv) $x(n-1).\delta(n-3)$

(v) $x(n).u(n-2)$

UQ 2

- For the signal given below, plot the following signals

$$x(n) = \{ \ 1, \quad \underset{\uparrow}{2}, \quad -1, \quad 5, \quad 0, \quad 4 \ \}$$

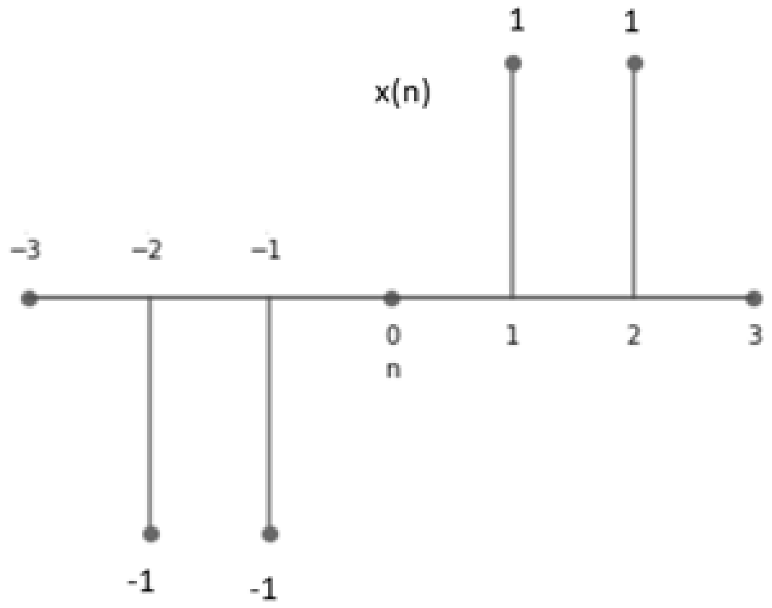
- (i) x(n+3)
- (ii)x(-n-2) (iii)x(n)u(n-1)
- (iv) x(n-2)δ(n-2) (v) x(2n)

Combo Operations Exercise Problem 1 :

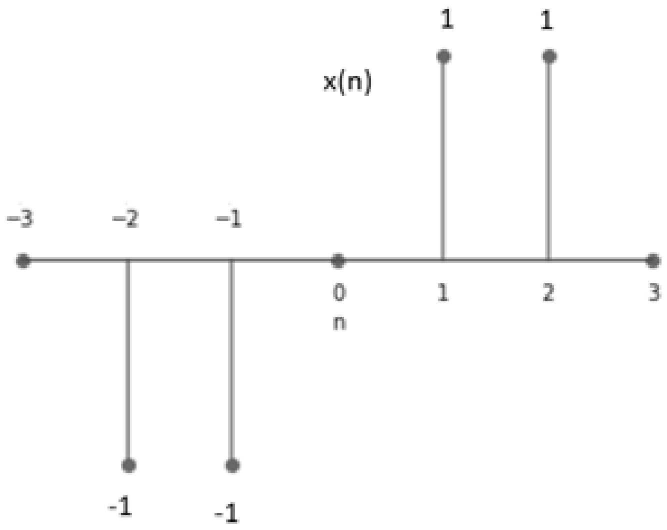
A discrete time signal is defined as follows. Find y(n) = x(2n+3).

$$x(n) = \begin{cases} 1 & n = 1,2 \\ -1 & n = -1,-2 \\ 0 & n = 0 \ \ and\ |n| > \ 2 \end{cases}$$

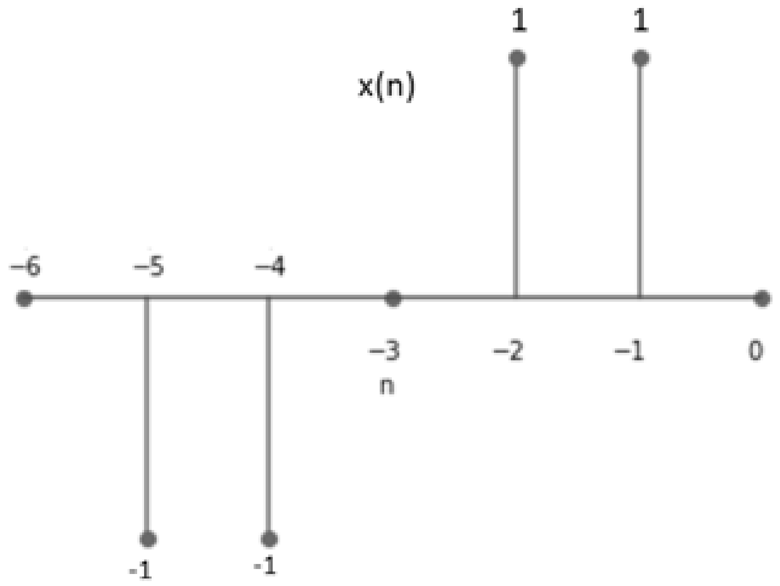
Solution



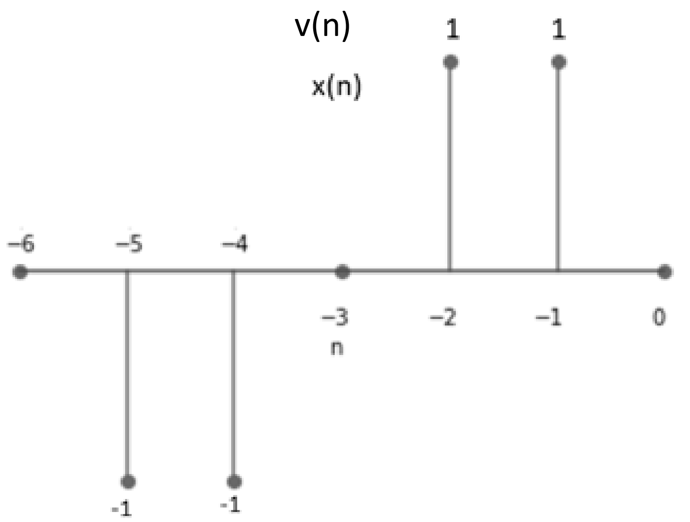
$v(n) = x(n + 3)$



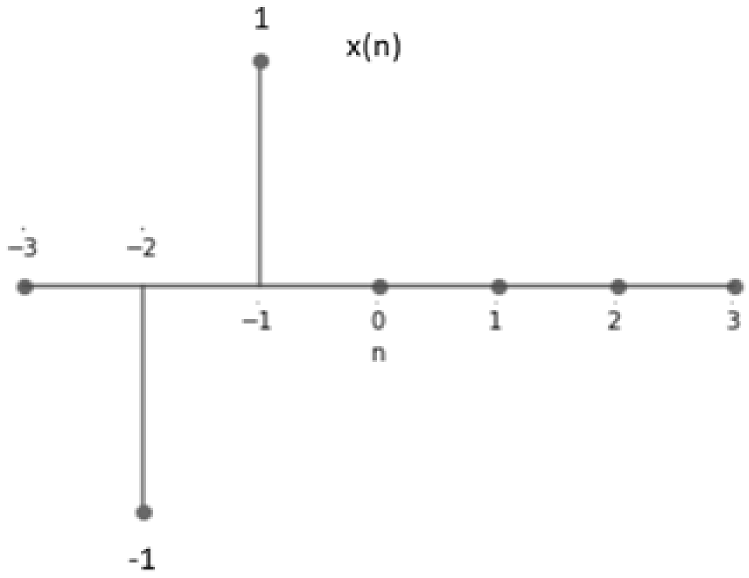
v(n) = x(n + 3)



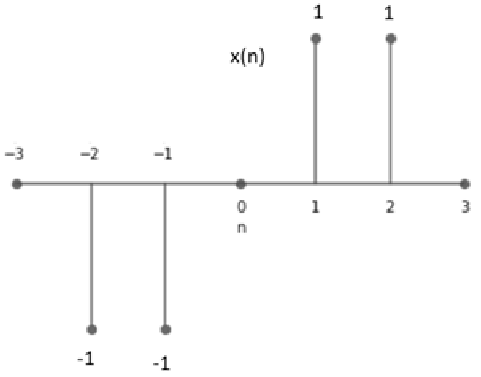
y(n) = v(2n) = x(2n+

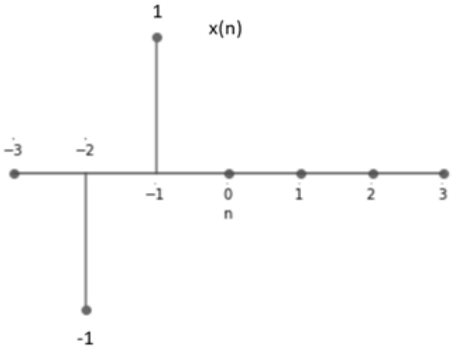


$y(n) = v(2n) = x(2n+3)$



Verification





Classification of Discrete Time Signals

Classification of Discrete Time Signals

- Discrete-time signals are further classified as follows:

1. Deterministic and random signals
2. Periodic and non-periodic signals
3. Energy and power signals
4. Causal and non-causal signals
5. Even and odd signals

Deterministic and Random signals

Deterministic Signals

- A signal exhibiting no uncertainty of its magnitude and phase at any given instant of time is called deterministic signal.
- A deterministic signal can be completely represented by a mathematical equation
- E.g. sine signal, cos signal, ramp signal etc.

Deterministic and Random signals

Random Signals

- A signal characterized by uncertainty about its occurrence is called a non-deterministic or random signal.
- A random signal cannot be represented by any mathematical equation.
- The behavior of such a signal is probabilistic in nature and can be analyzed only stochastically.
- E.g. Thermal noise.

Periodic and Non-periodic signals

- A signal which has a definite pattern and repeats itself at regular intervals of time is called a **periodic signal**
- A signal which does not repeat at regular intervals of time is called a **non-periodic or aperiodic signal**.
- $x(n)=x(n+N)$ → Periodic Signal
- $x(n)\neq x(n+N)$ → Non-Periodic Signal

- The angular frequency of a periodic signal is given by

$$\omega = \frac{2\pi}{N}$$

$\therefore$ Fundamental period $$N = \frac{2\pi}{\omega}$$

- If ratio of is rational number then the signal is periodic else it is aperiodic

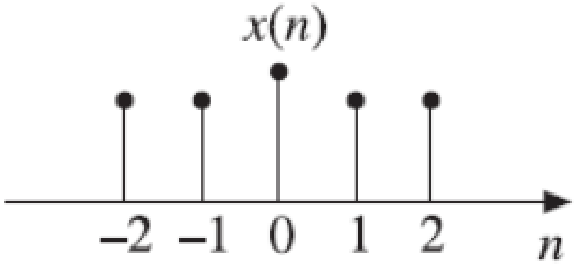
Even and Odd Signals

Even Signals (Symmetric Signals)

- A discrete-time signal x(n) is said to be an even signal if it satisfies the condition:

$$x(-n) = x(n) \quad \text{for all } n$$

- Even signals are symmetrical about the vertical axis or time origin.
- An even signal is identical to its reflection about the origin.
- E.g. Cosine wave

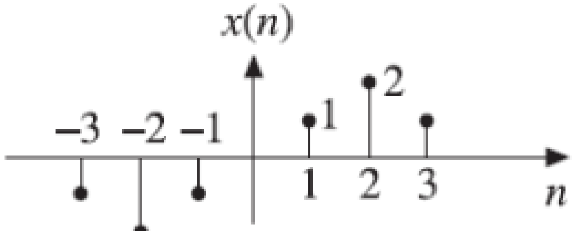


Odd Signals (Antisymmetric Signals)

- A discrete-time signal x(n) is said to be an odd (anti-symmetric) signal if it satisfies the condition:

$$x(-n) = -x(n) \quad \text{for all } n$$

- Odd signals are anti-symmetrical about the vertical axis.
- E.g. Sine wave



Even and Odd Signals

- Any arbitrary signal x(n) can be expressed as the sum of even and odd components.
- i.e. $x(n) = x_e(n) + x_o(n)$
- where $x_e(n)$ is even component and $x_o(n)$ is odd component of the signal.
- The even component $x_e(n)$ for the signal x(n) is given by

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

Even and Odd Signals

- The odd component $x_o(n)$ for the signal $x(n)$ is given by

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

- The product of two even or odd signals is an even signal and the product of even signal and odd signal is an odd signal.

