Course Project: Load Flow Studies

Candidate numbers: 10049, 10027, 10041, 10057 and 10061

April 9, 2022

TET4105
Norwegian University of Science and Technology

1 Summary

The purpose of the project was to gain a hands-on experience of load flow studies. The basic characteristic features of power flow was explored by using Matpower and Python software scripts. By observing the results from simulations of a base case, the direction of the active and reactive power flows in the system was determined. It was observed that the active and reactive power did not necessarily flow in the same direction. By simulating the power flow solution, it was observed how various changes in the system caused an effect to the parameters. An additional transmission line to the base case lead to a more stable network. Decreasing the voltage magnitude at a P-V bus forced the generator to absorb reactive power from the grid. Furthermore, a fault on the line lead to several overloads, and voltage magnitudes below the limit at load buses. Increasing the reactive power consumption at a load bus caused the voltage angles to increase in order to compensate for the increase in reactive power demand. Moreover, introducing a shunt capacitor increased the power delivered over the line. Neglecting the operating capacitance resulted in a decrease in voltage magnitudes and angles for all buses except the slack-bus. In addition, the total line losses increased when introducing this tweak. Decreasing the voltage magnitude by 5% at one of the P-V buses led to a overload on one of the transmission lines, as well as a voltage magnitude at a load bus below rated limits.

Lastly, the computed reactive power loss was compared to the result obtained from the simulation. The results differ since the operating capacitance related term was excluded when computing Q_{loss} in Python and Matlab.

Through this project, the group has gained a deeper understanding of the concept of load flow studies. We have learned how to observe the consequences of changes in the system, and how to use Matlab and Python in order to do a simulation of the network.

Table of Contents

2 Introduction

Electric power transmission networks are crucial for the supply of electric power all over the world. Both the regular consumer and large industries/companies depends on power supplied by such installations. With the increasing dependence on electricity and clean energy, the importance of effective and robust transmission networks is on the rise. However, in order to optimize such networks, one needs a basic understanding of the characteristics behind such systems.

This project is based on a 8-bus transmission line-system called the Telemarksnet. The purpose of the project is to study the load flows for a base case, in addition to observing impacts on the system after implementing different tweaks.

The rapport is organized chronological based on the order the problems are presented in the task description. In part 1 all the buses are classified and the load flow equations are presented. Further on, part 2 consists of a simulation of the base case in both Matlab and Python. Following, the results from the simulations are both presented and compared. Then, the line flows and losses are computed manually. In part 3, observations from the simulations and computations on the base case are discussed. For part 4, five different tweaks are implemented in the system, with simulation run for all the tweaks in both Matlab and Python. Following, observations on the impacts of the different tweaks are presented. Part 5 includes a comparison between computed and simulated values for the reactive power losses. Lastly, a conclusion is drawn based on observations done through all the different parts.

3 Part 1 - Compilation of the Input Data and Formulating the Load Flow Equations

3.1 a) Classification of the buses

Figure 1 shows the classification of the buses in a single line diagram. The green buses are buses where a generator is present, and at the red buses a load is connected. The per unit values shown in figure 1, real and reactive power were computed by dividing the actual value by the base value $S_{base} = 1000$ [MVA] [1].

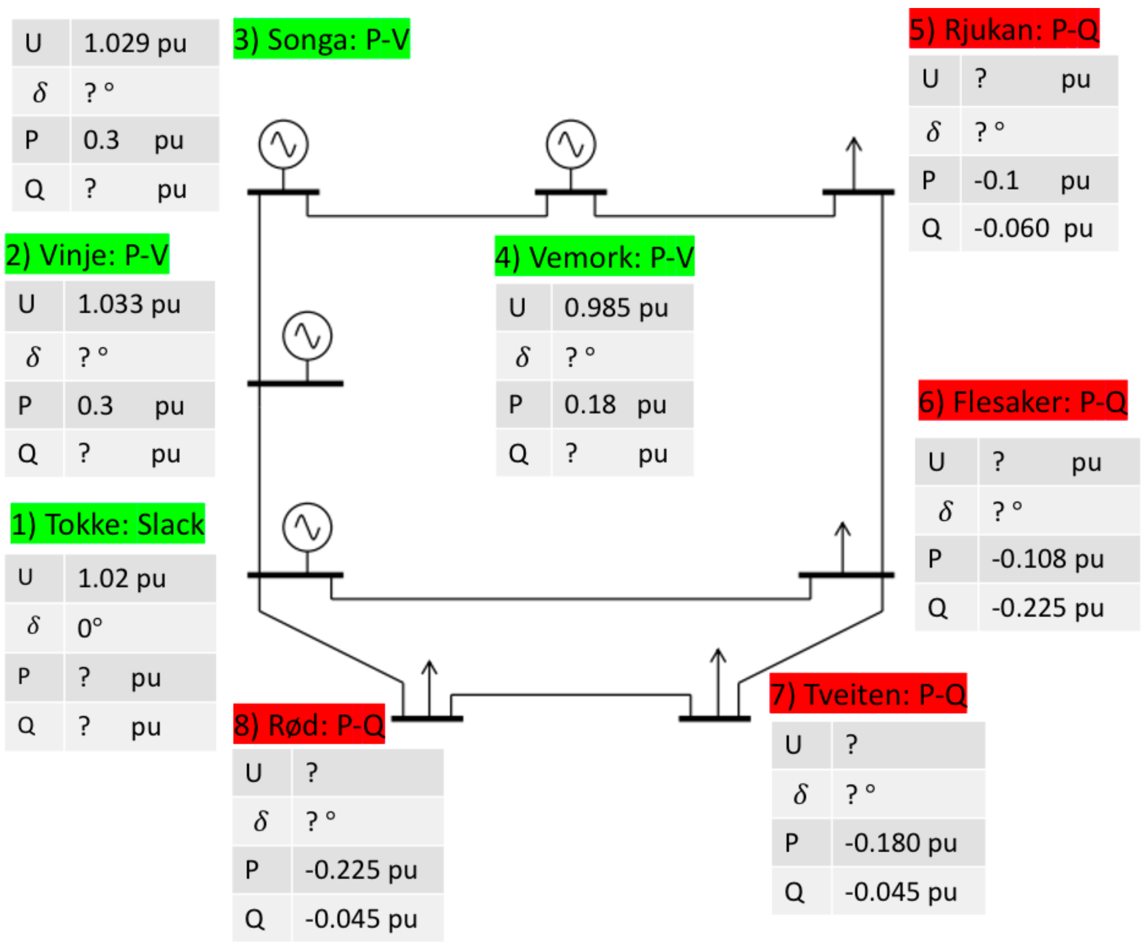


Figure 1: Classification of the buses on the single line diagram

3.2 b) Classification of the buses and line parameters in tabular form

Table 1 presents the same information as figure 1 in tabular form.

Table 1: Classification of the buses

Bus number	**Bus type**	**Known variables [pu]**	**Unknown variables**
1	Slack bus	$\lvert V_1 \rvert = 1.02; \delta_1 = 0°$	$P_1; Q_1$
2	P-V	$\lvert V_2 \rvert = 1.033; P_2 = 0.300$	$Q_2; \delta_2$
3	P-V	$\lvert V_3 \rvert = 1.029; P_3 = 0.300$	$Q_3; \delta_3$
4	P-V	$\lvert V_4 \rvert = 0.985; P_4 = 0.180$	$Q_4; \delta_4$
5	P-Q	$P_5 = -0.100; Q_5 = -0.060$	$\lvert V_5 \rvert; \delta_5$
6	P-Q	$P_6 = -0.108; Q_6 = -0.225$	$\lvert V_6 \rvert; \delta_6$
7	P-Q	$P_7 = -0.180; Q_7 = -0.045$	$\lvert V_7 \rvert; \delta_7$
8	P-Q	$P_8 = -0.225; Q_8 = -0.045$	$\lvert V_8 \rvert; \delta_8$

The line impedance was calculated by multiplying the relevant impedance [Ω/km] by the line length, and furthermore dividing this value by $Z_{base} = 90[\Omega]$ to get the pu-values. Z_{base} was calculated using equation (1). The half-line charging impedance was calculated by equation (2) and then converted to pu-values. Table 2 shows the line parameters.

$$Z_{base} = V_{base}^2 / S_{base} \tag{1}$$

$$Z_{ij-0} = \frac{1}{j2\pi f(C/2)} \tag{2}$$

Table 2: Line parameters

From bus - to bus	**Line**	**Impedance Z [pu]**	**Half-line charging impedance Z_{ij-0} [pu]**
1-2	Tokke-Vinje	$0.01018 + j0.10941$	-j355.04
2-3	Vinje-Songa	$0.00836 + j0.08982$	-j432.47
3-4	Songa- Vemork	$0.02124 + j0.22837$	-j170.09
4-5	Vemork- Rjukan	$0.00403 + j0.04437$	-j517.34
5-6	Rjukan- Flesaker	$0.01582 + j0.25316$	-j87.15
6-7	Fleskaer-Tveiten	$0.01209 + j0.19342$	-j114.06
7-8	Tveiten-Rød	$0.02196 + j0.23602$	-j164.59
1-8	Tokke-Rød	$0.04702 + j0.50549$	-j76.85
1-6	Tokke- Flesaker	$0.05187 + j0.55757$	-j69.67

3.3 c) The general structure of Y_{bus}

The general structure of Y_{bus} is shown in figure 2 [1].

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} & Y_{17} & Y_{18} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} & Y_{27} & Y_{28} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} & Y_{36} & Y_{37} & Y_{38} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} & Y_{46} & Y_{47} & Y_{48} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} & Y_{56} & Y_{57} & Y_{58} \\ Y_{61} & Y_{62} & Y_{63} & Y_{64} & Y_{65} & Y_{66} & Y_{67} & Y_{68} \\ Y_{71} & Y_{72} & Y_{73} & Y_{74} & Y_{75} & Y_{76} & Y_{77} & Y_{78} \\ Y_{81} & Y_{82} & Y_{83} & Y_{84} & Y_{85} & Y_{86} & Y_{87} & Y_{88} \end{bmatrix}$$

Figure 2: General Y_{bus}

The busses are not interconnected altogether, therefore some of the elements of the Y_{bus} were set to zero by visual inspection of the single line diagram. For two buses with no connection, their respective diagonal components were set to zero. The simplified Y_{bus} is shown in figure 3.

