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Analysis of Topological Dynamical Properties of Urban Road Traffic Network

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Abstract: The technologies and applications aiming at the dynamic properties of micro-level road traffic have made plenty of contributions for the development of Intelligent Transportation Systems (ITS) in past decades, which causes a lack of understanding and analysis for the dynamical properties of the whole structure at macro-level. Therefore, in this paper, we construct the Variable-Structure Dynamic Network modle (*VSDN*) and the Extended Variable-Structure Dynamic Network model (*EVSDN*) for urban road traffic systems, basing on the Level-Of-Service (LOS) and the traffic reachability. The *VSDN* model defines the edge set at any time includes only the edges whose LOS lies in un-congestion levels, which causes that the structure of the urban road traffic network varies with time. Furthermore, we extend *VSDN* to *EVSDN* that redefines the edge set to combine the micro-level time-varying characteristics into the macro-level topology. The experiments with traffic data and topology data of the Beijing road traffic system show that the *EVSDN* model is scale-free at some peak-time phases and the appearance of the scale-free property represents the significant change of topological features of urban road traffic networks. During rush hours, the scale-free property can be used to characterize urban road traffic networks.

Key Words: Traffic Reachability, *VSDN*, *EVSDN*, The Scale-Free Property

1 Introduction

Intelligent Transportation Systems (ITS) refers to transportation systems which apply emerging hard and soft information systems technologies to address and alleviate transportation congestion problems. Up till now, the most theories, technologies and applications in ITS are constructed based on the micro-level (roads) dynamical properties of urban road traffic systems. With the development of ITS, people realize that the methods relying on the dynamics of road traffic flow are inappropriate for describing and analyzing the complex character of urban road traffic systems. Therefore, to model and analyze the macro-level dynamic properties of urban road traffic systems with the perspectives of complex system theory and complex network theory is the trend in ITS.

Presently, the research on the complexity of transportation systems focuses on the topological dynamics, such as the scale-free property [1-2] and the small-world feature. The related references make contributions in inter-city highway networks [3-4], railway networks [5-7] and public transit networks [8-12], etc. Seldom work is subscribed in the complex dynamics of urban road traffic networks. The articles related to the scale-free property rapid increase in past decade. And the scale-free property has been come one of most concerned point in various kinds networks. A network is called scale-free when its degree distribution follows $P(\kappa) \propto A\kappa^{-\alpha}$, with exponents varying in the range $2 < \alpha < 3$ [1-2].

As for the research on the scale-free property, we conclude the features of urban road traffic networks, as followed:

(1) The amount of nodes (intersections, merging zones, etc.) and edges (arterials, freeways, etc.) keeps stable. It need a long time cycle to build infrastructures. So, the amount of roads and intersections won't increase significantly. Therefore, the models, such as BA model [1-2], WSF model [13], etc., that study the scale-free property by adding new nodes to the network are unsuitable for urban road traffic networks.

(2) The network structure varies significantly. In a 24h time cycle, the traffic state of the urban road traffic network has the "pendulum effect", which means the network emerges totally different behaviors at different time phases. It cannot use a single mathematic model to describe all these behaviors. The existing approaches are unable to model the evolving process of urban road traffic networks.

(3) It is hard to define the dynamical behaviors of the networks. The description of dynamical behaviors is tightly associated with the network topology. That is to say, the network modeling is important to define the dynamical properties. The definition of network models determines the exhibition of the concerned features of real systems. So, the modeling method is submitted to the motivation of network modeling and the indentified model is able to map the certain concerned characteristics of the real system. The existing network models that are low-level direct mapping models of the infrastructures of urban road traffic systems are inappropriate to describe the topological dynamical properties.

According to the elementary features of urban road traffic networks, we conclude some "abilities" that the models using to deal with the scale-free property should have, as followed.

(1) To reflect the traffic state on road accurately. It is the fundamental function of traffic models, which is also the basis for topology analysis.

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(2) To exhibit the topology and its dynamical change clearly. As stated above, urban road traffic networks have different characterizing topological properties at different time phases. Therefore, the identified network models are required to map the dynamic fluctuations on the roads and the structure.

In this paper, we focus on the scale-free property of urban road traffic networks by introducing the models describing the dynamical characteristics of traffic state on roads and structure, and analyzing the scale-free property with traffic data and topology data from the Beijing road traffic network.

