

# Combined AFS and DYC Control of Four-Wheel-Independent-Drive Electric Vehicles over CAN Network with Time-Varying Delays

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**Abstract**—This paper deals with the lateral motion control of four-wheel-independent-drive electric vehicles (4WID-EVs) subject to onboard network-induced time delays. It is well known that the in-vehicle network and x-by-wire technologies have considerable advantages over the traditional point-to-point communication. However, on the other hand, these technologies would also induce the probability of time-varying delays, which would degrade control performance or even deteriorate the system. To enjoy the advantages and deal with in-vehicle network delays, an  $H_\infty$ -based delay-tolerant linear quadratic regulator (LQR) control method is proposed in this paper. The problem is described in the form of an augmented discrete-time model with uncertain elements determined by the delays. Delay uncertainties are expressed in the form of a polytope using Taylor series expansion. To achieve a good steady-state response, a generalized proportional-integral control approach is adopted. The feedback gains can be obtained by solving a sequence of linear matrix inequalities (LMIs). Cosimulations with Simulink and CarSim demonstrate the effectiveness of the proposed controller. Comparison with a conventional LQR controller is also carried out to illustrate the strength of explicitly dealing with in-vehicle network delays.

**Index Terms**—Active front-wheel steering (AFS), direct yaw-moment control (DYC), four-wheel-independent-drive electric vehicle (4WID-EV),  $H_\infty$ -based linear quadratic regulator (LQR), time-varying network delays.

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## I. INTRODUCTION

**D**UE to concerns about environment pollution and energy shortages, development of electric vehicles (EVs) is currently at a rapidly growing phase. Meanwhile, there is great demand for vehicle driving safety, maneuverability, and driving comfort. Boosted by the fast development of electric motor, battery, and control technologies, the four-wheel-independent-drive electric vehicle (4WID-EV), as an emerging configuration of EVs, has attracted increasing research efforts from both the industry and academia. Equipped by in-wheel motors with quick, accurate, and flexible torque generations, 4WID-EVs possess considerable advantages in terms of vehicle dynamics control, energy optimization, and vehicle structural flexibility [1]–[6].

For vehicle lateral stability control, steering-based systems and direct yaw-moment control (DYC) systems are most effective, and there have been various research studies on the combination of the two systems. Yu and Moskwa designed a four-wheel steering and independent wheel torque control system to enhance vehicle maneuverability and safety [7]. Bedner *et al.* proposed a supervisory control approach to manage both braking and four-wheel steering systems for vehicle stability control [8]. A coordinated and reconfigurable vehicle dynamics control system that can coordinate the steering and braking actions of each wheel individually was designed in [9]. There are also some works focusing on combining active front-wheel steering (AFS) and DYC systems. An integrated front-wheel steering and individual wheel torque controller was proposed to govern the vehicle lateral position using frequency-weighted coordination [10]. Nagai also proposed an integrated control system of AFS and DYC to control the vehicle yaw rate and the sideslip angle using a model-matching controller [11]. A vehicle yaw stability control approach coordinating steering and individual wheel braking actuations was developed in [12]. A coordinated controller of AFS and DYC based on an optimal guaranteed cost method was designed in [13]. Mokhiamar and Abe compared different combinations of DYC with AFS, active rear-wheel steering (ARS), and AFS + ARS in simulation in [14]. Heinzl *et al.* also compared three different control strategies, namely, AFS, AFS plus unilateral braking, and ARS plus unilateral braking for vehicle dynamics control in a severe cornering and braking maneuver situation in simulation [15]. Among all the solutions coordinating the steering-based system and the DYC control system, the combination of AFS and DYC shows the best compromise between control performance

and system complexity. With in-wheel motors, each wheel of the 4WID-EV can generate not only individual braking torque but individual driving torque as well, are able to yield greater direct yaw-moment than the conventional vehicles. In addition, the 4WID-EV dynamics control capability can be further enhanced by the integration of an AFS. For example, Li *et al.* proposed an integrated model predictive control algorithm of AFS and DYC to improve the control performance of 4WID-EVs with in-wheel motors [16].

However, all of these aforementioned control methods for combined AFS and DYC assumed that the controllers, sensors, and actuators were directly connected by wires. In other words, the 4WID-EV was considered as a centralized control system. Rather, with the development and appearance of in-vehicle networks and x-by-wire technologies, the control signals from the controllers and the measurements from some sensors are exchanged using a communication network in modern vehicles [17], i.e., Controller Area Network (CAN) or FlexRay. Thus, a 4WID-EV is a networked control system rather than a centralized control system, which imposes the effects of network-induced delays into the control loop. The unknown and time-varying delays of the network communication between different controllers could degrade the control performance of the entire system or even make the system unstable. For example, according to the research of Caruntu *et al.*, time-varying delays of the CAN can lead to driveline oscillations in the control of a vehicle drivetrain [17].

Vehicle motion controls are also characterized by fast dynamics as drivetrain control. Motivated by the adverse influences of network-induced delays, this paper aims to design a combined AFS and DYC controller that is robust to time-varying network delays. As the most dominant and representative in-vehicle network nowadays, the CAN is selected for studying the characteristic of network-induced delays in this paper. Based on the theoretical research on CAN-induced delays [18]–[20], a practical result of the CAN-induced delay model was adopted for the study of automotive system control loop closed by CAN [17]. This CAN-induced delay model is also used to obtain the discrete model of the network-induced time-varying delays.

For networked control systems, an important issue is the appropriate handling of the nonlinearities from uncertain time-varying delays. There are numerous approaches for the appropriate handling based on polytopic inclusions, e.g., the method based on element-wise minimization–maximization (EMM) [21], the method based on Jordan normal form (JNF) [22], the method based on Taylor series expansion (TA) [23], and the method based on Cayley–Hamilton (CH) theorem [24]. The accuracy of EMM, TA, and CH is of the same order of magnitude, whereas for JNF, the polytope is much larger [24]. Moreover, TA is one of the approaches with the least complexity among the aforementioned methods [24] and, therefore, is chosen to cope with the nonlinearities of uncertain time-varying delays in this paper.

In the stabilization problem of networked control systems with time-varying network-induced delays, generally, there are two main control synthesis methods. One method is to construct the control Lyapunov function for an augmented system with

the original states and previous inputs as the augmented states and then use a linear matrix inequality (LMI) approach to yield a feedback controller offline [25]–[27]. The other method is to solve an online optimization problem for the original nonaugmented system [17], [28]. Based on the former idea, which has the advantages in real-time performance, we propose an  $H_\infty$ -based linear quadratic regulator (LQR) tracking control scheme to cope with the tracking problem in this paper. The sufficient condition of system stability can be provided in the form of a sequence of LMIs, and the control gains of the tracking controller can be obtained by solving the LMIs.

The main contributions of this work are twofold. First, the network-induced delays are explicitly considered in the vehicle lateral motion control problem. Different from the conventional AFS and DYC control, the network-induced time-varying delays lead to a challenging control problem for the vehicle lateral stability and handling. The network-induced delays are described via the polytopic technique. With an augmented technique, the delayed system is converted into a polytopic time-varying system. Second, to track the reference model, trade off between the performance and control effort, and attenuate the effect of external input, an  $H_\infty$ -based LQR tracking control scheme is designed for the combined AFS and DYC, which simultaneously guarantees the control performance and the robustness against time-varying delays.