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & 0 & 0 & 0 & Y_{16} & 0 & Y_{18} \\ Y_{21} & Y_{22} & Y_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{32} & Y_{33} & Y_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{43} & Y_{44} & Y_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_{54} & Y_{55} & Y_{56} & 0 & 0 \\ Y_{61} & 0 & 0 & 0 & Y_{65} & Y_{66} & Y_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_{76} & Y_{77} & Y_{78} \\ Y_{81} & 0 & 0 & 0 & 0 & 0 & Y_{87} & Y_{88} \end{bmatrix}$$

Figure 3: Simplified Y_{bus}

Two diagonal elements Y_{33} and Y_{44} were chosen from Y_{bus}, and their computation are shown in equation (3) and (4). For diagonal elements, all line impedances connected to the bus with same number as the index of Y_{bus}, were inverted and added. These impedances are shown in Table 2. The corresponding half-line charging impedance were also inverted and added to the line admittances. The off-diagonal elements with the same indexes as Y_{bus} were inverted and multiplied by (-1).

$$\begin{aligned} Y_{33} &= \frac{1}{Z_{23}} + \frac{1}{Z_{34}} + \frac{1}{Z_{23-0}} + \frac{1}{Z_{34-0}} \\ &= \frac{1}{0.0084 + j0.0898} + \frac{1}{0.0212 + j0.2284} + \frac{1}{-j432.4745} + \frac{1}{-j170.0946} \\ &= 1.4355 - j15.3720[pu] \end{aligned} \tag{3}$$

$$
\begin{aligned}
Y_{44} &= \frac{1}{Z_{34}} + \frac{1}{Z_{45}} + \frac{1}{Z_{34-0}} + \frac{1}{Z_{45-0}} \\
&= \frac{1}{0.0212 + j0.2284} + \frac{1}{0.0040 + j0.0444} + \frac{1}{-j170.0946} + \frac{1}{-j517.3373} \\
&= 2.4521 - j26.8741[pu]
\end{aligned} \tag{4}
$$

Two off-diagonal elements Y_{34} and Y_{45} were chosen from Y_{bus}, and their computation is shown in equation (5) and (6).

$$
Y_{34} = Y_{43} = -\frac{1}{Z_{34}} = -\frac{1}{0.0212 + j0.2284} = -0.4038 + j4.3411[pu] \tag{5}
$$

$$
Y_{45} = Y_{54} = -\frac{1}{Z_{45}} = -\frac{1}{0.0040 + j0.0444} = -2.0320 + j22.3546[pu] \tag{6}
$$

3.4 d) Power flow equations for any three buses

Power flow equations for bus 3, 4 and 5 are presented in equations (7), (9), (11), (13), (15) and (17). The known input quantities were then inserted, this is shown in equations (8), (10), (12), (14), (16) and (18) [1]. Voltage magnitudes and angles were retrieved from Table 1. Y_{bus} elements are shown in figure 3, and all diagonal and off-diagonal elements were calculated the same way as in (3) and (5) accordingly.

$$
\begin{aligned}
P_3 = \{&|V_3| \cdot |V_3| \cdot |Y_{33}| \cdot \cos(\delta_3 - \delta_3 - \theta_{33}) + \\
&|V_3| \cdot |V_2| \cdot |Y_{32}| \cdot \cos(\delta_3 - \delta_2 - \theta_{32}) + \\
&|V_3| \cdot |V_4| \cdot |Y_{34}| \cdot \cos(\delta_3 - \delta_4 - \theta_{34})\}
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
P_{3(pu)} = \{&1.029 \cdot 1.029 \cdot |(14.1932 + j0.00819)| \cdot \cos(\arctan(\tfrac{0.00819}{14.1932})) + \\
&1.029 \cdot 1.033 \cdot |(-1.0278 + j11.0402)| \cdot \cos(\delta_3 - \delta_2 - \arctan(-\tfrac{11.0402}{1.0278})) + \\
&1.029 \cdot 0.985 \cdot |(-0.40367 + j4.3407)| \cdot \cos(\delta_3 - \delta_4 - \arctan(-\tfrac{4.3407}{0.40367}))\}
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
P_4 = \{&|V_4| \cdot |V_4| \cdot |Y_{44}| \cdot \cos(\delta_4 - \delta_4 - \theta_{44}) + \\
&|V_4| \cdot |V_3| \cdot |Y_{33}| \cdot \cos(\delta_4 - \delta_3 - \theta_{34}) + \\
&|V_4| \cdot |V_5| \cdot |Y_{45}| \cdot \cos(\delta_4 - \delta_5 - \theta_{45})\}
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
0.3_{(pu)} = \{&0.985 \cdot 0.985 \cdot (2.4340 - j26.6862) \cdot \cos\arctan(-\tfrac{26.6862}{2.4340}) + \\
&0.985 \cdot 1.029 \cdot |(-0.40367 + j4.3407)| \cdot \cos(\delta_3 - \delta_2 - \arctan(-\tfrac{4.3407}{0.40367})) + \\
&0.985 \cdot |V_5| \cdot |(-0.40367 + j4.3407)| \cdot \cos(\delta_3 - \delta_4 - \arctan(-\tfrac{4.3407}{0.40367}))\}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
P_5 = \{&|V_5| \cdot |V_5| \cdot |Y_{55}| \cdot \cos(\delta_5 - \delta_5 - \theta_{55}) + \\
&|V_5| \cdot |V_4| \cdot |Y_{45}| \cdot \cos(\delta_5 - \delta_4 - \theta_{54}) + \\
&|V_5| \cdot |V_6| \cdot |Y_{56}| \cdot \cos(\delta_5 - \delta_6 - \theta_{56})\}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
-0.1_{(pu)} = \{&|V_5| \cdot |V_5| \cdot |(2.4281 - j26.6922)| \cdot \cos(\delta_5 - \delta_5 - \arctan(-\tfrac{26.6922}{2.4281})) + \\
&|V_5| \cdot 0.985 \cdot |(-2.0303 + j22.3533)| \cdot \cos(\delta_5 - \delta_4 - \arctan(-\tfrac{22.3533}{2.0303})) + \\
&|V_5| \cdot |V_6| \cdot |(-0.2459 + j3.9347)| \cdot \cos(\delta_5 - \delta_6 - \arctan(-\tfrac{3.9347}{0.2459}))\}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
Q_3 = \{&|V_3| \cdot |V_3| \cdot |Y_{33}| \cdot \sin(\delta_3 - \delta_3 - \theta_{33}) + \\
&|V_3| \cdot |V_2| \cdot |Y_{23}| \cdot \sin(\delta_3 - \delta_2 - \theta_{32}) + \\
&|V_3| \cdot |V_4| \cdot |Y_{34}| \cdot \sin(\delta_3 - \delta_4 - \theta_{34})\}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
Q_{3(pu)} = \{&1.029 \cdot 1.029 \cdot |(14.1932 + j0.00819)| \cdot \sin(\arctan(\tfrac{0.00819}{14.1932})) + \\
&1.029 \cdot 1.033 \cdot |(-1.0278 + j11.0402)| \cdot \sin(\delta_3 - \delta_2 - \arctan(-\tfrac{11.0402}{1.0278}) + \\
&1.029 \cdot 0.985 \cdot |(-0.40367 + j4.3407)| \cdot \sin(\delta_3 - \delta_4 - \arctan(-\tfrac{4.3407}{0.40367}))\}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
Q_4 = \{&|V_4| \cdot |V_4| \cdot |Y_{44}| \cdot \sin(\delta_4 - \delta_4 - \theta_{44}) + \\
&|V_4| \cdot |V_3| \cdot |Y_{34}| \cdot \sin(\delta_4 - \delta_3 - \theta_{43}) + \\
&|V_4| \cdot |V_5| \cdot |Y_{45}| \cdot \sin(\delta_4 - \delta_5 - \theta_{45})\}
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
Q_{4(pu)} = \{&0.985 \cdot 0.985 \cdot |(2.4340 - j26.6862)| \cdot \sin(\arctan(-\tfrac{26.6862}{2.4340})) + \\
&0.985 \cdot 1.029 \cdot |(-0.40367 + j4.3407)| \cdot \sin(\delta_3 - \delta_2 - \arctan(-\tfrac{4.3407}{0.40367})) + \\
&0.985 \cdot |V_5| \cdot |(-0.40367 + j4.3407)| \cdot \sin(\delta_3 - \delta_4 - \arctan(-\tfrac{4.3407}{0.40367}))\}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
Q_5 = \{&|V_5| \cdot |V_5| \cdot |Y_{55}| \cdot \sin(\delta_5 - \delta_5 - \theta_{55}) + \\
&|V_5| \cdot |V_4| \cdot |Y_{45}| \cdot \sin(\delta_5 - \delta_4 - \theta_{54}) + \\
&|V_5| \cdot |V_6| \cdot |Y_{56}| \cdot \sin(\delta_5 - \delta_6 - \theta_{56})\}
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
-0.06_{(pu)} = \{&|V_5| \cdot |V_5| \cdot (2.4281 - j26.6922) \cdot \sin(\delta_5 - \delta_5 - \arctan(-\tfrac{26.6922}{2.4281})) + \\
&|V_5| \cdot 0.985 \cdot |(-2.0303 + j22.3533)| \cdot \sin(\delta_5 - \delta_4 - \arctan(-\tfrac{22.3533}{2.0303})) + \\
&|V_5| \cdot |V_6| \cdot |(-0.2459 + j3.9347)| \cdot \sin(\delta_5 - \delta_6 - \arctan(-\tfrac{3.9347}{0.2459}))\}
\end{aligned} \tag{18}
$$

4 Part 2 - Computations based on the Simulation Results

4.1 a) Power flow simulation on the base case

The data from Table 1 and the impedance from Table 2 was gathered in a system data Excel document, see appendix C. This file was used in both Matlab and Python, where the data was read and used as input for the power flow simulation. The Matpower Matlab code used for the simulation is shown in appendix D.

4.2 b) Power flow simulation results

The voltage magnitude, and phase angle results from the simulation are presented in Table 3. See appendix A.1 for the full Matlab simulation results, and appendix B.1 for the full Python simulation results. The only difference observed between the two simulations was the unit in which the branch data was given. The branch data in Matlab was given in [MW] and [MVAr], while the branch data form Python was given in [pu]. The bus information, on the other hand, was presented in pu-values for both simulators. Overall the simulated values were the same for both Matlab and Python simulations.

Table 3: Python and Matlab results (P=Python and M=Matlab).

Bus	Voltage- P [pu]	Voltage- M [pu]	Phase angle- P [deg]	Phase angle- M [deg]
1	1.020	1.020	0.000	0.000
2	1.033	1.033	0.606	0.606
3	1.029	1.029	-0.289	-0.289
4	0.985	0.985	-6.379	-6.379
5	0.969	0.969	-8.068	-8.068
6	0.910	0.910	-17.070	-17.070
7	0.912	0.912	-16.646	-16.646
8	0.928	0.928	-13.309	-13.309

4.3 c) Manually computation of the real and reactive line flows

When calculating the line flows and line losses, equations (19) and (20) were used. For the computation of the current, the impact of the operating capacitance can either be included (22) or not (21). Excluding operating capacitance is an approximation often made by computer algorithms to save computational time. However, if a line has a large operating capacitance, it should not be neglected. [1]

To obtain the equations (21) and (22), KCL is applied. In this case, a Pi-line model is used. Therefore, the current flowing through the operating capacitance must be taken into account (22). [1]

$$S_{ij} = \overline{V}_i \cdot \overline{I}_{ij}^* \tag{19}$$

$$S_{loss} = S_{ij} + S_{ji} \tag{20}$$

$$\overline{I}_{ij} = y_{ij} \cdot (\overline{V}_i - \overline{V}_j) \tag{21}$$

$$\overline{I}_{ij} = y_{ij} \cdot (\overline{V}_i - \overline{V}_j) + y_{ij-0} \cdot (\overline{V}_i) \tag{22}$$

4.4 d) Calculation of the line losses

The values used for the following computations are listed in Table 3. The equations (19), (20) and (22) are used to calculate the line losses on line 3-4 and line 4-5.

