```
> #ex1
> ec1:=x*diff(y(x),x)-y(x)-ln(x^2+1)+Pi/2=0;
```

$$ec1 := x\left(\frac{d}{dx}y(x)\right) - y(x) - \ln(x^2+1) + \frac{1}{2}\pi = 0 \quad (1)$$

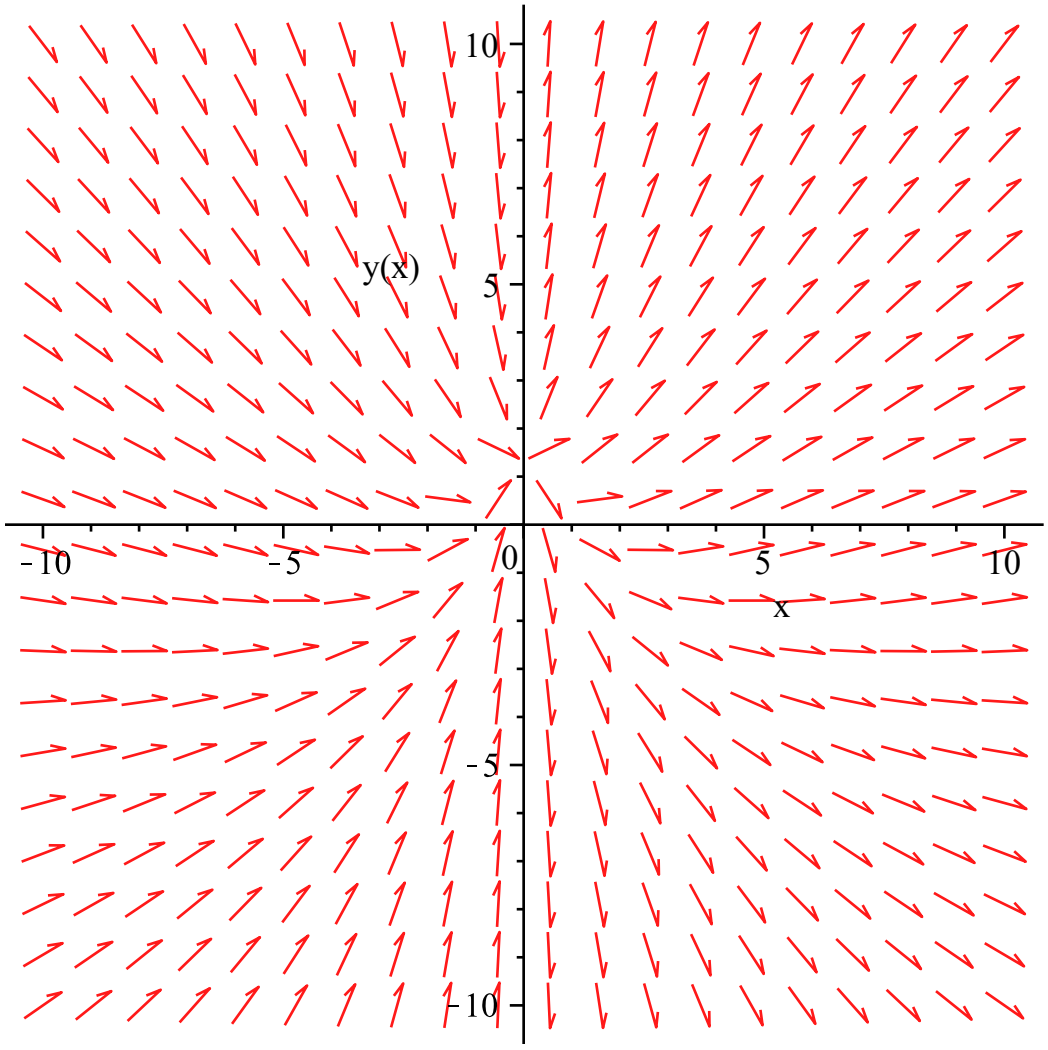
```
> sol1:=dsolve(ec1,y(x));
```

$$sol1 := y(x) = -\ln(x^2+1) + 2x\arctan(x) + \frac{1}{2}\pi + x\_C1 \quad (2)$$

```
> with(DEtools);
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*] (3)