The remaining sections of this paper are organized as follows: In Section II, we introduce the networked control architecture of combined AFS and DYC and mathematically formulate the problem to be studied. A continuous-time model is introduced with the reference model states as the tracking control targets. Then, the original system is augmented and discretized. With the analysis of CAN-induced delays, the discrete control-oriented model is reformed and augmented into the final formulation. In Section III, the nonlinear uncertainties induced by the time-varying delays are expressed as polytopes of matrices by using the Taylor series approximation. In Section IV, we present the control synthesis for this problem. The  $H_\infty$ -based LQR is used for the tracking control design procedure and solved by using the LMI theory. In Section V, the validity of the proposed control design procedure is demonstrated via cosimulation with Simulink and CarSim, which is a commercial full-vehicle dynamics simulation package. Section VI presents some concluding remarks.

## II. PROBLEM FORMULATION

### A. Networked Control Architecture of Combined AFS and DYC

In an AFS system, the front-wheel steering angle is determined as a sum of two contributions. One is directly determined by the driver from her/his steering wheel angle input, and the other is decided by the steer-by-wire controller [29]. The two parts are combined by a planetary gear set with two inputs and one output in an AFS system [30]. One input is from the steering wheel of the driver, whereas the other is from the servo motor controlled by the electronic controller of the AFS system, which is connected to the in-vehicle network via the CAN bus.

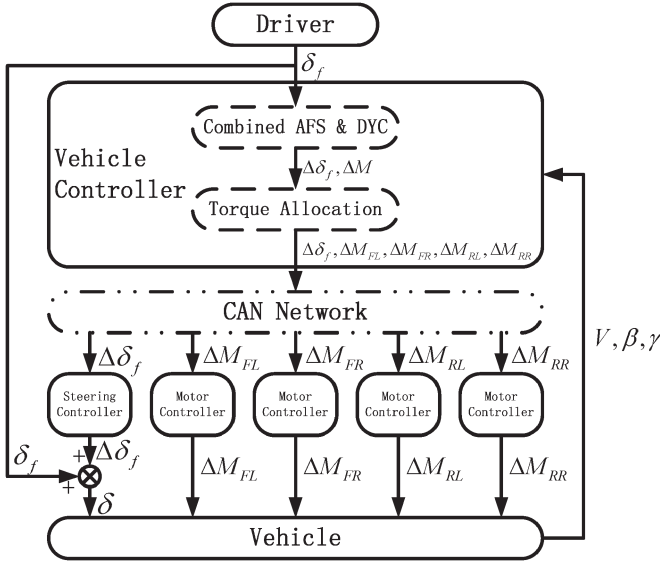


Fig. 1. Architecture of combined AFS and DYC over CAN.

The output shaft of the planetary gear set manipulates the front road wheels via a rack-and-pinion power steering system. (See [30] for detailed information on AFS systems.) In the DYC system of a 4WID-EV, the in-wheel motors are key components. With the development of in-wheel motor and integration technology, there have been several revolutionary electric wheel products such as the active wheel system from Michelin, which is highly integrated with in-wheel motor, braking system, and active suspension, and all these components are controlled by one control unit [31], which can be connected to the in-vehicle network via the CAN bus.

With the AFS system and integrated in-wheel motor systems connected via the CAN bus and coordinated by a higher level vehicle controller, networked control architecture of combined AFS and DYC for a 4WID-EV is shown in Fig. 1. The vehicle motion control algorithm in the vehicle controller is hierarchical. The upper-level controller decides the steering angle to be superposed to the front wheels and the direct yaw-moment to be imposed to the vehicle, whereas the lower-level controller distributes the total direct yaw-moment to the torque commands of the four in-wheel motors. This paper only studies the upper-level controller, which has the direct responsibility on the system robustness against time-varying network delays. (For control allocation of the total direct yaw-moment, see [32] and [33].) In most vehicle motion control systems, the yaw rate sensor and the longitudinal/lateral acceleration sensor are usually directly connected with the vehicle controller, from which the vehicle yaw rate and the sideslip angle can be measured or estimated by the upper-level controller directly without going through the in-vehicle network. Therefore, only the network-induced delays in the forward path are settled in this paper, whereas feedback delays are not considered here.

### B. Control-Oriented Model for Vehicle Lateral Dynamics

In this paper, a 2-DOF bicycle model of vehicle lateral dynamics is used for controller design, as shown in Fig. 2. The

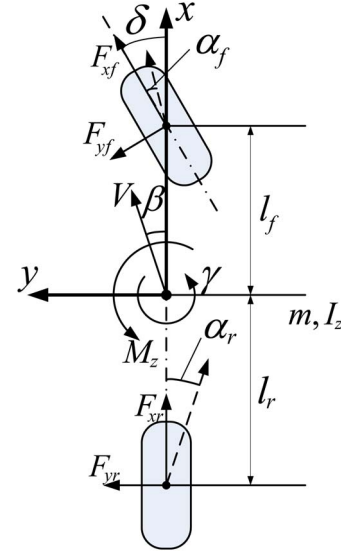


Fig. 2. Bicycle model of vehicle lateral dynamics.

vehicle has mass  $m$  and moment of inertia  $I_z$  about the yaw axis through its center of gravity (CG). The distances from the front and rear axles to the CG are  $l_f$  and  $l_r$ , respectively.  $\delta$  is the steering angle of the front wheels. Tire slip angles  $\alpha_f$  and  $\alpha_r$  lead to lateral force  $F_{yf}$  and  $F_{yr}$  while steering.  $V$  is the velocity of the vehicle CG with  $\beta$  being the vehicle sideslip angle and  $\gamma$  being the yaw rate of the vehicle.

According to the 2-DOF vehicle model, the lateral dynamics of a vehicle with AFS and DYC can be expressed as follows [13]:

$$\dot{x} = A_c x + B_{wc} w + B_{uc} u \quad (1)$$

where

$$x = [\beta \quad \gamma]^T \quad w = \delta_f \quad u = [\Delta\delta_f \quad \Delta M_z]^T$$

$$A_c = \begin{bmatrix} -2 \frac{C_f + C_r}{mV} & -2 \frac{C_f l_f - C_r l_r}{mV^2} - 1 \\ -2 \frac{C_f l_f - C_r l_r}{I_z} & -2 \frac{C_f l_f^2 + C_r l_r^2}{I_z V} \end{bmatrix}$$

$$B_{wc} = \begin{bmatrix} \frac{2C_f}{mV} \\ \frac{2l_f C_f}{I_z} \end{bmatrix}, \quad B_{uc} = \begin{bmatrix} \frac{2C_f}{mV} & 0 \\ \frac{2C_f l_f}{I_z} & \frac{1}{I_z} \end{bmatrix}.$$

Sideslip angle  $\beta$  and yaw rate  $\gamma$  are the two states of the vehicle lateral dynamics,  $\delta_f$  is the portion of the front-wheel steering angle that is directly manipulated by the driver, and  $\Delta\delta_f$  is the portion of the front-wheel steering angle that is controlled by the AFS controller. The actual front-wheel steering angle  $\delta$  is the summation of  $\delta_f$  and  $\Delta\delta_f$ .  $\Delta M_z$  is the external yaw moment generated by the longitudinal forces of the left and right-side wheels.  $C_f$  and  $C_r$  are the cornering stiffness of the front and rear tires, respectively.