Line 3-4

$$\begin{aligned} \overline{I}_{34} &= y_{34} \cdot (\overline{V}_3 - \overline{V}_4) + y_{34-0} \cdot (\overline{V}_3) \\ &= (-0.4038 + j4.3411) \cdot ((1.0290\angle - 0.28900°) \\ &- (0.9850\angle - 6.37867°) + (\tfrac{1}{-j170.0946}) \cdot (1.0290\angle - 0.28900°) \\ &= -0.4727 + j0.1814[pu] \end{aligned} \tag{23}$$

$$\begin{aligned} S_{34} &= \overline{V}_3 \cdot (\overline{I}_{34})^* \\ &= (1.0290\angle - 0.28900°) \cdot (-0.47272 + j0.18138) \\ &= -0.4874 - j0.1842[pu] \end{aligned} \tag{24}$$

$$\begin{aligned} \overline{I}_{43} &= y_{43} \cdot (\overline{V}_4 - \overline{V}_3) + y_{34-0} \cdot (\overline{V}_4) \\ &= (-0.4038 + j4.3411) \cdot ((0.9850\angle - 6.37867°)) \\ &- (1.0290\angle - 0.28900°) + (\tfrac{1}{-j170.0946}) \cdot (0.9850\angle - 6.37867°) \\ &= 0.4734 - j0.1696[pu] \end{aligned} \tag{25}$$

$$\begin{aligned} S_{43} &= \overline{V}_4 \cdot (\overline{I}_{43})^* \\ &= (0.9850\angle - 6.37867°) \cdot (0.47339 + j0.16957) \\ &= 0.4820 + j0.1142[pu] \end{aligned} \tag{26}$$

$$\begin{aligned} S_{loss} &= S_{34} + S_{43} \\ &= (0.48196 + j0.11419) + (-0.48736 - j0.18418) \\ &= 0.0054 - j0.0699[pu] \end{aligned} \tag{27}$$

$$P_{loss} = 0.0054[pu] \tag{28}$$

$$Q_{loss} = 0.0699[pu] \tag{29}$$

Line 4-5

$$\begin{aligned} \overline{I}_{45} &= y_{45} \cdot (\overline{V}_4 - \overline{V}_5) + y_{45-0} \cdot (\overline{V}_4) \\ &= (-2.0320 + j22.3456) \cdot ((0.9850\angle - 6.37867°) \\ &- (0.96902\angle - 8.06849°)) + (\tfrac{1}{-j517.3373}) \cdot (0.9850\angle - 6.37867°) \\ &= -0.6335 + j0.3831[pu] \end{aligned} \tag{30}$$

$$\begin{aligned} S_{45} &= \overline{V}_4 \cdot (\overline{I}_{45})^* \\ &= (0.9850\angle - 6.37867°) \cdot (-0.63346 - j0.38312) \\ &= -0.6620 - j0.3057[pu] \end{aligned} \tag{31}$$

$$\begin{aligned} \overline{I}_{54} &= y_{45} \cdot (\overline{V}_5 - \overline{V}_4) + y_{45-0} \cdot (\overline{V}_5) \\ &= (-2.0320 + j22.3456) \cdot ((0.96902\angle - 8.06849°) \\ &- (0.9850\angle - 6.37867°)) + (\tfrac{1}{-j517.3373}) \cdot (0.96902\angle - 8.06849°) \\ &= 0.6339 - j0.3794[pu] \end{aligned} \tag{32}$$

$$\begin{aligned} S_{54} &= \overline{V}_5 \cdot (\overline{I}_{54})^* \\ &= (0.96902\angle - 8.06849°) \cdot (0.63394 - j0.37938) \\ &= 0.6598 + j0.2777[pu] \end{aligned} \tag{33}$$

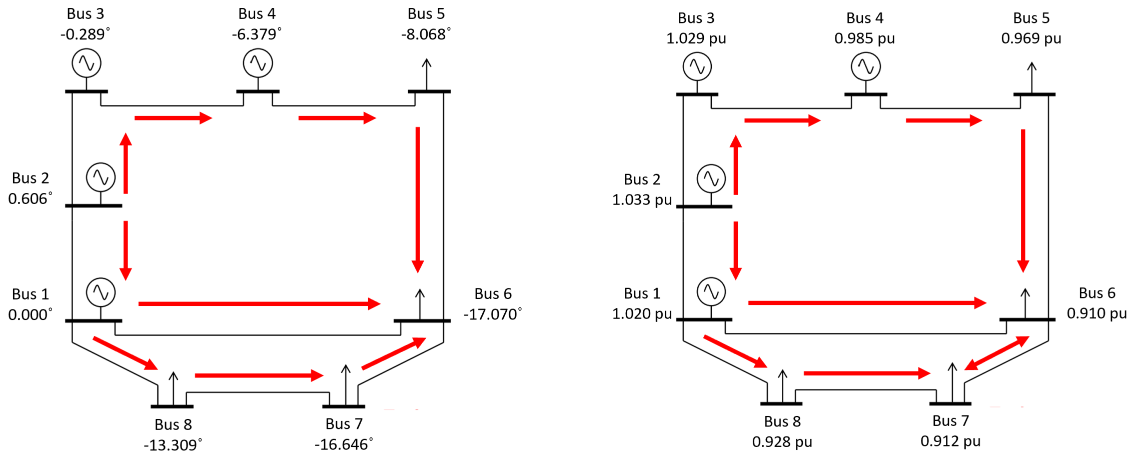
$$\begin{aligned} S_{loss} &= S_{45} + S_{54} \\ &= (-0.66203 - j0.30572) + (0.65982 + j0.27766) \\ &= -0.0022 - j0.0281[pu] \end{aligned} \tag{34}$$

$$P_{loss} = 0.0022[pu] \tag{35}$$

$$Q_{loss} = 0.0281[pu] \tag{36}$$

4.5 e) Single line diagram

The direction of the power flows are determined through the P-δ and Q-|V| coupling rules, and the branch data from the simulation (see appendix A.1 and B.1) [1]. In figure 4a, the active power flows from the highest to lowest voltage angle δ. The reactive power in figure 4b flows from highest to lowest voltage magnitude |V|. This is illustrated in figure 4 with red arrows.



(a) Active power flow

(b) Reactive power flow

Figure 4: Power flow on base case

5 Part 3 - Observations on the Base Case

5.1 a) Flow of real power

Typically, the real power flows from a bus with greater voltage angle to a bus with a smaller voltage angle. It can be observed from the data computed in Matlab and Python that this rule holds for the 8 bus-system. Let's use the values from bus 1 to 2 as an example, see table 3. The angle is 0 degrees at bus 1 and 0.606 degrees at bus 2. According to the rule, the real power should flow from bus 2 to 1. Furthermore the simulation showed a negative real power injection at bus 1, and a positive real power injection at bus 2, see appendix A.1. These results confirms that the real power flows from bus 2 to 1. The real power flow from bus 1 to 2 is illustrated in figure 4a.

5.2 b) Flow of reactive power

The reactive power tends to flow from a bus with greater voltage magnitude to a bus with smaller voltage magnitude. It can be observed that this holds for all lines, except for line 6-7 for the 8 bus-system. Table 3 shows that bus 6 has a voltage magnitude of

0.910 [pu], and that bus 7 has a voltage magnitude of 0.912 [pu]. This should result in a reactive power flow from bus 7 to 6. However, the simulation results show that the injected reactive power at both bus 6 and 7 are negative, se appendix A.1. This implies that reactive power flows in both directions in line 6-7, which is illustrated in figure 4b. This might be due to the small difference in voltage magnitude on this specific line.

5.3 c) Comparison of the flow direction

Real and reactive power doesn't necessarily flow in the same direction. As an example, it can be observed from figure 4 that the real power flows from bus 7 to 6, while the reactive power flows in both directions between the two busses. Reactive power flow is a measure of consumption and generation of VARs. There is no net transfer energy in the Q flow direction or reduction in current when Q flows both ways. Q flowing in both ways are therefore expected. [2]

6 Part 4 - Tweaks on the Base Case

Throughout multiple re-runs with different tweaks, there was not observed any significant deviation between the simulations in Matlab and Python. Similar to the base case the only difference between the two simulators was the unit for the presented branch data. Seeing that the difference between the two simulators were insignificant, only the Python results are presented. All results from both Matpower and Python are presented respectively in appendix A and appendix B.

6.1 a) Additional line between bus 1 and 8

The first tweak was added to the system by inserting an additional line between buses 1 and 8. The parameters for this line was set to the same as the original values: $R = 0.04702[pu]$, $X = 0.50549[pu]$ and $B = 0.02603[pu]$. All the simulation results for this tweak can be found in appendix A.2 and B.2. By comparing this output with the base case we found the following impacts for the different parameters:

Voltage magnitudes: There was no difference in the voltage magnitudes for buses 1, 2, 3, and 4 after the tweak. Which means that the voltage magnitudes for both the P-V buses and the slack bus were unaffected by the tweak. This makes sense as these values are set for these buses. In contrast, the voltage magnitudes increased for the P-Q buses 5, 6, 7 and 8. The impact on the voltage magnitudes are given in Table 4.

Table 4: Difference in voltage magnitudes

Bus	Difference in magnitude [pu]
1	0
2	0
3	0
4	0
5	+0.00287
6	+0.01622
7	+0.02482
8	+0.03799

Line flows: There was no difference what so ever in any of the load data after adding the tweak.

Line losses: By adding the line there was an overall decrease in the total line loss. Both real and reactive line loss decreased. The total real power line loss were decreased by 0.00766$[pu]$, while the total reactive power line loss were decreased by 0.08812$[pu]$. Looking further into the lines there was observed a decrease in both real and reactive power line losses for lines 2-3, 3-4, 4-5, 5-6, 1-6 and 1-8. Whereas the real and reactive power line losses increased for lines 1-2, 6-7 and 7-8. The difference in the line losses are given in Table 5.

