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H_∞ fuzzy Networked control for vehicle lateral dynamics

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***Abstract*— A vehicle dynamics control system (NCS) has been developed in this study for improving vehicle yaw rate dynamics under unreliable communication links with packet dropouts, and network-induced delay which are two typical network constraints of unreliable transmission. The NCS system consists of a fuzzy H_∞ static output feedback controller. After giving the nonlinear model of the vehicle, a Takagi-Sugeno (T-S) fuzzy model representation is first discussed. Next, based on the Lyapunov krasovskii functional approach and a parallel distributed compensation scheme, the gains of the fuzzy controller are determined in terms of Linear Matrix Inequality (LMI). Simulations have been conducted to evaluate performance of the closed loop system under limitation of the network resources caused by data transmission.**

Keywords: vehicle dynamics control system (NCS), (T-S) fuzzy model, H_∞ control, Linear Matrix Inequality (LMI) .

I. INTRODUCTION

In the late of last century, many active safety systems have been developed and installed on vehicles for real time monitoring and controlling the dynamic stability. Some of the systems have already been commercialized and been installed in passenger cars (ABS, ESP, TCS ...) [1], [3] and [9]. These systems are controlled by a variety of Electronic Control Units (ECU) that are connected to each other via different kinds of bus systems in order to reduce the amount of cables needed.

In today's premium automobiles, they can be many individual ECU communicating over multiplixed data networks such as Controller Area Network (CAN), Local Interconnect Network (LIN), Flex Ray for X-by-wire applications. However, as more features and ECUs are introduced, overall system complexity increases. This is why we are introducing the network control. [6]

In fact, in order to figure out the increasing communication between multi ECU and expanding wire harness, and so perform more complex tasks, the embedded distributed control system have been widely used in the vehicle electronic control system, with the advantage of good reliability and real time performance. The CAN (Controller Area Network) control system is a simple two-wire differential serial bus system, which was developed by Bosch for automotive applications in the early 1980s.

The most prominent feature of an Networked control system NCS is that its components (sensors, controllers, and actuators) are not connected directly by normal wires but through a network such as Controller Area Network (CAN). Figure 4

The use of NCSs has drawn enormous attention from current researchers because of its flexibility and robustness. However, because of the limitation of the network resources, the network-induced delays and data packet dropouts caused by data transmission will inevitably degrade the performance of the NCSs and even cause system instability, which make analysis and synthesis of NCSs complex. It is pointed out that the communication delay, which has time-varying characteristics, is one of the important issues to be considered in NCS analysis and synthesis. Fortunately, many methods and elegant results on NCSs have been reported in the literature [11], [13], [16], [17], [18], [19], [20] and [23] .

During the last decade, Takagi Sugeno (TS) fuzzy model has become a widespread approach to deal with complex nonlinear systems and provides a systematic and effective design strategy to complement other nonlinear control techniques [10], [12], [14].The typical approach for controller design is carried out via the so-called parallel distributed compensation (PDC) method [5].

In this paper, the TS fuzzy system is proposed to nonlinear NCSs and a design strategy for stabilization of NCSs is developed based on delay-dependent approach and linear matrix inequality for admissibility with H_∞ performance of a class of fuzzy systems subject to exogenous disturbances [22]. Furthermore, the networked-induced-delay and the packet dropout are considered.

The numerical simulation of the vehicle handling with and without the developed controller has been carried out to demonstrate the effectiveness of the proposed algorithm in terms of improving the vehicle stability using active safety system.

Notations:$W + W^T$ is denoted as $Sym(W)$.

The symbol (*) within a matrix represents the symmetric entries.

II. VEHICLE MODEL DESCRIPTION

The two-dimensional model with nonlinear tire characteristics of the four wheel vehicle behavior can be described by differential equations (cf.fig. 2)

$$\begin{cases} \dot{\beta} = \dfrac{2F_f + 2F_r}{mU} - r \\ \dot{r} = \dfrac{2a_f F_f - 2a_r F_r + M_z}{I_z}. \end{cases} \tag{1}$$

Where β denotes the side slip angle, r is the yaw rate, F_f is the cornering force of the two front tires, F_r is the cornering

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force of the two rear tires. U is the vehicle velocity, I_z is the yaw moment of inertia, m is the vehicle mass. The parameters of the vehicle are given in the following table :