### C. Reference State Responses

For the two states in the vehicle lateral dynamics control, the yaw rate reflects more on the handling performance, whereas

the sideslip angle is more concerned with the vehicle stability [13]. The vehicle yaw rate can be readily measured by a sensor in modern vehicles with electronic stability control systems, whereas the sideslip angle is often estimated due to the cost and complexity of direct measurement [34]. (On the sideslip angle estimation methods, see [34]–[39], and it is assumed that both states can be made available in this paper.)

Generally, the reference sideslip angle is given as zero to ensure stability [40], whereas the reference yaw rate is defined in terms of vehicle parameters, longitudinal speed, and steering input of the driver as [29]

$$\gamma_{ref} = \frac{V}{l_f + l_r + \frac{mV^2(C_r l_r - C_f l_f)}{2C_f C_r (l_f + l_r)}} \delta_f. \quad (2)$$

Therefore, the reference state responses can be expressed as follows:

$$r = Rw \quad (3)$$

with

$$r = \begin{bmatrix} \beta_{ref} \\ \gamma_{ref} \end{bmatrix} \quad R = \begin{bmatrix} 0 & \frac{V}{l_f + l_r + \frac{mV^2(C_r l_r - C_f l_f)}{2C_f C_r (l_f + l_r)}} \end{bmatrix}^T.$$

#### D. Augmented Model for Tracking Control

To track the reference state response, the original system described in (1) is augmented by defining a new vector  $\bar{x} = [x^T \int_0^t (r^T - x^T) dt]^T$ . Then, the control-oriented model can be revised as

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}_w w + \bar{B}_u u + \bar{B}_r r \quad (4)$$

where

$$\bar{A} = \begin{bmatrix} A_c & 0 \\ -I & 0 \end{bmatrix} \quad \bar{B}_w = \begin{bmatrix} B_{wc} \\ 0 \end{bmatrix} \\ \bar{B}_u = \begin{bmatrix} B_{uc} \\ 0 \end{bmatrix} \quad \bar{B}_r = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

By substituting (3) into (4), we obtain

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}_{wr} w + \bar{B}_u u \quad (5)$$

where

$$\bar{B}_{wr} = (\bar{B}_w + \bar{B}_r R).$$

As modern vehicles are controlled by digital controllers communicating and exchanging data over in-vehicle networks, in an ideal case, each controller obtains and updates its control commands at a fixed cycle period  $T_s$ . The discrete state-space equation can be written as follows:

$$\bar{x}_{k+1} = \bar{A}_d \bar{x}_k + \bar{B}_{wrd} w_k + \bar{B}_{ud} u_k \quad (6)$$

where

$$\bar{A}_d = e^{\bar{A}T_s} \\ \bar{B}_{wrd} = \int_0^{T_s} e^{\bar{A}(T_s-\theta)} d\theta \cdot \bar{B}_{wr} \\ \bar{B}_{ud} = \int_0^{T_s} e^{\bar{A}(T_s-\theta)} d\theta \cdot \bar{B}_u$$

with  $\bar{x}_k$ ,  $w_k$ , and  $u_k$  being the state vector and control input vectors at time  $kT_s$ .

#### E. Analysis of CAN-Induced Time Delays

It is reasonable to assume that within an in-vehicle network (e.g., CAN), without physical disconnection, the induced time-varying delays are bounded, and this bound can be known in advance by experimental tests [17]. There are a variety of theoretical research studies for calculating the upper bound of network-induced delays of each message sent on CAN in automotive applications, see, e.g., [18]–[20]. An explicit expression can be demonstrated as in [19] and [20], which is given as follows:

$$\tau_{l \arg e, j} = \frac{(j+2)l}{R - \sum_{i=0}^{j-1} \frac{l}{c_i}} \quad (7)$$

where  $\tau_{l \arg e, j}$  denotes the upper bound of the delay of the  $j$ th priority CAN message,  $l$  is the maximum frame length,  $R$  is the rate of a high-speed CAN, and  $c_i$  is the cycle length of the  $i$ th priority message, which represents the period after which the message is repeated.

Based on these CAN time-delay models, some practical presentations of CAN-induced delays were adopted in engineering control systems, e.g., in [17]. The delays of all messages introduced by CAN are assumed to be time varying and uniformly distributed in the interval  $[0, \tau_{l \arg e}]$ , where  $\tau_{l \arg e} = 1.7T_s$ , and  $T_s$  is the sampling period of the system, which has been proved to be convenient and reasonable for practical control designs.

Additionally, according to the protocol of CAN, messages within the same message ID group are queued while being sent, which leads to the fact that a message transmitted at a certain time can never arrive before a message with the same ID that was transmitted at a previous time [17]. Thus, the delay of a message sent at time  $kT_s$  can be finally depicted with the following expression:

$$\max\{0, \tau_{k-1} - T_s\} \leq \tau_k \leq \tau_{l \arg e}. \quad (8)$$

#### F. Control-Oriented Model With Time-Varying Delays

With the time-varying delays induced by the CAN, the ideal condition under which (6) is derived is no longer satisfied. The procedure for vehicle dynamics control can be interpreted with



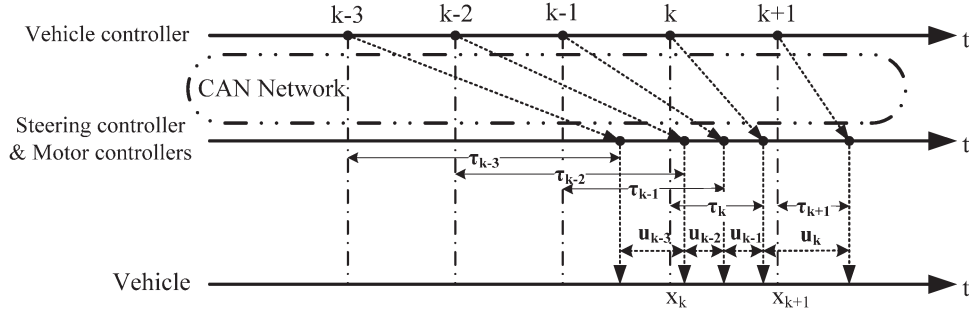


Fig. 3. Influence of network-induced time-varying delays on vehicle dynamics control.

Fig. 3. As control commands from the vehicle controller are delayed by CAN, the control input of the vehicle at time  $t$  can be written as

$$u(t) = u_k, \quad \forall t \in [kT_s + \tau_k, (k+1)T_s + \tau_{k+1}]. \quad (9)$$

It is reasonable to express the maximum delay as

$$\tau_{\text{large}} = (\Upsilon + v)T_s \quad (10)$$

where  $\Upsilon \in \mathbb{Z}_+$  and  $v \in \mathbb{R}_{[0,1)}$ .