```
> DEplot(ec1,y(x),x=-10..10,y=-10..10);
```




```
> cond1:=y(1)=a;
```

$$cond1 := y(1) = a \tag{4}$$

```
> sol1_C:=dsolve({ec1,cond1},y(x));
```

$$sol1\_C := y(x) = -\ln\left(x^2+1\right) + 2\,x\arctan(x) + \frac{1}{2}\,\pi + x\,(a + \ln(2) - \pi) \tag{5}$$

```
> f1:=unapply(rhs(sol1_C),x);
```

$$f1 := x \to -\ln\left(x^2+1\right) + 2\,x\arctan(x) + \frac{1}{2}\,\pi + x\,(a + \ln(2) - \pi) \tag{6}$$

```
> solve(f1(1)=0,a);
```

$$0 \tag{7}$$

```
> #ex2
> ec2:=diff(y(x),x,x)+3*diff(y(x),x)+2*y(x)=0;
```

$$ec2 := \frac{d^2}{dx^2}\,y(x) + 3\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0 \tag{8}$$

```
> sol2:=dsolve(ec2,y(x));
```

$$sol2 := y(x) = \_C1\,e^{-x} + \_C2\,e^{-2x} \tag{9}$$

```
> cond2:=y(0)=3,D(y)(0)=2;
```

(10)

$$cond2 := y(0) = 3, D(y)(0) = 2 \quad (10)$$

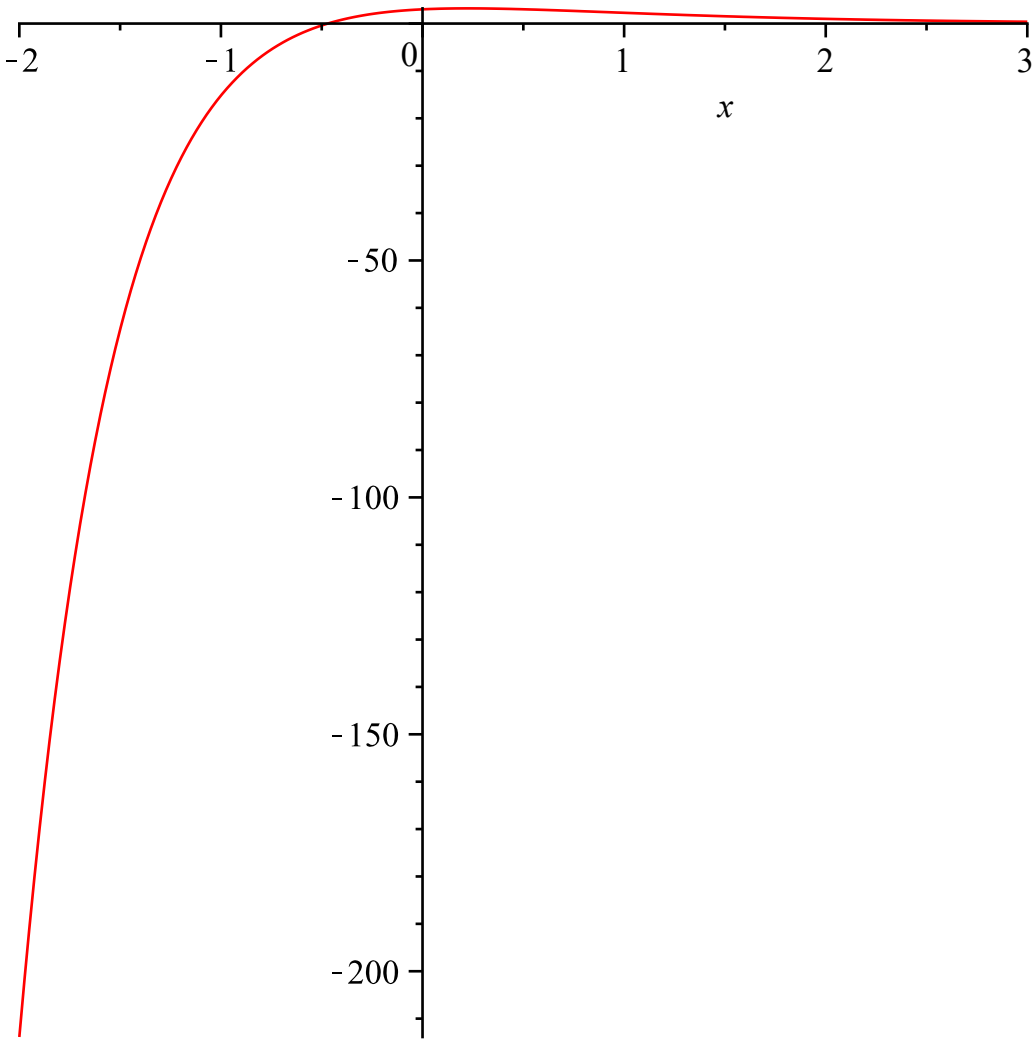
```
> sol2_C:=dsolve({ec2,cond2},y(x));
```

$$sol2\_C := y(x) = 8\,e^{-x} - 5\,e^{-2x} \quad (11)$$

