Lecture 19:
Liquid State Machines (LSMs)

Objectives

1. Understand the architecture of Liquid-state Machine
2. Understand the baseline working principle of Liquid-State Machines
3. Understand the properties of a good Liquid-state Machine

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- Essentially, the brain cortex or a specific area of it is treated as a "liquid." These cortical microcircuits exhibit remarkable computational abilities when confronted with perturbations. These microcircuits possess diverse elements, such as neurons and synapses, along with a variety of mechanisms and time constants that define their interactions, including recurrent connections.

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- To illustrate the LSM approach, consider a sequence of temporary disruptions applied to an excitable medium, similar to sounds, objects dropped into a liquid or wind gusts. By treating the liquid as an attractor neural network, its resting state becomes the sole attractor. However, perturbed states of the liquid represent both current and past inputs, carrying valuable information for analyzing the surrounding environment.

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- These functions are typically multidimensional because they capture spike trains from multiple external neurons providing inputs to the circuit and involve various readouts that extract output spike trains. Since an LSM maps input streams $u(\cdot)$ to output streams $y(\cdot)$, it is often described as implementing a functional or operator, akin to a filter. From a mathematical perspective, it represents a function that operates on objects of a higher type than numbers or bits.

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- One notable characteristic of this higher-type computational processing is that the output stream's target value, $y(t)$, at a given time t, may depend on the input stream's values, $u(s)$, at many (potentially even infinitely many) preceding time points s.

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- Various methods can be used to model this readout neuron, such as a linear gate, a perceptron with a threshold, a sigmoidal gate, or a spiking neuron. The main purpose of the LSM (referred to as the "Liquid") is to act as a pre-processor for this readout neuron, expanding the range of possible functions that it can learn from input streams $u(t)$.

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- This division of computational processing into the Liquid and the readout is highly efficient because the same Liquid can serve multiple readout neurons, each learning to extract a distinct "summary" of information from the same Liquid.
- The need to extract different information summaries from a cortical microcircuit arises due to different computational objectives, such as determining the movement direction of objects or identifying object identities when $u(t)$ represents visual inputs.

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- The need to extract different information summaries from a cortical microcircuit arises due to different computational objectives, such as determining the movement direction of objects or identifying object identities when $u(t)$ represents visual inputs.
- Neurophysiological data indicates that spike trains from different projection neurons in the same column are generally weakly correlated with natural stimuli. Therefore, the LSM serves as a model for multiplexing diverse computations on a shared input stream $u(t)$.

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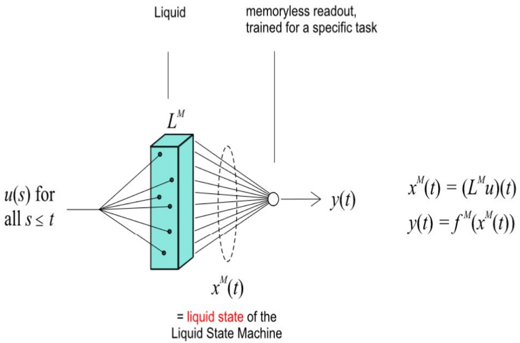
- The figure below shows only one input and output channel for simplicity. A liquid-state machine consists of three main components. The first component is a layer of input neurons where an external stimulus is introduced. The signal from this component is transmitted to the selected neurons of the second component, called the liquid column denoted that L^M, where the proper neural computation takes place.



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- The liquid component of the system receives a continuous input stream, denoted as $u(s)$. As time progresses, specifically at a later time $t > s$, the current internal state $x^M(t)$ of the liquid holds a significant amount of information regarding the recent inputs $u(s)$. This current liquid state $x^M(t)$ is then transmitted to the final component of the liquid-state machine. This component is referred to as a memory-less readout map, denoted as f^M, meaning that it has no access to any states prior to time t.

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- The readout map performs an analysis and interpretation of the liquid's calculations. Ultimately, the input signal is transformed by the readout map, resulting in the system's output $y(t)$ given by:

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- The structure of the readout map, f^M, is not explicitly provided, allowing for the utilization of various statistical analysis or pattern recognition methods that are available (e.g. linear regression, stochastic gradient descent, etc).

Properties of a good Liquid State Machines (LSMs)

- The fundamental characteristic of an LSM lies in its capacity to generate distinct responses to different input patterns. This ability, known as "the separation property" of the liquid, plays a pivotal role.
- On the other hand, the "approximation property" refers to the capability of the readout maps to differentiate these responses, generalize them, and establish a connection with the given output.

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- The approximation property primarily relies on the adaptive nature of the readout map, while the separation property is directly influenced by the complexity of the liquid itself.
- Differentiating between two liquid states or comparing the responses of a Liquid State Machine to two stimuli presents a challenging task. There is no definitive measure of the distance between liquid states or input patterns. Due to the highly dynamic and non-linear nature of liquid states, it becomes evident that a straightforward "one-to-one" mapping of two states is not feasible. Even a slight variation in the input pattern can result in a significant disparity in the liquid's response.

