ENGINEERING MATHEMATICS 521

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Problem Set 5

Due date: October 21

1. Locate and characterize all singular points of the functions:

   (a) $\dfrac{z^4}{1+z^4}$

   (b) $\dfrac{e^z}{1+z^2}$

   (c) $\dfrac{1}{(1+z^2)^{2/3}}$

   (d) $e^{\cot(1/z)}$

   (e) $\sin\left(\dfrac{1}{\sin(1/z)}\right)$

2. Find the residues of:

   (a) $\dfrac{\cot z \coth z}{z^3}$ at $z = 0$.

   (b) $\dfrac{z^2 - 2z}{(z+1)^2\,(z^2+4)}$ at each singular point.

   (c) $\dfrac{z^6}{(1+z)^3}$ at $z = -1$ and $z = \infty$.

3. Expand in a Laurent series valid in $1 < |z| < 2$ the function

$$f(z) = \left[\frac{z}{(z-1)\,(z-2)}\right]. \tag{1}$$

4. In what annular regions can $f_j(z)$ be so interpreted that it has a Laurent series about the origin (i.e., $z = 0$) in each of those annuli?

   (a) when $f_1(z) = \left(z^2+4\right)^{1/3}$

   (b) when $f_2(z) = \left(z^2+1\right)^{-1/2}$

   (c) when $f_3(z) = \log\left[\dfrac{(3-z)}{(3+z)}\right]$

   What are those expansions?

5. Use contour integration to evaluate the following real integrals:

   (a) $\displaystyle\int_0^\infty \frac{\cos \pi x}{1+4x^2}\, dx.$

   (b) $\displaystyle\int_0^{2\pi} \frac{1}{(a+b\cos x)^2}\, dx$, where $a > b > 0$.

6. Find the integral
$$\int_0^\infty \frac{x^2}{x^{200}+1}\, dx \tag{2}$$
by considering a complex contour C consisting of sector of a circle of angle $\pi/100$ and radius R.

7. Find the integral
$$\int_{-i\infty}^{+i\infty} \frac{a^{z+1}}{z+1}\, dz \tag{3}$$
where a is a real number. Consider separately two different cases: (i) $a > 1$, and (ii) $0 < a < 1$. Show all details of your calculations. Hint: Respectively close the contour on left and right sides with semi-circles of radius R.