Then, the control-oriented model with the time-varying delays can be derived as

$$\begin{aligned} \bar{x}_{k+1} = & \bar{A}_d \bar{x}_k + \bar{B}_{wr} w_k + \bar{B}_{ud} u_k + \Delta_{0,k} (u_{k-1} - u_k) \\ & + \Delta_{1,k} (u_{k-2} - u_{k-1}) + \cdots + \Delta_{\Upsilon,k} (u_{k-\Upsilon-1} - u_{k-\Upsilon}) \end{aligned} \quad (11)$$

where

$$\Delta_{i,k} = \begin{cases} 0, & \tau_{k-i} - iT_s \leq 0 \\ \int_0^{\tau_{k-i} - iT_s} e^{\bar{A}(T_s - \theta)} d\theta \cdot \bar{B}_u, & 0 \leq \tau_{k-i} - iT_s \leq T_s \\ \int_0^{T_s} e^{\bar{A}(T_s - \theta)} d\theta \cdot \bar{B}_u, & T_s \leq \tau_{k-i} - iT_s. \end{cases} \quad (12)$$

Defining a new state vector  $\xi_k = [\bar{x}_k^T \ u_{k-1}^T \ \cdots \ u_{k-\Upsilon-1}^T]^T$ , we have the augmented system as

$$\xi_{k+1} = A_{aug} \xi_k + B_{wr,aug} w_k + B_{u,aug} u_k \quad (13)$$

where

$$A_{aug} = \begin{bmatrix} \bar{A}_d & \Delta_{0,k} - \Delta_{1,k} & \cdots & \Delta_{\Upsilon-1,k} - \Delta_{\Upsilon,k} & \Delta_{\Upsilon,k} \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}$$

$$B_{wr,aug} = \begin{bmatrix} \bar{B}_{wr} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad B_{u,aug} = \begin{bmatrix} \bar{B}_{ud} - \Delta_{0,k} \\ I \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

Thus, we have described the problem in the form of an augmented discrete-time control-oriented model with uncertain

elements determined by the time-varying network delays. We will then propose the expression of the delay uncertainties in Section III and the control synthesis method in Section IV.

### III. DESCRIPTION OF NETWORK-INDUCED TIME-DELAY UNCERTAINTIES

As is shown in the problem formulation, the time-varying delays introduce system uncertainties to the control system. Therefore, to address the problem of time-varying delays, the first step is to express (13) with a corresponding format. There are several methods in robust control to describe and deal with system uncertainties, and the method that is most frequently used is polytopic inclusion.

For systems with linear uncertainties, it is a general way to express the uncertainties as a polytope of matrices. While for systems with time-varying delays, the uncertain terms  $(\Delta_{0,k}, \Delta_{1,k}, \dots, \Delta_{\Upsilon,k})$  are caused by integral operation of time delay  $\tau_k$ , as is described in (12). In other words, the uncertainties in (13) have nonlinear relationships with the time delay, which renders the polytopic inclusion method unable to be directly applied. To make use of the polytopic inclusion method, the uncertainties should be described in a linear formulation of time delay  $\tau_k$ . TA is a common approach for linearization and was used to describe the nonlinear uncertainties as a polytope of matrices by Hetel [23]. In this paper, the TA will also be employed.

Consider an integral term as

$$\Gamma(x) = \int_0^x e^{A_c(T_s - \theta)} d\theta \quad (14)$$

which is the general form of the uncertain terms in (12).

With the TA, (14) can be written as

$$\begin{aligned} \Gamma(x) &= \Gamma(0) + \dot{\Gamma}(0)x + \ddot{\Gamma}(0)\frac{x^2}{2!} + \cdots + \frac{d^q \Gamma}{dx^q}(0)\frac{x^q}{q!} + \cdots \\ &= - \sum_{q=1}^{\infty} \frac{(-x)^q}{q!} A_c^{q-1} e^{A_c T_s}. \end{aligned} \quad (15)$$

Focusing on the first  $h$  terms of (15),  $\Gamma(x)$  can be described as a finite summation and a remainder, i.e.,

$$\Gamma(x) = - \sum_{q=1}^h \frac{(-x)^q}{q!} A_c^{q-1} e^{A_c T_s} + \Theta^h. \quad (16)$$

With a proper selection of the number  $h$ , the high-order terms in the remainder can be relatively small. Neglecting the remainder  $\Theta^h$ , we can obtain the  $h$ -order approximation of (14) with sufficient accuracy as

$$\Gamma^h(x) = - \sum_{q=1}^h \frac{(-x)^q}{q!} A_c^{q-1} e^{A_c T_s}. \quad (17)$$

Now, we have transformed the integral relationship between  $\Gamma^h(x)$  and  $x$  into a polynomial relationship, and to acquire the linear relationship, we define the notations

$$\begin{aligned} G_q &= \frac{(-1)^{q+1}}{q!} A_c^{q-1} e^{A_c T_s} \\ \phi_{j,1} &= [\rho^h I \quad \rho^{h-1} I \quad \cdots \quad \rho^2 I \quad \rho I]^T \\ \phi_{j,2} &= [\rho^h I \quad \rho^{h-1} I \quad \cdots \quad \rho^2 I \quad \bar{\rho}_j I]^T \\ \phi_{j,3} &= [\rho^h I \quad \rho^{h-1} I \quad \cdots \quad \bar{\rho}_j^2 I \quad \bar{\rho}_j I]^T \\ &\vdots \\ \phi_{j,h} &= [\rho^h I \quad \bar{\rho}_j^{h-1} I \quad \cdots \quad \bar{\rho}_j^2 I \quad \bar{\rho}_j I]^T \\ \phi_{j,h+1} &= [\bar{\rho}_j^h I \quad \bar{\rho}_j^{h-1} I \quad \cdots \quad \bar{\rho}_j^2 I \quad \bar{\rho}_j I]^T \end{aligned} \quad (18)$$

where  $q = 1, 2, \dots, h, j = 0, 1, \rho = 0, \bar{\rho}_0 = T_s$ , and  $\bar{\rho}_1 = vT_s$ .

For all the possible integral expressions in our problem, variable “ $x$ ” should be always in interval  $[0, T_s]$  or  $[0, vT_s]$ ; then, we can include any integral term  $\Delta_{i,k}$  into a polytope with enough vertices.