Table 5: Difference in line losses

Line	Difference in line losses: P [pu]	Difference in line losses: Q [pu]
1-2	+0.00139	+0.00149
2-3	-0.00014	-0.00147
3-4	-0.00102	-0.01099
4-5	-0.00042	-0.00466
5-6	-0.00138	-0.02203
6-7	+0.00030	+0.00482
7-8	+0.00157	+0.01686
1-6	-0.00396	-0.04261
1-8	-0.01148	-0.12296
Added line: 1-8	+0.00404	+0.04341
Total	-0.00766	-0.08812

Observations: Seeing that the tweak resulted in an overall decrease in total line losses, one can argue that the system was improved by the tweak. In addition two parallel lines would make the system more resilient against faults on one of the two parallel lines between bus 1 and 8. However, constructing a transmission system with two parallel lines with a length of 105.8 km, is more expensive than a transmission system with just the one. Transmission lines are expensive, and the line between bus 1 and 8 is a particularly long one. For this tweak no overloads were observed at any of the lines, and all the voltage magnitudes are within the given parameters.

6.2 b) Decreasing voltage magnitude

The second tweak featured a 5% decrease in the voltage magnitude at bus 4.The voltage magnitude was thus altered from 0.985 [pu] to 0.93575 [pu] through the following equation: $(\frac{0.985[pu]}{100} \cdot 95 = 0.93575[pu])$. All the simulation results for this tweak can be found in appendix A.3 and B.3. The impact on the voltage magnitudes and angles are given in Table 6.

Table 6: Difference in voltage magnitudes and phase angles

Bus	Difference in voltage magnitude [pu]	Difference in phase angle [pu]
1	0	0
2	0	+0.04965
3	0	+0.09005
4	-0.04925	+0.16192
5	-0.04708	-0.00303
6	-0.03181	-0.69058
7	-0.02635	-0.60549
8	-0.01888	-0.38133

Voltage magnitudes and angles: Table 6 demonstrates that the voltage angles increased for all the P-V buses, an decreased for all the P-Q buses. Similarly the voltage magnitudes decreased for all P-Q buses in the system. However, for the P-V and slack buses, only bus 4 had a change in it's voltage magnitude. This is expected as the voltage tweak was done at bus 4.

Reactive power from generator 4: Table 7 shows that the generated reactive power decreased at bus 4. In addition, this bus saw a change of sign for its reactive power. This change signifies that generator 4 consumes reactive power after the tweak, and thus is behaving like a P-Q bus. This is as expected with a voltage drop of 5%. The generator will be forced to reduce its excitation and absorb reactive power from the grid [1].

Table 7: Difference in reactive power delivered by the generator at bus 4

Base case value	Tweaked value [pu]	Difference [pu]
+0.17639	-0.07970	-0.25608

Deviations from set parameters: When changing the voltage at a P-V bus, it is natural to check if all values still fulfill the limits given for the buses and lines. The voltage magnitudes at load buses should be between $270 - 310[kV]$ $(0.9 - 1.034[pu])$ for the power company to deliver as promised, and to avoid damage on the components. The simulations showed that $|V_6|$ and $|V_7|$ both were below 230V (0.9 pu) after the tweak, see Table 8. Too low of a voltage injected to the load can damage components on the transmission system and in the usual home [3].

Table 8: Voltage magnitude and limits

Load-bus	Voltage magnitude	Innside voltage limits of $0.9 < \|V\| < 1.034$
5	0.922	Yes
6	0.879	No
7	0.885	No
8	0.909	Yes

Apparent power delivered over the line should not exceed the rated values. The lines have different qualities, seen in Table 9. The decrease of voltage magnitude led to overload at line 3-4 of 3.235%, as shown in table 9.

Table 9: Line ratings

Line	Line rating [MVA]	Apparaent power flow in line [MVA]	Overload [%]
1-2	600	164.81	0
2-3	600	181.90	0
3-4	600	619.41	3.235%
4-5	900	694.28	0
5-6	1000	571.74	0
6-7	1000	51.02	0
7-8	600	230.52	0
1-6	600	584.29	0
1-8	600	506.27	0

6.3 c) Fault on a line

The third tweak was a constructed fault on one of the lines in the transmission system. Line 3-4 was chosen, and the tweak was implemented by removing all the parameters for this line. All the simulation results for this tweak can be found in appendix A.4 and B.4. Table 10 shows the difference in total active and reactive power after the tweak, compared to the base case 3.

Table 10: Difference in total power with and without line 3-4

Datatype	Difference in total 'Power in generator ' [%]		Difference in total 'Power in load' [%]		Difference in total 'Power loss in branch' [%]	
	P_g	Q_g	P_l	Q_l	P	Q
Busdata	0.226	53.5	0	0	-	-
Branchdata	-	-	-	-	87.7	77.9

Generated Power: There was a minimal change in the total generated real power for the system after the tweak. For the P-V buses 2, 3 and 4 the real power generated is fixed and, thus, saw no change. The generated real power at the slack bus on the other hand, increased by 0.226%. Making bus 1 the only bus responsible for the increase in total generated real power presented in table 10.

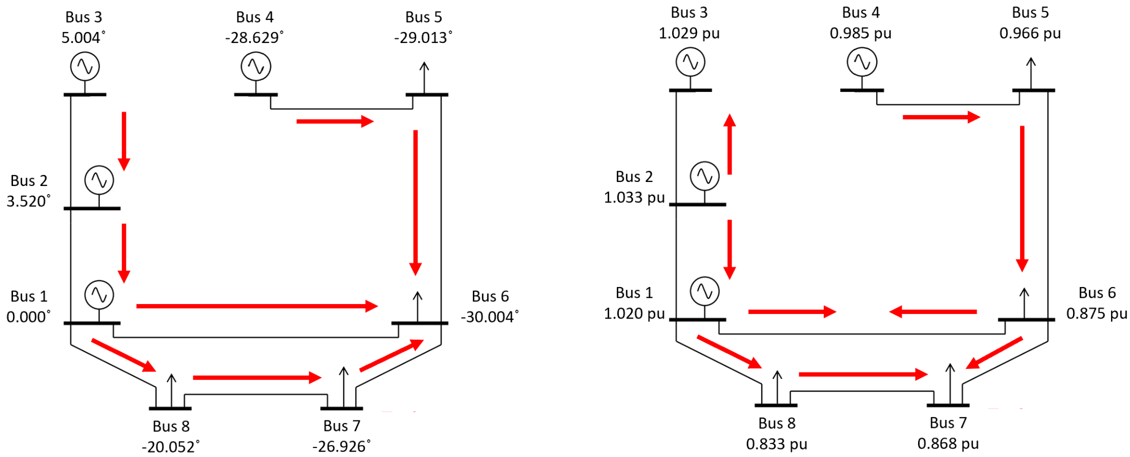
In contrast there was a significant increase in total reactive power supplied by the generators. Generated reactive power saw an increase of 53.5% in total. The reactive power supplied by the generators at buses 1,2 and 4 all increased. However, the reactive power at bus 3 decreased and turned negative. Which, as discussed before, suggests that generator 3 consumes reactive power and thus behaves like a P-Q bus.

Power consumed by the loads: There was no difference in the power consumed by the loads after the tweak. Both real and reactive power demands at the P-Q buses are fixed values, causing no change in total reactive power demand in the system.

Line losses: Both the real and reactive power line losses saw a significant increase after the tweak. The fault on line 3-4 altered the structure of the network, whereas the power demands at the load buses remained the same. Consequently the power flow in the system was altered, resulting in higher line losses.

Power flows: The power flow is shown for P and Q in figure 5. Figure 5a shows that the active power flow on line 2-3 has changed direction, and is now sending all of its power 'downward' to the system. The rest of the flows are the same as before the faulted line.

Figure 5b illustrates the direction of reactive power flow. Compared to the base case, this flow shows a one-directional flow on branch 6-7, but a two-directional flow on branch 1-6. This is an exception to the strong Q-|V| coupling. Observation of the big difference in power angles on line 1-6 (see figure 5a, indicates a highly stressed system.



(a) Active power flow

(b) Reactive power flow

Figure 5: Power flow with one line removed

Deviations from set parameters: The same check on deviations from set limits as done in 6.2 b), is also done for this tweak. Bus 6, 7 and 8 all have voltage below the set limits and may cause issues. This is shown in Table 11. A possible reason for this can be the altered flow pattern. As discussed, this lead to a greater power loss in the transmission lines. Which would result in a greater voltage drop in the lines.

Table 11: Voltage magnitude and limits

Load-bus	**Voltage magnitude**	**Innside voltage limits of** $0.9 < \|V\| < 1.034$
5	0.966	Yes
6	0.875	No
7	0.868	No
8	0.883	No

For this tweak apparent power flowing in line 1-2, 1-6 and 1-8 are all above rated value, and are therfor overloaded. This is shown in Table 12. The overload can lead to overheating on the line, and damage the components. This can again be a consequence of the flow pattern [3].

Table 12: Line ratings

Line	**Line rating [MVA]**	**Apparaent power flow in line [MVA]**	**Overload [%]**
1-2	600	604.85	0.808%
2-3	600	309.32	0
4-5	900	436.27	0
5-6	1000	341.63	0
6-7	1000	394.89	0
7-8	600	390.52	0
1-6	600	922.85	53.808%
1-8	600	712.54	18.757%

6.4 d) Increasing reactive power consumption

The fourth tweak included an 5% increase in the reactive power consumption at any load bus, in this case bus 6. All the simulation results for this tweak can be found in appendix A.5 and B.5.The original reactive power consumption for bus 6 at the base case was 60 [pu]. Thus the tweaked reactive power consumption for bus 6 was set to $60 \cdot 1.05 = 63$ [pu]. All the simulation results for this tweak can be found in appendix A.5 and B.5. All the impacts on the voltage magnitudes and angles after the tweak are given in Table 13.

Voltage magnitudes: There was no difference in the voltage magnitudes for buses 1, 2, 3 and 4 after the tweak. Although for the remaining buses, the P-Q buses, the voltage magnitudes all decreased after the tweak. Still, the voltage magnitudes stayed within the given parameters.

Voltage angles: There was no difference in the voltage angle for bus 1. However, the voltage angle increased for P-V buses 2, 3 and 4, as well as for the P-Q bus 5. Furthermore there was a decrease in the voltage angles for the P-Q buses 6, 7 and 8.

Table 13: Difference in voltage magnitudes and angles

Bus	Difference in voltage magnitude		Difference in voltage angle	
	[pu]	%	[pu]	%
1	0	0	0	0
2	0	0	0.00004	0.00604
3	0	0	0.00007	0.02297
4	0	0	0.00015	0.00228
5	-0.00013	-0.01350	0.00063	0.00786
6	-0.00009	-0.00959	-0.00143	-0.00840
7	-0.00007	-0.00788	-0.00130	-0.00780
8	-0.00005	-0.00550	-0.00081	-0.00611

Observations: The tweak had no impact whatsoever for both the voltage magnitude and angle for bus 1. This is not surprising seeing that bus 1 is the slack bus, and these values are fixed for this bus. All P-V buses saw no change in voltage magnitudes as this is a set value for these buses. However all the voltage angles for all the P-V buses increased after the tweak. This increase might be the result of the P-V buses compensating for the increased reactive power consumption at bus 6. In contrast to the P-V buses, all the P-Q buses, except bus 5, saw a decrease in voltage angles after the tweak. For all the P-Q buses the voltage magnitude decreased after the tweak. In addition, no overloads were observed after this tweak were implemented.