2 Level-of-Service and Traffic Reachability

The concepts and methods used to evaluate the traffic state on road are numerous, in which Level-Of-Service is adopted widely. Based on LOS, many cities have been constructed road traffic evaluation systems according to their specific characteristics of traffic systems. In this section, we introduce the basic LOS method and the traffic reachability relying on LOS.

2.1 Traffic Level-Of-Service

It is well known that the traffic state is dependent not only on the static parameters (lane number and width, road classes, etc.) of the infrastructures (l roads, intersections, etc.), but also on the continuous changing state of traffic flow on roads. In fact, it is the dynamical character that decides the behavior of road traffic flow. The detected traffic variables (volume, speed, and occupancy) reflect the traffic state with data. And then the traffic state determined by the traffic variables can be used to conduct the other concerned dynamic properties, such as traffic reachability.

Traffic Level-Of-Service, LOS, firstly denoted in Highway Capacity Manual (HCM) [14], is used to qualitatively judge the traffic state on roads.

Level of service (LOS) is a measure used by traffic engineers to determine the effectiveness of elements of transportation infrastructure, including six levels {A,…,F} with A being best and F being worst [14].

Essentially, LOS refers to the ratio of the volume on a road at some time to the maximum capacity of the road.

LOS may be calculated through one of the traffic variables or their compositions. The distinct of different calculation methods for LOS is from the traffic conditions of different cities. As well as, the classification of LOS levels relies on the requirement of traffic management. For examples, Cameron [15] classes LOS into 10 levels as {A,…,J}, Baumgaertner [16] into 9 levels using {A,…,I}. This paper provides a general description for LOS. The LOS of a road at any t time, named $\phi(t)$, can be calculated in the following equation.

$$\phi(t) = f(V(t), S(t), Q(t)),$$

in which, $V(t)$, $S(t)$ and $Q(t)$ are respectively the volume, the speed and the occupancy at t time; f is the qualitatively function with its value range $\Lambda = \{\lambda_1, \cdots, \lambda_m\}, m > 1$, λ_1 being best and λ_m being worst.

For any traffic state evaluation system, the critical level, $\lambda_{critical}$ ($1 < \lambda_{critical} \le m$), is selected for judging the traffic state of a road is congested or un-congested.

We divide the set of LOS levels (Λ) into two independent subsets, naming Un-congestion Subset (Λ_α) and Congestion Subset (Λ_β). The Un-congestion Subset is an ordered set $\Lambda_\alpha = \{\lambda_1, \cdots, \lambda_{critical-1}\}$, $\Lambda_\alpha \subseteq \Lambda$. And the Congestion Subset is $\Lambda_\beta = \{\lambda_{critical}, \cdots, \lambda_m\}$, $\Lambda_\beta \subseteq \Lambda$. When the LOS of a road at any time lies in Λ_α, the traffic state of the road at this time is called "un-congested", or, that is, in Λ_β, called "congested".

In general, LOS reflects the traffic state of roads, which provides a tool for modeling the dynamical properties of road traffic and a functional basis for defining traffic reachability and presenting the dynamical network models.

2.2 Traffic Reachability

Contrasting with LOS as an important concept in road traffic flow theory, the traffic reachability is introduced in transportation network domain for study on the dynamical properties of road traffic level.

In graph theory, reachability is the notion of being able to get from one vertex in a directed graph to some other vertex. Two vertices are called adjacent if they share a common edge. For any urban road traffic system in the world, there are hardly any isolated vertex. That is, we could find at least one path from one address to another. Therefore, it is meaningful for urban road traffic networks to discuss the traffic reachability between two adjacent vertexes. Through the traffic reachability, the dynamical properties on both micro- and macro-levels can be clearly and effectively represented in dynamic network models.

The traffic reachability sets up a kind of map between the qualitative evaluation on segments (roads) in real networks and the presence of edges in abstract networks. The traffic reachability of two adjacent vertexes refers to what extent does the capacity provided by one road satisfy the traffic requirement. From the perspective of computation, the traffic reachability means if the traffic state at any given time satisfies the critical LOS level. If it does, the two vertexes are traffic reachable, or, un-reachable. When the later happens, the segment (the head and the tail are traffic un-reachable.) is judged to lose the core function as a road. So, the corresponding edge in network models may be excluded in the networks according to the modeling strategies.