Problem

- Find the even and odd components of the following signals:

(a) $x(n)=\left\{-3, 1, \underset{\uparrow}{2}, -4, 2\right\}$

$x(n) = \{-3, 1, \underset{\uparrow}{2}, -4, 2\}$

$x(-n) = \{2, -4, \underset{\uparrow}{2}, 1, -3\}$

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$= \frac{1}{2}[(-3+2), (1-4), (2+2), (-4+1), (2-3)]$

$= \frac{1}{2}[-1, -3, 4, -3, -1]$

$= \left\{\frac{-1}{2}, \frac{-3}{2}, 2, -\frac{3}{2}, \frac{-1}{2}\right\}$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$= \frac{1}{2}[(-3-2), (1+4), (2-2), (-4-1), (2+3)]$

$= \frac{1}{2}[-5, 5, \underset{\uparrow}{0}, -5, 5]$

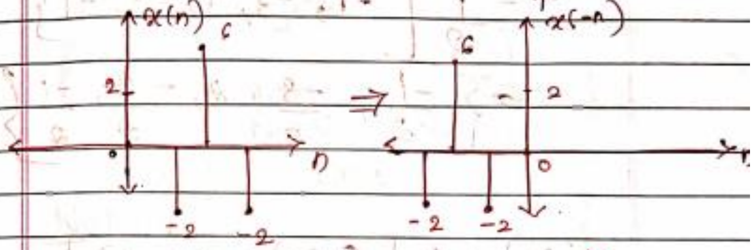
$= \left\{\frac{-5}{5}, \frac{5}{2}, 0, \frac{-5}{2}, \frac{5}{2}\right\}$

Problem 2:

X(n)={2, -2, 6, -2}

$$x(n) = \{\underset{\uparrow}{2}, -2, 6, -2\}$$

$$x(-n) = \{-2, 6, -2, \underset{\uparrow}{2}\}$$



$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \frac{1}{2}\left[(-2+0), (6+0), (-2+0), \underset{\uparrow}{(2+2)}, (-2+0), (6+0), (-2+0)\right]$$

$$= \frac{1}{2}\left[-2, 6, -2, \underset{\uparrow}{4}, -2, 6, -2\right]$$

$$= \{-1, 3, -1, \underset{\uparrow}{2}, -1, 3, -1\}$$

$$x_o(n) = \frac{1}{2}\left(x(n) - x(-n)\right)$$

$$= \frac{1}{2}\{2, -6, 2, \underset{\uparrow}{0}, -2, 6, -2\}$$

$$\{1, -3, 1, 0, -1, 3, -1\}$$

Problem 3:
X(n)={1,2,1,3}

Eg: $x(n) = \{\underset{\uparrow}{1}, 2, 1, 3\}$

$x(-n) = \{3, 1, 2, \underset{\uparrow}{1}\}$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \frac{\{\underset{\uparrow}{1}, 2, 1, 3\} + \{3, 1, 2, 1\}}{2}$$

$$x_e(n) = \frac{\{3, 1, 2, \underset{\uparrow}{2}, 2, 1, 3\}}{2}$$

$$= \left\{\frac{3}{2}, \frac{1}{2}, 1, 1, 1, 1, 3\right\}$$

Problem 4:
X(n)= e^jn

$x(n) = e^{jn}$

$x(n) = \cos n + j \sin n$

$x(n) = x_e(n) + x_o(n)$

$e^{jn} = x_e(n) + x_o(n)$

$\therefore \; x_e(n) = \cos n$

$x_o(n) = j \sin n$

OR

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \frac{e^{jn} + e^{-jn}}{2}$$

$$= \cos n$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$= \frac{1}{2}\left(e^{jn} - e^{-jn}\right)$$

Problem 4:

$$x(n) = 3\, e^{j\pi/5\, n}.$$

$$x(n) = 3\left[\cos\frac{\pi}{5}n + j\sin\frac{\pi}{5}n\right].$$

$$x(-n) = 3\, e^{-j\pi/5\, n}$$

$$= 3\left[\cos\frac{\pi}{5}n - j\sin\frac{\pi}{5}n\right]$$

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Even part $x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$

$$= \frac{1}{2}\left[\left[3\cos\frac{\pi}{5}n + 3j\sin\frac{\pi}{5}n\right] + \left[3\cos\frac{\pi}{5}n - 3j\sin\frac{\pi}{5}n\right]\right]$$

$$= \frac{1}{2} \times 6\cos\frac{\pi}{5}n$$

$$= 3\cos\frac{\pi}{5}n.$$

Odd part $x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$

$$= \frac{1}{2}\left[\left(3\cos\frac{\pi}{5}n + 3j\sin\frac{\pi}{5}n\right) - \left(3\cos\frac{\pi}{5}n - 3j\sin\frac{\pi}{5}n\right)\right]$$

$$= \frac{1}{2}\left[6j\sin\frac{\pi}{5}n\right]$$

$$= 3j\sin\frac{\pi}{5}n$$

Causal and Noncausal Signals

- A discrete-time signal x(n) is said to be **causal** if x(n) = 0 for n < 0
- A discrete-time signal x(n) is said to be **anti-causal** if x(n) = 0 for n > 0.
- A signal which exists in positive as well as negative time is also called a **non-casual** signal.
- E.g.

  u(n) is a causal signal

  u(– n) is an anti-causal signal

  x(n) = 1 for – 2 ≤ n ≤ 3 is a non-causal signal.

Problem

- Find which of the following signals are causal or non-causal

(a) $x(n) = u(n+4) - u(n-2)$

(b) $x(n) = 0.5^n u(n-2)$

(c) $x(n) = u(-n)$

(a) $x(n) = u(n+4) - u(n-2)$

Energy and Power Signals

- Signals may also be classified as energy signals and power signals.
- The energy E of a discrete-time signal x(n) is defined as:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

- The average power P of a discrete-time signal x(n) is defined as:

$$P = \underset{N \to \infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Energy Signals

- A signal is said to be an energy signal if and only if its total energy E over the interval $(-\infty, \infty)$ is finite i.e., $0 < E < \infty$
- For an energy signal, average power $P = 0$.

Power Signals

- A signal is said to be a power signal, if its average power P is finite (i.e.,0 < P< ∞).
- For a power signal, total energy E = ∞ .
- Periodic signals are the examples of power signals.
- The power of a periodic signal, with fundamental period N, can be computed as

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

Important Facts

- The signals that do not satisfy the above properties are neither energy signals nor power signals.

  E.g. $x(n) = u(n)$, $x(n) = nu(n)$, $x(n) = n^2u(n)$.

- Both energy and power signals are mutually exclusive, i.e. no signal can be both energy signal and power signal

Exercise Problem

- Find which of the following signals are energy signals, power signals, neither energy nor power signals:

(a) $\left(\frac{1}{2}\right)^n u(n)$

(b) $e^{j[(\pi/3)n+(\pi/2)]}$

(c) $\sin\left(\frac{\pi}{3}n\right)$

(d) $u(n)-u(n-6)$

(e) $nu(n)$

Important formulas

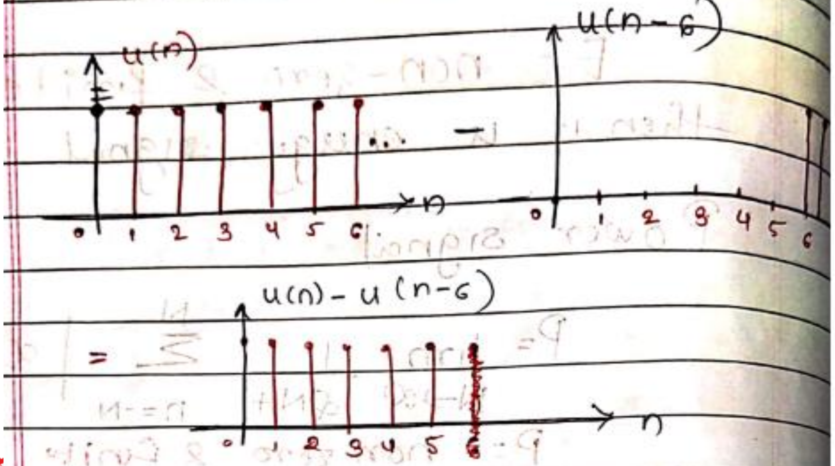
Problem 1:

x(n) = u(n)-u(n-6)

$$x(n) = u(n) - u(n-6)$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |\{u(n) - u(n-6)\}|^2$$



$$E = \sum_{n=-\infty}^{-1} |0|^2 + \sum_{n=0}^{5} |1|^2$$

$$= 0 + 1 + 1 + 1 + 1 + 1 + 1$$

$$\boxed{E = 6 \text{ joules}}$$

Problem 1:

x(n) = u(n)-u(n-6)

$$\text{Power of the signal } P = \underset{N\to\infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} [u(n) - u(n-6)]^2$$

$$= \underset{N\to\infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=0}^{5} 1 = 0$$

Energy is finite and power is zero. Therefore, it is an energy signal.

Problem 2:

x(n) = nu(n)

$$x(n) = n\,u(n)$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{-\infty}^{\infty} |n\,u(n)|^2$$

$$= \sum_{n=-\infty}^{-1} n^2 (u(n))^2 + \sum_{n=0}^{\infty} n^2 [u(n)]^2$$

$$= 0 + \sum_{n=0}^{\infty} n^2 \cdot (1)^2$$

$$= \sum_{n=0}^{\infty} n^2.$$

$$\boxed{E = \infty}$$

Problem 2:

x(n) = nu(n)

Power Signal

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |n \cdot u(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2 \cdot 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[0^2 + 1^2 + 2^2 + 3^2 + N^2 \right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[0 + 1 + 4 + \cdots + \infty \right]$$

$$= \infty$$

Energy is infinite and power is also infinite. Therefore, it is neither energy signal nor power signal.