Compensation for increased reactive power consumption: Connecting a shunt capacitor at the load-end will increase power delivered over the line. The shunt capacitor can deliver $|\overline{V_{end}}|^2/Z_c^*$ [VA] over the line to compensate for the increased power consumption. [4]

6.5 e) Neglecting the operating capacitances

This last tweak was implemented in the system by removing all the values for the full line shunt admittances. All the simulation results for this tweak can be found in appendix A.6 and B.6.

Reactive power supplied by the generators: Before the tweak, the operating capacitance admitted reactive power through the lines. Despite of the removal of the OC, the P-Q buses demand for reactive power still stands. In order to meet this demand, the reactive power delivered by generators has to increase to compensate for the removed supply of reactive power through the operating capacitances. The reactive power delivered by the generators increased after the tweak for all generators. This is shown in Table 14.

Table 14: Difference in reactive power from the generators

Generator	Difference in Q [pu]
1	0.07284
2	0.00548
3	0.00887
4	0.05214

Lines losses: When the operating capacitances were neglected, the total line losses increased. The total real power losses for the lines saw an increases by $0.00135[pu]$, whereas the total reactive power losses increased by $0.01617[pu]$. The impact on the line losses for each line is shown in Table 15.

Table 15: Difference in line losses

Line	Difference in line losses: P [pu]	Difference in line losses: Q [pu]
1-2	$-3.16 \cdot 10^{-6}$	$-33.94 \cdot 10^{-6}$
2-3	$+4.72 \cdot 10^{-6}$	$+50,73 \cdot 10^{-6}$
3-4	$+31.76 \cdot 10^{-6}$	$+341.59 \cdot 10^{-6}$
4-5	$+128.52 \cdot 10^{-6}$	$+1414.98 \cdot 10^{-6}$
5-6	$+306.19 \cdot 10^{-6}$	$+4899.72 \cdot 10^{-6}$
6-7	$-0.25 \cdot 10^{-6}$	$-3.97 \cdot 10^{-6}$
7-8	$+24.41 \cdot 10^{-6}$	$+262.29 \cdot 10^{-6}$
1-6	$+458.88 \cdot 10^{-6}$	$+4932.74 \cdot 10^{-6}$
1-8	$+400.43 \cdot 10^{-6}$	$+4304.92 \cdot 10^{-6}$
Total	$+1351.52 \cdot 10^{-6}$	$+16169.24 \cdot 10^{-6}$

Operating capacitance compensates for reactive power consumed by the lines. Consequently the line losses increased after an increase in reactive power flowing through the lines. However for lines 1-2 and 6-7, the reactive power line losses decreased after the tweak. This might be due to the design of the circuit. Perhaps the circuit is designed in such a way that the power demands for the P-Q buses is met without increasing the power flow in lines 1-2 and 6-7. Similarly to the reactive power line losses, the real power line losses also saw an increase for all the lines except for lines 1-2 and 6-7. However, the tweak did not result in any overloads at any of the lines.

Voltage magnitudes and angles: The tweak resulted in a drop in all the voltage angles except for bus 1. In addition the voltage magnitudes decreased for all the P-Q buses. Despite the decrease, all the voltage magnitudes stayed within the given parameters. Operating capacitance maintains voltages, and thus the removal of the OC's resulted in a decrease in all voltage magnitudes and angles for all the buses where these values are not fixed.

7 Part 5 - Differences in reactive power losses

The reactive power losses obtained from both the Matpower and Python simulation runs, are different than the ones calculated in part 4.4. A possible reason for this, is that line losses can be computed both with and without the impact of operating capacitance. In Matpower, Q_{ij} and Q_{ji} is computed with operating capacitances in the expression of S_{ij}. See equation (19) and (22). Q_{loss}, on the other hand, it is computed without operating capacitance in the expression $S_{ij}+S_{ji}$ in Python and Matlab, see equation (19) and (21). Excluding the OC is an approximation made by Python and Matlab to save computational time. Since Q_{loss} in part 4.4 is computed with the impact of operating capacitance, the losses will be different. [2]

8 Conclusion

Transmission systems are essential all over the world, every day. This project has shown how an alteration to the system, or tweak, can create large repercussions. Calculations and simulations on load flow studies on the Telemark net has presented important observations and perspectives. Different tweaks on the base case has made different concepts more apparent. An additional line in the transmission system functions as a parallel coupling. It improves the solutions, as well as making it more resilient to faults. A decrease in voltage magnitude illustrates that a generator will be forced to reduce its excitation and absorb reactive power from the grid, and thus become a P-Q bus. Moreover this tweak results in an overload on line 3-4. A faulted line in the system will create a bundle of consequences. A P-V bus will again become a P-Q bus, and the lines 1-2, 1-6, and 1-8 will experience an overload. Increased reactive power consumption at a P-Q bus will increase voltage angles to compensate for this tweak. However, introducing a shunt capacitor will improve the power flow results. Lastly, neglecting operating capacitance results in a decrease in all voltage magnitudes and angles for all the buses where these values are not fixed.

The project group agrees that this has been an overall good learning experience. Even though the project started with the group knowing little about the subject, it has been a positive experience. Seeing the effect of different tweaks on the system has provided a wider competence regarding load flow studies.

References

[1] S. Völler and V. V. Vadlamudi, *A partial compendium on fundamentals of power systems for tet 4105 (power system analysis 1)*, Distributed through Blackboard, Aug. 2021.

[2] S. Völler and V. V. Vadlamudi, *Faq project*, Published in folder Project in course TET4105 Elektriske kraftsystemer 1 on blacboard, Online; accessed 1 November 2021, 2021.

[3] N. G. Purobi Parowary, "Overload security and risk assessment of overhead lines," 2015.

[4] S. Völler and V. V. Vadlamudi, *Assignment 2 solution*, Published in folder Assignments in course TET4105 Elektriske kraftsystemer 1 on blacboard, Online; accessed 16 November 2021, 2021.

A Results from Matpower

A.1 Task 2.a

```
MATPOWER Version 6.0, 16-Dec-2016 -- AC Power Flow (Newton)

Newton's method power flow converged in 4 iterations.

Converged in 0.23 seconds
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses              8     Total Gen Capacity       0.0       -4000.0 to 4000.0
Generators         4     On-line Capacity         0.0       -4000.0 to 4000.0
Committed Gens     4     Generation (actual)   1626.9             735.4
Loads              4     Load                  1585.0             375.0
  Fixed            4       Fixed               1585.0             375.0
  Dispatchable     0       Dispatchable          -0.0 of -0.0      -0.0
Shunts             0     Shunt (inj)             -0.0               0.0
Branches           9     Losses (I^2 * Z)        41.90            483.56
Transformers       0     Branch Charging (inj)     -              123.2
Inter-ties         0     Total Inter-tie Flow     0.0               0.0
Areas              1


                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.910 p.u. @ bus 6          1.033 p.u. @ bus 2
Voltage Angle     -17.07 deg   @ bus 6          0.61 deg   @ bus 2
P Losses (I^2*R)             -                  15.52 MW    @ line 1-6
Q Losses (I^2*X)             -                 166.83 MVAr  @ line 1-6

================================================================================
|     Bus Data                                                                 |
================================================================================
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.020    0.000*   846.90    279.32       -         -
    2  1.033    0.606    300.00    137.34       -         -
    3  1.029   -0.289    300.00    142.35       -         -
    4  0.985   -6.379    180.00    176.39       -         -
    5  0.969   -8.068       -         -      100.00     60.00
    6  0.910  -17.070       -         -     1080.00    225.00
    7  0.912  -16.646       -         -      180.00     45.00
    8  0.928  -13.309       -         -      225.00     45.00
                        --------  --------  --------  --------
               Total:   1626.90    735.40   1585.00    375.00
```

```
================================================================================
|     Branch Data                                                              |
================================================================================
Brnch   From   To    From Bus Injection   To Bus Injection     Loss (I^2 * Z)
  #     Bus    Bus    P (MW)   Q (MVAr)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
-----  -----  -----  --------  --------  --------  --------  --------  --------
   1      1      2   -112.11   -113.15    112.35    109.82     0.242      2.60
   2      2      3    187.65     27.52   -187.36    -29.39     0.283      3.04
   3      3      4    487.36    171.74   -481.96   -125.61     5.400     58.06
   4      4      5    661.96    301.99   -659.76   -281.43     2.204     24.26
   5      5      6    559.76    221.43   -553.57   -142.70     6.187     99.01
   6      6      7    -31.99    -10.86     32.01     -3.44     0.015      0.24
   7      7      8   -212.01    -41.56    213.23     44.42     1.223     13.14
   8      1      8    449.06    181.05   -438.23    -89.42    10.825    116.37
   9      1      6    509.96    211.43   -494.44    -71.43    15.520    166.83
                                                             --------  --------
                                                    Total:    41.898    483.56
```

A.2 Task 4.a

```
Newton's method power flow converged in 4 iterations.

Converged in 0.09 seconds
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses                8   Total Gen Capacity       0.0       -4000.0 to 4000.0
Generators           4   On-line Capacity         0.0       -4000.0 to 4000.0
Committed Gens       4   Generation (actual)   1619.2             618.4
Loads                4   Load                  1585.0             375.0
  Fixed              4     Fixed               1585.0             375.0
  Dispatchable       0     Dispatchable          -0.0 of -0.0      -0.0
Shunts               0   Shunt (inj)             -0.0               0.0
Branches            10   Losses (I^2 * Z)        34.24            395.44
Transformers         0   Branch Charging (inj)     -              152.0
Inter-ties           0   Total Inter-tie Flow     0.0               0.0
Areas                1

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.927 p.u. @ bus 6          1.033 p.u. @ bus 2
Voltage Angle     -14.61 deg   @ bus 6          0.94 deg   @ bus 2
P Losses (I^2*R)             -                  11.56 MW    @ line 1-6
Q Losses (I^2*X)             -                 124.22 MVAr  @ line 1-6
```

```
================================================================================
|     Bus Data                                                                 |
================================================================================
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.020    0.000*   839.24    239.48       -         -
    2  1.033    0.938    300.00    137.35       -         -
    3  1.029    0.313    300.00    136.07       -         -
    4  0.985   -5.060    180.00    105.54       -         -
    5  0.972   -6.616       -         -      100.00     60.00
    6  0.927  -14.613       -         -     1080.00    225.00
    7  0.936  -12.782       -         -      180.00     45.00
    8  0.966   -8.005       -         -      225.00     45.00
                        --------  --------  --------  --------
               Total:   1619.24    618.44   1585.00    375.00

================================================================================
|     Branch Data                                                              |
================================================================================
Brnch   From   To    From Bus Injection   To Bus Injection     Loss (I^2 * Z)
  #     Bus    Bus    P (MW)   Q (MVAr)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
-----  -----  -----  --------  --------  --------  --------  --------  --------
   1      1      2   -167.35   -107.26    167.73    105.42     0.381      4.09
   2      2      3    132.27     31.93   -132.13    -35.27     0.146      1.57
   3      3      4    432.13    171.34   -427.75   -136.19     4.378     47.08
   4      4      5    607.75    241.73   -605.97   -225.83     1.781     19.61
   5      5      6    505.97    165.83   -501.16   -109.54     4.810     76.98
   6      6      7   -145.58    -43.14    145.90     32.99     0.316      5.06
   7      7      8   -325.90    -77.99    328.69     97.00     2.792     30.01
   8      1      6    444.81    169.28   -433.26    -72.32    11.556    124.22
   9      1      8    280.88     88.73   -276.85    -71.00     4.038     43.41
  10      1      8    280.88     88.73   -276.85    -71.00     4.038     43.41
                                                             --------  --------
                                                    Total:    34.237    395.44
```