The traffic reachability reflects the basic task of roads, maps the qualitative evaluation into network models, which causes a kind of changing structures of networks.

3 Urban Road Traffic Dynamic Network Models

Generally, the dynamics of road traffic has been over-studied by researcher from urban road traffic domain for a long time because the direct detected traffic variables support. Otherwise, the study on the dynamical properties of traffic network topologies has been paid little attention for its high abstraction. Firstly, the topologically dynamical properties are strong related to the dynamics on roads. And the second is there are special characters on the structure level emerged from but not existed in road level dynamics, such as the scale-free property. In this section, we try to construct the models to reflect the complex property of urban road traffic networks.

3.1 The Variable-Structure Dynamic Network Model

The traditional modeling strategies adding time-varying properties on weights ignore the dynamics on structure. Consequently, the models have stable structures. Based on the concepts of LOS and traffic reachability, we redefine the set of edges to make the network's structure being able to change with time.

Let E be the complete edge set corresponding to the segments in an urban road traffic network. The *VSND* model's edge set, marked as E', is a dynamic set. At any t time, the set $E'(t)$ include the edges that its head and tail are traffic reachable.

$$E'(t) = \{\forall e \in E | \phi_e(t) \in \Lambda_\alpha\},$$

in which, $\phi(t)$ is the LOS level of the road corresponding to e at t time.

As shown in the equation, the edge set E' has only the edges whose corresponding roads' LOS values are at a Un-Congestion level (in Λ_α), and $E' \subseteq E$. Therefore, E' contains different elements at different time.

The Variable-Structure Dynamic Network (*VSDN*) model can be described in the following way.

$$VSDN(t) = VSDN(N, E'(t)),$$

where, N is the set of vertexes.

The *VSDN* model has a time-varying topology by introducing traffic reachability. From the perspective of traffic reachability, *VSDN* is the functional sub-network of the urban road traffic network. And all the roads that provide all the transport service of the network are included in the model. Consequently, the different character is emerged when the topology is changed.

3.2 The Extended Variable-Structure Dynamic Network model

Because of lacking the direct mapping from the road level to the network level, the topology is changing only when qualitative fluctuations of LOS and traffic reachability happen to roads (congestion appears). So, we present the Extended Variable-Structure Network (*EVSDN*) model to describe the topological dynamics of urban road traffic networks.

Similarly to *VSDN* model, *EVSDN* model can be defined in the following equation.

$$EVSDN(t) = EVSDN(N, E''(t)),$$

where, N is the set of vertexes; $E''(t)$ is the edge set at t time.

In *EVSDN*, the edge set is defined in a totally different way. *EVSDN* defines an edge as one moving vehicle on the road whose head and tail are traffic reachable. That is to say, *EVSDN* replaces any edge between two vertexes in *VSDN* with a number of new edges. The number of edges at any time between two vertexes in *EVSDN* is the volume on the corresponding road at that time. The edge set is defined in the following equation.

$$E''(t) = E^*_{e1}(t) \cup E^*_{e2}(t) \cup \cdots \cup E^*_{em}(t), \quad ei \in E'(t), m = |E'(t)|$$

where,

$$E^*_e(t) = \{l_1, l_2, \cdots, l_n\}, n = V_e(t),$$

l is one edge in *EVSDN*; $V_e(t)$ is the t-time volume of the road corresponding to the edge $e \in E'(t)$ in *VSDN*.

Through a simple example, the modeling process of *EVSDN* is exhibited. As shown in Fig. 1, a simple urban road traffic network with 8 vertexes and 16 edges marked with the volume value at a certain time has been demonstrated. We suppose that the LOS values at the time on all roads are of un-congested levels, and then the figure represents the structure of *VSDN* model. According the volume values, the network structure at that time of *EVSDN* model is shown in Fig. 2. For the huge workload, we present the *VSDN* network structures in figures and the *EVSDN* with data analysis.