TABLE I
VEHICLE PARAMETERS

Parameters	$I_z(Kg^2m)$	m(Kg)	$a_f(m)$	$a_r(m)$	W(m/s)
Values	3000	1500	1.3	1.2	20

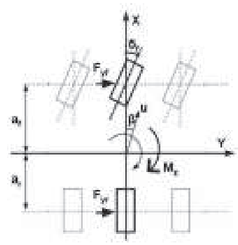

Fig. 1. Bicycle model (2ddl)

To obtain the TS fuzzy model, we have modelized front and rear lateral forces [8] by the following rules:

$$\text{IF } |\alpha_f| \text{ is } M_1 \text{ THEN} \begin{cases} F_f = C_{f1}\alpha_f \\ F_r = C_{r1}\alpha_r \end{cases} \tag{2}$$

$$\text{IF } |\alpha_f| \text{ is } M_2 \text{ THEN} \begin{cases} F_f = C_{f2}\alpha_f \\ F_r = C_{r2}\alpha_r \end{cases} \tag{3}$$

With :

$$\begin{cases} F_f = h_1(|\alpha_f|)C_{f1}\alpha_f + h_2(|\alpha_f|)C_{f2}\alpha_f \\ F_r = h_1(|\alpha_r|)C_{r1}\alpha_r + h_2(|\alpha_r|)C_{r2}\alpha_r \end{cases} \tag{4}$$

Where α_f is the front steer angle, α_r is the rear steer angle, C_{fi} and C_{ri} are the stiffness coeffitients, and $h_j(j = 1, 2)$ is the j^{th} bell curve membership function of fuzzy set M_j. The membership function parameters and consequence parameters of rules are obtained using an identification method based on the Levenberg-Marquadt algorithm [15].

For road friction coefficient $\mu = 0.7$, membership functions h_1 and h_2 are given in figure 2.

Front and rear Pacejka forces [8] compared to estimated front and rear forces described by equation (4) are shown in figure 3.

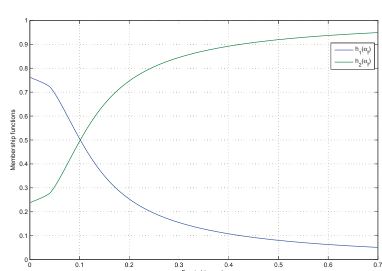

Fig. 2. Membership functions

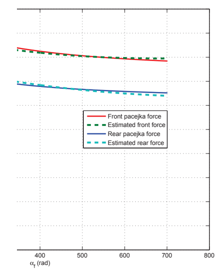

Fig. 3. Comparison of the estimated force and Pacejka force

Using the above approximation idea of nonlinear lateral forces by TS rules and by considering that $\alpha_f \cong \beta + \frac{a_f r}{U} - \delta_f$, $\alpha_r \cong \beta + \frac{a_r r}{U}$.

Nonlinear model (1) can be represented by the following TS fuzzy model:

$$\text{IF } |\alpha_f| \text{ is } M_1 \text{ THEN} \begin{cases} \dot{x} = A_1 x + B_{f1}\delta_f + BM_Z \\ y = C_2 x \end{cases} \tag{5}$$

$$\text{IF } |\alpha_f| \text{ is } M_2 \text{ THEN} \begin{cases} \dot{x} = A_2 x + B_{f2}\delta_f + BM_Z \\ y = C_2 x \end{cases} \tag{6}$$

Where

$$A_i = \begin{bmatrix} -2\frac{C_{fi}+C_{ri}}{mU} & -2\frac{C_{fi}a_f - C_{ri}a_r}{mU^2} - 1 \\ -2\frac{C_{fi}a_f - C_{ri}a_r}{I_z} & -2\frac{C_{fi}a_f^2 + C_{ri}a_r^2}{I_zU} \end{bmatrix},$$
$$B_{fi} = \begin{bmatrix} -2C_{fi}mU \\ \frac{2a_f C_{fi}}{I_z} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{I_z} \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{7}$$

Where

$$x = \begin{bmatrix} \beta \\ r \end{bmatrix} \tag{8}$$

Where $x(t)$ is the state vector, M_Z is the control input, δ_f is the external disturbance input, $z(t)$ is the controlled output, and $y(t)$ is the measured output.

