Lecture 13: Sources of noise in neurons, element of probability theory and applications to stochastic neuron models.

Objectives

1. Learn about the sources of noise (stochasticity) in neurons.
2. Recall basic concepts in probability theory, stochastic processes, and built stochastic neuron models.
3. How to simulate the stochastic neuron.

Sources and effects of noise in neurons

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- The sources of neuronal noise include synaptic noise, that is, the quasi-random release of neurotransmitters by synapses or random synaptic input from other neurons, and channel noise, that is, the random switching of ion channels.
- The addition of noise terms into the neuron model equation has to be therefore seen as the effects of the other neurons (synaptic noise) and the opening and closing of ion channels (channel noise) on this neuron.

Sources and effects of noise in neurons

- Depending on the deterministic parameter regime of a neuron model, adding noise to either the membrane potential variable (synaptic noise) and/or the recovery current variable (channel noise) can induce different dynamical effects. Some of these effects can be somewhat counterintuitive, as noise plays a constructive role in weak signal detection or coherence of spiking activity instead of being a nuisance. .

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- Neurons have been shown to use noise to improve information processes via phenomena such as stochastic resonance (SR), coherence resonance (CR), inverse stochastic resonance (ICR), stochastic synchronization (SS), etc

Mathematical modeling of stochastic neurons

- Mathematically, we want to perturb a dynamical system by adding new terms to it. The perturbation is done by adding a Wiener process to the dynamical system, which becomes a stochastic system. The study of stochastic dynamical systems is more abstract than deterministic ones (i.e., the ones without noise), and needs important definitions and results from probability theory which are, unfortunately beyond the scope of this course. .

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- Therefore, we only briefly define a few objects in probability theory, stochastic processes, Wiener and Ito processes, and how to numerically integrate stochastic neuron models.

Basic elements of probability theory

A probability space, denoted as $(\Omega, \mathcal{F}, P)$, is a mathematical construct used to model a random or stochastic process. It consists of three components:

1. The sample space (denoted as Ω) in a probability space is the set of all possible outcomes of a random experiment. Each element in Ω represents a unique outcome that can occur when the experiment is performed. The sample space can be:
   - Finite: When the number of possible outcomes is countable and limited. For example, the sample space of flipping a coin is $\Omega = \{\text{Heads}, \text{Tails}\}$.

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   - Uncountably Infinite: When the number of possible outcomes is uncountable. For example, the sample space for measuring the exact amount of rainfall in a day is $\Omega = [0, \infty)$.

Basic elements of probability theory

1. Sample space (Ω)
2. A σ-algebra ($\mathcal{F}$) is a collection of subsets of a given set Ω that satisfies certain properties, making it suitable for defining a probability measure. Formally, a σ-algebra $\mathcal{F}$ on a set Ω is a collection of subsets of Ω such that:
   - The set Ω itself is in $\mathcal{F}$: $\Omega \in \mathcal{F}$
   - Closed under complementation: If a set A is in $\mathcal{F}$, then its complement A^c (relative to Ω) is also in $\mathcal{F}$:$A \in \mathcal{F} \implies A^c \in \mathcal{F}$
   - Closed under countable unions: If $A_1, A_2, A_3, \ldots$ are in $\mathcal{F}$, then the union of all these sets is also in $\mathcal{F}$: $A_1, A_2, A_3, \ldots \in \mathcal{F} \implies \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$

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    Since the σ-algebra is closed under complementation and countable unions, it is also closed under countable intersections by De Morgan's laws. Specifically:

$$\bigcap_{i=1}^{\infty} A_i = \left(\bigcup_{i=1}^{\infty} A_i^c \right)^c \in \mathcal{F}$$

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   Example: Consider a simple set $\Omega = \{a, b, c\}$. One possible σ-algebra on Ω is:

$$\mathcal{F} = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$$

   This collection $\mathcal{F}$ is a σ-algebra because it includes Ω, is closed under complementation, and is closed under countable unions.

Basic elements of probability theory

1. Sample space (Ω)
2. σ-algebra ($\mathcal{F}$)
3. Probability measure (P): This is a function that assigns a probability to each event in $\mathcal{F}$. The probability measure must satisfy the following properties:

- $P(\Omega) = 1$: The probability of the entire sample space is 1.
- $P(A) \geq 0$ for any event $A \in \mathcal{F}$: Probabilities are non-negative.
- P is countably additive: For any countable collection of mutually exclusive events $A_1, A_2, \ldots \in \mathcal{F}$,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

In summary, a probability space $(\Omega, \mathcal{F}, P)$ provides a formal framework for assigning probabilities to events and studying stochastic phenomena.