The vertices of the convex polytope can be written as

$$\bar{\Delta}_{0,i} = [G_h \quad G_{h-1} \quad \cdots \quad G_2 \quad G_1] \phi_{0,i} B_{uc} \quad (20)$$

$$\bar{\Delta}_{1,i} = [G_h \quad G_{h-1} \quad \cdots \quad G_2 \quad G_1] \phi_{1,i} B_{uc}. \quad (21)$$

Then, for an arbitrary delay  $\tau_k \in [0, \tau_{l \arg e}]$ , the uncertain terms in (13) can be expressed as a linear combination of the vertices in (20) or (21), i.e.,

$$\Delta_{i,k} = \sum_{l=1}^{h+1} \mu_{i,l}(k) \bar{\Delta}_{1,l}, \quad \text{for } i = 0, 1, \dots, \Upsilon - 1 \quad (22)$$

$$\Delta_{i,k} = \sum_{l=1}^{h+1} \mu_{i,l}(k) \bar{\Delta}_{0,l}, \quad \text{for } i = \Upsilon \quad (23)$$

where  $\mu_{i,l}(k)$  is a time-varying coefficient determined by  $\tau_k$  and is subject to

$$\sum_{l=1}^{h+1} \mu_{i,l}(k) = 1 \quad \text{and} \quad \mu_{i,l}(k) > 0.$$

**Remark 1:** For a CAN whose delay upper bound is  $\tau_{l \arg e} = (\Upsilon + v)T_s$ , there are  $(\Upsilon + 1)$  uncertain terms  $(\Delta_{0,k}, \Delta_{1,k}, \dots, \Delta_{\Upsilon,k})$  in the system matrices of the augmented system in (13). For each  $\Delta_{i,k}$  ( $i = 0, 1, \dots, \Upsilon$ ), an  $h$ -order Taylor series expansion generates  $(h + 1)$  terms  $(\Delta_{0,l}$  or  $\Delta_{1,l}, l = 1, 2, \dots, h + 1)$  in the approximate expression of (22) and (23). Therefore, based on the idea of the polytopic inclusion method, there will be a convex polytope with  $(h + 1)^{\Upsilon+1}$  vertices to properly describe the network-induced time-delay uncertainties.

#### IV. CONTROL SYNTHESIS

This section presents the control synthesis of the system with time-varying delays using the  $H_\infty$ -based LQR tracking control. In Section III, the delay uncertainties are described in the form of polytopic uncertainty. Based on this expression, the  $H_\infty$ -based LQR tracking control is obtained and solved in the form of a sequence of LMIs for the combined AFS and DYC controller of a 4WID-EV.

##### A. $H_\infty$ -Based LQR Tracking Control

With the system in (13), the control objective is to minimize the tracking error and the control input signals. A performance index is formulated as a combination of the tracking error and the control signals. In this paper, we adopt a quadratic form of the tracking error and the control signals as

$$J = \sum_{i=0}^{\infty} (e_i^T Q e_i + u_i^T R u_i) \quad (24)$$

where  $Q$  and  $R$  are two positive definite weighting matrices to regulate the weight of active steering angle correction and direct yaw-moment as well as the tolerance of tracking control error. In practical controller design,  $Q$  and  $R$  may vary by driving conditions such as vehicle speed to balance between vehicle handing, comfort, and safety, which is not our research emphasis here. Therefore,  $Q$  and  $R$  are selected as reasonable constants for control synthesis and simulation in this paper.

The  $H_\infty$ -based LQR tracking controller is obtained by finding the optimized state-feedback gain  $K$  ( $u_k = K \xi_k$ ) to minimize index  $J$ , which is also equal to the 2-norm of the following constructed signal:

$$z_k = E \xi_k + F u_k \quad (25)$$

where

$$E = \begin{bmatrix} 0 & Q^{1/2} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \text{and} \quad F = \begin{bmatrix} 0 \\ R^{1/2} \end{bmatrix}.$$

Since  $w = \delta_f$  is bounded in  $l_2$  space, we introduce an  $H_\infty$  performance index  $\eta$  such that  $J < \eta^2 \|w\|_2^2$ . Then, the optimization problem becomes an optimal  $H_\infty$  control problem for the following system:

$$\begin{aligned} \xi_{k+1} &= A_{aug} \xi_k + B_{wr,aug} w_k + B_{u,aug} u_k \\ z_k &= E \xi_k + F u_k. \end{aligned} \quad (26)$$

With the state-feedback control, the closed-loop system is

$$\begin{aligned} \xi_{k+1} &= (A_{aug} + B_{u,aug} K) \xi_k + B_{wr,aug} w_k \\ z_k &= (E + F K) \xi_k. \end{aligned} \quad (27)$$

The design objective is to determine control gain  $K$  in (27) such that the closed-loop system in (27) is asymptotically stable, and the following performance inequality is satisfied:

$$\|z\|_2 < \eta \|w\|_2. \quad (28)$$

To solve the problem, we introduce the following lemmas.

*Lemma 1:* Suppose that the controller is designed. The closed-loop system in (27) is stable with a given  $H_\infty$  performance  $\eta$ , if there exists a positive definite matrix  $P = P^T > 0$  satisfying [41]

$$\begin{bmatrix} -P & 0 & P(A_{aug} + B_{u,aug}K) & PB_{wr,aug} \\ * & -I & E + FK & 0 \\ * & * & -P & 0 \\ * & * & * & -\eta^2 I \end{bmatrix} < 0. \quad (29)$$

When the control gains are given, Lemma 1 provides an  $H_\infty$  criterion for the closed-loop system. However, it cannot be directly applied to the controller design because of the bilinear term  $P(A_{aug} + B_{u,aug}K)$  in the equality. In [42], the bilinear term can be removed by using a congruence transformation with free matrices being introduced.

*Lemma 2:* Suppose that the controller is designed. The closed-loop system in (27) is stable with a given  $H_\infty$  performance  $\eta$  if there exist positive definite matrices  $\Omega = \Omega^T > 0$ ,  $M$  satisfying [42]

$$\begin{bmatrix} -\Omega & 0 & (A_{aug} + B_{u,aug}K)M & B_{wr,aug} \\ * & -I & (E + FK)M & 0 \\ * & * & \Omega - M - M^T & 0 \\ * & * & * & -\eta^2 I \end{bmatrix} < 0. \quad (30)$$

Next, based on Lemma 2, we propose the controller design for combined AFS and DYC of the 4WID-EV.

### B. Controller Design for Combined AFS and DYC

For the controller design of the considered problem, with the  $(h+1)^{Y+1}$  vertices of the convex matrix polytope obtained in Section III, we have  $(h+1)^{Y+1}$  systems, i.e.,

$$\begin{aligned} \xi_{k+1} &= A_{aug,i}\xi_k + B_{wr,aug}w_k + B_{u,aug,i}u_k, \\ i &= 1, 2, \dots, (h+1)^{Y+1}. \end{aligned} \quad (31)$$