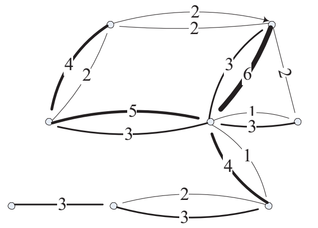


Fig. 1: A Simple Urban Traffic Road Network

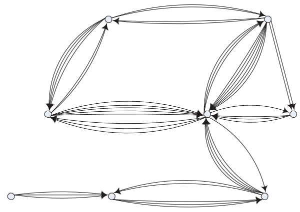


Fig. 2 : The Network Structure of the *EVSDN* Model

The *EVSDN* model denotes the micro-level dynamic properties on the macro-level features by implementing the relation between topological dynamics and traffic data. From the perspective of complexity, topological features of urban road traffic networks are emerging from the dynamical qualitative and quantitative properties of traffic stream on roads. The *EVSDN* model provides a promising tool for complex feature research on urban road traffic networks as presented in the following experiments with data from the Beijing road traffic network.

4 Experiments and Results Analysis

The *VSDN* model and the *EVSDN* model presented in this paper are aiming at describing the dynamical properties of network structure. Based on the traffic data from the Beijing road traffic network, we analyze the models through some experiments.

4.1 The Beijing Road Traffic System

The Beijing road traffic system, as a typical huge urban traffic system, has 140000 various kinds roads and several million vehicles, and faces more and more traffic jams and unbalance which causes that the traffic management encounters plenty of problems and traditional analytical approaches cannot explain complex phenomena effectively.

Fig. 3 is the full view of the roads in which the thicker lines are freeways and arterial. All the freeways and arterials have been equipped with detecting sensors, which means the real-time detection data of traffic flow on them can be provided for supporting the dynamic network modeling.

Moreover, the sensor-equipped road is dominant to the adjacent roads whose design class is lower. So, we deliver the major-road network in Fig. 4 based on the sensor-equipped roads.

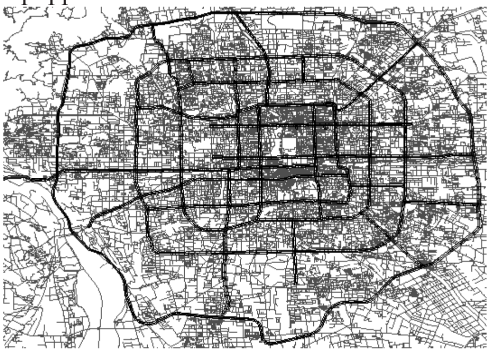


Fig. 3 : Overview of Beijing Road Network

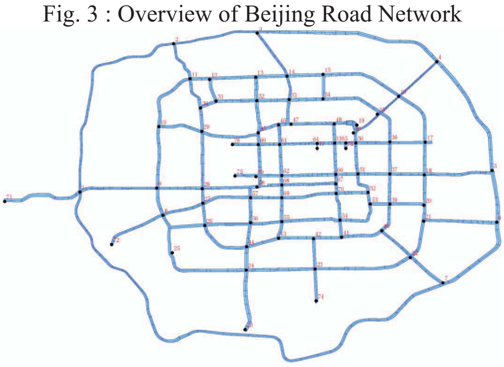


Fig. 4 : Freeways and Arterials of Beijing Road Network

The directed network structure of the basic road network of the Beijing road traffic system (Fig. 4) is demonstrated in Fig. 5, in which the complete set of vertexes is $N = \{n_1, \cdots, n_{76}\}$, the edge set is $E = \{e_1, e_2, \cdots, e_{252}\}$.

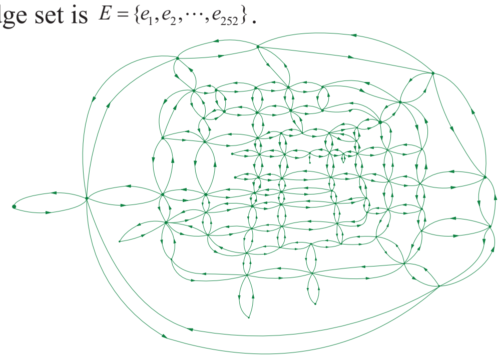


Fig. 5 : Digraph of the Beijing road network

4.2 Analysis of VSDN

In the technical report [17] for the scientific project of "Evaluation of Traffic State and Level-Of-Service of the Beijing Area Road Traffic", the calculating methods of LOS for the Beijing road traffic system are studied and introduced in detail. We demonstrate the network structure of the *VSDN* model by using the methods in [17] with the traffic data and the topology data.