Problem 3:

$x(n) = u(n)$

$$x(n) = u(n)$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |u(n)|^2$$

$$= \sum_{n=-1}^{-\infty} |u(n)|^2 + \sum_{n=0}^{\infty} |u(n)|^2$$

$$= 0 + \sum_{n=0}^{\infty} 1^2$$

$$= \sum_{n=1}^{\infty} 1$$

$$= 1 + 2 + 3 + \cdots + \infty$$

$$= \infty$$

Problem 3:

x(n) = u(n)

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[\sum_{n=-1}^{-N} |u(n)|^2 + \sum_{n=0}^{N} |u(n)|^2 \right]$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1$$

Problem 3:

x(n) = u(n)

$$\because \boxed{\sum_{n=0}^{N} 1 = N+0+1 = N+1}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} * N+1$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$= \lim_{N\to\infty} \frac{1+1/N}{2+1/N} \quad \text{(Taking out N common)}$$

$$\boxed{\lim_{N\to\infty} \frac{1}{N} = \frac{1}{\infty} = 0}$$

$$= \cancel{\lim_{N\to\infty}} \frac{1+0}{2+0}$$

$$\boxed{P = \frac{1}{2} \text{ Watt}}$$

Problem 4:

$x(n) = a^n\, u(n)$

$$E = \sum_{-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{-\infty}^{\infty} |a^n u(n)|^2$$

$$= \sum_{-\infty}^{\infty} a^{2n} u(n)^2$$

$$= \sum_{-\infty}^{-1} 0 + \sum_{0}^{\infty} a^{2n}$$

$$* \quad \boxed{\sum_{0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow \sum_{0}^{\infty} a^{2n} = \frac{1}{1-a^2}}$$

$$= \frac{1}{1-a^2} \text{ Joules}$$

∴ Energy is finite

∴ It is Energy Singal

Hence Power = 0.

In General, if $x(n) = a^n u(n)$

then $E = \dfrac{1}{1-a^2}$, $P = 0$

if $x(n) = -(0.5)^n u(n$

then $E = \dfrac{1}{1-0.5^2}$

$= \dfrac{1}{1-0.25}$ J

if $x(n) = \left(\frac{1}{4}\right)^n u(n)$

$E = \dfrac{1}{1-(1/4)^2}$ J

Problem 5:

$$x(n) = \sin\left(\frac{\pi}{3} n\right)$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left|\sin\left(\frac{\pi}{3} n\right)\right|^2$$

$$= \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{\pi}{3} n\right)$$

$$\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$$

Trignometric Formula

$$= \sum_{n=-\infty}^{\infty} \frac{1-\cos 2\pi/3\, n}{2}$$

$$= \frac{1}{2}\left[\sum_{n=-\infty}^{\infty}\left(1 - \cos \frac{2\pi}{3} n\right)\right]$$

$$= \frac{1}{2}\left[\underbrace{\sum_{n=-\infty}^{\infty} 1}_{A} - \underbrace{\sum_{n=-\infty}^{\infty} \cos \frac{2\pi}{3} n}_{B}\right]$$

Problem 5:

$\cdot A =$ Sum of infinite terms is ∞.

$\therefore A = \infty$

$$B = \sum_{n=-\infty}^{\infty} \cos \frac{2\pi}{3} \cdot n$$

$n = 0, \quad \cos 0 = 1$

$n = 1, \quad \cos 2\pi/3 = -1/2$

$n = 2, \quad \cos 4\pi/3 = -1/2$

Add 1^{st} 3 values $\Rightarrow$ you will get 0

$\therefore$ if you add all values from $-\infty$ to ∞

$\Longrightarrow$ you will 0

Hence $B = 0$

$\therefore E = \infty$.

Problem 5:

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\sin^2 \frac{\pi}{3} n\right|$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1 - \cos 2\pi/3\, n}{2}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1}{2} \left[\underbrace{\sum_{n=-N}^{N} 1}_{A} - \underbrace{\sum_{n=-N}^{N} \cos 2\pi/3\, n}_{B} \right]$$

$$\therefore B = 0$$

$$A = \sum_{n=-N}^{N} 1 = N - (-N) + 1 = 2N+1$$

$$\therefore P = \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1}{2} \left[2N+1 - 0 \right]$$

$$P = \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} \cdot \frac{1}{2} \cdot \cancel{2N+1}$$

$$= \frac{1}{2} \text{ Watt}$$

∴ it is P. signal

Discrete Correlation

- Correlation is mathematical operation used to compare two signals.
- It is a measure of similarity between signals.
- It is used in radar and sonar systems to find the location of the target by comparing the transmitted and reflected signal.
- It occupies a significant role in signal processing.

Types of Correlation

- The correlation is of two types:
  (i) Cross correlation
  (ii) Auto-correlation.

Cross correlation

- The cross correlation between a pair of sequences x(n) and y(n) is given by

$$R_{xy}(n) = \sum_{k=-\infty}^{\infty} x(k)\, y(k-n)$$

- The expression for $R_{xy}(n)$ can be written as:

$$R_{xy}(n) = x(n) * y(-n)$$

Auto-correlation

- The autocorrelation of a sequence is correlation of a sequence with itself.
- It gives a measure of similarity between a sequence and its shifted version.
- The autocorrelation of a sequence x(n) is defined as:

$$R_{xx}(n) = \sum_{k=-\infty}^{\infty} x(k)\,x(k-n)$$

- Its given by $R_{xx}(n) = x(k) * x(-k)$

Problem 1:

$x(n)=\{2,3,1,4\}$

$y(n)= \{1,3,2,1\}$

① Cross Correlation

$x(n) = \{2, 3, 1, 4\}$

$y(n) = \{1, 3, 2, 1\}$

$y(-n) = \{1, 2, 3, 1\}$

$$\boxed{R_{xy}(n) = x(n) * y(-n)}$$

$x(n)$ \ $y(-n)$	1	2	3	1
2	2	4	6	2
3	3	6	9	3
1	1	2	3	1
4	4	8	12	4

$$R_{xy}(n) = \{2,\ 3+4,\ 1+6+6,\ 4+2+9+2,\ 8+3+3,\ 12+1,\ 4\}$$

$$= \{2, 7, 13, 17, 14, 13, 4\}$$

Problem 2:

$x(n)=\{2,3,1,4\}$

(ii) Auto correlation:

$$x(n) = \{2, 3, 1, 4\}$$

$$x(n) = \{4, 1, 3, 2\}$$

$$R_{xx}(n) = x(n) * x(-n)$$

$x(n)$ \ $x(-n)$	4	1	3	2
2	8	2	6	4
3	12	3	9	6
1	4	1	3	2
4	16	4	12	8

$$R_{xx}(n) = \{8, 14, 13, 30, 13, 14, 8\}$$

Problem 3:

$x(n)=\{3,5,1,2\}$

$y(n)= \{1,4,3\}$

$x(n) = \{3, \underset{\uparrow}{5}, 1, 2\}$

$h(n) = \{\underset{\uparrow}{1}, 4, 3\}$

$h(-n) = \{3, 4, \underset{\uparrow}{1}\}$

	3	4	1
3	9	12	3
5	15	20	5
1	3	4	1
2	6	8	2

$R_{xy}(n) = x(n) * h(n)$

$= \{9, \underset{\uparrow}{27}, 26, 15, 9, 2\}$

Additional Problems: HomeWork

Eg: H.W $x(n) = \{2, 5, -4\}$

Ans: $R_{xx}(n) = \{-8, -10, 45, -10, -8\}$

UQ Problem: Homework

- Find the cross correlation of the sequence

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\} \qquad h(n) = \{2, 4, 6\}$$

UQ Problem: Homework

- Perform the cross correlation of the causal sequences

$$x(n) = \{3, 3, 1, 1\} \qquad h(n) = \{3, 2, 1, 2\}$$

UQ Problem: Homework

- Determine the cross correlation of two causal sequences

$x(n) = \{ 2, 3, 1, 4 \}$ and $y(n) = 3\delta(n-3) - 2\delta(n) + \delta(n-1) + 4\delta(n-2)$

UQ Problem

$x(n) = \{ 2, 3, 1, 4 \}$ and $y(n) = 3\delta(n-3) - 2\delta(n) + \delta(n-1) + 4\delta(n-2)$

UQ Problem: Homework

- For the given causal sequence x(n) = {8, 9, 2, 3} and h(n) = {4, 3, 6} find the cross correlation

UQ Problem:Homework

- For the given causal sequence x(n) = {3, 3, 1, 1} and h(n) = {1, 2, 1} find the cross correlation

UQ Problem: Homework

- Find the autocorrelation of the causal sequence $x(n) = \{2, 4, 6, 8\}$

Convolution of Finite Duration Sequences

Convolution

- It is a mathematical operation to combine 2 signals to form the third signal.

Types of convolution

1. Linear convolution
2. Circular convolution
3. Linear using circular convolution

1. Linear Convolution

- It is denoted by *
- y(n)=x(n)*h(n)
- There are various methods to find linear convolution of two sequences.

  Method 1: Graphical Method

  Method 2: Tabular array Method

  Method 3: Tabular Method

  Method 4: Matrix Method

  Method 5: Sum by Column Method

  Method 6: Flip, Shift, Multiply and Sum Method

Problem

- Determine the convolution sum of two sequences:

$$x(n) = \{4, 2, 1, 3\}, \qquad h(n) = \left\{\begin{matrix} 1, 2, 2, 1 \\ \uparrow \end{matrix}\right\}$$

Method 1 : Linear Convolution using Graphical Method

1. Choose the initial value of n. If x(n) starts at n_1 and h(n) starts at n_2 , then $n=n_1+n_2$ will be a good choice.

2. Express both sequence in terms of k

3. Fold h(k) about k=0 to obtain h(-k) and shift it by n

4. Multiply x(k) and h(n-k) and sum up the product to get y(n)

5.Increment n, shift h(n-k) to right by one and do step 4

6. Repeat 5 until sum of products for all remaining n is zero.

Problem: $x(n) = \{4, 2, 1, 3\}$, $h(n) = \left\{ \begin{matrix} 1, & 2, & 2, & 1 \\ & \uparrow & & \end{matrix} \right\}$

Solution:

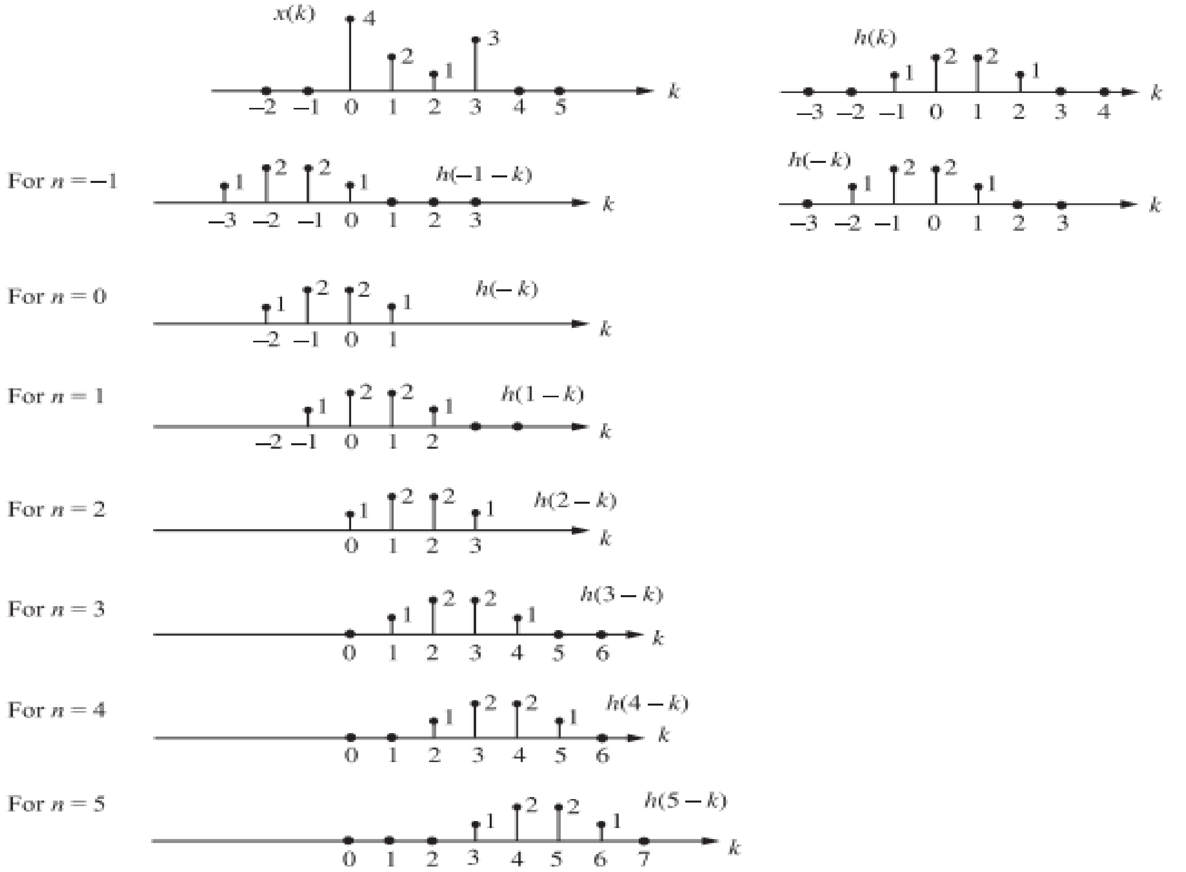
$x(n)$ starts at $n_1 = 0$ and $h(n)$ starts at $n_2 = -1$. Therefore, the starting sample of $y(n)$ is at

$$n = n_1 + n_2 = 0 - 1 = -1$$

$x(n)$ has 4 samples, $h(n)$ has 4 samples. Therefore, $y(n)$ will have $N = 4 + 4 - 1 = 7$ samples, i.e., from $n = -1$ to $n = 5$.

Problem:

$$x(n) = \{4, 2, 1, 3\}, \qquad h(n) = \left\{1, \underset{\uparrow}{2}, 2, 1\right\}$$



Problem: $x(n) = \{4, 2, 1, 3\}$, $\quad h(n) = \left\{ \underset{\uparrow}{1}, 2, 2, 1 \right\}$

We know that $$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

From Figure 2.6, we get

For $n = -1$ $\quad y(-1) = \sum_{k=-\infty}^{\infty} x(k)\,h(-1-k) = 4\cdot1 = 4$

For $n = 0$ $\quad y(0) = \sum_{k=-\infty}^{\infty} x(k)\,h(-k) = 4\cdot2 + 2\cdot1 = 10$

For $n = 1$ $\quad y(1) = \sum_{k=-\infty}^{\infty} x(k)\,h(1-k) = 4\cdot2 + 2\cdot2 + 1\cdot1 = 13$

For $n = 2$ $\quad y(2) = \sum_{k=-\infty}^{\infty} x(k)\,h(2-k) = 4\cdot1 + 2\cdot2 + 1\cdot2 + 3\cdot1 = 13$

For $n = 3$ $\quad y(3) = \sum_{k=-\infty}^{\infty} x(k)\,h(3-k) = 2\cdot1 + 1\cdot2 + 3\cdot2 = 10$

For $n = 4$ $\quad y(4) = \sum_{k=-\infty}^{\infty} x(k)\,h(4-k) = 1\cdot1 + 3\cdot2 = 7$

For $n = 5$ $\quad y(5) = \sum_{k=-\infty}^{\infty} x(k)\,h(5-k) = 3\cdot1 = 3$

$\therefore$ $\quad y(n) = \left\{ 4, \underset{\uparrow}{10}, 13, 13, 10, 7, 3 \right\}$

Method 2 : Linear Convolution using Tabular Array

Step 1: Change the index from n to k, and write x(k) and h(k).

Step 2: Represent the sequences x(k) and h(k) as two rows of tabular array.

Step 3: Fold one of the sequences. Let us fold h(k) to get h(– k).

Step 4: Shift the sequence h(– k), q times to get the sequence h(q – k). If q is positive, then shift the sequence to the right and if q is negative, then shift the sequence to the left.

Step 5 : Determine the product sequence x(k) h(q – k) for one period.

Step 6 : The sum of the samples of the product sequence gives the sample at n = q

Problem: $x(n) = \{4, 2, 1, 3\}, \qquad h(n) = \left\{1, \underset{\uparrow}{2}, 2, 1\right\}$

k		-4	-3	-2	-1	0	1	2	3	4	5	6	7
$x(k)$		–	–	–	–	4	2	1	3	–	–	–	–
$h(-k)$		–	–	1	2	2	1	–	–	–	–	–	–
$n = -1$	$h(-1-k)$	–	1	2	2	1	–	–	–	–	–	–	–
$n = 0$	$h(-k)$	–	–	1	2	2	1	–	–	–	–	–	–
$n = 1$	$h(1-k)$	–	–	–	1	2	2	1	–	–	–	–	–
$n = 2$	$h(2-k)$	–	–	–	–	1	2	2	1	–	–	–	–
$n = 3$	$h(3-k)$	–	–	–	–	–	1	2	2	1	–	–	–
$n = 4$	$h(4-k)$	–	–	–	–	–	–	1	2	2	1	–	–
$n = 5$	$h(5-k)$	–	–	–	–	–	–	–	1	2	2	1	–

The starting value of $n = -1$. From the table, we can see that

For $n = -1$ $$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k) = 4\cdot 1 = 4$$

For $n = 0$ $$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = 4\cdot 2 + 2\cdot 1 = 10$$

For $n = 1$ $$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = 4\cdot 2 + 2\cdot 2 + 1\cdot 1 = 13$$

For $n = 2$ $$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = 4\cdot 1 + 2\cdot 2 + 1\cdot 2 + 3\cdot 1 = 13$$

For $n = 3$ $$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = 2\cdot 1 + 1\cdot 2 + 3\cdot 2 = 10$$

For $n = 4$ $$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = 1\cdot 1 + 3\cdot 2 = 7$$

For $n = 5$ $$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k) = 3\cdot 1 = 3$$

$\therefore$ $$y(n) = \left\{\begin{matrix} 4, & 10, & 13, & 13, & 10, & 7, & 3 \\ & \uparrow & & & & & \end{matrix}\right\}$$

Method 3 : Linear Convolution using Tabular Method

Step 1: Write down the sequences x(n) and h(n) as shown below

		$x(n)$			
		x_1	x_2	x_3	x_4
	h_1				
	h_2				
$h(n)$	h_3				
	h_4				

Method 3 : Linear Convolution using Tabular Method

Step 2: Multiply each and every sample in h(n) with the samples of x(n) and tabulate the values as shown below.

$$
\begin{array}{cc|cccc}
 & & & x(n) & & \\
 & & x_1 & x_2 & x_3 & x_4 \\
\hline
 & h_1 & x_1h_1 & x_2h_1 & x_3h_1 & x_4h_1 \\
 & h_2 & x_1h_2 & x_2h_2 & x_3h_2 & x_4h_2 \\
h(n) & h_3 & x_1h_3 & x_2h_3 & x_3h_3 & x_4h_3 \\
 & h_4 & x_1h_4 & x_2h_4 & x_3h_4 & x_4h_4
\end{array}
$$

Method 3 : Linear Convolution using Tabular Method

Step 3: Group the elements in the table by drawing diagonal lines as shown in table.

		$x(n)$			
		x_1	x_2	x_3	x_4
	h_1	x_1h_1	x_2h_1	x_3h_1	x_4h_1
	h_2	x_1h_2	x_2h_2	x_3h_2	x_4h_2
$h(n)$	h_3	x_1h_3	x_2h_3	x_3h_3	x_4h_3
	h_4	x_1h_4	x_2h_4	x_3h_4	x_4h_4

Method 3 : Linear Convolution using Tabular Method

Step 4: Starting from the left sum all the elements in each strip and write down in the same order.

Step 5: Mark the symbol $\uparrow$ at time origin (n = 0).

Problem: $x(n) = \{4, 2, 1, 3\}, \qquad h(n) = \{1, \underset{\uparrow}{2}, 2, 1\}$

$x(n)$

$h(n)$		4	2	1	3
	1	4	2	1	3
	2	8	4	2	6
	2	8	4	2	6
	1	4	2	1	3

$$y(n) = 4,\ 8 + 2,\ 8 + 4 + 1,\ 4 + 4 + 2 + 3,\ 2 + 2 + 6,\ 1 + 6,\ 3$$
$$= 4,\ 10,\ 13,\ 13,\ 10,\ 7,\ 3$$

The starting value of n is equal to –1, mark the symbol ↑ at time origin ($n = 0$).

$$\therefore \qquad y(n) = \{4, \underset{\uparrow}{10}, 13, 13, 10, 7, 3\}$$

Additional Problems: Homework

Find the convolution of the signals

$$x(n) = \begin{cases} 2 & n = -2, 0, 1 \\ 3 & n = -1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \delta(n) - 2\delta(n-1) + 3\delta(n-2) - \delta(n-3)$$

Additional Problems: Homework

Find the convolution of the following sequences:

$$x(n) = 3\delta(n+1) - 2\delta(n) + \delta(n-1) + 4\delta(n-2)$$

$$h(n) = 2\delta(n-1) + 5\delta(n-2) + 3\delta(n-3)$$

Additional Problems: Homework

Find the discrete convolution of

$$u(n) * u(n-2)$$

For Solutions of homework problems, refer Anandkumar ebook..Page no 108 onwards

UQ Problem

- Perform convolution operation between given function in time domain if

$x(n) = 2^{-n} \qquad -2 \leq n \leq 2$

$\quad\;\; = 0 \qquad \text{Otherwise}$

and $h(n) = u(n+2) - u(n-2)$

UQ Problem

- Perform the convolution operation between the given function in time domain if

$x_1(n) = (-3)^n$ for n = 0, 1, 2, 3

$= 0$ Otherwise

and

- $x_2(n) = u(n) - u(n-4)$

2. Circular Convolution

- Circular convolution is performed on periodic signals.
- It is denoted by symbol
- y(n) = x(n) h(n)
- Circular convolution of 2 signals can be solved using following approaches.

  Method 1: Concentric Circle Method

  Method 2: Matrix Method

- Circular Convolution can be performed if both the sequences have same number of samples.
- If the sequences have different number of samples, then convert the smaller size sequence to the size of larger size sequence by appending zeros.
- The circular convolution produces a sequence whose length is same as that of input sequences.