Our objective is to find an optimized control gain  $K$ , such that the following conditions are achievable:

$$\begin{bmatrix} -\Omega & 0 & (A_{aug,i} + B_{u,aug,i}K)M & B_{wr,aug} \\ * & -I & (E + FK)M & 0 \\ * & * & \Omega - M - M^T & 0 \\ * & * & * & -\eta^2 I \end{bmatrix} < 0$$

$$\forall i = 1, 2, \dots, (h+1)^{Y+1}. \quad (32)$$

Defining a new variable  $Y = KM$ , conditions in (32) become

$$\begin{bmatrix} -\Omega & 0 & A_{aug,i}M + B_{u,aug,i}Y & B_{wr,aug} \\ * & -I & EM + FY & 0 \\ * & * & \Omega - M - M^T & 0 \\ * & * & * & -\eta^2 I \end{bmatrix} < 0$$

$$\forall i = 1, 2, \dots, (h+1)^{Y+1}. \quad (33)$$

For given matrices  $Q$  and  $R$ , a smaller performance index  $\eta$  means that the controlled output  $z$  (a combination of the tracking error and the control signal) is also smaller. Therefore, the

controller design problem for the combined AFS and DYC can be finally expressed as

$$\begin{aligned} \min_{\Omega, M, Y, \eta} \quad & \eta^2 \\ \text{subject to} \quad & \begin{bmatrix} -\Omega & 0 & A_{aug,i}M + B_{u,aug,i}Y & B_{wr,aug} \\ * & -I & EM + FY & 0 \\ * & * & \Omega - M - M^T & 0 \\ * & * & * & -\eta^2 I \end{bmatrix} < 0 \\ & \forall i = 1, 2, \dots, (h+1)^{Y+1}. \end{aligned} \quad (34)$$

The problem described in (34) is a typical minimization problem of a linear objective function with constraints of LMIs and can be solved with the LMI Toolbox in MATLAB. Once this minimization under constraints is solved, the  $H_\infty$ -based LQR tracking controller can be obtained by  $K = YM^{-1}$ . The proposed controller has a fixed gain matrix  $K$ , which could be calculated offline. Hence, it is easier to be applied in a practical system, and real-time performance can be guaranteed.

## V. SIMULATION RESULTS

To study the performance of the proposed controller, simulations are conducted in Simulink with a full-vehicle model constructed by CarSim. The simulation diagram is shown in Fig. 4. The vehicle model parameter values were measured from a prototype in-wheel motor EV, and some key parameters are listed in Table I [2], [3].

The vehicle controller structure is composed of two levels as shown in Fig. 1, where a combined AFS and DYC controller is at the upper level, and a torque allocation strategy is at the lower level. For comparison, the proposed controller and a conventional LQR controller are adopted at the upper levels. For the two control systems to be compared, the same static torque allocation logic is employed as the lower-level controller, which distributes the direct yaw-moment equally to the driving/braking torque of the four wheels, such that each wheel has the same contribution to the total yaw-moment.

The network model simulates the message transfer delays induced by the CAN bus. With reference to the results in [17]–[20], CAN-induced delays are assumed to be time varying and uniformly distributed in interval  $[0, 1.7 T_s]$ , where  $T_s = 10$  ms is the sampling period of the closed-loop system. Meanwhile, delays of different messages should be subjected to the restraint introduced by (8).

The AFS system and four in-wheel motor blocks are modeled to transfer network control commands to the actual control outputs of actuators, with actuator dynamics properly taken into account. For 4WID-EVs studied in this paper, the AFS system is controlled by a servo motor, and the four wheels are directly driven or braked by in-wheel motors. Therefore, dynamics of an electric motor is mainly modeled here, including the rate limit and amplitude saturation of the driving/braking torque, the response time of the motor controller, motor inertia, and the AFS angle correction constraint. (For the DYC procedure accomplished by hydraulic braking units, see [43] for actuator dynamics.)

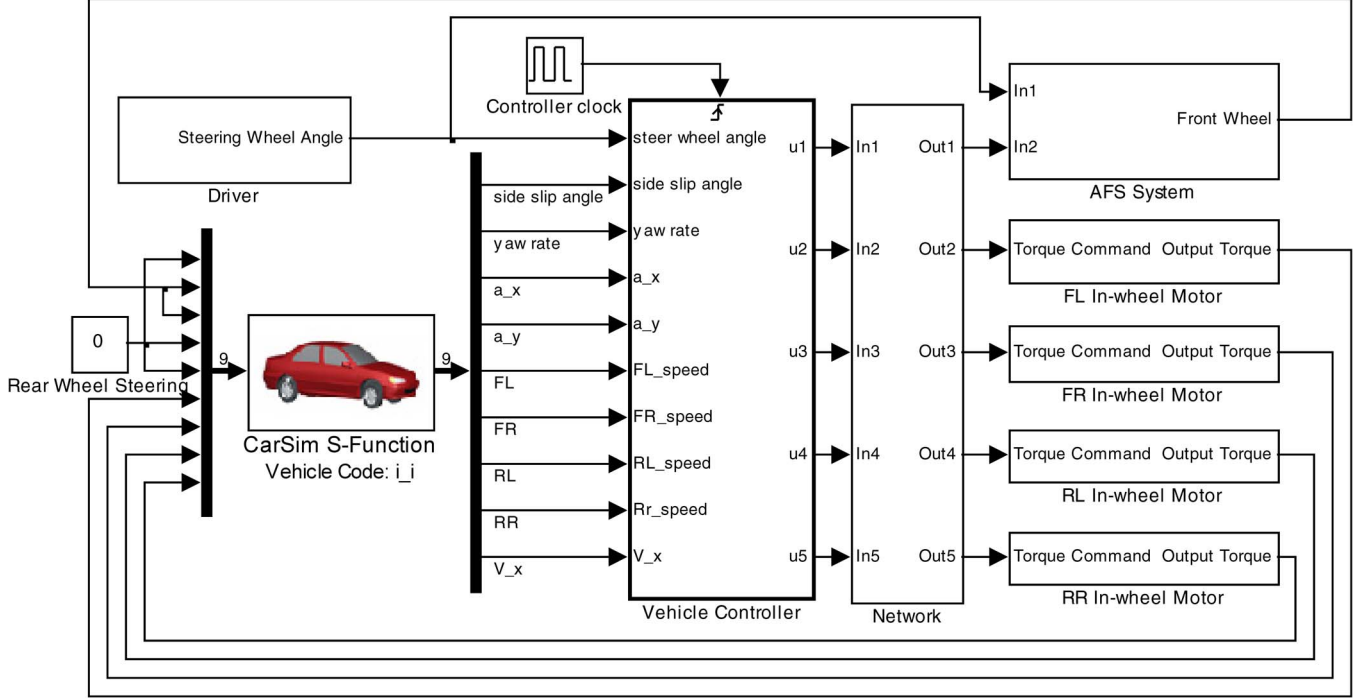


Fig. 4. Simulation diagram with vehicle model constructed by CarSim.

TABLE I  
VEHICLE MODEL KEY PARAMETERS

Parameter	Value (Unit)
$m$	800(kg)
$I_z$	728.6(kg·m <sup>2</sup> )
$l_f$	0.85(m)
$l_r$	1.04(m)
$C_f$	10000(N/rad)
$C_r$	10000(N/rad)
Steer gear ratio	18

The driver model gives steering wheel angle input for each driving maneuver. Three different maneuvers are considered here, with the respective steering wheel angle signals shown in Fig. 5. The first maneuver is ramp steering, which is usually adopted in a J-turn maneuver. The second maneuver is single lane changing with a single sinusoidal steering input. The third maneuver is double lane changing, which is produced from a prescribed steering input. All of the three maneuvers are commonly used in vehicle tests and can well imitate vehicles' performance in extreme cases, such as high-speed overtaking and obstacle avoidance. In addition, with the given steering signals, the AFS and DYC systems have the opportunities to cooperate well to track the driver's driving intention, which makes the evaluation of the proposed control system more convincing. In the simulations, the vehicle longitudinal speed is set to be 100 km/h, and the tire-road friction coefficient is 0.85 in all the maneuvers.