In this paper, we select several times from the 24h time cycle of June 2, 2010, to exhibit the changing process of the network structure of the *VSDN* model.

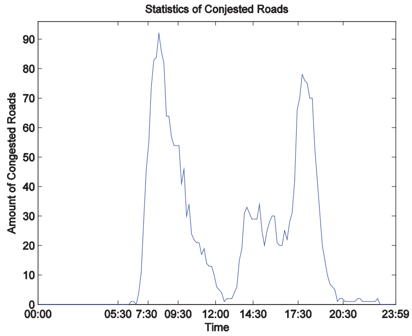


Fig. 6 : Statistics of Congested Roads (24h)

By calculating the LOS values of all roads in all times, we count all congested roads of each time, as shown in Fig. 6, in which the number of congested roads humps in the 24h time cycle. Obviously, with the number of congested roads, the network structure of *VSDN* changes with time as well. The snapshots of the network structure at "7:20", "7:50", "8:00", "10:00", "17:00", "18:10" and "19:00" are shown in the following figures.

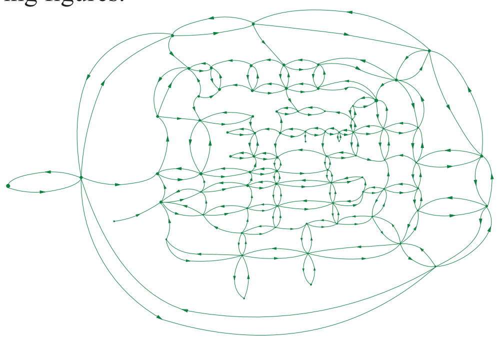


Fig. 7 : Network structure of *VSDN* at '7:20'

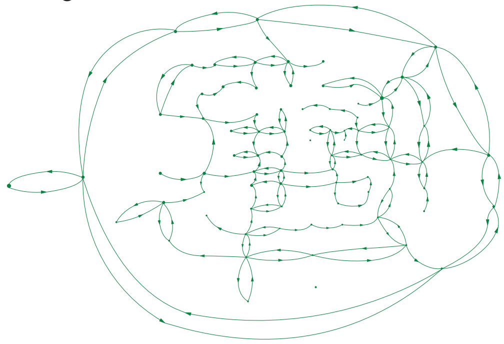


Fig. 8 : Network structure of *VSDN* at '7:50'

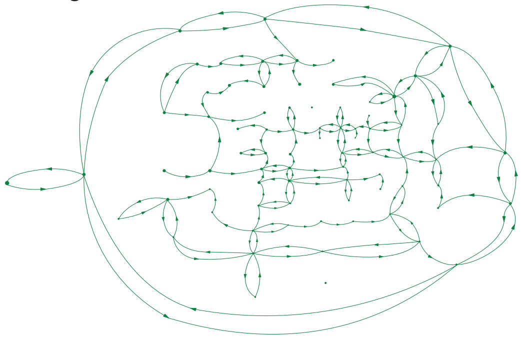


Fig. 9 : Network structure of *VSDN* at '8:00'

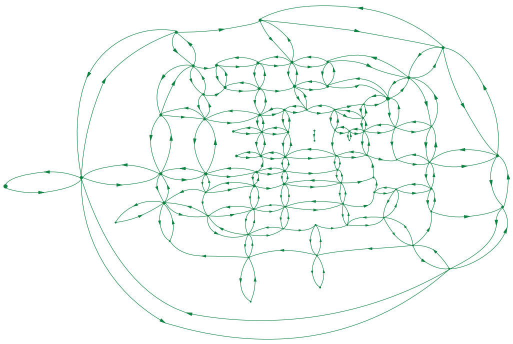


Fig. 10: Network structure of *VSDN* at '10:00'