A.3 Task 4.b

```
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses                  8     Total Gen Capacity       0.0       -4000.0 to 4000.0
Generators             4     On-line Capacity         0.0       -4000.0 to 4000.0
Committed Gens         4     Generation (actual)   1632.4             800.2
Loads                  4     Load                  1585.0             375.0
  Fixed                4       Fixed               1585.0             375.0
  Dispatchable         0       Dispatchable          -0.0 of -0.0      -0.0
Shunts                 0     Shunt (inj)             -0.0               0.0
Branches               9     Losses (I^2 * Z)        47.44            543.06
Transformers           0     Branch Charging (inj)     -              117.9
Inter-ties             0     Total Inter-tie Flow     0.0               0.0
Areas                  1

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.879 p.u. @ bus 6          1.033 p.u. @ bus 2
Voltage Angle     -17.76 deg   @ bus 6          0.66 deg   @ bus 2
P Losses (I^2*R)             -                 17.44 MW    @ line 1-6
Q Losses (I^2*X)             -                187.44 MVAr  @ line 1-6

================================================================================
|     Bus Data                                                                 |
================================================================================
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.020    0.000*   852.44    380.24       -         -
    2  1.033    0.656    300.00    137.30       -         -
    3  1.029   -0.199    300.00    362.32       -         -
    4  0.936   -6.217    180.00    -79.69       -         -
    5  0.922   -8.072       -         -      100.00     60.00
    6  0.879  -17.761       -         -     1080.00    225.00
    7  0.885  -17.251       -         -      180.00     45.00
    8  0.909  -13.690       -         -      225.00     45.00
                        --------  --------  --------  --------
               Total:   1632.44    800.17   1585.00    375.00
```

```
================================================================================
|     Branch Data                                                              |
================================================================================
Brnch   From   To    From Bus Injection   To Bus Injection     Loss (I^2 * Z)
  #     Bus    Bus    P (MW)   Q (MVAr)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
-----  -----  -----  --------  --------  --------  --------  --------  --------
   1      1      2   -120.38   -112.29    120.64    109.14     0.259      2.78
   2      2      3    179.36     28.16   -179.10    -30.28     0.259      2.79
   3      3      4    479.10    392.60   -471.31   -320.16     7.795     83.82
   4      4      5    651.31    240.47   -649.09   -219.34     2.222     24.47
   5      5      6    549.09    159.34   -542.94    -79.64     6.144     98.31
   6      6      7    -37.48    -34.58     37.52     21.49     0.034      0.55
   7      7      8   -217.52    -66.49    218.95     72.11     1.432     15.40
   8      1      6    517.01    272.20   -499.57   -110.78    17.437    187.44
   9      1      8    455.81    220.33   -443.95   -117.11    11.862    127.52
                                                             --------  --------
                                                    Total:    47.445    543.06
```

A.4 Task 4.c

```
Converged in 0.01 seconds
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses                8     Total Gen Capacity       0.0        -4000.0 to 4000.0
Generators           4     On-line Capacity         0.0        -4000.0 to 4000.0
Committed Gens       4     Generation (actual)   1663.6            1129.1
Loads                4     Load                  1585.0             375.0
  Fixed              4       Fixed               1585.0             375.0
  Dispatchable       0       Dispatchable          -0.0 of -0.0      -0.0
Shunts               0     Shunt (inj)             -0.0               0.0
Branches             8     Losses (I^2 * Z)        78.64            860.37
Transformers         0     Branch Charging (inj)     -              106.3
Inter-ties           0     Total Inter-tie Flow     0.0               0.0
Areas                1
```

```
                         -------------------------  -------------------------------
Voltage Magnitude   0.868 p.u. @ bus 7          1.033 p.u. @ bus 2
Voltage Angle     -30.00 deg   @ bus 6           5.00 deg   @ bus 3
P Losses (I^2*R)             -                   43.05 MW    @ line 1-6
Q Losses (I^2*X)             -                  462.72 MVAr  @ line 1-6

================================================================================
|     Bus Data                                                                 |
================================================================================
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.020    0.000*   883.64    646.36       -         -
    2  1.033    3.520    300.00    157.51       -         -
    3  1.029    5.004    300.00    -72.22       -         -
    4  0.985  -28.629    180.00    397.41       -         -
    5  0.966  -29.013       -         -      100.00     60.00
    6  0.875  -30.004       -         -     1080.00    225.00
    7  0.868  -26.926       -         -      180.00     45.00
    8  0.883  -20.052       -         -      225.00     45.00
                        --------  --------  --------  --------
               Total:   1663.64   1129.05   1585.00    375.00


================================================================================
|     Branch Data                                                              |
================================================================================
Brnch   From   To    From Bus Injection   To Bus Injection     Loss (I^2 * Z)
  #     Bus    Bus    P (MW)   Q (MVAr)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
-----  -----  -----  --------  --------  --------  --------  --------  --------
   1      1      2   -595.76    -50.52    599.25     82.15     3.495     37.56
   2      2      3   -299.25     75.36    300.00    -72.22     0.749      8.05
   3      4      5    180.00    397.41   -179.20   -392.32     0.797      8.77
   4      5      6     79.20    332.32    -77.10   -318.22     2.100     33.61
   5      6      7   -207.91     42.90    208.64    -44.68     0.721     11.54
   6      7      8   -388.64     -0.32    393.03     38.27     4.399     47.27
   7      1      6    838.03    386.46   -794.98     50.33    43.046    462.72
   8      1      8    641.37    310.42   -618.03    -83.27    23.334    250.85
                                                             --------  --------
                                                    Total:    78.640    860.37
```

A.5 Task 4.d

```
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses              8     Total Gen Capacity       0.0        -4000.0 to 4000.0
Generators         4     On-line Capacity         0.0        -4000.0 to 4000.0
Committed Gens     4     Generation (actual)   1626.9             738.6
Loads              4     Load                  1585.0             378.0
  Fixed            4       Fixed               1585.0             378.0
  Dispatchable     0       Dispatchable          -0.0 of -0.0      -0.0
Shunts             0     Shunt (inj)             -0.0               0.0
Branches           9     Losses (I^2 * Z)        41.91            483.71
Transformers       0     Branch Charging (inj)     -              123.1
Inter-ties         0     Total Inter-tie Flow     0.0               0.0
Areas              1

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.910 p.u. @ bus 6          1.033 p.u. @ bus 2
Voltage Angle     -17.07 deg   @ bus 6          0.61 deg   @ bus 2
P Losses (I^2*R)             -                 15.52 MW    @ line 1-6
Q Losses (I^2*X)             -                166.87 MVAr  @ line 1-6

================================================================================
|     Bus Data                                                                 |
================================================================================
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.020    0.000*   846.91    279.59       -         -
    2  1.033    0.606    300.00    137.34       -         -
    3  1.029   -0.289    300.00    142.35       -         -
    4  0.985   -6.379    180.00    179.28       -         -
    5  0.969   -8.068       -         -      100.00     63.00
    6  0.910  -17.071       -         -     1080.00    225.00
    7  0.912  -16.647       -         -      180.00     45.00
    8  0.928  -13.309       -         -      225.00     45.00
                        --------  --------  --------  --------
               Total:   1626.91    738.56   1585.00    378.00
```

```
================================================================================
|     Branch Data                                                              |
================================================================================
Brnch   From   To    From Bus Injection   To Bus Injection     Loss (I^2 * Z)
  #     Bus    Bus    P (MW)   Q (MVAr)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
-----  -----  -----  --------  --------  --------  --------  --------  --------
   1      1      2   -112.12   -113.15    112.36    109.82     0.242      2.60
   2      2      3    187.64     27.52   -187.36    -29.39     0.283      3.04
   3      3      4    487.36    171.74   -481.96   -125.61     5.400     58.06
   4      4      5    661.96    304.89   -659.75   -284.24     2.211     24.34
   5      5      6    559.75    221.24   -553.56   -142.52     6.187     99.01
   6      6      7    -32.00    -10.93     32.02     -3.37     0.015      0.24
   7      7      8   -212.02    -41.63    213.24     44.50     1.223     13.15
   8      1      6    509.96    211.59   -494.44    -71.55    15.524    166.87
   9      1      8    449.07    181.15   -438.24    -89.50    10.827    116.40
                                                             --------  --------
                                                    Total:    41.912    483.71
```

A.6 Task 4.e

```
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses                  8     Total Gen Capacity       0.0        -4000.0 to 4000.0
Generators             4     On-line Capacity         0.0        -4000.0 to 4000.0
Committed Gens         4     Generation (actual)   1628.2             874.7
Loads                  4     Load                  1585.0             375.0
  Fixed                4       Fixed               1585.0             375.0
  Dispatchable         0       Dispatchable          -0.0 of -0.0      -0.0
Shunts                 0     Shunt (inj)             -0.0               0.0
Branches               9     Losses (I^2 * Z)        43.25            499.73
Transformers           0     Branch Charging (inj)     -                0.0
Inter-ties             0     Total Inter-tie Flow     0.0               0.0
Areas                  1

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.900 p.u. @ bus 7          1.033 p.u. @ bus 2
Voltage Angle     -17.22 deg   @ bus 6          0.60 deg   @ bus 2
P Losses (I^2*R)             -                  15.98 MW    @ line 1-6
Q Losses (I^2*X)             -                 171.76 MVAr  @ line 1-6
```

