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Figure 1a shows a standard CMOS inverter. However, during manufacturing, the circuit was contaminated with a particle and the gate of the PMOS transistor got shorted to GND. The contaminated inverter circuit could be modeled as shown in Figure 1b.

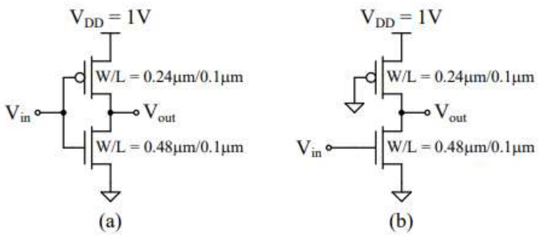


Figure 1: CMOS Inverter.

(*Note:* L indicates drawn channel length).

1A Find $\mathbf{V_{OH}}$, $\mathbf{V_{OL}}$, and $\mathbf{V_M}$ for the inverters in Figures 1a and 1b. (Don't assume that $V_M = V_{DD}/2$, do some hand calculation)

1A

Assume typical value for low output voltage and high output voltage of CMOS Inverter and given that VCC = VDD = 1V

$V_{OH} = 1\,V$: $V_{OH} = V_{CC} \rightarrow V_{OL} = 0$

$V_{OL} = 0\,V$

NMOS stronger: $\frac{K_n}{K_p} = \frac{K_n'}{K_p'} \cdot \frac{\left(\frac{W}{L}\right)_N}{\left(\frac{W}{L}\right)_P}$: $\left(\frac{W}{L}\right)_N > \left(\frac{W}{L}\right)_P$

Find V_M, assume transistors are saturated.

$$I_{dN} = \frac{1}{2} K_N' \left(\frac{W}{L}\right)_N \left(V_{GS} - V_{TH}\right)_N^2 \left(1 + \lambda V_{DS}\right)_N$$

$$(0.5)(140)(5.64)(V_M - .17)^2(1 + .75V_M)$$

$$394.8\left[V_M^2 - .34V_M + .0289\right]\left[1 + .75V_M\right]$$

$$394.8\left[V_M^2 - .34V_M + .0289 + .75V_M^3 - .255V_M^2 + .021675V_M\right]$$

$$394.8\left[.75V_M^3 + .745V_M^2 - .3183V_M + .0289\right]$$

$$I_{dP} = \frac{1}{2} K_P' \left(\frac{W}{L}\right)_P \left(V_{GS} - V_{TH}\right)_P^2 \left(1 + \lambda V_{DS}\right)_P$$

$$= (0.5)(100)(2.82)(1 - V_M - .2)^2(1 + .62(1 - V_M))$$

$$141\left[.8 - V_M\right]^2\left[1.62 - .62V_M\right]$$

.5*140*5.64 = 394.8 , .5*100*2.82 = 141

$I_{dP} = I_{dN} \rightarrow$ using Software $V_M \approx \boxed{0.40911}$ or 2 complex results

1B

Let input voltage be $V_{OH} = 1V$, then assume $V_{OL} < V_{DSAT} \rightarrow$ NMOS is in Linear region while PMOS is saturated.

assume $V_{OL} < V_{DSAT}$ → NMOS is in Linear region while PMOS is saturated.

Sat Lin VSAT

$$M_n: \min(V_{gt}, V_{DS}, V_{DSAT}) = \min(V_M - V_T, V_M, 0.3)$$

$$= \min(V_M - .17, \underline{V_M}, 0.3)$$

$$M_p: \min(|V_{gt}|, |V_{DS}|, |V_{DSAT}|) = \min(1 - V_m - V_T, 1 - V_M, 0.4)$$

$$= \min(1 - V_M - .2, 1 - V_M, .4) = \min(\underline{.8 - V_M}, 1 - V_M, .4)$$

NMOS off during V_{OL} → $V_{OH} = 1V$, $V_{OL} = 0.12\ V$

$$I_{dN} = k_N'\left(\frac{W}{L}\right)_N\left[(V_{GS} - V_{TH})V_{DSAT} - \frac{V_{DSAT}^2}{2}\right]_N (1 + \lambda V_{DS})_N$$

$$(140)(5.64)\left[(V_M - .17)0.3 - 0.045\right](1 + .75 V_M)$$

$$I_{dp} = k_p'\left(\frac{W}{L}\right)_p\left[(V_{GS} - V_{TH})_p V_{DS} - \frac{V_{DS}^2}{2}\right]_p (1 + \lambda V_{DS})_p$$

$$(100)(2.82)\left[(1 - V_M - .2)(1 - V_M) - \tfrac{1}{2}(1 - V_M)^2\right](1 + \lambda(1 - V_M))$$