A conventional LQR controller without explicitly considering the network-induced time delays is designed as a compari-

son with the proposed controller. The weighting matrices in the performance index of the conventional LQR are chosen as

$$Q_c = \begin{bmatrix} 2000 & 0 \\ 0 & 100\,000 \end{bmatrix} \quad R_c = \begin{bmatrix} 8000 & 0 \\ 0 & 0.00001 \end{bmatrix}. \quad (35)$$

By using the *lqr* command in MATLAB, the control gain matrix of the conventional LQR controller can be obtained as

$$K_c = \begin{bmatrix} 0.099 & 0.945 \\ 1716.6 & 44\,485 \end{bmatrix}. \quad (36)$$

Simulation results and analyses are presented in the following sections. For each driving maneuver, the proposed controller and the compared controller are evaluated in two stages: The first stage is under the ideal in-vehicle network condition, where the controllers work as if in a centralized control system, with no time delays, to evaluate their tracking performance; then, the network-induced time-varying delays previously described in the network model are introduced into the system to assess the robustness of the controllers.

#### A. Ramp Steering Maneuver

Fig. 6 shows the simulation results of vehicle yaw rate response in the ramp steering maneuver. Under the ideal network condition without time delays, both the conventional LQR controller and the proposed controller give satisfactory results, and the proposed controller performs better without a steady-state tracking error, which is the effect of the generalized proportional-integral control approach achieved by the system augmentation in (4). However, with the time-varying delays induced by the CAN, the conventional LQR controller yields oscillations in the transient process and error in steady state, whereas the proposed controller can still track the desired yaw rate, as well as what it does under the ideal network condition.



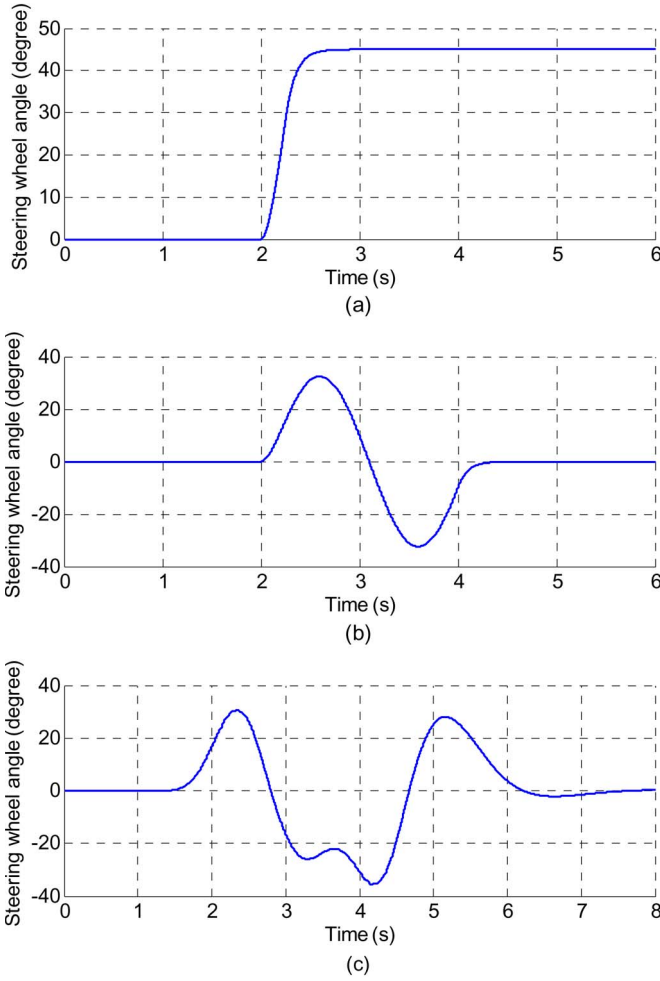


Fig. 5. Steering wheel angle of simulation maneuvers. (a) Ramp steering maneuver. (b) Single lane-changing maneuver. (c) Double lane-changing maneuver.

### B. Single Lane-Changing Maneuver

Fig. 7 shows the simulation results of the vehicle yaw rate under a single lane-changing maneuver. Under the ideal network condition without delays, both the conventional LQR controller and the proposed controller are able to track the desired yaw rate well. Nevertheless, with the time-varying delays induced by CAN, there are significant oscillations in the vehicle yaw rate with the conventional LQR controller, and the oscillations still exist even when the steering wheel angle returns to zero in the last 2 s, which indicates that the vehicle system is almost at the criticality of instability. While for the proposed controller, there is almost no negative influence on the tracking performance even when the network delays are introduced. Intuitively, one can consider that the proposed robust controller based on the augmented system model containing the information of possible delays in the past few sample periods has some function similar to a low-pass filter, which can degrade the adverse impacts of the network-induced time-delay uncertainties.

### C. Double Lane-Changing Steering Maneuver

In the double lane-changing maneuver, the comparative results shown in Fig. 8 are almost the same as those of the

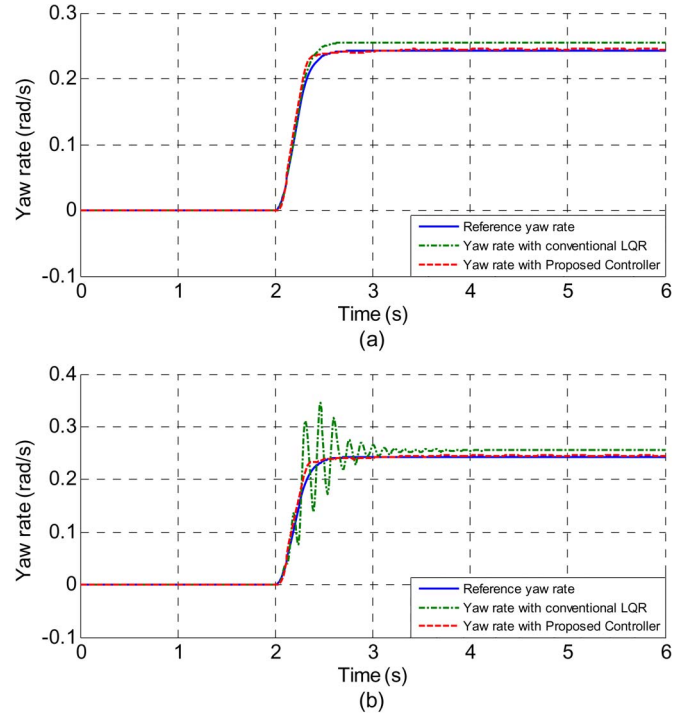


Fig. 6. Vehicle yaw rate response in ramp steering maneuver. (a) Under the ideal network condition. (b) With CAN-induced delays.

