H5:

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Questions:

1. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:
   a. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

i.

		P	~P	Q	P -> Q
S	F	S -> F ≡ ~S V F	~(~S v F) ≡ S ^ ~F	~S -> ~F	~P V Q
F	F	T	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
T	T	T	F	T	T

   ii. This statement is neither: valid, unsatisfiable
   iii. It is satisfiable for some interpretations, but not valid for all interpretations

   b. (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

i.

			P	T	Q	P -> Q
S	H	F	S -> F	S V H	~T V F	~P V Q
F	F	F	T	F	T	T
F	F	T	T	F	T	T
F	T	F	T	T	F	F
F	T	T	T	T	T	T
T	F	F	F	T	F	T
T	F	T	T	T	T	T
T	T	F	F	T	F	T
T	T	T	T	T	T	T

   ii. This statement is neither: valid, unsatisfiable
   iii. It is satisfiable for some interpretations, but not valid for all interpretations

   c. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

i.

			A	P	Q	P <-> Q
S	H	F	S ^ H	~A V F	(S -> F) V (H -> F)	P ≡ Q
F	F	F	F	T	T	T
F	F	T	F	T	T	T
F	T	F	F	T	T	T
F	T	T	F	T	T	T
T	F	F	F	T	T	T
T	F	T	F	T	T	T

T	T	F	T	F	F	T
T	T	T	T	T	T	T

ii. P and Q are logically equivalent.
iii. The statement is satisfiable and valid.

Justify your answer using truth tables.

2. Consider
   a. Knowledge base:
      Define: -> for logical implication, = for assigning a statement {P,Q,R,S }to the propositional logic sentence, ^ for AND/Conjunction , V for OR/Disjunction, ~ for NOT/Negation of statement
      i. P = Mythical -> Immortal
      ii. Q = ~Mythical -> ~Immortal ^ Mammal
      iii. R = Immortal V Mammal -> Horned
      iv. S = Horned -> Magical
   b. CNF
      Definition: Clauses connected by conjunction and have disjunction inside to connect literals. EX: (A V ~B) ^ (A V C V ~D)
      I will highlight the CNF form
      i. (~Mythical) V (Immortal)
      ii. Mythical V (~Immortal ^ Mammal) ≡ ~(~Mythical ^ (Immortal V ~Mammal)) ≡ ~((~Mythical ^ Immortal) V (~Mythical ^ ~Mammal)) ≡ (Mythical V ~Immortal) ^ (Mythical V Mammal)
      iii. (~Immortal ^ ~Mammal) V (Horned) ≡ (~Immoral V Horned) ^ (~Mammal V Horned)
      iv. (~Horned V Magical) ^ 1 ≡ (~Horned V Magical)
   c. We are unable to prove the unicorn is magical. It can be proven that it is mythical or horned.
      i.

1. ~Immortal -> ~Mythical	Contrapositive of P
2. ~Immortal -> ~Immortal ^ Mammal	Syllogism of (1) and Q
3. Immortal V (~Immortal ^ Mammal)	Implication on (2)
4. (Immortal V ~Immortal) ^ (Immortal V Mammal)	Distribution
5. (Immortal V Mammal) ≡ 1 ^ (Immortal V Mammal)	Identity and Simplification
6. Horned	Modus ponens on (5) and R
7. Mythical	Modus ponens on (6) and S

3. Consider
   a. Basis
      i. P(oil) = .5
      ii. P(gas) = .2
      iii. P(neither) = .3
      iv. P(oil ^ gas) = 0
      v. P(+ | oil) = .9
      vi. P(+ | gas) = .3
      vii. P(+ | neither) = .1
   b. Test comes back positive, what is probability of oil?

i. P(+) = P(+|gas)P(gas) + P(+|oil)P(oil) + P(+|neither)P(neither)
    1) P(+) = .5*.9 + .2*.3 + .3*.1 = 0.54
ii. P(oil | +) = P(oil)P(+|oil) / P(+)
    1) P(oil | +) = (.5*.9)/(.54) = 0.8333
    2) About 83% probability of oil