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Definition: A filtration on a probability space $(\Omega, \mathcal{F}, P)$ is a family of σ-algebras $\{\mathcal{F}_t\}_{t\geq 0}$ indexed by time t such that:

1. $\mathcal{F}_0 \subset \mathcal{F}_t$ for all $t \geq 0$: This implies that $\mathcal{F}_0$ is the initial σ-algebra, representing the initial information, and this information is included in all future σ-algebras.

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2. $\mathcal{F}_s \subset \mathcal{F}_t$ for all $0 \leq s \leq t$: This means that the σ-algebras are increasing, reflecting the idea that as time progresses, the amount of available information can only increase or stay the same. This property is known as the filtration property.

Shorter formal Definition: Let $(\Omega, \mathcal{F}, P)$ be a probability space. A filtration $\{\mathcal{F}_t\}_{t\geq 0}$ is a family of σ-algebras $\mathcal{F}_t \subset \mathcal{F}$ such that for all $0 \leq s \leq t$,

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Example: Consider a probability space $(\Omega, \mathcal{F}, P)$ and a stochastic process $\{X_t\}_{t\geq 0}$, where X_t represents the value of a stock at time t. The filtration $\{\mathcal{F}_t\}_{t\geq 0}$ can be interpreted as follows:

- $\mathcal{F}_0$ represents the information available at the initial time (e.g., initial stock price).
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Importance: Filtrations are crucial in the study of random processes and provide a rigorous way to handle the evolution of information and ensure that stochastic processes are properly aligned with the available information at each point in time.

A Borel set is a set that can be formed through countable operations (such as countable unions, countable intersections, and relative complements) starting from open sets in a given topological space. In the context of probability theory, Borel sets are crucial because they form the Borel σ-algebra, which is the smallest σ-algebra containing all open sets.

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Examples:

1. In $\mathbb{R}$: Every open interval (a, b) is a Borel set. Every closed interval $[a, b]$ is a Borel set because it can be constructed from open intervals. Countable sets (like $\mathbb{Q}$, the set of rational numbers) and their complements (like $\mathbb{R} \setminus \mathbb{Q}$, the set of irrational numbers) are Borel sets.

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2. In $\mathbb{R}^n$: Open balls and open rectangles are Borel sets. More complex sets formed by countable unions and intersections of open sets, such as the union of countably many open balls.

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Borel sets are important because they provide the framework within measures (like the Lebesgue measure) and integrals. In probability theory, random variables are typically defined to be measurable with respect to the Borel σ-algebra, ensuring that the preimage of any Borel set is an event in the underlying probability space.

Basic elements of probability theory

Consider a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\{\mathcal{M}_t\}_{t\geq 0}$, the term $\mathcal{M}_t$-measurable is used to describe a random variable (or more generally, a stochastic process) that is measurable with respect to the σ-algebra $\mathcal{M}_t$. This concept is crucial for defining when a random variable is "adapted" to a given filtration, meaning that the variable's value at time t only depends on the information available up to that time.

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Definition: A random variable $X_t : \Omega \to \mathbb{R}^n$ is said to be $\mathcal{M}_t$-measurable if, for every Borel set $B \subset \mathbb{R}^n$,

$$\{\omega \in \Omega : X_t(\omega) \in B\} \in \mathcal{M}_t.$$

In other words, the preimage of any Borel set under X_t belongs to the σ-algebra $\mathcal{M}_t$. This ensures that the value of X_t is determined by the information contained in $\mathcal{M}_t$.

Example: Consider a filtration $\{\mathcal{M}_t\}_{t\geq 0}$ and a stochastic process $\{X_t\}_{t\geq 0}$. If X_t represents the price of a stock at time t, saying that X_t is $\mathcal{M}_t$-measurable implies that the stock price at time t is fully determined by the information available up to time t.

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- Definition 1: Let $(\Omega, \mathcal{F}, P)$ be a probability space. A *filtration* on $(\Omega, \mathcal{F})$, is a family $\mathcal{M} = \{\mathcal{M}_t\}_{t\geq 0}$ of σ-algebras $\mathcal{M}_t \subset F$ such that $0 \leq s < t \Rightarrow \mathcal{M}_s \subset \mathcal{M}_t$.
  A stochastic process $\{X_t\}_{t\geq 0}$, $X_t : \Omega \longrightarrow \mathbb{R}^n, \forall t$, is called adapted to the filtration $\{\mathcal{M}_t\}$ if for each $t \geq 0$, X_t is $\mathcal{M}_t$-measurable.

