BRI509: Introduction to Brain Signal Processing
Assignment No. 2

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1. Explain the following terms.

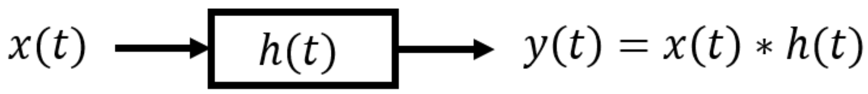
(a) Impulse response

Meaning:

The impulse response is the system's response to a unit impulse occurring at $t = 0$. If the input signal is $x(t) = \delta(t)$, then the impulse response is $y(t) = h(t)$.

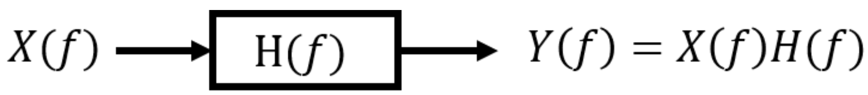
Convolution

The response of a system to an arbitrary input signal $x(t)$ can be calculated by convolving the input signal with the impulse response, that is, $y(t) = x(t) * h(t)$.



CTFT

In the Fourier-domain, the response of a system to an arbitrary input signal $x(t)$ can be calculated by taking the inverse Fourier transform of the product of the Fourier transforms of the input signal and the impulse response, that is, $y(t) = \mathcal{F}^{-1}\{Y(f)\} = \mathcal{F}^{-1}\{X(f)H(f)\}$.



(b) Harmonic functions in Fourier series

> The Fourier series of $x(t)$ is given by
>
> $$x(t) = \sum_{k=0}^{+\infty} c_x[k] e^{\frac{j2\pi kt}{T}}, \tag{1}$$
>
> where $k = [0, +\infty)$ is the harmonic number, T is the period, and $c_x[k]$ is the harmonic function. The **harmonic function** can be represented as
>
> $$c_x[k] = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{\frac{-j2\pi kt}{T}} dt, \tag{2}$$
>
> where t_0 is any arbitrary time.

(c) Unit-sync function

> $$\texttt{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \tag{3}$$

(d) How to approximate CTFT using DFT

> CTFT can be approximated at discrete frequencies by
>
> $$\begin{aligned} X(kf_s/N) &\cong T_s \sum_{n=0}^{N-1} x(nT_s) e^{-j2\pi kn/N} \\ &\cong T_s \times \mathcal{DFT}(x(nT_s)), \quad |k| << N \end{aligned} \tag{4}$$
>
> where $T_s = 1/f_s$ is chosen such that the signal $x(t)$ does not change much with this amount of time, and N is chosen such that the signal energy of the signal $x(t)$ can be covered within the time range 0 to NT_s.

(e) Graph the CTFT of cosine function $\cos(2\pi f_0 t)$ and sine function $\sin(2\pi f_0 t)$

$$\cos(2\pi f_0 t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right] \tag{5}$$

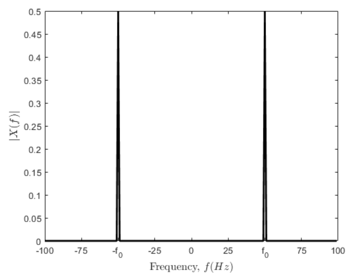


Figure 1: The CTFT of $\cos(2\pi f_0 t)$

$$\sin(2\pi f_0 t) \overset{\mathcal{F}}{\leftrightarrow} \frac{j}{2}\left[\delta(f+f_0) - \delta(f-f_0)\right] \tag{6}$$

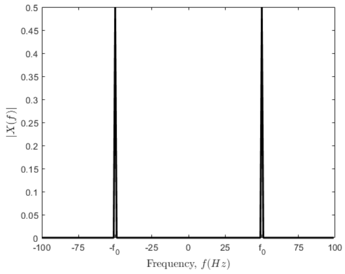


Figure 2: The CTFT of $\cos(2\pi f_0 t)$

2. Solve the following problems.

(a) Find the impulse response $h[n]$ of the system described by the difference equation

$$5y[n] + 2y[n-1] - 3y[n-2] = x[n] \tag{7}$$

Solution:

(i) Let $x[n] = \delta[n] \quad \rightarrow \quad y[n] = h[n]$. The difference equation now becomes

$$5h[n] + 2h[n-1] - 3h[n-2] = \delta[n]. \tag{8}$$

(ii) Homogeneous Solution:

(1.) Let $h[n] = K_h z^n$.

(2.) Substituting this into the difference equation will give

$$\begin{aligned}
5h[n] + 2h[n-1] - 3h[n-2] &= 0 \\
5K_h z^n + 2K_h z^{n-1} - 3K_h z^{n-2} &= 0 \\
K_h z^{n-2}\left(5z^2 + 2z - 3\right) &= 0 \\
5z^2 + 2z - 3 &= 0 \\
z_{1,2} &= \frac{-2 \pm \sqrt{4 - 4(5)(-3)}}{2(5)}
\end{aligned} \tag{9}$$

$$\boxed{z_1 = \frac{-2+8}{10} = \frac{3}{5}} \qquad \boxed{z_2 = \frac{-2-8}{10} = -1}$$