```
> f2:=unapply(rhs(sol2_C),x);
```

$$f2 := x \to 8\,e^{-x} - 5\,e^{-2x} \quad (12)$$

```
> plot(f2(x),x=-2..3);
```




```
> #ex3
> ec3_1:=diff(x(t),t)=x(t)-y(t);
```

$$ec3\_1 := \frac{d}{dt}x(t) = x(t) - y(t) \quad (13)$$

```
> ec3_2:=diff(y(t),t)=x(t)+y(t);
```

$$ec3\_2 := \frac{d}{dt}y(t) = x(t) + y(t) \quad (14)$$

```
> sist_:=ec3_1,ec3_2;
```

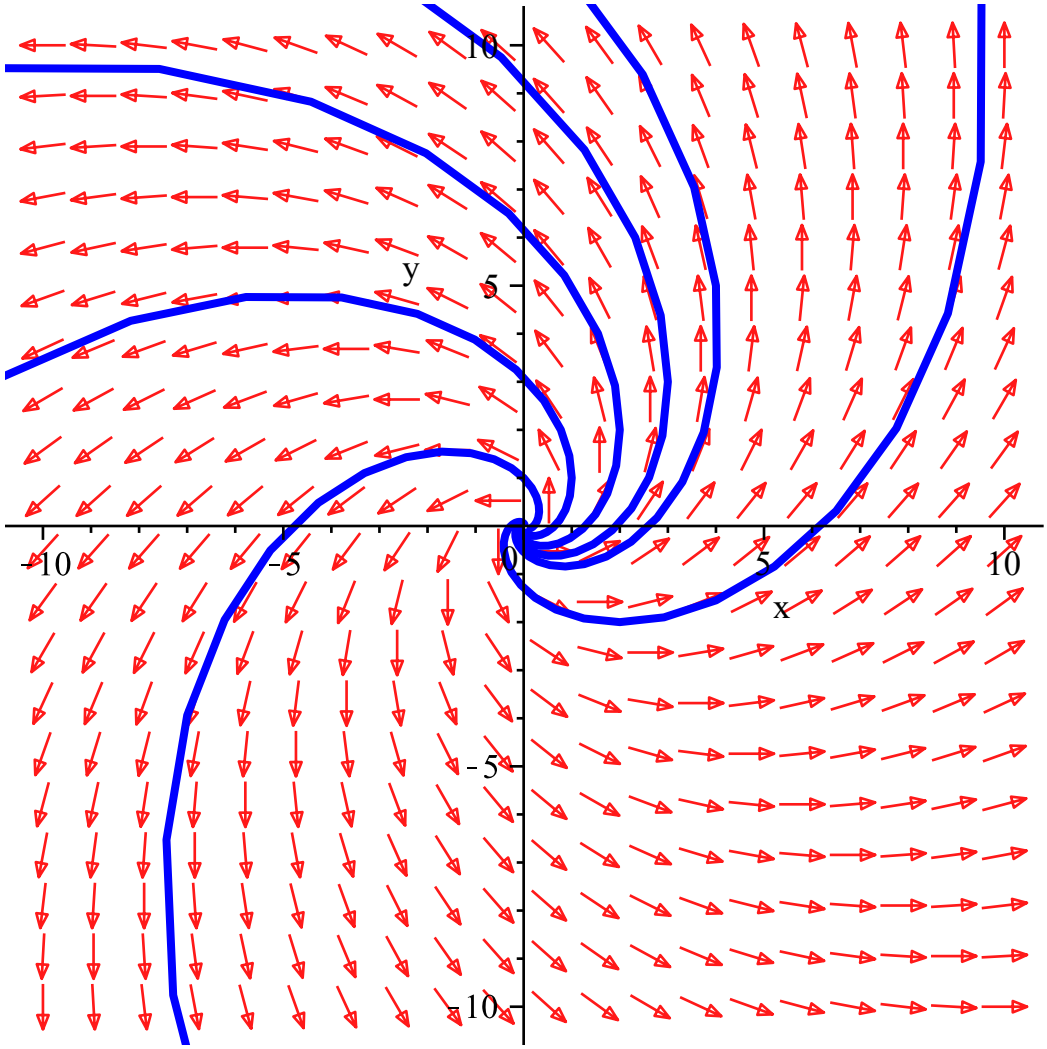
$$sist\_ := \frac{d}{dt}x(t) = x(t) - y(t),\ \frac{d}{dt}y(t) = x(t) + y(t) \quad (15)$$

```
> sol3:=dsolve({sist_},{x(t),y(t)});
```

(16)

$$sol3 := \{x(t) = e^t\,(\_C1\sin(t) + \_C2\cos(t)),\ y(t) = e^t\,(-\_C1\cos(t) + \_C2\sin(t))\} \quad (16)$$

```
> DEplot([sist_],[x(t),y(t)],t=-5..5,x=-10..10,y=-10..10,[[x(0)=0,y
  (0)=1],[x(0)=1,y(0)=1],[x(0)=2,y(0)=2],[x(0)=3,y(0)=3],[x(0)=4,y
  (0)=5],[x(0)=2,y(0)=-2]], arrows=medium, linecolor=blue);
```




```
> # conditia x(t)->0 si y(t)->0 cand t->infinit nu are loc. putem
  observa din directia sagetilor portretului fazic ca acestea tind
  catre infinit cand valoarea lui t creste;
```

```
> cond3_:=x(0)=-1,y(0)=1;
```

$$cond3\_ := x(0) = -1,\ y(0) = 1 \quad (17)$$

```
> sol3_C:=dsolve({sist_,cond3_},{x(t),y(t)});
```

$$sol3\_C := \{x(t) = e^t\,(-\sin(t) - \cos(t)),\ y(t) = e^t\,(\cos(t) - \sin(t))\} \quad (18)$$

```
> #ex4
```

```
> ec_:=diff(x(t),t)=r*x(t);
```

$$ec\_ := \frac{d}{dt}x(t) = r\,x(t) \quad (19)$$