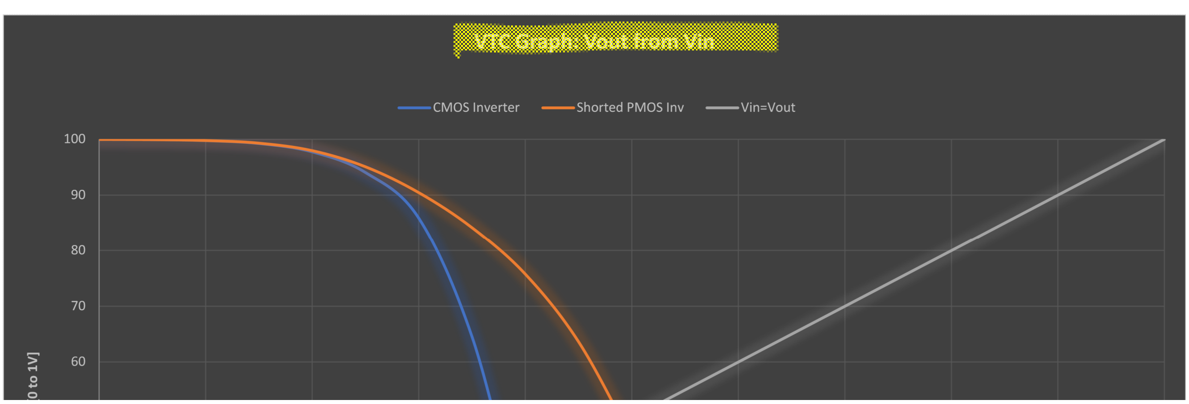
$$282(1 - V_M)\left[(1 - .2) - \tfrac{1}{2}(1 - V_M)\right](1.62 - .62V_M)$$