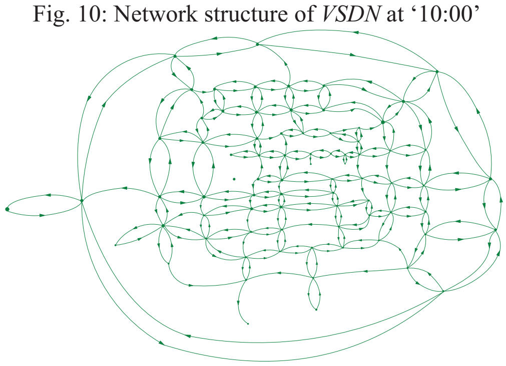


Fig. 11: Network structure of *VSDN* at '17:00'

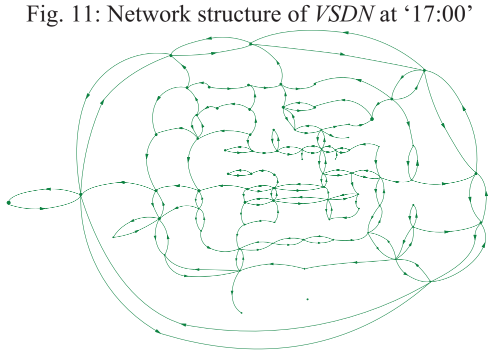


Fig. 12: Network structure of *VSDN* at '18:10'

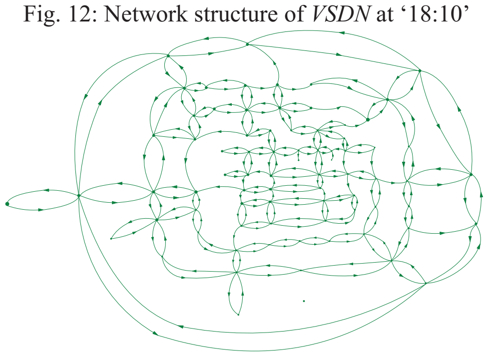


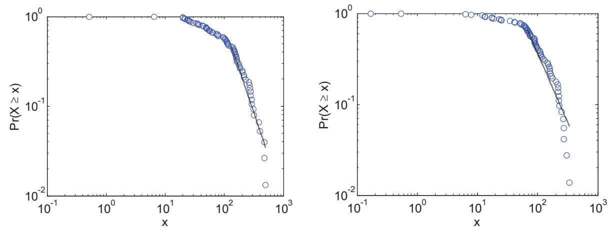
Fig. :13 Network structure of *VSDN* at '19:00'

The behavioral dynamics modeling is one of hot issues in urban road traffic systems. In the network-like modeling thinking, the behavioral dynamics refers to the topological dynamical character of urban road traffic networks. Aiming to describe the topological dynamics, this paper defines the *VSDN* to exhibit emerging features caused by congestion on roads.

4.3 Experimental Analysis of EVSDN

As stated above, the *EVSDN* model extendedly redefines the edge set of *VSDN* to make the network topology being able to significantly reflect the dynamical properties of traffic state on roads. To analyze the scale-free property of the *EVSDN* model for the Beijing road traffic network, we adopt the calculation method provided by Clauset et al. in the reference [18]. We use the 24h traffic data at June 2, 2010 to do the experiment and select several times to exhibit.

Through the complete data analysis, the experiment shows that the node degree distribution of the dynamic network at "7:50", "8:00" and "18:10" satisfies a power-law, as shown below.



(a) (b)

Fig. 14 : In-degree (a) and Out-degree (b) at '7:50' fitting power-law distribution

In Fig. 14, the out-degree values at "7:50" satisfy the following equation:

$$p(\kappa > 67.75) \propto \kappa^{-2.5236}.$$

And, the in-degree values at "7:50" satisfy:

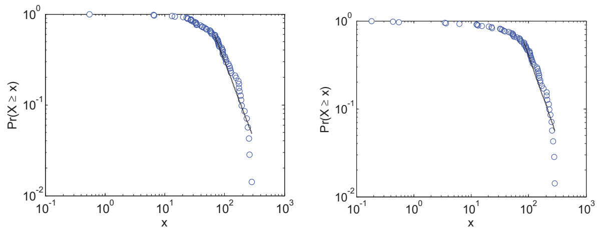
$$p(\kappa > 132.8166) \propto \kappa^{-2.9462}.$$