```
> cond_:=x(0)=x0;
```

$$cond\_ := x(0) = x0 \tag{20}$$

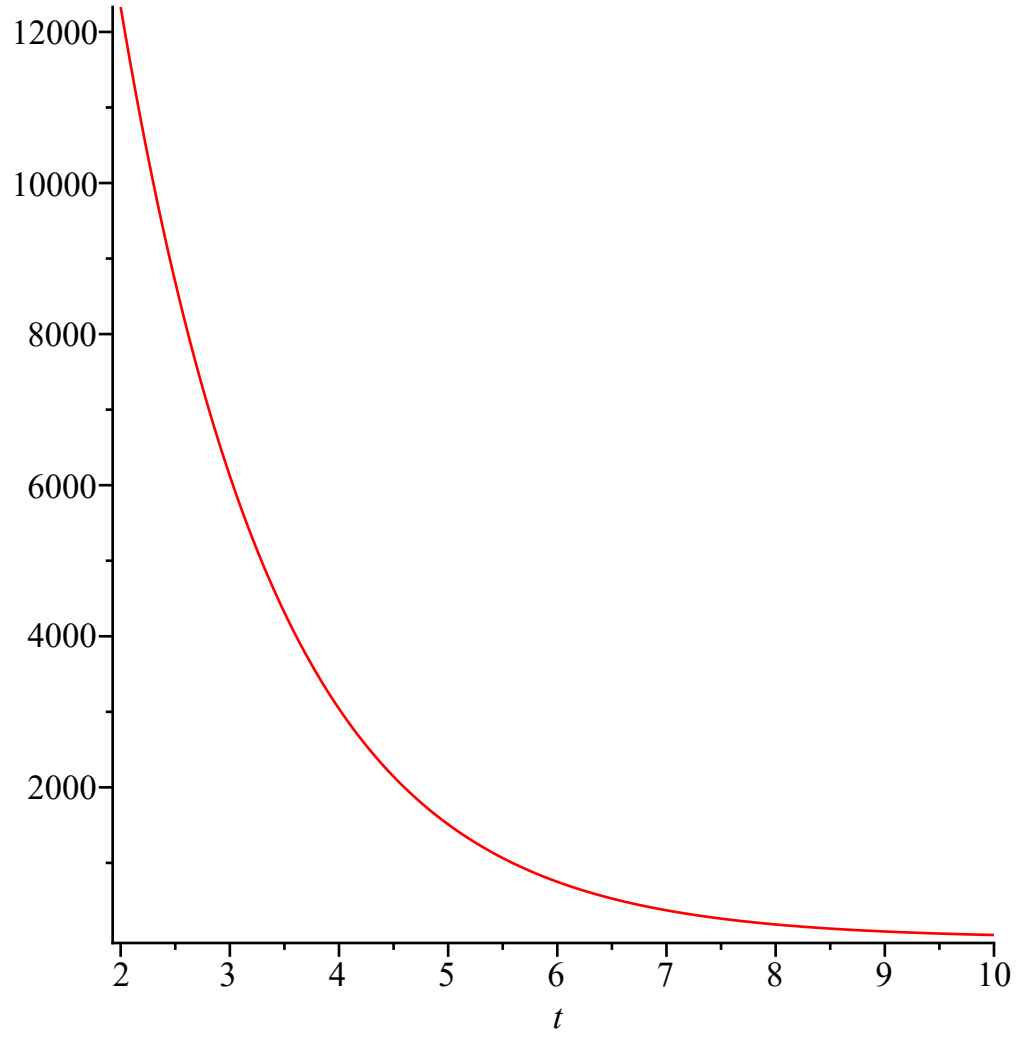
```
> solM:=dsolve({ec_,cond_},x(t));
```

$$solM := x(t) = x0\, e^{r\,t} \tag{21}$$

```
> f_M:=unapply(rhs(solM),t,x0,r);
```

$$f\_M := (t, x0, r) \rightarrow x0\, e^{r\,t} \tag{22}$$

```
> plot(f_M(t,5*10^4,-0.7),t=2..10);
```




```
> #determinam rata de crestere

> tFinal:=7; populatieInit:=5*10^4; populatieFin:=5.5*10^4;
```

$$tFinal := 7$$

$$populatieInit := 50000$$

$$populatieFin := 55000.0 \tag{23}$$