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- Definition 2: A stochastic process, $\{W_t\}_{t\geq 0}$, $W_t : \Omega \longrightarrow \mathbb{R}^n, \forall t$, defined for a filtration $\{\mathcal{M}_t\}_{t\geq 0}$ and characterized by
  1. $W_0 = 0$ almost surely
  2. $W_t - W_s$ is independent of $\mathcal{M}_t$ and $W_t - W_s \sim \mathcal{N}(0, t-s)$

  is called a *n-dimensional Wiener process* or *Brownian motion*.
  The application $t \rightarrow W_t(\omega)$ is called a *sample path* or *random trajectory* of the Wiener process for a fixed $\omega \in \Omega$.

Theorem

For almost every $\omega \in \Omega$, $\mathbb{P}(W_t(\omega)$*is nowhere differentiable*$) = 1$.

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- Let $\{W_t\}_{t\geq 0}$ be a n-dimensional Wiener process w.r.t the filtration $\{\mathcal{M}_t\}_{t\geq 0}$. A stochastic process $\{X_t\}_{t\geq 0}$ is called an Ito process if it can be written in the form

$$X_t = X_0 + \int_0^t a(s)ds + \int_0^t b(s)dW_s, \qquad (1)$$

where $b \in M^2$, a is adapted to the filtration $\{\mathcal{M}_t\}_{t\geq 0}$ and $\int\limits_0^\infty |a(t)|dt < +\infty$ almost surely. We also write the stochastic process in a differential form as

$$dX_t = a(t)dt + b(t)dW_t. \tag{2}$$

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- The Euler–Maruyama method is a numerical method used for approximating solutions to stochastic differential equations (SDEs). In Python, you can use the *sdeint* package to implement the Euler–Maruyama method and solve SDEs like the one given in Eq.(2).

- For example, a stochastic version of the FHN model is obtained by adding two Wiener processes: $W_{1,t}$ with amplitude σ_1 is added to the membrane potential v to model the synaptic noise and $W_{2,t}$ with amplitude σ_2 is added to the current recovery variable w to model the channel noise. The two-dimensional Ito process associated with the FHN model is

$$\begin{cases} dv_t = (v_t - \frac{v_t^3}{3} - w_t + I)dt + \sigma_1 dW_{1,t}, \\ dw_t = \varepsilon(d + v_t)dt + \sigma_2 dW_{2,t}. \end{cases} \quad (3)$$

- On the next slide, Figure (a) and (c) shows the phase portraits of the random trajectory and their respective time series in (b) and (d). In both cases, the deterministic equation (i.e, Eq.(4) with $\sigma_1 = 0$ and $\sigma_2 = 0$) is in the excitable state with $d = 1.05$, and therefore, there is no possibility of spiking.
- Switching on either noise source can induce spiking activity. The smoothness of the trajectory is distorted by the presence of noise as opposed to the trajectory when there is no noise.

Mathematical modeling of stochastic neurons

$$\begin{cases} dv_t &= (v_t - \frac{v_t^3}{3} - w_t + I)dt + \sigma_1 dW_{1,t}, \\ dw_t &= \varepsilon(d + v_t)dt + \sigma_2 dW_{2,t}. \end{cases} \tag{4}$$

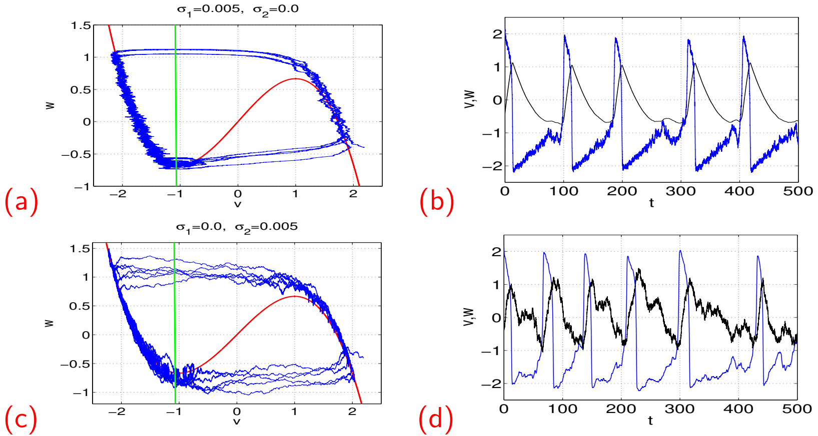

Figure: Phase portraits in (a) and (c) and corresponding time-series in (b) and (d) with v in blue and w in black. $d = 1.05$, $\varepsilon = 0.05$, $I = 0.001$.