CONSIDER $\quad f_\epsilon(x) = \dfrac{1}{\sqrt{\pi\epsilon}} e^{-\frac{x^2}{\epsilon}}$

$$F_\epsilon(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\lambda x} \frac{1}{\sqrt{\pi\epsilon}} e^{-x^2/\epsilon}\, dx$$

$$= \frac{1}{(\sqrt{2\epsilon})\pi} \int_{-\infty}^{\infty} e^{i\lambda x - \frac{x^2}{\epsilon}}\, dx \; e^{-\frac{\lambda^2\epsilon}{4}} e^{\frac{\lambda^2\epsilon}{4}}$$

$$= \frac{e^{-\frac{\lambda^2\epsilon}{4}}}{(\sqrt{2\epsilon})\pi} \int_{-\infty}^{\infty} e^{-\left(\frac{x}{\sqrt{\epsilon}} - \frac{i\lambda\sqrt{\epsilon}}{2}\right)^2} dx$$

$$= \frac{e^{-\frac{\lambda^2\epsilon}{4}}}{\sqrt{2}\,\pi} \int_{-\infty}^{\infty} e^{-\left(\frac{x}{\sqrt{\epsilon}} - \frac{i\lambda\sqrt{\epsilon}}{2}\right)^2} \frac{dx}{\sqrt{\epsilon}}$$

$$= \frac{e^{-\frac{\lambda^2\epsilon}{4}}}{\sqrt{2}\,\pi} \int_{-\infty}^{\infty} e^{-k^2}\, dk$$

$$= \frac{e^{-\frac{\lambda^2\epsilon}{4}}}{\sqrt{2}\,\pi} \sqrt{\pi} = \frac{e^{-\frac{\lambda^2\epsilon}{4}}}{\sqrt{2\pi}}$$

NOTE AS $\epsilon \to 0$

$$F_\epsilon(\lambda) \to \frac{e^0}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi}}$$

NOW, CONSIDER $\quad g_\epsilon(x) = \dfrac{1}{\sqrt{\pi\epsilon}} e^{-\frac{(x-a)^2}{\epsilon}}$

SHOW THAT $\quad G_\epsilon(\lambda) = \dfrac{1}{\sqrt{2\pi}} e^{i\lambda a} e^{-\frac{\epsilon\lambda^2}{4}}$ (CHECK)

THEN, COMPUTE $\quad \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{\infty} G_\epsilon(\lambda) e^{-i\lambda x}\, d\lambda$

$$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\lambda(a-x)} e^{-\frac{\epsilon\lambda^2}{4}}\, d\lambda$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left(\frac{\sqrt{\epsilon}}{2}\lambda - \frac{(a-x)i}{\sqrt{\epsilon}}\right)^2} d\lambda \; e^{-\frac{(a-x)^2}{\epsilon}}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left(\frac{\sqrt{\epsilon}}{2}\lambda - \frac{(a-x)i}{\sqrt{\epsilon}}\right)^2} d\left(\frac{\sqrt{\epsilon}\lambda}{2}\right) \times \frac{2}{\sqrt{\epsilon}} e^{-\frac{(a-x)^2}{\epsilon}}$$

$$= \frac{2}{2\pi}\frac{\sqrt{\pi}}{\sqrt{\epsilon}} e^{-\frac{(x-a)^2}{\epsilon}} = \frac{1}{\sqrt{\pi\epsilon}} e^{-\frac{(x-a)^2}{\epsilon}} = f_\epsilon(x-a)$$

AS $\epsilon \to 0 \quad f_\epsilon(x-a) \to \delta(x-a)$

AND SO

$$\lim_{\epsilon\to 0} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} G_\epsilon(\lambda) e^{-i\lambda x}\, d\lambda = \delta(x-a)$$

WHERE $\lim_{\epsilon\to\infty} G_\epsilon(\lambda) = \frac{1}{\sqrt{2\pi}} e^{i\lambda a}$)

WHICH IS F.T OF $\delta(x-a)$