```
> ecPopulatie:=f_M(tFinal,populatieInit,r)=populatieFin;
```

$$ecPopulatie := 50000\, e^{7\,r} = 55000.0 \tag{24}$$

```
> rataC:=solve(ecPopulatie,r);
```

$$rataC := 0.01361573997 \tag{25}$$

```
> #calculam populatia dupa 10 ani
> populatieCautata:=f_M(10,populatieInit,rataC);
```

$$populatieCautata := 57293.11190 \tag{26}$$

```
> #ex5
> f5_1:=(x,y)->x-y-1;
```

$$f5\_1 := (x, y) \rightarrow x - y - 1 \tag{27}$$

```
> f5_2:=(x,y)->x^2*y+x*y;
```

$$f5\_2 := (x, y) \rightarrow x^2 y + y x \tag{28}$$

```
> ec5_1:=diff(x(t),t)=f5_1(x(t),y(t));
```

$$ec5\_1 := \frac{d}{dt} x(t) = x(t) - y(t) - 1 \tag{29}$$

```
> ec5_2:=diff(y(t),t)=f5_2(x(t),y(t));
```

$$ec5\_2 := \frac{d}{dt} y(t) = x(t)^2 y(t) + x(t)\, y(t) \tag{30}$$

```
> sist5_:=ec5_1,ec5_2;
```

$$sist5\_ := \frac{d}{dt} x(t) = x(t) - y(t) - 1, \frac{d}{dt} y(t) = x(t)^2 y(t) + x(t)\, y(t) \tag{31}$$

```
> pctEch:=solve({f5_1(x,y)=0,f5_2(x,y)=0},{x,y});
```

$$pctEch := \{x = 0, y = -1\}, \{x = -1, y = -2\}, \{x = 1, y = 0\} \tag{32}$$

```
> with(linalg);
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*] (33)

```
> J:=jacobian([f5_1(x,y),f5_2(x,y)],[x,y]);
```

$$J := \begin{bmatrix} 1 & -1 \\ 2\,y\,x + y & x^2 + x \end{bmatrix} \tag{34}$$

```
> A1:=subs(pctEch[1,1],pctEch[1,2],eval(J));
```

$$A1 := \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \tag{35}$$

```
> vals_:=eigenvals(A1);
```

(36)

$$vals\_ := \frac{1}{2} + \frac{1}{2}\sqrt{5},\ \frac{1}{2} - \frac{1}{2}\sqrt{5} \tag{36}$$

```
> evalf(vals_[1]); evalf(vals_[2]);
```

$$1.618033988$$
$$-0.6180339880 \tag{37}$$

```
> # => pct de echilibru de tip sa
```

```
> A2:=subs(pctEch[2,1],pctEch[2,2],eval(J));
```

$$A2 := \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \tag{38}$$

```
> eigenvals(A2);
```

$$\frac{1}{2} + \frac{1}{2}\,I\sqrt{7},\ \frac{1}{2} - \frac{1}{2}\,I\sqrt{7} \tag{39}$$

```
> #1/2>0 => pct de echilibru de tip focus instabil
```

```
> A3:=subs(pctEch[3,1],pctEch[3,2],eval(J));
```

$$A3 := \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \tag{40}$$

```
> eigenvals(A3);
```

$$1, 2 \tag{41}$$

```
> #punct de echilibru de tip nod instabil
> with(DEtools);
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*] **(42)**