Bus Data

Bus	Voltage		Generation		Load	
#	Mag(pu)	Ang(deg)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)
1	1.020	0.000*	848.25	352.16	-	-
2	1.033	0.596	300.00	142.82	-	-
3	1.029	-0.307	300.00	151.22	-	-
4	0.985	-6.417	180.00	228.53	-	-
5	0.967	-8.104	-	-	100.00	60.00
6	0.901	-17.217	-	-	1080.00	225.00
7	0.900	-16.789	-	-	180.00	45.00
8	0.916	-13.387	-	-	225.00	45.00
		Total:	1628.25	874.73	1585.00	375.00

Branch Data

Brnch	From	To	From Bus Injection		To Bus Injection		Loss (I^2 * Z)	
#	Bus	Bus	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)
1	1	2	-110.50	-110.39	110.74	112.96	0.239	2.57
2	2	3	189.26	29.86	-188.98	-26.77	0.288	3.09
3	3	4	488.98	177.99	-483.54	-119.58	5.432	58.40
4	4	5	663.54	348.11	-661.21	-322.43	2.332	25.68
5	5	6	561.21	262.43	-554.72	-158.52	6.493	103.91
6	6	7	-30.86	6.70	30.88	-6.46	0.015	0.24
7	7	8	-210.88	-38.54	212.12	51.94	1.247	13.41
8	1	6	510.40	244.94	-494.42	-73.18	15.979	171.76
9	1	8	448.35	217.62	-437.12	-96.94	11.225	120.68
						Total:	43.250	499.73

B Results from Python

B.1 Task 2.a

Bus information

	Voltage		Generator data		Load data	
#Bus	V [pu]	Angle [deg]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	+1.02000000	+0.00000000	+0.84689814	+0.27932285	+0.0000000	+0.0000000
2	+1.03300000	+0.60600859	+0.30000000	+0.13733785	+0.0000000	+0.0000000
3	+1.02900000	-0.28900125	+0.30000000	+0.14235170	+0.0000000	+0.0000000
4	+0.98500000	-6.37866828	+0.18000000	+0.17638580	+0.0000000	+0.0000000
5	+0.96902496	-8.06849160	+0.00000000	+0.00000000	+0.1000000	+0.0600000
6	+0.91043594	-17.07000878	+0.00000000	+0.00000000	+1.0800000	+0.2250000
7	+0.91165026	-16.64583660	+0.00000000	+0.00000000	+0.1800000	+0.0450000
8	+0.92778180	-13.30864491	+0.00000000	+0.00000000	+0.2250000	+0.0450000
		Total:	+1.62689814	+0.73539820	+1.5850000	+0.3750000

Branch information

From	To	From Bus Injection		To Bus Injection		Line loss	
bus	bus	P [pu]	Q [pu]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	2	-0.11211301	-0.11315406	+0.11235488	+0.10982096	+0.00024187	+0.00259948
2	3	+0.18764512	+0.02751689	-0.18736222	-0.02938834	+0.00028290	+0.00303945
3	4	+0.48736222	+0.17174004	-0.48196228	-0.12560889	+0.00539994	+0.05806206
4	5	+0.66196228	+0.30199469	-0.65975862	-0.28142694	+0.00220366	+0.02426212
5	6	+0.55975862	+0.22142694	-0.55357141	-0.14270249	+0.00618722	+0.09901113
6	7	-0.03199308	-0.01086376	+0.03200819	-0.00344427	+0.00001512	+0.00024187
7	8	-0.21200819	-0.04155573	+0.21323104	+0.04442032	+0.00122284	+0.01314279
1	8	+0.44905575	+0.18104793	-0.43823104	-0.08942032	+0.01082472	+0.11637146
1	6	+0.50995539	+0.21142898	-0.49443552	-0.07143375	+0.01551988	+0.16682894
					Total:	+0.04189814	+0.48355930

B.2 Task 4.a

Convergence in 4 iterations.

Bus information

	Voltage		Generator data		Load data	
#Bus	V [pu]	Angle [deg]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	+1.02000000	+0.00000000	+0.83923726	+0.23948290	+0.0000000	+0.0000000
2	+1.03300000	+0.93790375	+0.30000000	+0.13735120	+0.0000000	+0.0000000
3	+1.02900000	+0.31298550	+0.30000000	+0.13606768	+0.0000000	+0.0000000
4	+0.98500000	-5.06049934	+0.18000000	+0.10553597	+0.0000000	+0.0000000
5	+0.97189822	-6.61584519	+0.00000000	+0.00000000	+0.1000000	+0.0600000
6	+0.92665604	-14.61294003	+0.00000000	+0.00000000	+1.0800000	+0.2250000
7	+0.93646698	-12.78185861	+0.00000000	+0.00000000	+0.1800000	+0.0450000
8	+0.96577666	-8.00451739	+0.00000000	+0.00000000	+0.2250000	+0.0450000
		Total:	+1.61923726	+0.61843775	+1.5850000	+0.3750000

Branch information

From	To	From Bus Injection		To Bus Injection		Line loss	
bus	bus	P [pu]	Q [pu]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	2	-0.16734534	-0.10726341	+0.16772586	+0.10542058	+0.00038053	+0.00408975
2	3	+0.13227414	+0.03193062	-0.13212779	-0.03526921	+0.00014634	+0.00157231
3	4	+0.43212779	+0.17133689	-0.42774951	-0.13619094	+0.00437828	+0.04707686
4	5	+0.60774951	+0.24172691	-0.60596882	-0.22582679	+0.00178069	+0.01960528
5	6	+0.50596882	+0.16582679	-0.50115851	-0.10954239	+0.00481030	+0.07697702
6	7	-0.14558361	-0.04313884	+0.14589988	+0.03298551	+0.00031627	+0.00505975
7	8	-0.32589988	-0.07798551	+0.32869167	+0.09699701	+0.00279179	+0.03000540
1	6	+0.44481421	+0.16928067	-0.43325787	-0.07231878	+0.01155633	+0.12422335
1	8	+0.28088420	+0.08873282	-0.27684584	-0.07099850	+0.00403836	+0.04341453
1	8	+0.28088420	+0.08873282	-0.27684584	-0.07099850	+0.00403836	+0.04341453
					Total:	+0.03423726	+0.39543877

B.3 Task 4.b

Bus information

	Voltage		Generator data		Load data	
#Bus	V [pu]	Angle [deg]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	+1.02000000	+0.00000000	+0.85244461	+0.38024154	+0.0000000	+0.0000000
2	+1.03300000	+0.65566277	+0.30000000	+0.13730261	+0.0000000	+0.0000000
3	+1.02900000	−0.19895358	+0.30000000	+0.36231700	+0.0000000	+0.0000000
4	+0.93575000	−6.21675229	+0.18000000	−0.07969158	+0.0000000	+0.0000000
5	+0.92194558	−8.07152085	+0.00000000	+0.00000000	+0.1000000	+0.0600000
6	+0.87862489	−17.76059039	+0.00000000	+0.00000000	+1.0800000	+0.2250000
7	+0.88529869	−17.25132163	+0.00000000	+0.00000000	+0.1800000	+0.0450000
8	+0.90890648	−13.68997370	+0.00000000	+0.00000000	+0.2250000	+0.0450000
		Total:	+1.63244461	+0.80016957	+1.5850000	+0.3750000

Branch information

From	To	From Bus Injection		To Bus Injection		Line loss	
bus	bus	P [pu]	Q [pu]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	2	−0.12037837	−0.11229313	+0.12063719	+0.10914223	+0.00025882	+0.00278168
2	3	+0.17936281	+0.02816038	−0.17910342	−0.03028441	+0.00025939	+0.00278688
3	4	+0.47910342	+0.39260141	−0.47130817	−0.32015874	+0.00779526	+0.08381734
4	5	+0.65130817	+0.24046716	−0.64908592	−0.21933943	+0.00222225	+0.02446679
5	6	+0.54908592	+0.15933943	−0.54294228	−0.07963780	+0.00614363	+0.09831368
6	7	−0.03748320	−0.03458084	+0.03751732	+0.02149068	+0.00003412	+0.00054586
7	8	−0.21751732	−0.06649068	+0.21894977	+0.07210635	+0.00143245	+0.01539560
1	6	+0.51701150	+0.27220121	−0.49957452	−0.11078136	+0.01743698	+0.18743659
1	8	+0.45581148	+0.22033346	−0.44394977	−0.11710635	+0.01186172	+0.12751975
					Total:	+0.04744461	+0.54306417

B.4 Task 4.c

Convergence in 5 iterations.

Bus information

	Voltage		Generator data		Load data	
#Bus	V [pu]	Angle [deg]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	+1.02000000	+0.00000000	+0.88364022	+0.64635753	+0.0000000	+0.0000000
2	+1.03300000	+3.52030849	+0.30000000	+0.15751141	+0.0000000	+0.0000000
3	+1.02900000	+5.00436812	+0.30000000	−0.07222383	+0.0000000	+0.0000000
4	+0.98500000	−28.62948582	+0.18000000	+0.39740983	+0.0000000	+0.0000000
5	+0.96629908	−29.01339339	+0.00000000	+0.00000000	+0.1000000	+0.0600000
6	+0.87526152	−30.00415373	+0.00000000	+0.00000000	+1.0800000	+0.2250000
7	+0.86842328	−26.92609594	+0.00000000	+0.00000000	+0.1800000	+0.0450000
8	+0.88344123	−20.05240776	+0.00000000	+0.00000000	+0.2250000	+0.0450000
		Total:	+1.66364022	+1.12905494	+1.5850000	+0.3750000

Branch information

From	To	From Bus Injection		To Bus Injection		Line loss	
bus	bus	P [pu]	Q [pu]	P [pu]	Q [pu]	P [pu]	Q [pu]
1	2	−0.59575597	−0.05052081	+0.59925097	+0.08215088	+0.00349500	+0.03756265
2	3	−0.29925097	+0.07536053	+0.30000000	−0.07222383	+0.00074903	+0.00804760
4	5	+0.18000000	+0.39740983	−0.17920320	−0.39232127	+0.00079680	+0.00877272
5	6	+0.07920320	+0.33232127	−0.07710320	−0.31822142	+0.00210000	+0.03360525
6	7	−0.20791397	+0.04289500	+0.20863503	−0.04468431	+0.00072105	+0.01153561
7	8	−0.38863503	−0.00031569	+0.39303354	+0.03826680	+0.00439851	+0.04727397
1	6	+0.83802880	+0.38645883	−0.79498282	+0.05032642	+0.04304598	+0.46271731
1	8	+0.64136739	+0.31041951	−0.61803354	−0.08326680	+0.02333385	+0.25085131
					Total:	+0.07864022	+0.86036642