The defuzzified output of this TS fuzzy system is :

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(|\alpha_f|)[A_i x(t) + B_{fi}\delta_f + BM_Z] \tag{9}$$
$$y(t) = C_2 x(t)$$
$$z(t) = C_1 x(t) \tag{10}$$

The membership functions are defined as

$$h_1(|\alpha_f|) = \frac{1}{(1 + abs(\frac{|\alpha_f| - c_1}{a_1}))^{2b_1}} \tag{11}$$
$$, h_2(|\alpha_f|) = \frac{1}{(1 + abs(\frac{|\alpha_f| - c_2}{a_2}))^{2b_2}}. \tag{12}$$

With $a_1 = 0.5077$, $b_1 = 0.4748$, $c_1 = 3.1893$, $a_2 = 5.3907$, $b_2 = 0.4356$, $c_2 = 0.5633$.

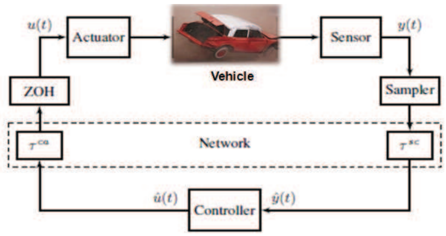

Fig. 4. Framework of networked control system

A typical NCS model with network-induced delays is shown in Fig. 1, where τ_{sc} is the sensor-to-controller delay and τ_{ca} is the controller-to-actuator delay. It is assumed that the controller computational delay can be absorbed into either τ_{sc} or τ_{ca}.
The following assumptions, which are common for NCSs research in the open literature , are also made in this work:

1) The sensors are clock driven, the controller and the actuators are event driven.
2) Data, either from measurement or for control, are transmitted with a single packet.
3) The effect of signal quantization and wrong code in communication are not considered.
4) The real input M_Z, realized through a zero-order hold, is a piecewise constant function.

It is worth mentioning that the sampling period of a sensor is pre-determined for control algorithm design, and thus the sensor can be assumed to be clock driven. However, an actuator does not change its output to the plant under control until an updated control signal is received, implying that the actuator is event driven.

L_2 is the space of square integrable functions over $[0, \infty)$, and $||.||_2$ denotes the L_2-norm.

III. Output feedback design

In order to stabilize the closed-loop, we consider the following controller Control rule :
IF $|\alpha_f|$ is M_1 THEN $M_Z = K_1 y(i_k h)$

IF $|\alpha_f|$ is M_2 THEN $M_Z = K_2 y(i_k h)$
Hence, the inferred fuzzy controller is given by

$$M_Z = \sum_{i=1}^{2} h_i(|\alpha_f|) K_i y(i_k h), t \in [i_k h + \tau_{ik}, i_{k+1} h + \tau_{ik+1}). \tag{13}$$

In the sequel, we note $h_i = h_i(\alpha_f)$ and

$$A(t) = \sum_{i=1}^{r} h_i A_i,\ B_f(t) = \sum_{i=1}^{r} h_i B_{fi},$$
$$H(t) = \sum_{i=1}^{r} h_i H_i,\ H_i = BK_iC_2. \tag{14}$$

From the above assumptions, using a similar modelling technique as the one employed in [2], [21], we model the closed-loop control system for (9) as

$$\dot{x}(t) = A(t)x(t) + H(t)x(i_k h) + B_f(t)\delta_f, \tag{15}$$
$$t \in [i_k h + \tau_{ik}, i_{k+1} h + \tau_{ik+1})$$
$$y(t) = \sum_{i=1}^{r} h_i(|\alpha_f|) C_2 x(t) \tag{16}$$
$$z(t) = \sum_{i=1}^{r} h_i(|\alpha_f|) C_1 x(t) \tag{17}$$

where h denotes the sampling period, $i_k (k = 1, 2, 3, ...)$ are some integers such that $\{i_1, i_2, i_3, ...\} \subset \{0, 1, 2, 3, ...\}$, the network-induced delay τ_{i_k} is the time from instant $i_k h$ when sensors sample from the plant to the instant when actuators send control actions to the plant. Here, we have assumed that the control computation and other overhead delays are included in τ_{i_k}.

