Theorem 0.1 (Cantor)**.** *For all sets X (including infinite), $X \not\gtrsim \mathcal{P}(X)$. That is, there does not exist an injection from $\mathcal{P}(X)$ to X.*

Theorem 0.2 (Informal statement of the axiom of choice)**.** *Given a family $\mathscr{F}$ of nonempty sets, it is possible to pick out an element from each set in the family.*

Theorem 0.3 (Zorn's Lemma)**.** *Let $(A, \triangleright)$ be a partial order such that the following condition is satisfied:*

($\mathcal{Z}$) In words, if every chain has an upper bound then there is a maximal element. More precisely, ff C is a chain, then $\exists x$ such that $x \triangle$

Theorem 0.4 (Collection $\mathcal{T}$ generated by a basis $\mathcal{B}$ is a topology on X)**.**

Claim 0.1. *τ_1 has a countable basis.*

Claim 0.2. *τ_2 doesn't have a countable basis.*

Claim 0.3. *$B_\varepsilon(b) \subset \mathbb{R}^2 - f(A)$*

Claim 0.4 (Inclusion is Continuous)**.** *If (X, τ) is a topological space and $A \subseteq X$, then $f : A \to X : a \mapsto a$ is continuous (with respect to τ_A and τ).*

Claim 0.5. *Let (X, τ) and (Y, σ) be topological spaces and $f : X \to Y$ be a continuous function (with respect to τ and σ). Let $A \subset X$ be nonempty and τ_A its subspace topology. Let $B \subset Y$ be nonempty and σ_B its subspace topology. Suppose further that $f(A) \subseteq B$ (that is, for $x \in A$, we have that $f(x) \in B$). Define $\hat{f} : A \to B : x \mapsto f(x)$ (that is, the restriction of f). **Then** $\hat{f} : A \to B$ is continuous (with respect to τ_A and τ_B).*

Theorem 0.5 ("Continuous" = "Continuous")**.** *If (X_1, d_1) and (X_2, d_2) are metric spaces with induced topologies τ_{d_1} and τ_{d_2}, then for $f : X_1 \to X_2$, the following are equivalent:*

(i) f is continuous with respect to τ_{d_1} and τ_{d_2}.

(ii) For all $a \in X_1$ and $\varepsilon > 0$, there exists a $\delta > 0$ such that for all $b \in X_1$ for which $d_1(a, b) < \delta$, we have that $d_2(f(a), f(b)) < \varepsilon$.

Theorem 0.6 (Separation Axiom)**.** *Metric spaces are always T_2.*

Claim 0.6. *$(a_i)_{i \in \mathbb{N}} \to b$*

Claim 0.7. *A sequence with no infinite repetition converges to every point.*

Claim 0.8 (Metric space: closed $\iff$ sequentially closed)**.** *In a metric space, a set is closed if and only if it is sequentially closed. More formally: If (X, d) is a metric space and $\mathcal{T}_d$ is the induced topology on X, then $S \subset X$ is sequentially closed if and only if S is closed (with respect to $\mathcal{T}$, that is $X - S \in \mathcal{T}$).*

Claim 0.9. *Let π_1 be π_2 projection maps (as above). Then*

(i) π_1 is continuous with respect to $\tau_1 \otimes \tau_2$ and τ_1.

(ii) π_2 is continuous with respect to $\tau_1 \otimes \tau_2$ and τ_2.

Theorem 0.7 (Bounded Value Theorem (BVT))**.** *If $f : [0, 1] \to \mathbb{R}$ is continuous, then $\operatorname{rng} f$ is bounded (above and below).*

Theorem 0.8 (Set-Theoretic Rules)**.** *We have that, for any sets A, B, C,* *(i) First distributive law for the operations* $\cap$ *and* $\cup$*:*

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \tag{0.1}$$

(ii) Second distributive law for the operations $\cap$ *and* $\cup$*:*

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \tag{0.2}$$

(iii) DeMorgan's laws:

$$A - (B \cup C) = (A - B) \cap (A - C) \tag{0.3}$$

"The complement of the union equals the intersection of the complements."

$$A - (B \cap C) = (A - B) \cup (A - C) \tag{0.4}$$

"The complement of the intersection equals the union of the complements."

Claim 0.10. *A set is open in the metric topology induced by* d *if and only if for each* $y \in U$ *there is a* $\delta > 0$ *such that* $B_d(y, \delta) \subset U$*.*