Method 1: Concentric Circle Method

In graphical method, also called concentric circle method, the given sequences are represented on concentric circles. Given two sequences $x_1(n)$ and $x_2(n)$ the circular convolution of these two sequences, $x_3(n) = x_1(n) \oplus x_2(n)$ can be found using the following steps:

Step 1: Graph N samples of $x_1(n)$, as equally spaced points around an outer circle in anticlockwise direction.

Step 2: Starting at the same point as $x_1(n)$, graph N samples of $x_2(n)$ as equally spaced points around an inner circle in the clockwise direction. This corresponds to $x_2(-n)$.

Step 3: Multiply the corresponding samples on the two circles and sum the products to produce output, $x_3(0) = \sum_{n=0}^{N-1} x_1(n)\, x_2(-n)$

Step 4: Rotate the inner circle one sample at a time in anticlockwise direction and go to Step 3 to obtain the next value of output. If it is rotated by q samples,

$$x_3(q) = \sum_{n=0}^{N-1} x_1(n)\, x_2(q-n).$$

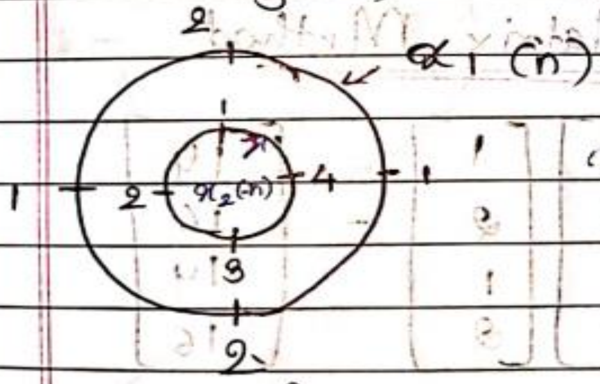
Step 5: Repeat Step 4 until the inner circle first sample lines up with the first sample of the outer circle once again.

Problem:

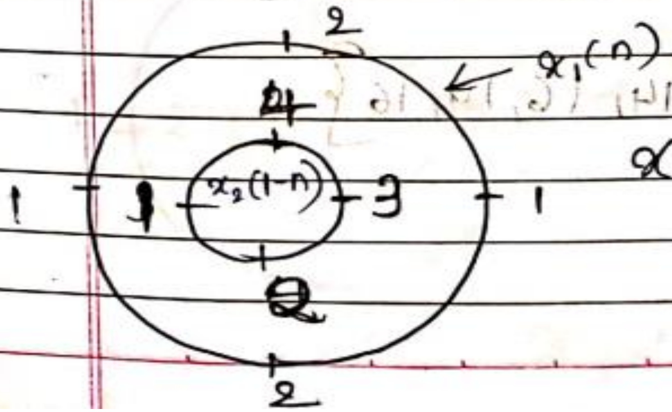
eg: $x_1(n) = \{1, 2, 1, 2\}$

$x_2(n) = \{4, 3, 2, 1\}$

$$x_3(n) = x_1(n) \oplus x_2(n)$$

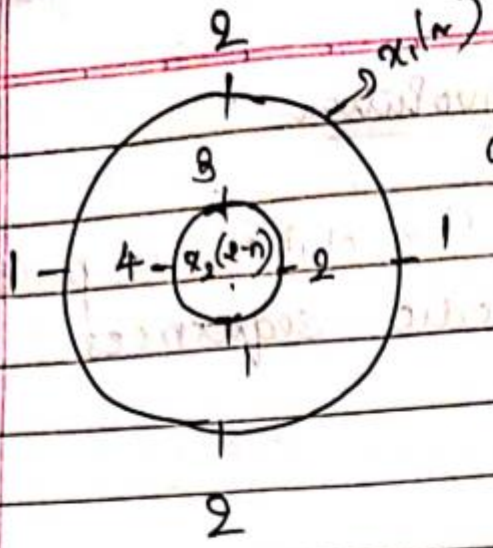


$$x_3(0) = 1\times4 + 2\times1 + 2\times1 + 3\times2 = 14$$

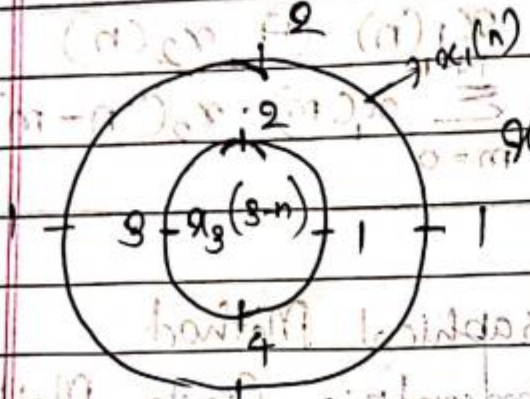


$$x_3(1) = 1\times3 + 4\times2 + 1\times1 + 2\times2 = 16$$

Note: Same problem solved in anandkumar page 138



$$x_3(2) = 2\times1 + 3\times2 + 1\times4 + \ldots$$
$$= 14$$



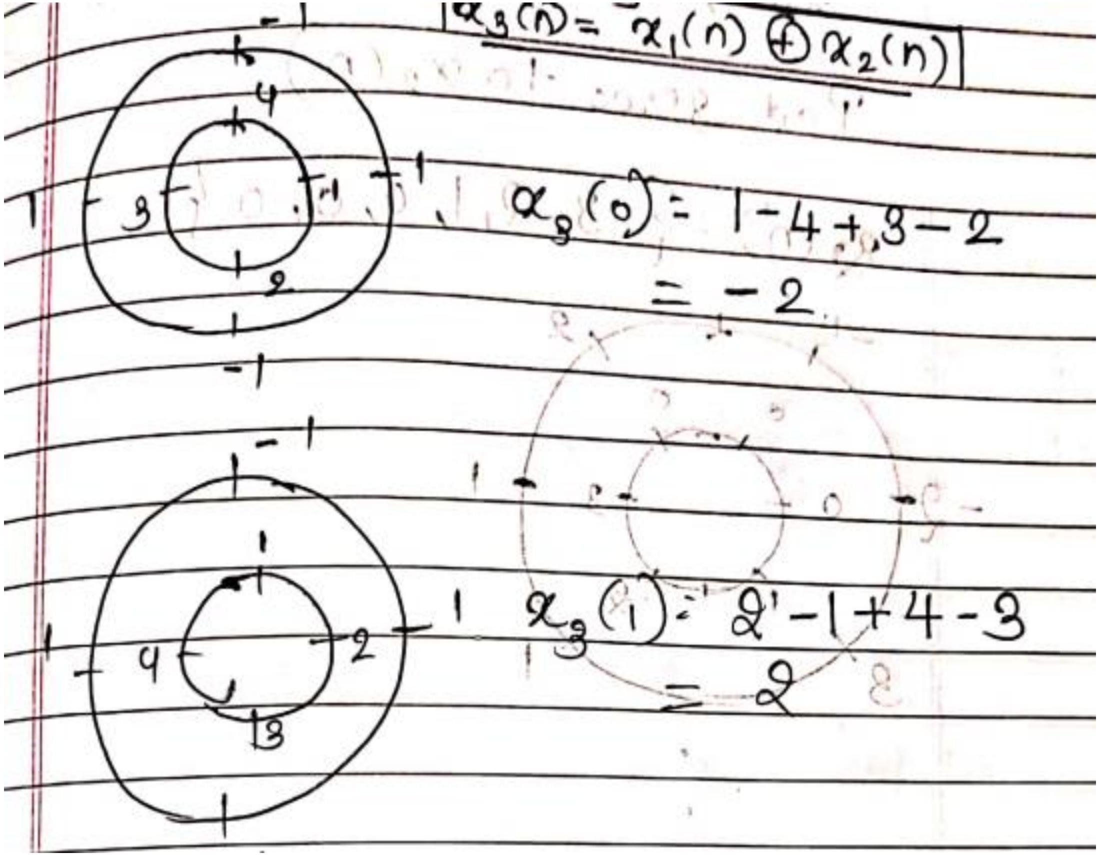
$$x_4(2) = 1\times1 + 2\times2 + 3\times1 + 4\times2$$
$$= 16$$

$$x_3(n) = \{14, 16, 14, 16\}$$

Problem:

x1(n) = {1,-1, 1, -1}

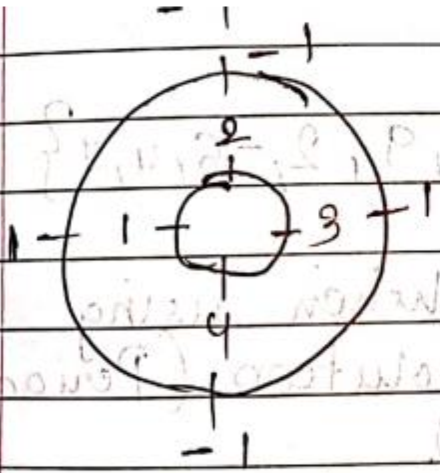
x2(n) = {1, 2, 3, 4}



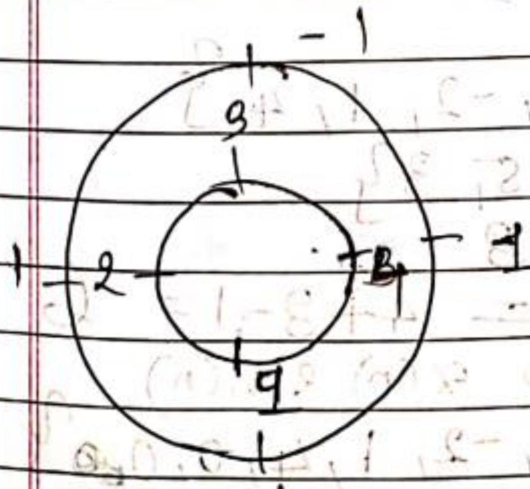
Problem:

$x1(n) = \{1,-1, 1, -1\}$

$x2(n) = \{1, 2, 3, 4\}$



$$x_3(2) = -2+3+(-4)+1$$
$$= -2$$



$$x_3(3) = 4-3+2-1$$
$$= 2$$

$$x_3(n) = \{-2, 2, -2, 2\}$$

Problem:

$x1(n) = \{1, 2, 1, -2, 3, 1\}$

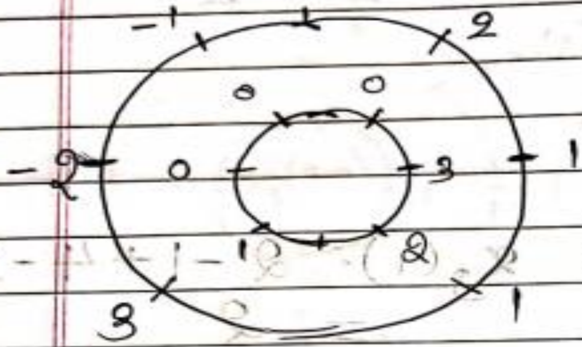
$x2(n) = \{3, 2, 1\}$

Eg. $x_1(n) = \{1, 2, -1, -2, 3, 1\}$

$x_2(n) = \{3, 2, 1\}$

Pad zeros to $x_2(n)$.