As in [11], define $\eta(t) = t - i_k h$, $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$, $k = 1, 2, 3, ...,$ in every $[\ i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ interval, we have

$$\tau_{i_k} \le \eta(t) \le (i_{k+1} - i_k)h + \tau_{i_{k+1}} \tag{18}$$

From (18), we have

$$\eta_1 \le \eta(t) \le \eta_2 \text{ and } \dot{\eta}(t) \le h_d \tag{19}$$

Where $\eta_2 = sup_k[(i_{k+1} - i_k)h + \tau_{i_{k+1}}]$. Since $x(t - (t - i_k h))$, Equation (15) becomes

$$\begin{cases} \dot{x}(t) = A(t)x(t) + H(t)x(t - \eta(t)) + B_f(t)\delta_f, \\ \quad t \in [i_k h + \tau_{ik}, i_{k+1} h + \tau_{ik+1}) \\ \quad x(t) = \phi(t), t \in [t_0 - \eta_2, t_0] \\ \quad y(t) = \sum_{i=1}^{r} h_i(|\alpha_f|) C_2 x(t) \\ \quad z(t) = \sum_{i=1}^{r} h_i(|\alpha_f|) C_1 x(t) \end{cases} \tag{20}$$

Where $\phi(t)$ can be viewed as the initial condition of the closed-loop control system. The system disturbance, δ_f is assumed to belong to $L_2[0, \infty)$, that is, $\int_0^\infty \delta_f^T \delta_f dt < \infty$. This implies that the disturbance has finite energy. Then based on (19) it is noted that NCSs (20) are equivalent to a system with an interval time-varying delay.

Our main objective is to develop a robust fuzzy static output control scheme that uses a communication network to exchange sensor and actuator data transmission between the vehicle and its stability control system. The NCS must ensure the global stability in presence the H_∞ performance against the driver actions ($\|z\| \le \gamma \|\delta_f\|$), with γ is attenuation level).

In order to obtain the main results in this paper, the following lemmas are needed:

Lemma 3.1: [7] For any scalars $M > 0$, $N > 0$, $h(t)$ is a continuous function and satisfies $h_m < h(t) < h_M$, then

$$\begin{aligned} & -\frac{h_M - h_m}{h(t) - h_m} M - \frac{h_M - h_m}{h_M - h(t)} N \\ & \le max(-(M + 3N), -(3M + N)) \end{aligned} \tag{21}$$

Based on Lyapunov-Krasovskii functional, we establish a pratically computable criterion for asymptotic stability of closed-loop NCS system (20).

Theorem 3.1: For given scalars $\eta_1 > 0$, $\eta_2 > 0$ and $\gamma > 0$, μ_1, μ_2, and μ_3, the closed-loop system (9) is asymptotically stable with H_∞ norm bounded γ, if there exist positive matrices $\bar{P}$, $\bar{Q}_1$, $\bar{Q}_2$, $\bar{Q}_3$, $\bar{Z}_1$, $\bar{Z}_2$ and , matrices $\hat{G}_1 > 0$, $\hat{G}_2 > 0$ and Y_i with appropriate dimensions, such that the following conditions hold

$$\bar{\Phi}_i + \bar{\Phi}_1(\bar{Z}_2) < 0 \tag{22}$$

$$\bar{\Phi}_i + \bar{\Phi}_2(\bar{Z}_2) < 0 \tag{23}$$

where

$$\bar{\Phi}_i = \begin{bmatrix} \bar{\Phi}_{11i} & \bar{\Phi}_{12i} & \bar{Z}_1 & 0 & \bar{\Phi}_{15i} & -\mu_1 B_{fi} & \bar{\Phi}_{17} \\ * & \bar{\Phi}_{22i} & 0 & 0 & \bar{\Phi}_{25i} & -\mu_2 Bfi & 0 \\ * & * & -\bar{Q}_2 - \bar{Z}_1 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{Q}_3 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Phi}_{55} & -\mu_3 B_{fi} & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} \tag{24}$$

$$\bar{\Phi}_1(\bar{Z}_2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -4\bar{Z}_2 & 3\bar{Z}_2 & \bar{Z}_2 & 0 & 0 & 0 \\ * & * & -3\bar{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} \tag{25}$$

$$\bar{\Phi}_2(\bar{Z}_2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -4\bar{Z}_2 & \bar{Z}_2 & 3\bar{Z}_2 & 0 & 0 & 0 \\ * & * & -\bar{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & -3\bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} \tag{26}$$

