FAIR ALGORITHMS FOR CLUSTERING

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Why do we care?

ARTIFICIAL INTELLIGENCE

Bias isn’t the only problem with credit scores—and no, AI can’t help

The biggest-ever study of real people’s mortgage data shows that predictive tools used to approve or reject loans are less accurate for minorities.

By Will Douglas Heaven June 17, 2021

NEWS | 24 October 2019 | Update 26 October 2019

Millions of black people affected by racial bias in health-care algorithms

Study reveals rampant racism in decision-making software used by US hospitals – and highlights ways to correct it.

TECH

Google Photos labeled black people 'gorillas'

Jessica Guynn USA TODAY

Published 1:15 p.m. ET July 1, 2015 | **Updated 2:10 p.m. ET July 1, 2015**

Technical Background

- Motivated by the Disparate Impact (DI) doctrine
- Minority Protection
  - Protect (include) under-represented group in a cluster
- Restricted Dominance
  - Restrict a certain group to overpower in a cluster
- Goal: Minimize inequality of outcome
- Use the output of the variety of vanilla (conventional) clustering algorithms to and use it to balance the groups
  - The algorithm can have overlapping groups
  - Any vanilla (k, p)-clustering problem can be extended to accommodate fairness.

Definitions

- C = set of all (v) points of dimension d we want to cluster in a metric space (χ)
- $F \subseteq \chi$ be the set of possible 'k' cluster centers, $k \in \mathbb{Z}^+$
- d(v, S) = minimum distance between a point y $\in$ set S and point v
- ϕ = Assignment C → S, ϕ^ = Fair assignment C → S
- L_p (S,ϕ) = Objective function
- Δ = maximum number of groups a point can be a part of
- α_i = restricted dominance (RD) constraint for each group i
- β_i = minority protection (MP) constraint for each group i
- λ = additive violation of RD/MP constraint in the fair clustering

Fair (k, p) clustering

Assume (l) number of groups (C): $C_1, C_2, \ldots, C_l$

The following constraints have to be satisfied:

$$|\{v \in C_i : \phi(v) = f\}| \leq \alpha_i |\{v \in C : \phi(v) = f\}|$$ (Upper Bound – Restricted dominance)

$$|\{v \in C_i : \phi(v) = f\}| \geq \beta_i |\{v \in C : \phi(v) = f\}|$$ (Lower Bound – Minority protection)

The RD and MP constraint prevent under-representation / over-representation of any particular group within a cluster.

Linear programming formulation results in λ-additive violation for fair clustering:

$$\beta_i|\{v \in C : \phi(v) = f\}| - \lambda \leq |\{v \in C_i : \phi(v) = f\}| \leq \alpha_i|\{v \in C : \phi(v) = f\}| + \lambda$$

Solution Technique

1. Use any Vanilla (k, p) clustering algorithm on the given data set (C)
2. Use LP 1 to solve the algorithm:

$$\min \sum_{v \in C, f \in S} d(v,f)^p x_{v,f}$$

This is an LP relaxation of the assignment problem

s.t: $$\beta_i \sum_{v \in C} x_{v,f} \leq \sum_{v \in C_i} x_{v,f} \leq \alpha_i \sum_{v \in C} x_{v,f} \; \forall f \in S, i \in [l]$$

$$\sum_{f \in S} x_{v,f} = 1 \; \forall v \in C$$

$$0 \leq x_{v,f} \leq 1$$

3. For each $x^*_{v,f} = 1$, set $\phi\hat{}(v) = f$ and remove v from C [and relevant (C_i)s].
4. Define: $T_f = \sum_{v \in C} x^*_{v,f} \; \forall f \in S$ & $T_{f,i} = \sum_{v \in C_i} x^*_{v,f} \; \forall f \in S, i \in [l]$

