ENGINEERING MATHEMATICS 521

P. Ponte C. Fall 2020

Problem Set 4

Due date: October 7

1. Find Taylor (Maclaurin) series expansions about $z = 0$ for each of the following:

   (a) $(1+z)^{\alpha}$ (α complex).

   (b) $\left(\dfrac{z-3}{z-i}\right)^{1/2}$

   (c) $\left(z^2+1\right)^{-1/2}$

   (d) $\log\left[(z-3)(3+z)\right]$

   In each case, indicate the radius of convergence and, when the function is multiple-valued, the particular branch.

2. Evaluate the following:

   (a) $\displaystyle\oint_{|z|=1} \frac{1}{|z|}\, dz$

   (b) $\displaystyle\oint_{|z|=1} \frac{\bar{z}}{z}\, dz$

   (c) $\displaystyle\oint_{|z|=1} \frac{1}{z}\, |dz|$

   (d) $\displaystyle\oint_{|z|=\frac{1}{2}} \frac{z+1}{z^2+z+1}\, dz$

   (e) $\displaystyle\oint_{C} \bar{z}\, dz$, where C bounds a domain D.

3. Let

$$f(a) = \oint_{|z|=3} \frac{z^3+2z}{(z-a)^3}\, dz. \tag{1}$$

   Evaluate: (a) $f(1+i)$, (b) $f(5)$.

4. Let C be a path joining $z = 0$ to $z = 1$ and not passing through $z = \pm i$. Show that

$$\int_{C} \frac{1}{1+z^2}\, dz = \frac{\pi}{4} + k\pi, \quad \text{where } k = 0, \pm 1, \pm 2, ... \tag{2}$$

   Exhibit paths appropriate to $k = 0, -2$ and $+4$.

5. Evaluate

$$\frac{1}{2\pi i} \oint\limits_C \frac{\cos \pi z}{z^2 - 1} \, dz, \tag{3}$$

where C is the rectangle with vertices at $-i$, $2 - i$, $2 + i$ and i.

6. Compute

$$\int_C \frac{z}{z^2 + 4} \, dz, \tag{4}$$

where C is a circle of radius 2.5.

7. If n and m are positive integers, show that

$$\int_C \frac{(n-1)! e^z}{(z - z_0)^n} \, dz = \int_C \frac{(m-1)! e^z}{(z - z_0)^m} \, dz, \tag{5}$$

where C is any closed contour containing z_0.

8. If $P(z)$ is a polynomial of degree n, show that

$$\int_{|z|=2} \frac{P(z)}{(z-1)^{n+2}} \, dz = 0. \tag{6}$$