$$\begin{aligned}
\bar{\Phi}_{11i} &= \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 - \mu_1 sym(A_i\bar{G}) - \bar{Z}_1, \\
\bar{\Phi}_{12i} &= -\mu_2 \bar{G}^T A_i^T - \mu_1 BY_iC_2, \\
\bar{\Phi}_{22i} &= -\mu_2 sym(BY_iC_2) - (1-h_d)\bar{Q}_1, \\
\bar{\Phi}_{15i} &= \bar{P} + \mu_1\bar{G} - \mu_3\bar{G}^T A_i^T, \\
\bar{\Phi}_{25i} &= \mu_2\bar{G} - \mu_3 C_2^T Y_i^T B^T, \\
\bar{\Phi}_{55} &= d_m^2\bar{Z}_1 + d_r^2\bar{Z}_2 + \mu_3 sym(\bar{G}), \\
\bar{\Phi}_{17} &= \bar{G}^T C_1^T, d_r = \eta_2 - \eta_1 \\
\bar{G} &= V\begin{bmatrix} \hat{G}_1 & 0 \\ 0 & \hat{G}_2 \end{bmatrix} V^T
\end{aligned}$$

Where $K_i = Y_i W S \hat{G}_1^{-1} S^{-1} W^T$

Proof:

$$\begin{aligned}
V(t) &= x^T(t)Px(t) + \int_{t-\eta(t)}^t x^T(s)Q_1x(s)\,\mathrm{d}s \\
&+ \int_{t-\eta_1}^t x^T(s)Q_2x(s)\,\mathrm{d}s + \int_{t-\eta_2}^t x^T(s)Q_3x(s)\,\mathrm{d}s \\
&+ \eta_1\int_{-\eta_1}^0\int_{t+s}^t \dot{x}^T(\upsilon)Z_1\dot{x}(\upsilon)\,\mathrm{d}\upsilon\mathrm{d}s \\
&+ d_r\int_{-\eta_2}^{-\eta_1}\int_{t+s}^t \dot{x}^T(\upsilon)Z_2\dot{x}(\upsilon)\,\mathrm{d}\upsilon\mathrm{d}s
\end{aligned} \tag{27}$$

According to the closed-loop fuzzy system (20) the time derivative of $V(t)$ satisfies

$$\begin{aligned}
&\dot{V}(t) + z^Tz - \gamma^2\delta_f^T\delta_f \\
\le& 2\dot{x}^T(t)Px(t) + x^T(t)(Q_1+Q_2+Q_3)x(t) \\
&- (1-h_d)x^T(t-\eta(t))Q_1x(t-\eta(t)) \\
&- x^T(t-\eta_1)Q_2x(t-\eta_1) - x^T(t-\eta_2)Q_3x(t-\eta_2) \\
&+ \dot{x}^T(t)(\eta_1^2Z_1 + d_r^2Z_2)\dot{x}(t) - \eta_1\int_{t-\eta_1}^t \dot{x}^T(\upsilon)Z_1\dot{x}(\upsilon)\,\mathrm{d}\upsilon \\
&- d_r\int_{t-\eta_2}^{t-\eta_1} \dot{x}^T(\upsilon)Z_2\dot{x}(\upsilon)\,\mathrm{d}\upsilon + x^T(t)C_1^TC_1x(t) - \gamma^2\delta_f^T\delta_f
\end{aligned} \tag{28}$$

Denoting $\psi_1 = x(t) - x(t-\eta_1)$, $\psi_2 = x(t-\eta_1) - x(t-\eta(t))$ and $\psi_3 = x(t-\eta(t)) - x(t-\eta_2)$, by Jensen inequality, one can obtain

$$-\eta_1\int_{t-\eta_1}^t \dot{x}^T(\upsilon)Z_1\dot{x}(\upsilon)\,\mathrm{d}\upsilon \le -\psi_1^TZ_1\psi_1 \tag{29}$$

and

$$\begin{aligned}
&- d_r\int_{t-\eta_2}^{t-\eta_1} \dot{x}^T(\upsilon)Z_2\dot{x}(\upsilon)\,\mathrm{d}\upsilon \\
&= -d_r\int_{t-\eta_2}^{t-\eta(t)} \dot{x}^T(\upsilon)Z_2\dot{x}(\upsilon)\,\mathrm{d}\upsilon \\
&- d_r\int_{t-\eta(t)}^{t-\eta_1} \dot{x}^T(\upsilon)Z_2\dot{x}(\upsilon)\,\mathrm{d}\upsilon \\
&\le -\frac{d_r}{(\eta(t)-\eta_1)}\psi_2^TZ_2\psi_2 - \frac{d_r}{(\eta_2-\eta(t))}\psi_3^TZ_2\psi_3
\end{aligned} \tag{30}$$