In Fig. 15, the out-degree values at "8:00" obey

$$p(\kappa > 83.5169) \propto \kappa^{-2.8687}.$$

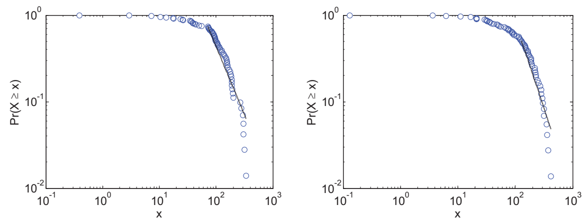
And the in-degree values at "8:00" follow

$$p(\kappa > 68.9472) \propto \kappa^{-2.7545}.$$



(a) (b)

Fig. 15: In-degree (a) and Out-degree (b) at '8:00' fitting power-law distribution



(a) (b)

Fig. 16 : In-degree (a) and Out-degree (b) '18:10' fitting power-law distribution

At "18:10", as shown in Fig. 16, the out-degree and the in-degree follow $p(\kappa > 71.8899) \propto \kappa^{-2.5802}$ and $p(\kappa > 128.8375) \propto \kappa^{-2.9841}$ respectively.

In conclusion, the Beijing road traffic network is scale-free at some times in a 24h time cycle. According to the experiments that show the scale-free property of the *EVSDN* model, we split a 24h time cycle into 9 time phases, "00:00-05:00", "05:00-07:30", "07:30-08:30", "08:30-11:30", "11:30-14:00", "14:00-16:50", "16:50-18:30", "18:30-20:30" and "20:30-24:00", to illustrate the percentages that the power-law distribution appears at the 9 phases. Through the data analysis about 60 days from April to June, we make the statistics of the appearance percentages as shown in Fig. 17. Obviously, the appearance percentages during peak times, "07:30-08:30" and "16:50-18:30" with 30% and 37% respectively, are much bigger than other time phases. In evidence, with the number of congested roads adding, kind of topological changes happen to urban road traffic networks. During rush hours, the scale-free property can be used to characterize urban road traffic networks.

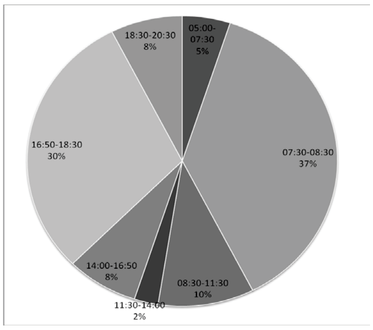


Fig. 17 : Percentages of the scale-free property appearing in time phases.

According the daily character of the Beijing road traffic network, the appearance of the scale-free property keeps consistent with the tide phenomena of urban road traffic systems.

The experimental results show that the *EVSDN* model has being able to exhibit the topological properties caused by the qualitative and quantitative changes of the traffic state on roads. As well, in a time cycle the urban road traffic network is scale-free at some times. Most frequently, the times are commonly conceived as peak times.

5 Conclusions

The work covered in this paper focuses on the topological properties of urban road traffic networks by constructing the variable-structure dynamic network model and the extended variable-structure dynamic network model.

Based on the LOS and the traffic reachability, the *VSDN* mode defines the qualitatively dynamical properties of network structure of urban road traffic networks. Furthermore, the *EVSDN* model reconstructs the dynamic edge set of *VSDN* to make the quantitative changes on roads be expressed on topology.

The experiments with traffic data and topology data from the Beijing road traffic systems show that *VSDN's* network structure changes obviously in a 24h time cycle and *EVSDN* has the scale-free property at some times.

Consequently, we conclude the two features of urban road traffic networks as following.

1) Random, but not totally random. The topological dynamics of urban road traffic networks comes from the dynamical features of traffic state on roads, which is caused by random behavior of vehicles and people on routing etc. However, from real data analysis, there is no one random model can reveal the dynamics of urban road traffic systems.

2) Structured, but not highly-structured. It is the former feature that indicates the urban road traffic system is running under some kinds of certain rules or mechanisms, which may be described in the scale-free model or else others. In this paper, we prove that the network is scale-free at some times, not all time. Therefore, the dynamics of the urban road traffic network cannot be represented in a curtained or uncertain model.

In conclusion, the study on the dynamics of urban road traffic networks is one of hot issues concerned by researchers from complexity science and network science. This paper provides a promising way of behavioral analysis of urban road traffic networks for ITS.

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