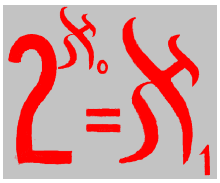

(http://rinterested.github.io/statistics/index.html)

COMPARING BETWEEN MIXED-EFFECTS MODELS AND WITH OLS:

I will consider three different models from the `sleepstudy` dataset. First off, we need to load two packages: `library(lme4); library(lmerTest)`.

MODEL 1: Ordinary Least Squares:

Syntax:

```
lm(Reaction ~ Days, sleepstudy)
```

MODEL 2: Random effects intercepts for each level of `Subject` as they deviate from a global intercept, and a global slope:

Syntax:

```
lmer(Reaction ~ Days + (1|Subject), sleepstudy)
```

MODEL 3: Random intercepts and slopes with correlation between spread intercepts and slopes:

The continuous variable `Days` is treated as a fixed effect, and its effect on each level of the categorical variable `Subject`, treated as a random effect, is considered *allowing correlation between the spread of the intercepts across `Subjects` and the `Days` effect deviations across `Subjects` levels*.

Syntax:

`lmer(Reaction ~ Days + (Days|Subject), sleepstudy)`, which is the same as `lmer(Reaction ~ Days + (1 + Days|Subject), sleepstudy)` as defined in this entry in Cross Validated (http://stats.stackexchange.com/a/13173/67822).

RETRIEVING RESULTS:

```
library(lme4)
library(lmerTest)
```

```
fm1 <- lm(Reaction ~ Days, sleepstudy)
summary(fm1)
```

```
##
## Call:
## lm(formula = Reaction ~ Days, data = sleepstudy)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -110.848  -27.483    1.546   26.142  139.953
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  251.405      6.610  38.033  < 2e-16 ***
## Days          10.467      1.238   8.454 9.89e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 47.71 on 178 degrees of freedom
## Multiple R-squared:  0.2865, Adjusted R-squared:  0.2825
## F-statistic: 71.46 on 1 and 178 DF,  p-value: 9.894e-15
```

```
coef(fm1)
```

```
## (Intercept)        Days
##   251.40510    10.46729
```

```
anova(fm1)
```

```
## Analysis of Variance Table
##
## Response: Reaction
##            Df Sum Sq Mean Sq F value    Pr(>F)
## Days        1 162703  162703  71.464 9.894e-15 ***
## Residuals 178 405252    2277
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
fm2 <- lmer(Reaction ~ Days + (1|Subject), sleepstudy)
summary(fm2)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approxim
ations
##   to degrees of freedom [lmerMod]
## Formula: Reaction ~ Days + (1 | Subject)
##    Data: sleepstudy
##
## REML criterion at convergence: 1786.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.2257 -0.5529  0.0109  0.5188  4.2506
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  Subject  (Intercept) 1378.2   37.12
##  Residual              960.5   30.99
## Number of obs: 180, groups:  Subject, 18
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
## (Intercept) 251.4051     9.7467  22.8100   25.79   <2e-16 ***
## Days         10.4673     0.8042 161.0100   13.02   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## Days -0.371
```

```
coef(fm2)
```

```
## $Subject
##     (Intercept)     Days
## 308    292.1888 10.46729
## 309    173.5556 10.46729
## 310    188.2965 10.46729
## 330    255.8115 10.46729
## 331    261.6213 10.46729
## 332    259.6263 10.46729
## 333    267.9056 10.46729
## 334    248.4081 10.46729
## 335    206.1230 10.46729
## 337    323.5878 10.46729
## 349    230.2089 10.46729
## 350    265.5165 10.46729
## 351    243.5429 10.46729
## 352    287.7835 10.46729
## 369    258.4415 10.46729
## 370    245.0424 10.46729
## 371    248.1108 10.46729
## 372    269.5209 10.46729
##
## attr(,"class")
## [1] "coef.mer"
```

```
fm3 <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy)
summary(fm3)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approxim
ations
##   to degrees of freedom [lmerMod]
## Formula: Reaction ~ Days + (Days | Subject)
##    Data: sleepstudy
##
## REML criterion at convergence: 1743.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9536 -0.4634  0.0231  0.4634  5.1793
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  Subject  (Intercept) 612.09   24.740
##           Days         35.07    5.922   0.07
##  Residual             654.94   25.592
## Number of obs: 180, groups:  Subject, 18
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  251.405      6.825  17.000  36.838  < 2e-16 ***
## Days          10.467      1.546  17.000   6.771 3.26e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## Days -0.138
```

```
coef(fm3)
```

```
## $Subject
##     (Intercept)       Days
## 308    253.6637 19.6662579
## 309    211.0065  1.8475828
## 310    212.4449  5.0184061
## 330    275.0956  5.6529547
## 331    273.6653  7.3973914
## 332    260.4446 10.1951153
## 333    268.2455 10.2436615
## 334    244.1725 11.5418620
## 335    251.0714 -0.2848731
## 337    286.2955 19.0955699
## 349    226.1950 11.6407002
## 350    238.3351 17.0814910
## 351    255.9829  7.4520288
## 352    272.2687 14.0032993
## 369    254.6806 11.3395026
## 370    225.7922 15.2897506
## 371    252.2121  9.4791309
## 372    263.7196 11.7513157
##
## attr(,"class")
## [1] "coef.mer"
```

MODEL SELECTION:

To compare different `lmer` models it's best to avoid `REML` when the fixed effects are different between models (http://stats.stackexchange.com/a/116796/67822). Even though it is not the case in our models I will redefine the models to steer clear of this potential issue:

```
fm2 <- lmer(Reaction ~ Days + (1|Subject), REML = F, sleepstudy)
fm3 <- lmer(Reaction ~ Days + (Days|Subject), REML = F, sleepstudy)
```

The Akaike Information Criteria is a good criterion of the quality of the model (https://en.wikipedia.org/wiki/Akaike_information_criterion). It tends to penalize adding extra predictors (overfitting). The models with the lowest AIC values are best (http://stats.stackexchange.com/a/9185/67822).

```
AIC(fm1, fm2, fm3)
```

```
##     df      AIC
## fm1  3 1906.293
## fm2  4 1802.079
## fm3  6 1763.939
```

It seems as though the last model is best in terms of its lowest AIC.

Alternatively we can run ANOVA tests on the models (http://stats.stackexchange.com/a/56157/67822):

```
anova(fm2, fm3)
```

```
## Data: sleepstudy
## Models:
## object: Reaction ~ Days + (1 | Subject)
## ..1: Reaction ~ Days + (Days | Subject)
##        Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## object  4 1802.1 1814.8 -897.04   1794.1
## ..1     6 1763.9 1783.1 -875.97   1751.9 42.139      2  7.072e-10
***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Home Page (http://rinterested.github.io/statistics/index.html)