According to lemma 3.1, we have

$$\begin{aligned}
&- d_r\int_{t-\eta_2}^{t-\eta_1} \dot{x}^T(\upsilon)Z_2\dot{x}(\upsilon)\,\mathrm{d}\upsilon \\
&\le max\{-\psi_2^TZ_2\psi_2 - 3\psi_3^TZ_2\psi_3, -3\psi_2^TZ_2\psi_2 - \psi_3^TZ_2\psi_3\}
\end{aligned} \tag{31}$$

From (20), we construct for any appropriately dimensional matrices G the following null equation

$$\begin{aligned}
&2\left[x^T(t)G_1^T + x^T(t-\eta(t))G_2^T + \dot{x}^T(t)G_3^T\right] \\
&\times\left[\dot{x}(t) - A(t)x(t) - H(t)x(t-\eta(t)) - B_f(t)\delta_f\right] = 0
\end{aligned} \tag{32}$$

where $\Psi^T(t) =$
$\begin{bmatrix} x^T(t) & x^T(t-\eta(t)) & x^T(t-\eta_1) & x^T(t-\eta_2) & \dot{x}^T(t) & \delta_f \end{bmatrix}$. By letting $G_1 = \mu_1 G$, $G_2 = \mu_2 G$, and $G_3 = \mu_3 G$, with μ_1, μ_2, μ_3 being positive scalars, the derivative of (27) along the trajectory of the closed-loop system (20) can be written as $\dot{V}(t) \le max\Big(\Psi^T(t)\sum_{i=1}^r h_i(\Phi_i + \Phi_1(Z_2))\Psi(t), \Psi^T(t)\sum_{i=1}^r h_i(\Phi_i + \Phi_2(Z_2))\Psi(t)\Big)$. where

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & \Phi_{12i} & Z_1 & 0 & \Phi_{15i} & -G_1^T B_{fi} & \Phi_{17} \\ * & \Phi_{22i} & 0 & 0 & \Phi_{25i} & -G_2^T B_{fi} & 0 \\ * & * & -Q_2 - Z_1 & 0 & 0 & 0 & 0 \\ * & * & * & Q_3 & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & -G_3^T B_{fi} & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} \tag{33}$$

$$\begin{aligned} \Phi_{11i} &= Q_1 + Q_2 + Q_3 - sym(G_1^T A_i) - Z_1, \\ \Phi_{12i} &= -A_i^T G_2 - G_1^T B K_i, \\ \Phi_{22i} &= -sym(G_2^T B K_i) - (1 - h_d) Q_1, \\ \Phi_{15i} &= P - G_1^T - A_i^T G_3, \\ \Phi_{25i} &= G_2^T - K_i^T B G_3, \\ \Phi_{55} &= \eta_1^2 Z_1 + d_r^2 Z_2 + sym(G_3), \quad \Phi_{17} = C_1^T \end{aligned}$$

Under the conditions of the Theorem 3.1, a feasible solution satisfies the condition $\bar{\Phi}_{55} < 0$ which implies that $\bar{G}$ is nonsingular. Define $G = \bar{G}^{-1}$, $\bar{P} = \bar{G}^T P \bar{G}$, $\bar{Q}_1 = \bar{G}^T Q_1 \bar{G}$, $\bar{Q}_2 = \bar{G}^T Q_2 \bar{G}$, $\bar{Q}_3 = \bar{G}^T Q_3 \bar{G}$, $\bar{Z}_1 = \bar{G}^T Z_1 \bar{G}$,and $\bar{Z}_2 = \bar{G}^T Z_2 \bar{G}$.

Checking a congruence transformation to (22)-(23) by $diag\{G, G, G, G, G, I, I\}$, we obtain $max\Big(\Psi^T(t)\sum_{i=1}^r h_i(\Phi_i + \Phi_1(Z_2))\Psi(t), \Psi^T(t)\sum_{i=1}^r h_i(\Phi_i + \Phi_2(Z_2))\Psi(t)\Big) \le z^T z - \gamma^2 \delta_f^T \delta_f$, which implies

$$\dot{V}(t) < z^T z - \gamma^2 \delta_f^T \delta_f \tag{34}$$

for $t \in [i_k h + \tau_{ik}, i_{k+1} h + \tau_{ik+1})$, if $h_i \ge 0$, therefore, the system (20) is asymptotically stable.