$x_2(n) = \{3, 2, 1, 0, 0, 0\}$

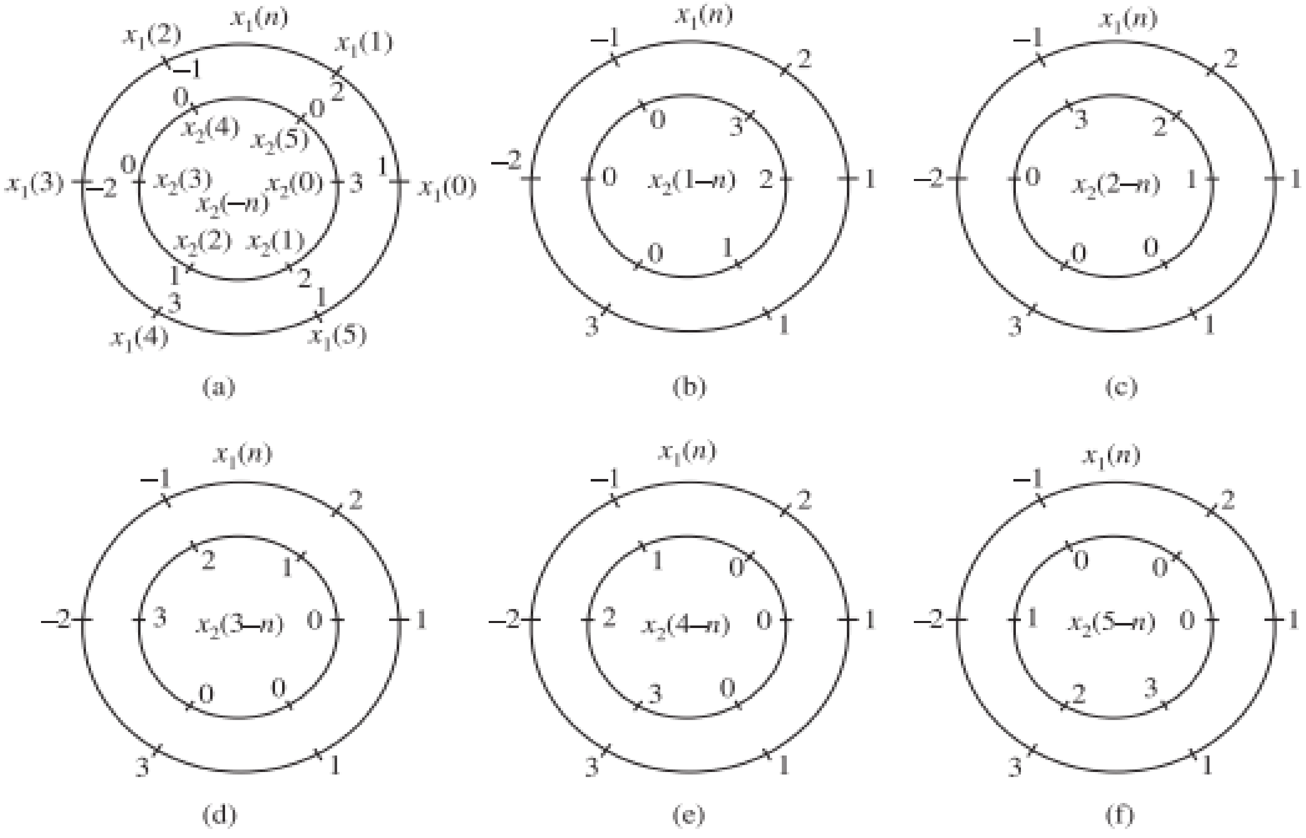


Ans: $x_3(n) = \{8, 9, 2, -6, 4, 7\}$

Problem:

$x1(n) = \{1, 2, 1, -2, 3, 1\}$

$x2(n) = \{3, 2, 1\}$



Problem:
$x1(n) = \{1,2, 1, -2, 3, 1\}$
$x2(n) = \{3, 2, 1\}$

From Figure 2.21(b), $x_3(1) = (1)(2) + (2)(3) + (-1)(0) + (-2)(0) + (3)(0) + (1)(1) = 9$

From Figure 2.21(c), $x_3(2) = (1)(1) + (2)(2) + (-1)(3) + (-2)(0) + (3)(0) + (1)(0) = 2$

From Figure 2.21(d), $x_3(3) = (1)(0) + (2)(1) + (-1)(2) + (-2)(3) + (3)(0) + (1)(0) = -6$

From Figure 2.21(e), $x_3(4) = (1)(0) + (2)(0) + (-1)(1) + (-2)(2) + (3)(3) + (1)(0) = 4$

From Figure 2.21(f), $x_3(5) = (1)(0) + (2)(0) + (-1)(0) + (-2)(1) + (3)(2) + (1)(3) = 7$

Therefore, the circular convolution of $x_1(n)$ and $x_2(n)$ is:

$$x_3(n) = x_1(n) \oplus x_2(n) = \{8, 9, 2, -6, 4, 7\}$$

Method 2: Matrix Method

Problem:
x1(n) = { 4, 3, 2, 1}
x2(n) = {1, 2, 1, 2}

$$\begin{bmatrix} 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

Therefore, the circular convolution of $x_1(n)$ and $x_2(n)$ is:

$$x_3(n) = x_1(n) \oplus x_2(n) = \{14,\ 16,\ 14,\ 16\}$$

3. Linear using Circular Convolution

Problem:

$x(n) = \{3, -2, 1, 4\}$

$h(n) = \{2, 5, 3\}$

$M = 4 \qquad N = 3$

$\therefore M + N - 1 = 4 + 3 - 1 = 6$

Add zeros to $x(n)$ & $h(n)$

$\therefore x(n) = \{3, -2, 1, 4, 0, 0\}$

$h(n) = \{2, 5, 3, 0, 0, 0\}$

Problem:

$$\begin{bmatrix} 3 & 0 & 0 & 4 & 1 & -2 \\ -2 & 3 & 0 & 0 & 4 & 1 \\ 1 & -2 & 3 & 0 & 0 & 4 \\ 4 & 1 & -2 & 3 & 0 & 0 \\ 0 & 4 & 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 1 \\ 7 \\ 23 \\ 12 \end{bmatrix}$$

$$y(n) = \{3, -2, 1, 4\} * \{2, 5, 3\}$$

$$= \{3, -2, 1, 4, 0, 0, 0\} \circledast \{2, 5, 3, 0, 0, 0\}$$

$$= \{6, 11, 1, 7, 23, 12\}$$

Problem:

$$\begin{bmatrix} 3 & 0 & 0 & 4 & 1 & -2 \\ -2 & 3 & 0 & 0 & 4 & 1 \\ 1 & -2 & 3 & 0 & 0 & 4 \\ 4 & 1 & -2 & 3 & 0 & 0 \\ 0 & 4 & 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 1 \\ 7 \\ 23 \\ 12 \end{bmatrix}$$

$$y(n) = \{3, -2, 1, 4\} * \{2, 5, 3\}$$

$$= \{3, -2, 1, 4, 0, 0, 0\} \circledast \{2, 5, 3, 0, 0, 0\}$$

$$= \{6, 11, 1, 7, 23, 12\}$$

Classification of Discrete Time Signals

- Discrete-time signals are further classified as follows:

1. Deterministic and random signals
2. Periodic and non-periodic signals
3. Energy and power signals
4. Causal and non-causal signals
5. Even and odd signals

Classification of Discrete Systems

Classification of Discrete Systems

- A system is defined as an entity that acts on an input signal and transforms it into an output signal.
- A system produces an output in response to an input signal.
- The response or output of the system depends on the transfer function of the system.
- A discrete time system is a device or an algorithm that performs some well-defined operations on a discrete time input signal to produce another discrete time output signal.

Classification of Discrete Systems

- The input signal x(n) is transformed into an output signal y(n) and it is given as follows:

$y(n) = T\{ x(n)\}$ where T is the transformation or operation.

- The operation T is a mathematical expression which defines the input-output relation.

Classification of Discrete Systems

- The discrete-time systems are classified as follows:

  1. Static and Dynamic systems
  2. Causal and Non-causal systems
  3. Linear and Nonlinear systems
  4. Time-Invariant and Time-Variant systems
  5. Stable and Unstable systems.

Static and Dynamic Systems

Static Systems

- A system is said to be static or memoryless if the output depends on the present input only.
- For a static or memoryless system, the output at any instant n depends only on the input applied at that instant n but not on the past or future values of input or past values of output.
- Examples:
  - $y(n) = x(n)$
  - $y(n) = 2x^2(n)$

Dynamic Systems

- A system is said to be dynamic or memory system if the response depends upon past or future inputs or past outputs.
- Examples:

- $y(n) = x(2n)$
- $y(n) = x(n) + x(n - 2)$
- $y(n)=x(n)+4\,y(n-1)+4y(n-2)$

Problem

- Find whether the following systems are dynamic or not:
- (a) $y(n) = x(n+1)$ ---Dynamic
- (b) $y(n) = x^2(n)$----Static
- (c) $y(n) = x(n-3) + x(n) + x(n+2)$ ----Dynamic

Additional Problems

EXAMPLE 1.12 Find whether the following systems are dynamic or not:

(a) $y(n) = x(n + 2)$ (b) $y(n) = x^2(n)$

(c) $y(n) = x(n - 2) + x(n)$

Solution:

(a) Given $y(n) = x(n + 2)$

The output depends on the future value of input. Therefore, the system is dynamic.

(b) Given $y(n) = x^2(n)$

The output depends on the present value of input alone. Therefore, the system is static.

(c) Given $y(n) = x(n - 2) + x(n)$

The system is described by a difference equation. Therefore, the system is dynamic.

Causal and Non-causal Systems

Causal Systems

- A system is said to be causal (or non-anticipative) if the output of the system at any instant n depends only on the present and past values of the input but not on future inputs.
- Causal systems are real systems
- They are physically realisable
- Examples:
  - $y(n) = nx(n)$
  - $y(n) = x(n - 2) + x(n - 1) + x(n)$

Non-causal Systems

- A system is said to be non-causal (anticipative) if the output of the system at any instant n depends on future inputs. They are anticipatory systems. They produce an output even before the input is given.
- These systems does not exists in real time
- Example:
- $y(n) = x(n) + x(2n)$
- $y(n) = x^2(n) + 2x(n + 2)$

Problem

- Check whether the following systems are causal or not:

1. y(n) = x(2n)—Non causal systems
2. y(n) = sin[x(n)]----Causal system

Additional Problems

EXAMPLE 1.13 Check whether the following systems are causal or not:

(a) $y(n) = x(n) + x(n-2)$ (b) $y(n) = x(2n)$

(c) $y(n) = \sin[x(n)]$ (d) $y(n) = x(-n)$

Solution:

(a) Given $y(n) = x(n) + x(n-2)$

For $n = -2$ $y(-2) = x(-2) + x(-4)$

For $n = 0$ $y(0) = x(0) + x(-2)$

For $n = 2$ $y(2) = x(2) + x(0)$

For all values of n, the output depends only on the present and past inputs. Therefore, the system is causal.

(b) Given $y(n) = x(2n)$

For $n = -2$ $y(-2) = x(-4)$

For $n = 0$ $y(0) = x(0)$

For $n = 2$ $y(2) = x(4)$

For positive values of n, the output depends on the future values of input. Therefore, the system is non-causal.