```
> cond_portret:=[x(0)=i,y(0)=-i]$i=-2..3,[x(0)=i,y(0)=i]$i=-2..3,[x
  (0)=1/i,y(0)=1/i]$i=1..5,[x(0)=-i,y(0)=i]$i=-2..3;
```

$$cond\_portret := [x(0) = -2, y(0) = 2], [x(0) = -1, y(0) = 1], [x(0) = 0, y(0) = 0], [x(0) = 1, y(0) = -1], [x(0) = 2, y(0) = -2], [x(0) = 3, y(0) = -3], [x(0) = -2, y(0) = -2], [x(0) = -1, y(0) = -1], [x(0) = 0, y(0) = 0], [x(0) = 1, y(0) = 1], [x(0) = 2, y(0) = 2], [x(0) = 3, y(0) = 3], [x(0) = 1, y(0) = 1], \left[x(0) = \frac{1}{2}, y(0) = \frac{1}{2}\right], \left[x(0) = \frac{1}{3}, y(0) = \frac{1}{3}\right], \left[x(0) = \frac{1}{4}, y(0) = \frac{1}{4}\right], \left[x(0) = \frac{1}{5}, y(0) = \frac{1}{5}\right], [x(0) = 2, y(0) = -2], [x(0) = 1, y(0) = -1], [x(0) = 0, y(0) = 0], [x(0) = -1, y(0) = 1], [x(0) = -2, y(0) = 2], [x(0) = -3, y(0) = 3] \quad (43)$$

```
> restart;
```

```
> with(DEtools);
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*] (44)

```
> ec51:=diff(x(t),t)=x(t)-y(t)-1;
```

$$ec51 := \frac{d}{dt} x(t) = x(t) - y(t) - 1 \quad (45)$$

```
> ec52:=diff(y(t),t)=(x(t))^2*y(t)+x(t)*y(t);
```

$$ec52 := \frac{d}{dt} y(t) = x(t)^2 y(t) + x(t)\, y(t) \quad (46)$$

```
> sistem5:=ec51,ec52;
```

$$sistem5 := \frac{d}{dt} x(t) = x(t) - y(t) - 1, \frac{d}{dt} y(t) = x(t)^2 y(t) + x(t)\, y(t) \quad (47)$$

```
> cond_portret:=[x(0)=i,y(0)=1]$i=-2..2,[x(0)=i,y(0)=2]$i=-2..2,[x
  (0)=i,y(0)=0]$i=-2..2,[x(0)=i,y(0)=-i]$i=-2..2,[x(0)=-i,y(0)=-i]
  $i=-2..2,[x(0)=1/i,y(0)=-1/i]$i=1..5;
```

$$cond\_portret := [x(0) = -2, y(0) = 1], [x(0) = -1, y(0) = 1], [x(0) = 0, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 2, y(0) = 1], [x(0) = -2, y(0) = 2], [x(0) = -1, y(0) = 2], [x(0) = 0, \quad (48)$$

$y(0)=2],\ [x(0)=1, y(0)=2],\ [x(0)=2, y(0)=2],\ [x(0)=-2, y(0)=0],\ [x(0)=-1,$
$y(0)=0],\ [x(0)=0, y(0)=0],\ [x(0)=1, y(0)=0],\ [x(0)=2, y(0)=0],\ [x(0)=-2,$
$y(0)=2],\ [x(0)=-1, y(0)=1],\ [x(0)=0, y(0)=0],\ [x(0)=1, y(0)=-1],\ [x(0)=2,$
$y(0)=-2],\ [x(0)=2, y(0)=2],\ [x(0)=1, y(0)=1],\ [x(0)=0, y(0)=0],\ [x(0)=-1,$
$y(0)=-1],\ [x(0)=-2, y(0)=-2],\ [x(0)=1, y(0)=-1],\ \left[x(0)=\frac{1}{2}, y(0)=-\frac{1}{2}\right],$
$\left[x(0)=\frac{1}{3}, y(0)=-\frac{1}{3}\right],\ \left[x(0)=\frac{1}{4}, y(0)=-\frac{1}{4}\right],\ \left[x(0)=\frac{1}{5}, y(0)=-\frac{1}{5}\right]$

```
> DEplot([sistem5],[x(t),y(t)],t=-5..5,x=-2..2,y=-2..2,
  [cond_portret], linecolor=violet, stepsize=0.1);
```