**Factor**

**Exact**

*Exact*

**Partial Integration:**

*Treat as a constant*

Find

Then integrate to find and

**Linear, first order**

Integrating factor

**Homogeneous Function**

**Homogeneous DE**

and

or

Separate variables and use integrating factor

**Wronskian**

**Superposition for linear DE**

If and are solutions of so is .

**2nd order, constant coefficient**

Find solving with

Find roots , of

repeated times:

Find , then

**Undetermined coefficients**

Convert to -form

Find roots of the characteristic equation

Find roots of the RHS by inverse inspection

*Limitation: RHS must be such that we can find*

Write considering every and

Identify and

Subs in the DE to find the constants

constants are found using the initial conditions

**Variation of parameters**

Find as in undetermined coefficients. Write:

Find derivatives for and solve the system for

Find integrating

**Bernoulli**

**Euler**

ODEs

, where

Solve the resulting linear equation.

**Power series solution**

is an **analytic function** if it has a power series represent. around .

**Initial Value Problem (IVP)**

analytic about

, in

Subs the IC to find

Differentiate the DE and subs and to find

*Drawback: differentiating the DE might not be practical.*

**Power series solution using recurrence relation**

Subs in the equation.

Shift indices in the summations so that the power of is the same in all

*Be careful not to loose terms!*

The coefficients of each power must be equal in LHS and RHS

Find recurrence relation for the each

Subs in and expand the summations as needed.

**Shifting**

**Singularities and the method of Frobenius**

analytical about ; ordinary

**Singular point of the DE**: , or has zero denominator about .

**Regular singular:** and are analytical

**Ordinary:** not singular. **Irregular:** Not regular

If equation has a regular singular point at , use a Frobenius series:

, where

Subs in the equation, shift indices etc. (same as pwr series)

Assume to find values for (.

Find the recurrence using . Write

For the **second solution**, find the recurrence as:

If

If

If

Subs into the DE and obtain an equation for

*1 Subs into the DE and the term vanishes as the terms it multiplies are equal to .*

*2 We use for convenience*

*3 Subs the found solution after simplifications*

Final solution:

**Bessel’s equation of order**

*, ,*

Bessel function of the 1st kind:

Bessel function of the 2nd kind:

**Matrices and vectors**

LIN ALG

LIN ALG

: row :column

: row matrix : column matrix

*(component of* ***a*** *in* ***b****)*  *(projection of* ***a*** *in* ***b****)*

**Definitions**

Conjugate: :

Rank: largest non zero determinant; number of independent vectors;

Trace: *t*

**Identities**

If is symmetric, then so is ,

**Square matrices**

Symmetric: Skew-Symmetric:

Positive definite: Non negative definite:

Indefinite:

Orthogonal

Nilpotent: and

Idempotent:

Involutory: Unitary:

Positive: Non-negative:

Diagonal Dominant: Strictly Diag Dom:

Associate: Hermitian: Skew Hermitian:

**Determinants**

A is a matrix, then:

Minor: Cofactor:

, for any row

Adjoint matrix is the transpose of the cofactor’s matrix.

**Inverse** *Inverses are unique*

**Properties of determinants**

If any col or row is null, then

If operate columns or rows, then does not change

If swap columns or rows, then changes sign

If two columns or rows are proportional then

If one column or row is the linear combination of others then

Multiply column or row by then

If , is singular and has no inverse.

**Set of vectors**

are linearly independent for at least one set of .

is a base if exists a unique choice of scalars for every vector . That is, are independent

is orthogonal

is orthonormal

Normalization:

**Gram-Schmidt orthogonalization of {}** are linearly independent

**Systems of linear equations**

Cramer’s rule: , where and is with column replaced by vector .

unique solution

infinite solutions

infinite solutions . Otherwise, no solution.

is diagonal: system is called uncoupled or the variables are called separated

and are similar: and , whare is the set of eigenvectors.

**Gaussian Elimination**

Operate equations/rows to *uptriangularize* the system.

Back substituition to find variables.

**Augmented matrix**

Operate rows to find

Operate to find solution and inverse as

**LU Factorization**

**Iterative method: Jacobi**

**Eigenvalues and Eigenvectors**

Characteristic equation (CE): ,

Find eigenvalues as the scalar roots of the CE

Replace every in the CE to find eigenvectors associated to each

must be linearly independent

is a zero of its characteristic equation *(Cayley Hamilton)*.

If, for any , is singular

If is real and symmetric

If is diagonal and

If is upper or lower triangular and

**Companion Matrix**

**Partitioned Matrix**

LAPLACE

**Workflow:** IVP 🡪 🡪 Algebra 🡪

**Properties - linearity**

**Operations**

**Periodic function:**

**Partial fractions**

Factor in linear factors and quadratic factors .

Solve for ,

**Remark:**

Ex:

Ex: Avoid imaginary factors:

**Step Function**

,

**Impulse Function**

**Convolution**

Ex:

**Polynomial coefficients**

Let and

Ex: New linear DE! Solve with Integration Factor to find .

**Systems of DE using Laplace Transforms**

Ex:

Solve for and . Invert to find and .

**Integral by parts:**

**Euler**

**Baskhara**

INTEGRALS