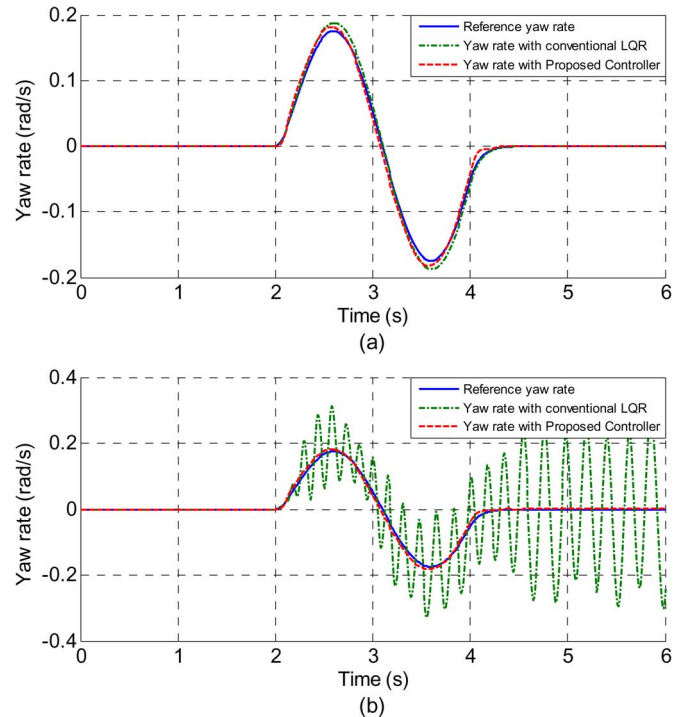


Fig. 7. Vehicle yaw rate response in single lane-changing maneuver. (a) Under the ideal network condition. (b) With CAN-induced delays.

previous two maneuvers. Both controllers perform well under the ideal network condition. However, once there are network-induced time-varying delays in the closed-control loop, the yaw rate of the vehicle controlled by the conventional LQR controller significantly oscillates after a nonzero steering wheel angle trigger, whereas the proposed controller demonstrates good

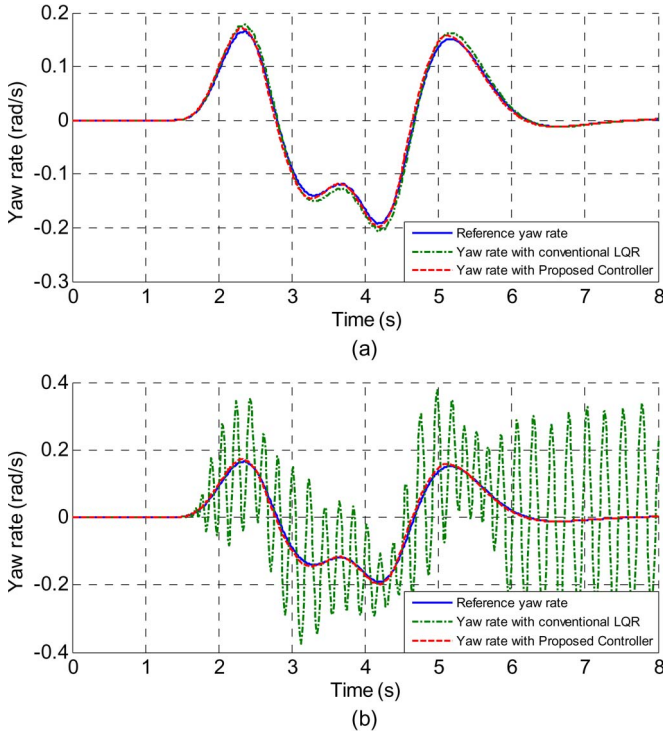


Fig. 8. Vehicle yaw rate response in double lane-changing maneuver. (a) Under the ideal network condition. (b) With CAN-induced delays.

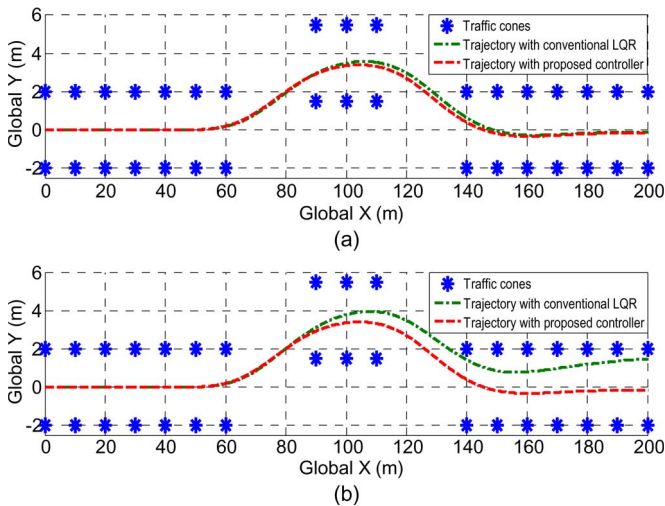


Fig. 9. Vehicle CG trajectory in double lane-changing maneuver. (a) Under the ideal network condition. (b) With CAN-induced delays.

robustness against the CAN-induced network time-varying delays as no evident control performance degradations can be observed.

In addition, as the steering action lasts long enough (from 1.5 to 6 s) in the double lane-changing maneuver, the effect of network-induced delays is hence significant enough to influence the vehicle global trajectory with the conventional LQR controller. Comparatively, for the proposed controller, the trajectory is just as the driver's expectation, which is displayed in Fig. 9.

From the given three comparative vehicle maneuver simulations, it is evident that the in-vehicle network-induced actuator

time delays considerably influence vehicle lateral dynamics control performance. Systematic and explicit incorporations of such time delays in the vehicle controller design can substantially attenuate the undesired effects and, thus, improve the robustness of the vehicle control systems implemented through in-vehicle networks.

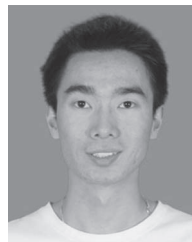
## VI. CONCLUSION

In this paper, a combined AFS and DYC controller with good robustness against in-vehicle network-induced time-varying delays has been proposed for the lateral motion and stability control of 4WID-EVs. The main idea is to augment the original system with uncertain terms induced by time-varying delays and then describe the nonlinear uncertainties as polytopic inclusions with the help of TA. An  $H_\infty$ -based LQR tracking controller is introduced and adopted in the control synthesis. Three simulation maneuvers are carried out on a full-vehicle model constructed by CarSim to verify the performance of the proposed controller. Simulation results show that the proposed controller not only achieves a good control effect under an ideal network condition (no time delays) but guarantees enough robustness and performance when there are network-induced time-varying delays in the closed-control loop as well. Comparisons with a typical LQR vehicle lateral stability controller without explicit consideration on the in-vehicle network delays further evidenced the effectiveness of the proposed controller. In addition to the results of vehicle motion control, this paper also indicates that time-varying delays in networked control systems would induce adverse impacts to system performance, and additional attention should be paid to system analysis and control synthesis of networked control systems to enhance the robustness against network-induced time delays, particularly for systems with high-frequency dynamics.

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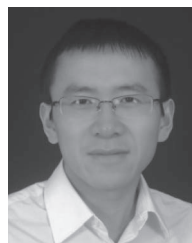
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