(c) Given $y(n) = \sin[x(n)]$

For $n = -2$ $y(-2) = \sin[x(-2)]$

For $n = 0$ $y(0) = \sin[x(0)]$

For $n = 2$ $y(2) = \sin[x(2)]$

For all values of n, the output depends only on the present value of input. Therefore, the system is causal.

(d) Given $y(n) = x(-n)$

For $n = -2$ $y(-2) = x(2)$

For $n = 0$ $y(0) = x(0)$

For $n = 2$ $y(2) = x(-2)$

For negative values of n, the output depends on the future values of input. Therefore, the system is non-causal.

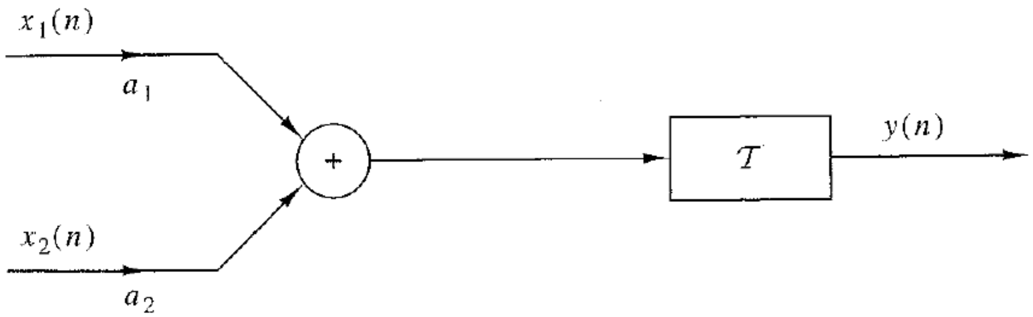
Linear and Nonlinear Systems

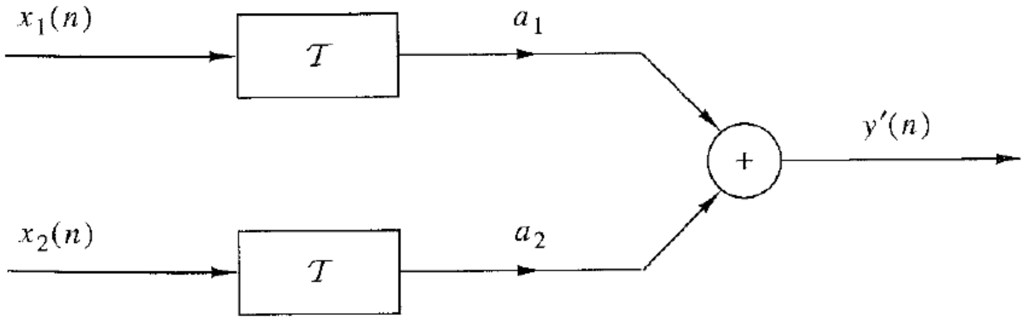
Linear and Nonlinear Systems

- A system which obeys the principle of superposition is called a linear system and one which does not obey the principle is a nonlinear system.
- The superposition principle states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of the outputs of the system to each of the individual signals.
- A system T is linear if and only if

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

- for any arbitrary input sequences $x_1(n)$ and $x_2(n)$, and any arbitrary constants a_1 and a_2





T is linear only if y(n)=y’(n)

Problem

- Check whether the following systems are linear or

(a) $y(n) = n^2 x(n)$

(b) $y(n) = x(n) + \dfrac{1}{2x(n-2)}$

(a) $y(n) = n^2 x(n)$

$y(n) = T[x(n)] = n^2 x(n)$

$T[ax_1(n) + bx_2(n)] = aT[x_1(n)] + bT[x_2(n)]$

Let an input $x_1(n)$ produce an output $y_1(n)$.

$$\therefore \quad y_1(n) = T[x_1(n)] = n^2 x_1(n)$$

Let an input $x_2(n)$ produce an output $y_2(n)$.

$$\therefore \quad y_2(n) = T[x_2(n)] = n^2 x_2(n)$$

The weighted sum of outputs is:

$$ay_1(n) + by_2(n) = a[n^2 x_1(n)] + b[n^2 x_2(n)] = n^2[ax_1(n) + bx_2(n)]$$

The output due to weighted sum of inputs is:

$$y_3(n) = T[ax_1(n) + bx_2(n)] = n^2[ax_1(n) + bx_2(n)]$$

$$y_3(n) = ay_1(n) + by_2(n)$$

The weighted sum of outputs is equal to the output due to weighted sum of inputs. The superposition principle is satisfied. Therefore, the given system is linear.

(b) $y(n) = x(n) + \dfrac{1}{2x(n-2)}$

$T[ax_1(n) + bx_2(n)] = aT[x_1(n)] + bT[x_2(n)]$

$$y(n) = T[x(n)] = x(n) + \frac{1}{2x(n-2)}$$

For an input $x_1(n)$,

$$y_1(n) = T[x_1(n)] = x_1(n) + \frac{1}{2x_1(n-2)}$$

For an input $x_2(n)$,

$$y_2(n) = T[x_2(n)] = x_2(n) + \frac{1}{2x_2(n-2)}$$

The weighted sum of outputs is:

$$ay_1(n) + by_2(n) = a\left[x_1(n) + \frac{1}{2x_1(n-2)}\right] + b\left[x_2(n) + \frac{1}{2x_2(n-2)}\right]$$

$$= [ax_1(n) + bx_2(n)] + \frac{a}{2x_1(n-2)} + \frac{b}{2x_2(n-2)}$$

(b) $$y(n) = x(n) + \frac{1}{2x(n-2)}$$

$T[ax_1(n) + bx_2(n)] = aT[x_1(n)] + bT[x_2(n)]$

The output due to weighted sum of inputs is:

$$y_3(n) = T[ax_1(n) + bx_2(n)] = [ax_1(n) + bx_2(n)] + \frac{1}{2[ax_1(n-2) + bx_2(n-2)]}$$

$$y_3(n) \neq ay_1(n) + by_2(n)$$

The weighted sum of outputs is not equal to the output due to weighted sum of inputs. The superposition principle is not satisfied. Therefore, the given system is non-linear.

Additional Problems

(c) $y(n) = 2x(n) + 4$

(d) $y(n) = x(n)\cos \omega n$

(e) $y(n) = |x(n)|$

(f) $y(n) = \dfrac{1}{N}\sum_{k=0}^{N-1} x(n-k)$

For Solution refer Anandkumar page no 38 onwards

Time-Invariant and Time-Variant Systems

Time-Invariant and Time-Variant Systems

- A system is said to be time-invariant if its input/output characteristics do not change with time, i.e., if a time shift in the input results in a corresponding time shift in the output.
- If $\quad T[\, x(n)] = y(n)$

  Then $\quad T[\, x(n-k)] = y(n-k)$
- A system not satisfying the above requirements is called a time-variant system.

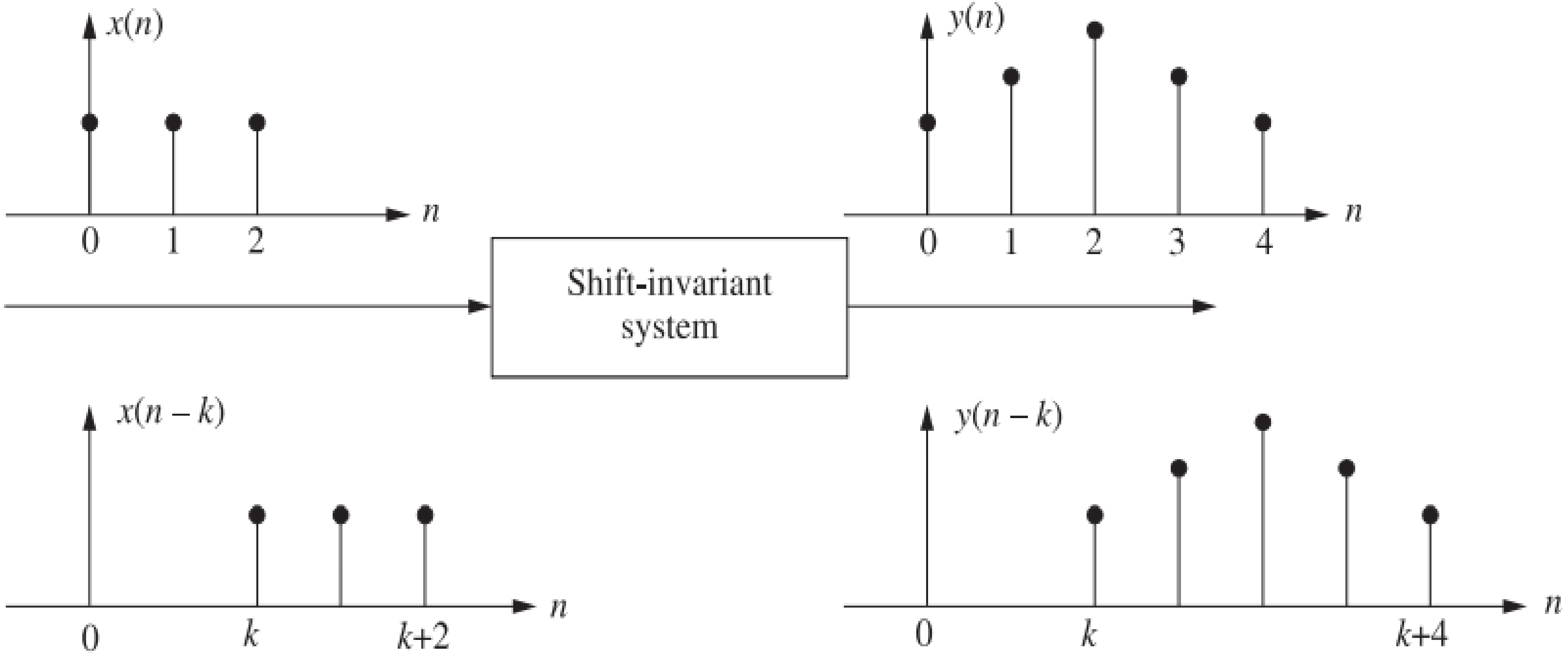


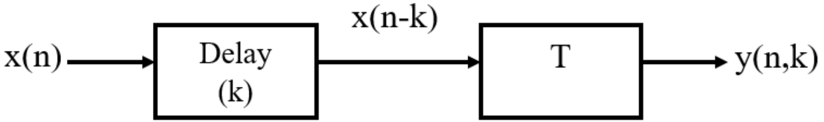
Figure 1.23 Time-invariant system.

Time-Invariant and Time-Variant

- Let x(n) be the input and let x(n–k) be the input delayed by k units and y(n)= T[x(n)] be the output for the input x(n).

$$y(n,k) = T[x(n-k)] = y(n)\Big|_{x(n)=x(n-k)}$$

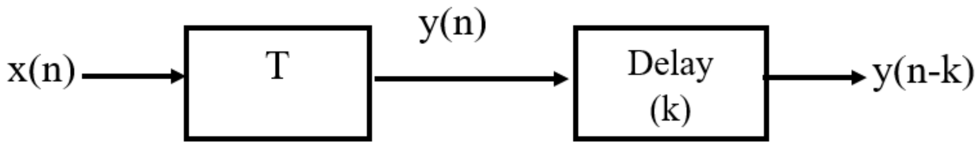
- Let the output for the delayed input x(n – k) be



Time-Invariant and Time-Variant Systems

- Let the output delayed by k units be

$$y(n-k) = y(n)\Big|_{n=n-k}$$



- If y(n, k) = y(n – k), then the system is said to be Time Invariant (Shift Invariant).

