Transformări geometrice

Formula generală pentru produsul tensorial a doi vectori este

$$\mathbf{a} \otimes \mathbf{b} = \mathbf{b} \cdot \mathbf{a}^t.$$

Astfel, în plan

$$\mathbf{a} \otimes \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \cdot \begin{pmatrix} a_1 & a_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_2 b_1 \\ a_1 b_2 & a_2 b_2 \end{pmatrix},$$

iar în spaţiu

$$\mathbf{a} \otimes \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \cdot \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_2 b_1 & a_3 b_1 \\ a_1 b_2 & a_2 b_2 & a_3 b_2 \\ a_1 b_3 & a_2 b_3 & a_3 b_3 \end{pmatrix}.$$

Transformări plane

Translaţia în plan

$$\mathrm{Trans}(\mathbf{w}) = \begin{bmatrix} 1 & 0 & w_1 \\ 0 & 1 & w_2 \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

Rotaţia

$$\mathrm{Rot}(Q, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & q_1(1-\cos\theta) + q_2 \sin\theta \\ \sin\theta & \cos\theta & -q_1 \sin\theta + q_2(1-\cos\theta) \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

Scalarea uniformă

$$\mathrm{Scale}(Q, s) = \begin{pmatrix} s & 0 & (1-s)q_1 \\ 0 & s & (1-s)q_2 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

Scalarea neuniformă

$$\mathrm{Scale}(Q, s_x, s_y) = \begin{pmatrix} s_x & 0 & (1-s_x)q_1 \\ 0 & s_y & (1-s_y)q_2 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

Reflexia

$$\mathrm{Mirror}(Q, \mathbf{w}) = \begin{pmatrix} I_2 - 2\left(\mathbf{w}^\perp \otimes \mathbf{w}^\perp\right) & 2\left(\mathbf{w}^\perp \otimes \mathbf{w}^\perp\right) \cdot Q \\ 0 & 1 \end{pmatrix}. \tag{5}$$

Forfecarea

$$\operatorname{Shear}(Q,\mathbf{w},\theta)=\begin{pmatrix} I_2+\operatorname{tg}\theta\left(\mathbf{w}^{\perp}\otimes\mathbf{w}\right) & -\operatorname{tg}\theta\left(\mathbf{w}^{\perp}\otimes\mathbf{w}\right)\cdot Q \\ 0 & 1\end{pmatrix}. \tag{6}$$

Transformări în spaţiu

Translaţia

$$\operatorname{Trans}(\mathbf{w})=\begin{pmatrix} 1 & 0 & 0 & w_1 \\ 0 & 1 & 0 & w_2 \\ 0 & 0 & 1 & w_3 \\ 0 & 0 & 0 & 1\end{pmatrix}. \tag{7}$$

Rotaţia

$$\operatorname{Rot}(Q,\mathbf{u},\theta)=\begin{pmatrix} \operatorname{Rot}(\mathbf{u},\theta) & (I_3-\operatorname{Rot}(\mathbf{u},\theta))\cdot\mathbf{Q} \\ 0 & 1\end{pmatrix}, \tag{8}$$

unde

$$\operatorname{Rot}(\mathbf{u},\theta)=\cos\theta\cdot I_3+(1-\cos\theta)(\mathbf{u}\otimes\mathbf{u})+\sin\theta\cdot(\mathbf{u}\times-) \quad \text{şi} \quad \mathbf{u}\times-=\begin{pmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0\end{pmatrix}.$$

Scalarea uniformă

$$\operatorname{Scale}(Q,s)=\begin{pmatrix} s & 0 & 0 & (1-s)\cdot q_x \\ 0 & s & 0 & (1-s)\cdot q_y \\ 0 & 0 & s & (1-s)\cdot q_z \\ 0 & 0 & 0 & 1\end{pmatrix}. \tag{9}$$

Scalarea neuniformă

$$\operatorname{Scale}(Q,s_x,s_y,s_z)=\begin{pmatrix} s_x & 0 & 0 & (1-s_x)\cdot q_x \\ 0 & s_y & 0 & (1-s_y)\cdot q_y \\ 0 & 0 & s_z & (1-s_z)\cdot q_z \\ 0 & 0 & 0 & 1\end{pmatrix}. \tag{10}$$

Reflexia

$$\operatorname{Mirror}(Q,\mathbf{n})=\begin{pmatrix} I_3-2(\mathbf{n}\otimes\mathbf{n}) & 2(\mathbf{n}\otimes\mathbf{n})\cdot Q \\ 0 & 1\end{pmatrix} \tag{11}$$

Forfecarea

$$\operatorname{Shear}(Q,\mathbf{n},\mathbf{u},\theta)=\begin{pmatrix} I_3+\operatorname{tg}\theta\cdot(\mathbf{n}\otimes\mathbf{u}) & -\operatorname{tg}\theta\cdot(\mathbf{n}\otimes\mathbf{u})\cdot Q \\ 0 & 1\end{pmatrix}. \tag{12}$$