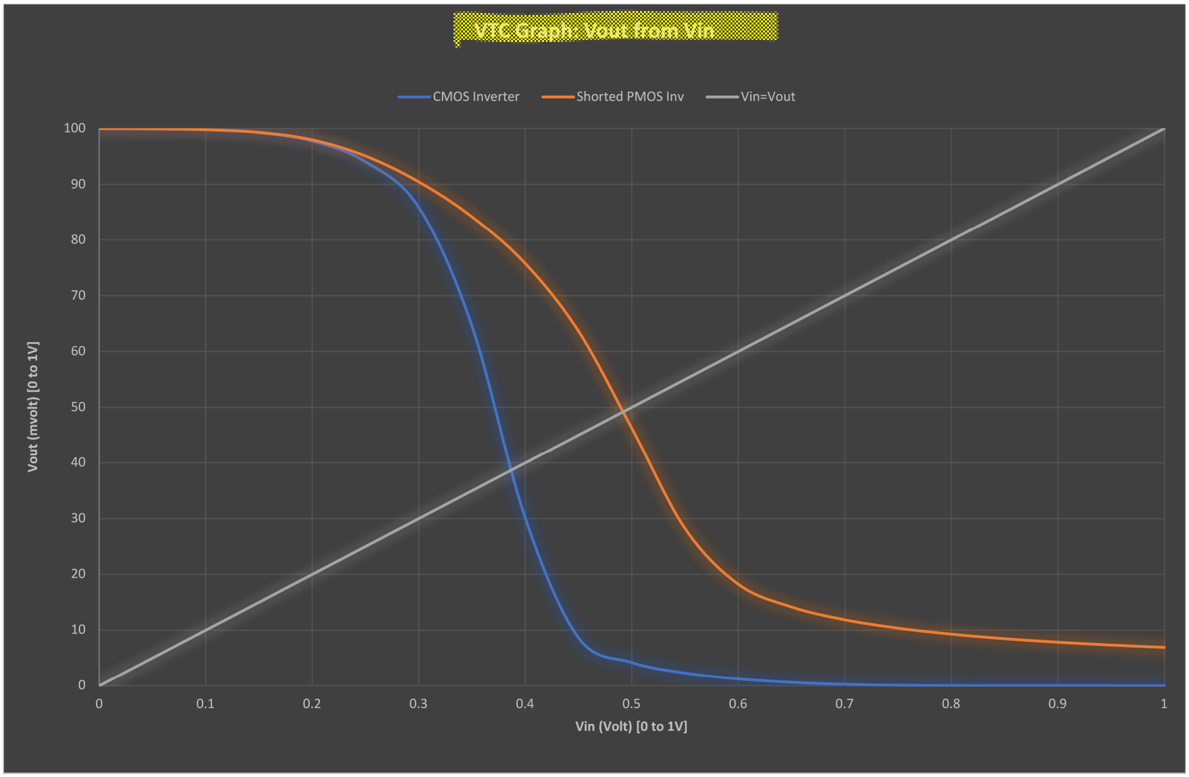
140*5.64=789.6

$I_{dN} = I_{Dp}$ using Software → $V_M \simeq -0.79, 0.578, 5.26$

$V_M \simeq 0.578\ V$

For the below curve I had to scale the left side as millivolts to get the graph values to show correctly in Excel





VTC Curves: The Maximum Low Output Voltage of the inverter is actually not zero in our PMOS shorted to ground circuit. We also see a right shift due to this transistor shorting to ground compared with our CMOS Inverter, so the drop in output voltage occurs at higher input voltages. This may indicate that the PMOS never fully shut off as we would expect both NMOS and PMOS to change states throughout the graph, but the output voltage never went completely to 0.

Robustness: The PMOS shorted figure is considered less robust. Consider NM_L *such that* $V_{IL} - V_{OL}$ *can be analyzed from either line curve such that* $NM_L < NM_H$ *proving this analyis of robustness.*

Regeneration: Both curves appear to closely represent the graphs we studied in class in both shape and a nice steepness where we would expect slopes to approximate to -1. However, it is clear upon analysis that the CMOS inverter does have a steeper dropoff for the undefined region where we expect the MOSFETs to start switching . Because of the larger gain for the first curve, there is faster regeneration there.

2)

1. $$I_{DSAT} = K'\left(\frac{W}{L}\right)\left((V_{DD}-V_{TH})V_{DSAT} - \frac{V_{DSAT}^2}{2}\right)$$

2. $$R_{on-approx} = \frac{3}{4}\frac{V_{DD}}{I_{DSAT}}\left(1 - \frac{5}{6}\lambda V_{DD}\right)$$

Ron VDD	Idsat (uA)	Analytical Model (Req)	Simulation
.4	7.6	29.1	27.8
.6	26.29	10.3	9
.8	44.9	6.2	6.5
[illegible]	[illegible]	[illegible]	[illegible]

.6	26.29	10.3	9
.8	44.9	6.2	6.5
1	63.6	3.9	5

Lambda is held as a constant in these calculations but often this value can change along with changes in Vgs . So a better depiction of the results could be obtained from looking at different values of Vgs since Vgs is linked to the Vcc or Vdd that is typically fed into the inverter. It is likely that if we used the adjusted lambda values for each Vgs corresponding to it, then we may get more accurate values to Ron simulated.

We also note that the values for Vdd and possibly Vgs of 0.8 and 1 have a smaller percent error from the expected simulation values. Therefore, there is likely an error that arises from using our calculations for lower values of Vdd and Vgs with our formulas. It is likely that Vdsat is not necessarily in the saturation region this early and is giving slightly off results. This would justify why our later values in analytical model are closer to simulation values.

3)

3a

Vil	.3
Vih	.6
Vol	.2
Voh	.9
NMl	.1
NMh	.3

3b

(i) $E_{H(i)} = C_L \left[1 V_{DD} - \frac{1}{2} \cdot 1^2\right] = 100\left(1-\frac{1}{2}\right) = 50 fJ$

(ii) $E_{H(ii)} = C_L \left[\frac{1}{2} \cdot 1^2 - \frac{1}{2}(0.3)^2\right] = 50(1-0.09) = 45.5 fJ$

(iii) $E_H(iii) = C_L \left[V_{DD}(1-0.3) - \frac{1}{2}(1^2 - 0.3^2)\right] = 100\left[1(0.7) - \frac{1}{2}(.91)\right] = 24.5 fJ$

i	50 fJ
ii	45.5 fJ
iii	24.5 fJ