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- However, certain basic comparisons, such as the Euclidean norm, can offer insights into the distance between liquid trajectories for two input patterns. The Euclidean distance, formally calculated using the following formula, determines the distance between liquid states for all recorded time steps:

$$E_{I_1 I_2}(t) = \sqrt{\sum_{i=1}^{n} \left(y_i^{I_1}(t) - y_i^{I_2}(t) \right)^2} \tag{3}$$

where n stands for the number of neurons in the liquid, $y_i^{I_1}(t)$ and $y_i^{I_2}(t)$ are the ith neuron activity (the neuron activity is usually the spiking times) measured in time t after presenting I_1 and I_2 inputs, respectively.

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- In order to investigate the information flow in LSMs a global entropy (denoted as S) concept based on the classical definition of Shannon's informational entropy, can be used. Simulating liquid for a duration of T and obtaining N spikes on the readout, the probability of having n_i spikes occurring during a period t_i of e.g., $t_i = T/100$ is:

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- Such a probability can be interpreted as a chance of giving the whole information in t_i part of time T.

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- In general, the measure of global entropy helps in evaluating the capability of the system to represent the information contained in the data properly, and its high index is a cue that the system is able to capture the useful information available in input.
- A neural network model should have a large Euclidean distance $E_{I_1 I_2}(t)$ (good separability property) and a large global entropy S (to capture the useful information from the input) for it to use as a liquid. Research has shown that leaky integrate-and-fire, Morris-Lecar, resonate-and-fire, and Hindmarsh-Rose neuron models are good candidates for LSMs.

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- Decorrelation (to induce suitable separability property) between liquid neurons is introduced by drawing synaptic delays, current bias, and connection strengths from a probability distribution.
- We connect n_{rec} excitatory liquid neurons all-to-all to perfect linear readout neurons in a feedforward fashion.

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- The connection strengths between input and liquid neurons and within the liquid can be tuned to make the liquid memory-driven or input-driven.
- For example, by increasing the connection strengths between input and liquid neurons, the activity regime of the liquid will depend more on the inputs than its memory.

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- We sample the liquid state at discrete timesteps of Δ_{sample} for $t \in [0, t_{\text{train}}]$. They are accumulated into a matrix $X \in \mathbb{R}^{n_{\text{rec}} \times n_{\text{sample}}}$ where $n_{\text{sample}} = t_{\text{train}}/\Delta_{\text{sample}}$

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  where $y_i \in \mathbb{R}^{n_{\text{sample}}}$ is the sampled target (output) signals of readout neuron i.
- In other words, each readout neuron solves a linear regression (just as with ESN) to map the accumulated liquid states to accumulate target signals for this neuron.

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- In other words, each readout neuron solves a linear regression (just as with ESN) to map the accumulated liquid states to accumulate target signals for this neuron.
- To reduce overfitting, we use a regularized linear regression that penalizes strong synaptic weights. The parameter λ is selected with respect to a cross-validation dataset.

Training of a Liquid State Machine

- The training part of an LSM consists of finding the proper weights of the connections between the liquid and the readout neurons.
- Let $w_i \in \mathbb{R}^{n_{\text{rec}}}$ be the weights between a readout neuron i and the set of recorded liquid neurons. Training consists of finding the proper weights for all readout neurons.
- The liquid state $x(t) \in \mathbb{R}^{n_{\text{rec}}}$ is a vector containing the post-synaptic membrane potentials of the n_{rec} recorded liquid neurons at time t.
- We sample the liquid state at discrete timesteps of Δ_{sample} for $t \in [0, t_{\text{train}}]$. They are accumulated into a matrix $X \in \mathbb{R}^{n_{\text{rec}} \times n_{\text{sample}}}$ where $n_{\text{sample}} = t_{\text{train}}/\Delta_{\text{sample}}$
- The matrix X contains all the liquid states encountered during training. We, therefore, have the linear system for all readout neurons i:

$$y_i = X \cdot w_i \tag{7}$$

  where $y_i \in \mathbb{R}^{n_{\text{sample}}}$ is the sampled target (output) signals of readout neuron i.
- In other words, each readout neuron solves a linear regression (just as with ESN) to map the accumulated liquid states to accumulate target signals for this neuron.
- To reduce overfitting, we use a regularized linear regression that penalizes strong synaptic weights. The parameter λ is selected with respect to a cross-validation dataset.
- Once the weights $w_i \in \mathbb{R}^{n_{\text{rec}}}$ are sufficiently well trained, the test and prediction phases are similar to the ones described for the ESN.

Evaluation of performance of LSM

- We evaluate the performance of the method with respect to the predictions generated by the liquid. We define a metric to evaluate the predictions. This metric is general and consists of computing the normalized error for all tests or predictions:

$$E(W) = \frac{1}{n_{\text{samples}}^{\text{test}}} \sum_{i=1}^{n_{\text{samples}}^{\text{test}}} \left\| X^{\text{test}} \cdot w_i - y_i \right\|, \tag{8}$$

where X^{test} the accumulated liquid states during the test or prediction period, and $n_{\text{samples}}^{\text{test}}$ the number of samples in the test set. The residual error is the one we implicitly minimize when solving the linear equations for all readout neurons, see Eq. (7) above.