Integrating both sides of (34) from $i_k h + \tau_{ik}$ to $t \in [i_k h + \tau_{ik}, i_{k+1} h + \tau_{ik+1})$, we get

$$V(t) - V(i_k h + \tau_{ik}) < \int_{i_k h + \tau_{ik}}^t z^T(t) z(t) dt - \int_{i_k h + \tau_{ik}}^t \gamma^2 \delta_f^T(t) \delta_f(t) dt. \tag{35}$$

Since $V(t)$ is continuous in $t \in [t_0, \infty)$, it can be seen explicitly that

$$V(t) - V(t_0) < - + \int_{t_0}^t z^T(t) z(t) dt - \int_{t_0}^t \gamma^2 \delta_f^T(t) \delta_f(t) dt.$$

Letting $t \to \infty$ and under zero initial condition, the H_∞ performance can be satisfied, i.e., $||z||_2 < \gamma ||\delta_f||_2$, for any nonzero $\delta_f(t) \in L_2$. ■

IV. Vehicle simulation results

To show the effectiveness of the proposed controller, we have carried the following series of simulations, which are controlled through a network. In the design, the nominal stiffness coefficients considered are [4]:

Nominal stiffness coefficients	Cf1	Cf2	Cr1	Cr2
Values	60712	4812	60088	3455

The network-related parameters are assumed: $h = 5ms$, the maximum delay $\eta_1 = 6ms$, $\eta_2 = 20ms$, the maximum number of data packet dropouts $\sigma = 2$, the minimum allowable γ is 4, $h_d = 0.1$, $\mu_1 = 1$, $\mu_2 = 0.1$ and $\mu_3 = 0.2$ by Theorem 3.1 we find a feasible solution as follows

$$K_1 = -2.6235 \quad 10^4, \qquad K_2 = -3.6011 \quad 10^4 \tag{36}$$

The steering angle applied is shown in figure (5). In the first one, we considered the case without control, the behaviour of the vehicle is unstable as shown in figure (6) and figure (7).

To overcome this problems, we have tested the proposed method with considering delay and packet dropout in the control design with (36), figure (6) and figure (7) show state variable evolutions and figure (8) shows the input control in this case. We remark that our approach is efficient and can stabilise the vehicle through network communication in the control case with delay in design.

For simulation, the initial condition is assumed to be $x_0 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$. The state responses of the NCS with control input are depicted in figures (6) and (7) which we can see that all the states component converge to zero. The simulation results are in accordance with the analysis and support the effectiveness of the developed design strategy.

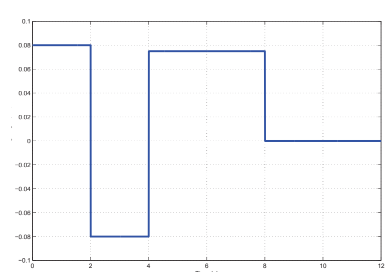

Fig. 5. Front steering angle δ_f

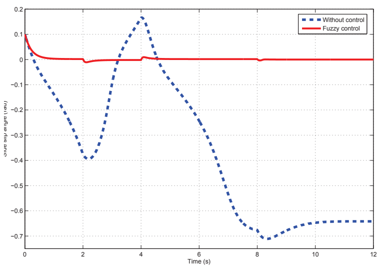

Fig. 6. Response of the sideslip angle β.

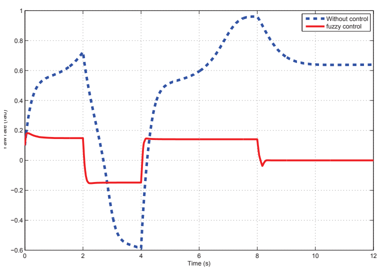

Fig. 7. Response of the yaw rate r.

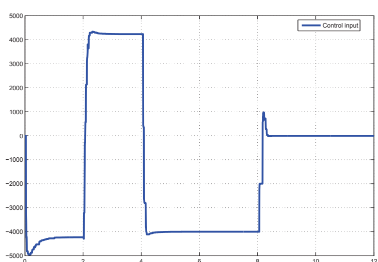

Fig. 8. Curve of moment M_Z.

V. Conclusion

In this paper, we have presented new conditions for H_∞ output feedback controller design of NCSs with considering network induced delay and data packet dropout. With this model, the optimal allowable delay bound and the feedback gain of a memoryless controller can be derived by solving a set of LMIs based on the Lyapunov functional method.

By vehicle simulations, we know that based on the LMIs, the controller designed method for stability improvement is very efficient and practical.

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