5. Construct LP 2 with decision variables for $x^*_{v,f} > 0$

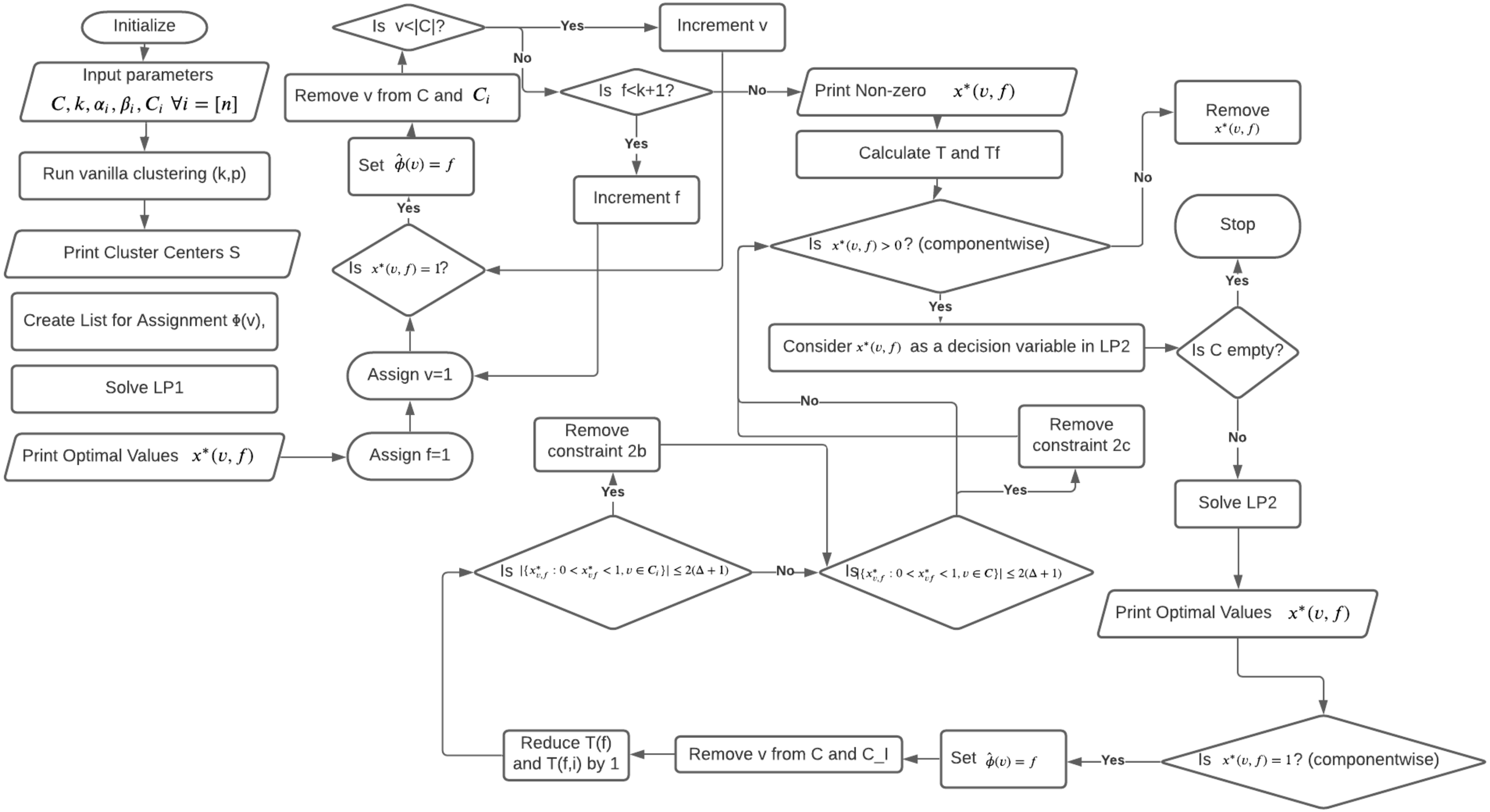
6. While there exists $v \in C$ such that $\hat{\phi}(v) = \emptyset$:

- Solve LP2 and get the optimal value $(x^*_{v,f})$

$$
\begin{aligned}
\mathrm{LP2} := \min \sum_{v \in C, f \in S} & d(v,f)^p x_{v,f} && x_{v,f} \in [0,1], \ \forall v \in C, f \in S && \text{(2a)} \\
\lfloor T_f \rfloor \ \leq \ & \sum_{v \in C} x_{v,f} \ \leq \ \lceil T_f \rceil && \forall f \in S, \forall i \in [\ell] && \text{(2b)} \\
\lfloor T_{f,i} \rfloor \ \leq \ & \sum_{v \in C_i} x_{v,f} \ \leq \ \lceil T_{f,i} \rceil && \forall f \in S, \forall i \in [\ell] && \text{(2c)} \\
& \sum_{f \in S} x_{v,f} \ = \ 1 && \forall v \in C && \text{(2d)}
\end{aligned}
$$

- For each $x^\star_{v,f} = 0$, delete the $x^\star_{v,f}$ variable from LP2
- For each $x^\star_{v,f} = 1$, set $\hat{\phi}(v) = f$ and remove v from C
- for every $i \in [\ell]$ and $f \in S$, if $| x^\star_{v,f} : 0 < x^\star_{v,f} < 1, v \in C_i | \leq 2(\Delta + 1)$ remove the respective constraint in eq. (2c)
- for every $f \in S$, if $| x^\star_{v,f} : 0 < x^\star_{v,f} < 1, v \in C| \leq 2(\Delta + 1)$ remove the respective constraint in eq. (2b)

Algorithm Flowchart



Application Example

- Worked with ”bank” dataset from the original paper.
  - Number of clusters: k = 3, 4, 5 and 6
  - Protected groups: marital (single, married, and divorced) and default (Y/N)
  - Vanilla clustering approach used: k-means++, with three attributes

Metrics for evaluation of model:

- r_i = proportion of each group (i)
- $r_i(f)$ = proportion of each group (i) with cluster center f
- $r_i(f) = |C_i(f)|/|C(f)|$, $r_i = |C_i|/|C|$
- Balance (f) = $\min\{r_i/r_i(f), r_i(f)/r_i\}$

- Cost of fairness (COF) = objective value of fair clustering / objective value of vanilla clustering

Results

- Cost of fairness is observed to be slightly greater than 1 → Fair clustering is not expensive compared to vanilla k-means++ !

- Balance values are close to one for most clusters → fairness is ensured!

- Although theoretical prediction of λ additive violation is 4Δ+3, it is observed that it does not cross 1 in practice → MD and RD constraints are violated only by a small margin!