ENGINEERING MATHEMATICS 521

P. Ponte C. Fall 2019

Problem Set 7

Due date: November 18

1. Show that

$$\Gamma\left(\frac{1}{2}+z\right)\Gamma\left(\frac{1}{2}-z\right)=\frac{\pi}{\cos\pi z}. \tag{1}$$

2. Prove that the Beta function β satisfies the identity

$$\beta(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}. \tag{2}$$

3. If α is an arbitrary complex number and $|z|<1$, show that

$$(1+z)^{\alpha}=\sum_{n=0}^{\infty}\frac{\Gamma(\alpha+1)}{n!\,\Gamma(\alpha-n+1)}z^{n}. \tag{3}$$

4. Develop series solutions for Hermite's differential equation

$$y''(x)-2xy'(x)+2\alpha y(x)=0. \tag{4}$$

   Show that by appropriate choice of α the series solution may be cut off and converted to finite polynomials.

5. Solve Chebyshev's equation

$$(1-x^2)T_n''(x)-xT_n'(x)+n^2T_n(x)=0 \tag{5}$$

   (n is a non-negative integer) by series substitution. What restrictions are imposed on n in order for the series to converge at $x=\pm1$?