ENM 521 Midterm Review

Fall 2019

1. Find the Laurent series expansion for $f(z) = \sinh\left(\frac{1}{z^2}\right)$ in $0 < |z| < \infty$.

2. Find the branch points and the corresponding branch cuts for

   (a) $f(z) = \left(1 + z^{1/2}\right)^2$,

   (b) $g(z) = \left(1 + z^{1/2}\right)^{1/2}$

3. Make use of the Residue Theorem to show that

   (a) $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{x^n + 1} = \frac{\pi}{n \sin(\pi/n)}$, where $n \geq 2$ is an integer.

   (b) $\displaystyle\int_0^\infty \frac{x^{2m}}{x^{2n} + 1}\mathrm{d}x = \frac{\pi}{2n} \csc\left(\frac{2m+1}{2n}\pi\right)$, where $0 \leq m < n$ are two integers.

4. Find the integrals

   (a) $\displaystyle\int_{-\infty}^{+\infty} \frac{\sin x}{x^2 + 4x + 5}\mathrm{d}x = -\frac{\pi}{e}\sin 2$,

   (b) $\displaystyle\int_{-\infty}^{+\infty} \frac{(x+1)\cos x}{x^2 + 4x + 5}\mathrm{d}x = \frac{\pi}{e}(\sin 2 - \cos 2)$.

5. Show that $\displaystyle\int_0^\infty \frac{\ln x}{(x^2+4)^2}\,\mathrm{d}x = \frac{\pi}{32}(\ln 2 - 1)$

6. Show that

   (a) $\displaystyle\int_0^{2\pi} e^{\cos\theta}\cos(n\theta - \sin\theta)\,\mathrm{d}\theta = \frac{2\pi}{n!}$

   (b) $\displaystyle\int_0^{2\pi} e^{\cos\theta}\sin(n\theta - \sin\theta)\,\mathrm{d}\theta = 0$

7. By use of contour integrals, show that, if $-\pi < \alpha < \pi$ and $\alpha \neq 0$,

$$\int_0^{+\infty} \frac{\cosh(\alpha x)}{\cosh(\pi x)}\,\mathrm{d}x = \frac{1}{2}\sec\frac{\alpha}{2}$$