B.5 Task 4.d

```
Convergence in 5 iterations.

Bus information
-----------------
        |        Voltage          |       Generator data        |        Load data            |
#Bus    |    V [pu]    | Angle [deg] |    P [pu]    |    Q [pu]    |    P [pu]    |    Q [pu]    |
1       |  +1.02000000 | +0.00000000 | +0.84691221 | +0.27959285 | +0.0000000 | +0.0000000 |
2       |  +1.03300000 | +0.60604520 | +0.30000000 | +0.13733782 | +0.0000000 | +0.0000000 |
3       |  +1.02900000 | -0.28893486 | +0.30000000 | +0.14235096 | +0.0000000 | +0.0000000 |
4       |  +0.98500000 | -6.37852294 | +0.18000000 | +0.17928231 | +0.0000000 | +0.0000000 |
5       |  +0.96889415 | -8.06785717 | +0.00000000 | +0.00000000 | +0.1000000 | +0.0630000 |
6       |  +0.91034866 | -17.07144259 | +0.00000000 | +0.00000000 | +1.0800000 | +0.2250000 |
7       |  +0.91157840 | -16.64713388 | +0.00000000 | +0.00000000 | +0.1800000 | +0.0450000 |
8       |  +0.92773074 | -13.30945803 | +0.00000000 | +0.00000000 | +0.2250000 | +0.0450000 |
---------------------------------------------------------------------------------------------------
                          Total:      | +1.62691221 | +0.73856393 | +1.5850000 | +0.3780000 |

Branch information
------------------
From | To  |      From Bus Injection       |       To Bus Injection        |          Line loss          |
bus  | bus |    P [pu]     |    Q [pu]     |    P [pu]     |    Q [pu]     |    P [pu]    |    Q [pu]    |
  1  |  2  |  -0.11211911 | -0.11315343 | +0.11236099 | +0.10982046 | +0.00024188 | +0.00259961 |
  2  |  3  |  +0.18763901 | +0.02751736 | -0.18735613 | -0.02938900 | +0.00028288 | +0.00303926 |
  3  |  4  |  +0.48735613 | +0.17173996 | -0.48195631 | -0.12561009 | +0.00539982 | +0.05806077 |
  4  |  5  |  +0.66195631 | +0.30489240 | -0.65974534 | -0.28424359 | +0.00221098 | +0.02434268 |
  5  |  6  |  +0.55974534 | +0.22124359 | -0.55355816 | -0.14251501 | +0.00618718 | +0.09901054 |
  6  |  7  |  -0.03200221 | -0.01093376 | +0.03201735 | -0.00337143 | +0.00001514 | +0.00024217 |
  7  |  8  |  -0.21201735 | -0.04162857 | +0.21324063 | +0.04449924 | +0.00122328 | +0.01314750 |
  1  |  6  |  +0.50996363 | +0.21159308 | -0.49443963 | -0.07155124 | +0.01552400 | +0.16687327 |
  1  |  8  |  +0.44906768 | +0.18115320 | -0.43824063 | -0.08949924 | +0.01082705 | +0.11639657 |
-------------------------------------------------------------------------------------------------------
                                                            Total:    | +0.04191221 | +0.48371237 |
```

B.6 Task 4.e

```
Convergence in 5 iterations.

Bus information
-----------------
        |        Voltage          |       Generator data        |        Load data            |
#Bus    |    V [pu]    | Angle [deg] |    P [pu]    |    Q [pu]    |    P [pu]    |    Q [pu]    |
1       |  +1.02000000 | +0.00000000 | +0.84824966 | +0.35216456 | +0.0000000 | +0.0000000 |
2       |  +1.03300000 | +0.59630338 | +0.30000000 | +0.14281510 | +0.0000000 | +0.0000000 |
3       |  +1.02900000 | -0.30660101 | +0.30000000 | +0.15122158 | +0.0000000 | +0.0000000 |
4       |  +0.98500000 | -6.41719728 | +0.18000000 | +0.22852730 | +0.0000000 | +0.0000000 |
5       |  +0.96702331 | -8.10401529 | +0.00000000 | +0.00000000 | +0.1000000 | +0.0600000 |
6       |  +0.90050553 | -17.21728959 | +0.00000000 | +0.00000000 | +1.0800000 | +0.2250000 |
7       |  +0.89950607 | -16.78930896 | +0.00000000 | +0.00000000 | +0.1800000 | +0.0450000 |
8       |  +0.91638201 | -13.38657319 | +0.00000000 | +0.00000000 | +0.2250000 | +0.0450000 |
---------------------------------------------------------------------------------------------------
                          Total:      | +1.62824966 | +0.87472854 | +1.5850000 | +0.3750000 |

Branch information
------------------
From | To  |      From Bus Injection       |       To Bus Injection        |          Line loss          |
bus  | bus |    P [pu]     |    Q [pu]     |    P [pu]     |    Q [pu]     |    P [pu]    |    Q [pu]    |
  1  |  2  |  -0.11049741 | -0.11039277 | +0.11073612 | +0.11295832 | +0.00023871 | +0.00256554 |
  2  |  3  |  +0.18926388 | +0.02985678 | -0.18897626 | -0.02676660 | +0.00028762 | +0.00309018 |
  3  |  4  |  +0.48897626 | +0.17798819 | -0.48354455 | -0.11958453 | +0.00543171 | +0.05840365 |
  4  |  5  |  +0.66354455 | +0.34811183 | -0.66121237 | -0.32243473 | +0.00233218 | +0.02567710 |
  5  |  6  |  +0.56121237 | +0.26243473 | -0.55471896 | -0.15852371 | +0.00649341 | +0.10391102 |
  6  |  7  |  -0.03086269 | +0.00669914 | +0.03087756 | -0.00646125 | +0.00001487 | +0.00023790 |
  7  |  8  |  -0.21087756 | -0.03853875 | +0.21212480 | +0.05194383 | +0.00124725 | +0.01340508 |
  1  |  6  |  +0.51039712 | +0.24493712 | -0.49441835 | -0.07317543 | +0.01597876 | +0.17176168 |
  1  |  8  |  +0.44834996 | +0.21762022 | -0.43712480 | -0.09694383 | +0.01122515 | +0.12067638 |
-------------------------------------------------------------------------------------------------------
                                                            Total:    | +0.04324966 | +0.49972854 |
```

C System data for base case

From Line	To Line	R [pu]	X [pu]	Full-Line B [pu]
1	2	0,01018	0,10941	0,00563
2	3	0,00836	0,08982	0,00462
3	4	0,02124	0,22838	0,01176
4	5	0,00403	0,04437	0,00387
5	6	0,01582	0,25316	0,02295
6	7	0,01209	0,19342	0,01753
7	8	0,02196	0,23602	0,01215
1	8	0,04702	0,50549	0,02603
1	6	0,05187	0,55757	0,02871

Figure 6: Branch data

Bus Num	BusType	V [V]	Angle [rad]	P_gen	Q_gen	P_load	Q_load		S_base [MVA]	V_base
1	0	306	0	0	0	0	0		1000	300
2	1	309,9	0	300	0	0	0			
3	1	308,7	0	300	0	0	0			
4	1	295,5	0	180	0	0	0			
5	2	300	0	0	0	100	60			
6	2	300	0	0	0	1080	225			
7	2	300	0	0	0	180	45			
8	2	300	0	0	0	225	45			

Figure 7: Bus data

D Matpower code for base case

```
function mpc = System
%CASE4GS  Power flow data for 4 bus, 2 gen case from Grainger &
↪  Stevenson.
%   Please see CASEFORMAT for details on the case file format.
%
%   This is the 4 bus example from pp. 337-338 of "Power System
↪  Analysis",
%   by John Grainger, Jr., William Stevenson, McGraw-Hill, 1994.

%   MATPOWER

%         ####  Change name of input file (Excel) here   ####
Bus_data = xlsread('System_data.xlsx','BusData');
Branch_data = xlsread('System_data.xlsx','BranchData');


%% MATPOWER Case Format : Version 2
mpc.version = '2';
```

```
%%-----  Power Flow Data  -----%%
%% system MVA base
mpc.baseMVA = Bus_data(1,11);

%% bus data

Bus = zeros(length(Bus_data(:,1)),13);
for i = 1:length(Bus_data(:,1))
    Bus(i,1) = round(Bus_data(i,1)); % bus_i
    if Bus_data(i,2) == 0            % type
        Bus(i,2) = 3;
    elseif Bus_data(i,2) == 1
        Bus(i,2) = 2;
    else
        Bus(i,2) = 1;
    end
    Bus(i,3) = Bus_data(i,7);        % Pd
    Bus(i,4) = Bus_data(i,8);        % Qd
    % Bus(i,5) = 0                    % Gs
    % Bus(i,6) = 0                    % Bs
    Bus(i,7) = 1;                    % Area
    Bus(i,8) = 1;                    % Vm
    % Bus(i,9) = 0;                   % Va
    Bus(i,10) = Bus_data(1,12);      % Base kV
    Bus(i,11) = 1;                   % zone
    Bus(i,12) = 1.1;                 % Vmax
    Bus(i,13) = 0.9;                 % Vmin
end
mpc.bus = Bus;

%% generator data

numgen = sum(Bus_data(:,2) == 1);
numgen = numgen + 1;
Gen = zeros(numgen,21);
count = 1;
for i = 1:length(Bus_data(:,1))
   if Bus_data(i,2) == 0                             % Pmax
       Gen(count,8) = Bus_data(i,9)*10;
   elseif Bus_data(i,2) == 1
       Gen(count,8) = Bus_data(i,5);
   end
   if Bus_data(i,2) == 0 || Bus_data(i,2) == 1
      Gen(count,1) = Bus_data(i,1);                  % bus
      Gen(count,2) = Bus_data(i,5);                  % Pg
      Gen(count,3) = Bus_data(i,6);                  % Qg
      Gen(count,4) = 1000; %Bus_data(i,9);                   % Qmax
```

```
        Gen(count,5) = -1000; %Bus_data(i,10);                  % Qmin
        Gen(count,6) = Bus_data(i,3)/Bus_data(1,12);   % Vg
        Gen(count,7) = Bus_data(1,11);                 % mBase
        Gen(count,8) = 1;                              % status
        count = count + 1;
    end

end
mpc.gen = Gen;

%% branch data
Branch = zeros(length(Branch_data(:,1)),13);
for i = 1:length(Branch_data(:,1))
     Branch(i,1) = round(Branch_data(i,1));   %fbus
     Branch(i,2) = round(Branch_data(i,2));   %tbus
     Branch(i,3) = Branch_data(i,3);          %r
     Branch(i,4) = Branch_data(i,4);          %x
     Branch(i,5) = Branch_data(i,5);          %b
     Branch(i,6) = 1000;                      %rateA
     Branch(i,7) = 1000;                      %rateB
     Branch(i,8) = 1000;                      %rateC
     % ratio = 0
     % angle = 0
     Branch(i,11) = 1;                        %status
     Branch(i,12) = -360;                     %angmin
     Branch(i,13) = 360;                      %angmin
end
mpc.branch = Branch;
```