Problem: $y(n)= x(n)-x(n-1)$

$$y(n) = T[x(n)]$$
$$= x(n) - x(n-1)$$

Delay i/p by k units & apply to system

$$\therefore y(n,k) = x(n-k) - x(n-k-1) \quad \text{(I)}$$

i.e o/p

Now delay y(n) by k units in time

$$y(n-k) = \overbrace{x(n-k)}^{} - x(n-k-1) \quad \text{(II)}$$

From (I) & (II)

$$y(n,k) = y(n-k)$$

$\Rightarrow$ Time Invariant

Problem: y(n)= n x(n)

$y(n) = n\,x(n)$

$y(n) = T[x(n)]$

$= n\,x(n)$

Delay i/p $x(n)$ by k units in time

$y(n,k) = n\,x(n-k)$ — ①

Now, delay o/p $y(n)$ by k units

$y(n-k) = (n-k)\,x(n-k)$

$= n\,x(n-k) - k\,x(n-k)$ — II

From ① & II

$y(n,k) \neq y(n-k)$

∴ System is time-variant system.

Problem: y(n)= x(-n)

$$y(n) = x(-n)$$

$$y(n) = T[x(n)] = x(-n)$$

Delay i/P by k units

$$y(n,k) = x(-n-k) \quad \text{—— I}$$

Delay o/P y(n) by k units.

$$y(n-k) = x(-(n-k))$$

$$= x(-n+k). \quad \text{—— II}$$

$$y(n,k) \neq y(n-k)$$

∴ System is time variant

Problem: y(n)= x(n) cos n

$$y(n) = x(n)\ \cos \omega_0 n$$

$$y(n) = T[x(n)]$$

$$= x(n) \cdot \cos \omega_0 n$$

Delay i/p by k units:

$$\therefore y(n,k) = x(n-k) \cdot \cos \omega_0 n \quad \text{— I}$$

Delay o/p by k units

$$y(n-k) = x(n-k) \cdot \cos \omega_0 (n-k) \quad \text{— II}$$

From I & II, $y(n,k) \neq y(n-k)$

System is time variant.

Problem: y(n)= e

$$y(t) = e^{x(t)}$$

$$\therefore y(t) = T[x(t)]$$

$$y(t,k) = e^{x(t-k)} \quad \text{—— I}$$

$$y(t-k) = e^{x(t-k)} \quad \text{—— II}$$

From I & II

$$y(t,k) = y(t-k)$$

$\therefore$ System is time invariant

Problem:

- Determine whether the following systems are time-invariant or not:

  (a) $y(n) = x(n/2)$

  (b) $y(n) = x^2(n-2)$

Problem: y(n) = x(n/2)

(a) Given

$$y(n) = x\left(\frac{n}{2}\right)$$

$$y(n) = T[x(n)] = x\left(\frac{n}{2}\right)$$

The output due to input delayed by k units is:

$$y(n, k) = T[x(n-k)] = y(n)\big|_{x(n)=x(n-k)} = x\left(\frac{n}{2} - k\right)$$

The output delayed by k units is:

$$y(n-k) = y(n)\big|_{n=n-k} = x\left(\frac{n-k}{2}\right)$$

$$y(n, k) \neq y(n-k)$$

i.e. the delayed output is not equal to the output due to delayed input. Therefore, the system is time-variant.

Problem: $y(n) = x^2(n-2)$

Given $$y(n) = x^2(n-2)$$

$$y(n) = T[x(n)] = x^2(n-2)$$

The output due to input delayed by k units is:

$$y(n,k) = T[x(n-k)] = y(n)\big|_{x(n)=x(n-k)} = x^2(n-2-k)$$

The output delayed by k units is:

$$y(n-k) = y(n)\big|_{n=n-k} = x^2(n-2-k)$$

$$y(n,k) = y(n-k)$$

Time invariant system

Additional Problems

(c) $y(n) = x^2(n-2)$

(d) $y(n) = x(n) + nx(n-2)$

Refer Anandkumar book page 42 for solution

Stable and Unstable Systems

- The system is said **to be stable only when the output is bounded for bounded input**.
- For a **bounded input, if the output is unbounded** in the system then it is said to be **unstable.**
- Bounded means finite in amplitude.
- Some examples of bounded inputs are functions of sine, cosine, DC and unit step.

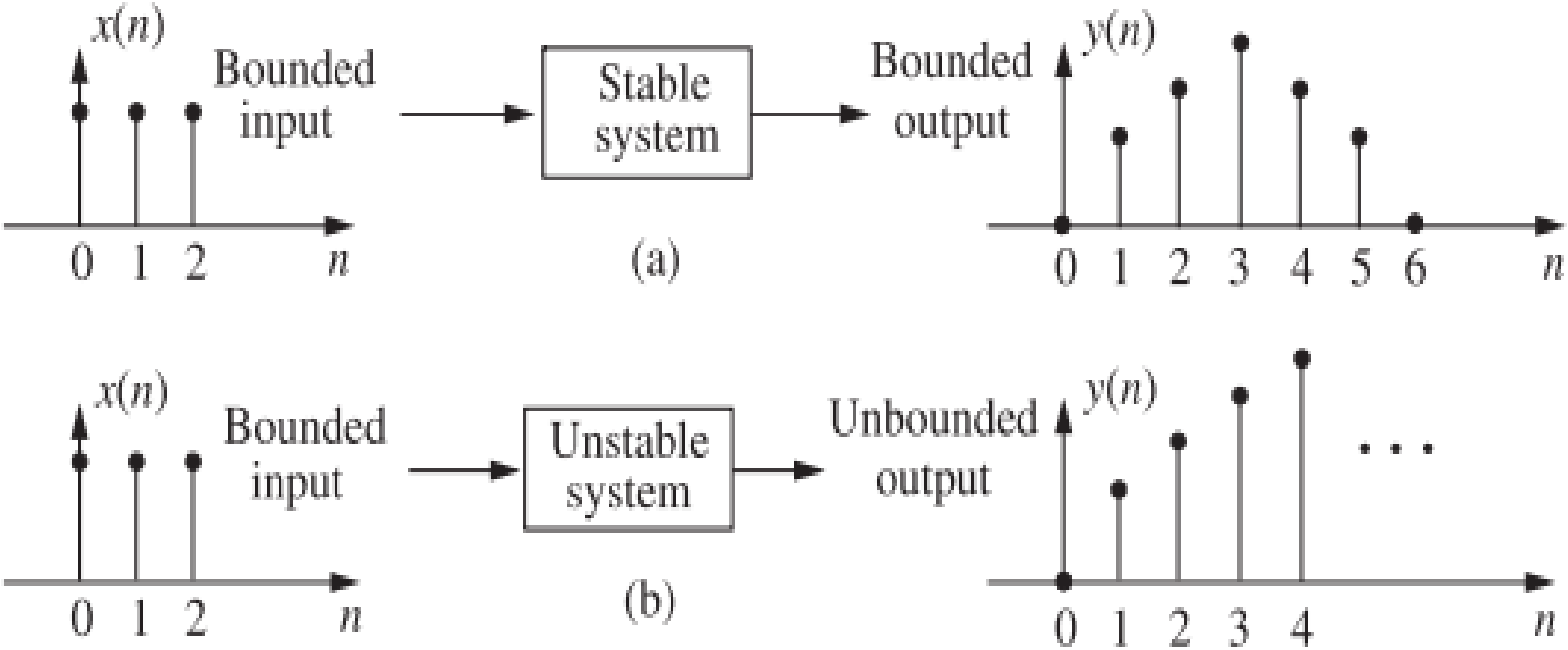


Figure 1.25 (a) Stable system (b) Unstable system.

Condition for Stability

- Stability indicates the usefulness of the system.
- The stability can be found from the impulse response of the system
- Impulse response means the output of the system for a unit impulse input.
- If the impulse response is absolutely summable for a discrete-time system, then the system is stable.

BIBO stability Criterion

- The necessary and sufficient condition for a discrete-time system to be BIBO stable is given by the expression:

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

h(n) is the impulse response of the system.

- This is called BIBO stability criterion.

Problem: y(n)=ax(n-7)

Given $y(n) = ax(n-7)$

Let $x(n) = \delta(n)$

Then $y(n) = h(n)$

$\therefore$ $h(n) = a\delta(n-7)$

$\therefore$ $h(n) = a$ for $n = 7$

$= 0$ for $n \neq 7$

A system is stable if its impulse response $h(n)$ is absolutely summable

i.e. $$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

In this case,

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} a\delta(n-7) = a$$

Hence the given system is stable if the value of a is finite.

The material in t

Problem:

$$y(n) = x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$$

Given $\quad y(n) = x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

Let $\quad x(n) = \delta(n)$

Then $\quad y(n) = h(n)$

$\therefore \quad h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$

A discrete-time system is stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

The given $h(n)$ has a value only at $n = 0$, $n = 1$ and $n = 2$. For all other values of n from $-\infty$ to ∞, $h(n) = 0$.

At $n = 0$, $h(0) = \delta(0) + \frac{1}{2}\delta(0-1) + \frac{1}{4}\delta(0-2) = \delta(0) + \frac{1}{2}\delta(-1) + \frac{1}{4}\delta(-2) = 1$

At $n = 1$, $h(1) = \delta(1) + \frac{1}{2}\delta(1-1) + \frac{1}{4}\delta(1-2) = \delta(1) + \frac{1}{2}\delta(0) + \frac{1}{4}\delta(-2) = \frac{1}{2}$

At $n = 2$, $h(2) = \delta(2) + \frac{1}{2}\delta(2-1) + \frac{1}{4}\delta(2-2) = \delta(2) + \frac{1}{2}\delta(1) + \frac{1}{4}\delta(0) = \frac{1}{4}$

$\therefore \quad \sum_{n=-\infty}^{\infty} |h(n)| = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4} < \infty$ a finite value.

Hence the system is stable.

Problem: $h(n) = a^n \quad \text{for } 0 < n < 11$

(c) Given $\quad h(n) = a^n \quad \text{for } 0 < n < 11$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |a^n| = \sum_{n=0}^{11} a^n = \frac{1-a^{12}}{1-a}$$

This value is finite for finite value of a. Hence the system is stable if a is finite.

For additional Problems refer Anandkumar book Page no: 51 and 52

EXAMPLE 1.17 Check whether the following systems are:

1. Static or dynamic
2. Linear or non-linear
3. Causal or non-causal, and
4. Shift-invariant or shift-variant

(a) $y(n) = \text{ev}\{x(n)\}$

(b) $y(n) = x(n)\,x(n-2)$

(c) $y(n) = \log_{10}|x(n)|$

(d) $y(n) = a^n u(n)$

(e) $y(n) = x^2(n) + \dfrac{1}{x^2(n-1)}$

For additional Problems refer Anandkumar book Page no: 54