(3.) Therefore, the impulse response can now be represented as

$$h[n] = K_{h,1}\left(\frac{3}{5}\right)^n + K_{h,2}\,(-1)^n \tag{10}$$

(4.) Initial conditions:

n	$x[n]$	$y[n-2]$	$y[n-1]$	$y[n]$
0	$x[0] = 1$	$h[-2] = 0$	$h[-1] = 0$	$5h[0] + 2h[-1] - 3h[-2] = 1$ $h[0] = \frac{1}{5}$
1	$x[1] = 0$	$h[-1] = 0$	$h[0] = \frac{1}{5}$	$5h[1] + 2h[0] - 3h[-1] = 0$ $h[1] = \frac{-2}{25}$

(5.) Applying the initial conditions:

Initial Condition 1:

$$\begin{aligned} h[0] = \frac{1}{5} &= K_{h,1}\left(\frac{3}{5}\right)^{0} + K_{h,2}\left(-1\right)^{0} \\ \frac{1}{5} &= K_{h,1} + K_{h,2} \end{aligned} \tag{11}$$

Initial Condition 2:

$$\begin{aligned} h[1] = \frac{-2}{25} &= K_{h,1}\left(\frac{3}{5}\right)^{1} + K_{h,2}\left(-1\right)^{1} \\ \frac{-2}{25} &= K_{h,1}\left(\frac{3}{5}\right) + K_{h,2}(-1) \end{aligned} \tag{12}$$

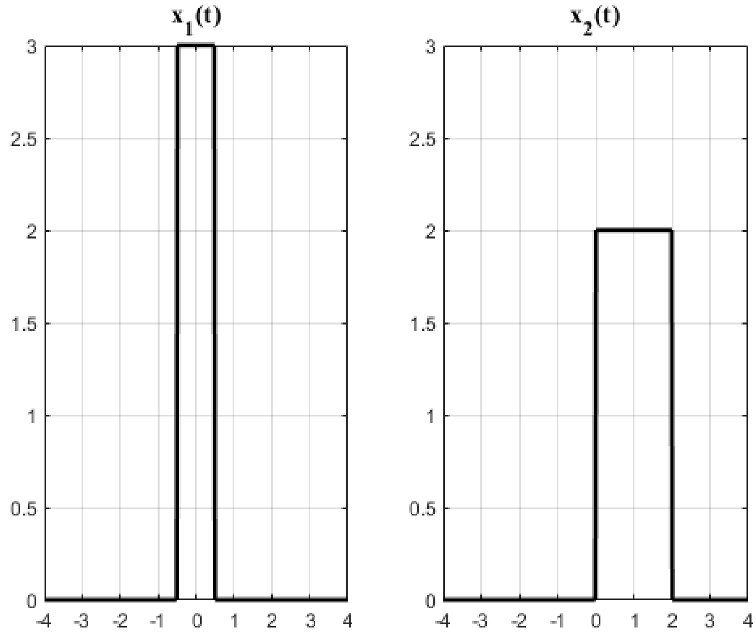
These system of equations can be written in matrix form as

$$\begin{aligned} \begin{bmatrix} \frac{1}{5} \\ \frac{-2}{25} \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ \frac{3}{5} & -1 \end{bmatrix} \begin{bmatrix} K_{h,1} \\ K_{h,2} \end{bmatrix} \\ \begin{bmatrix} K_{h,1} \\ K_{h,2} \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ \frac{3}{5} & -1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{5} \\ \frac{-2}{25} \end{bmatrix} \\ \begin{bmatrix} K_{h,1} \\ K_{h,2} \end{bmatrix} &= \begin{bmatrix} \frac{3}{40} \\ \frac{5}{40} \end{bmatrix} \end{aligned} \tag{13}$$

(iii.) Therefore, the impulse response is given by

$$\boxed{h[n] = \frac{3}{40}\left(\frac{3}{5}\right)^{n} + \frac{5}{40}\left(-1\right)^{n}} \tag{14}$$

(b) Find the convolution of the two functions $x_1(t)$ and $x_2(t)$.



Solution

(i.) The functions $x_1(t)$ and $x_2(t)$ can be represented as

$$x_1(t) = 3\texttt{rect}(t) = 3\left[\texttt{u}\left(t+\frac{1}{2}\right) - \texttt{u}\left(t-\frac{1}{2}\right)\right] \tag{15}$$

$$\begin{aligned} x_2(t) = 2\texttt{rect}\left(\frac{t-1}{2}\right) &= 2\left[\texttt{u}\left(\frac{t}{2}\right) - \texttt{u}\left(\frac{t-2}{2}\right)\right] \\ &= 2\left[\texttt{u}\left(t\right) - \texttt{u}\left(t-2\right)\right] \end{aligned} \tag{16}$$

(ii.) From the differentiation property of convolution, $y^{'}(t) = x^{'}(t) * h(t)$ or $y^{'}(t) = x(t) * h^{'}(t)$. Therefore,

$$\begin{aligned} y(t) &= x_1(t) * x_2(t) \\ y^{'}(t) &= x_1^{'}(t) * x_2(t) \\ y^{''}(t) &= x_1^{'}(t) * x_2^{'}(t) \\ y^{''}(t) &= 2\left[\delta\left(t+\frac{1}{2}\right) - \delta\left(t-\frac{1}{2}\right)\right] * 3\left[\delta\left(t\right) - \delta\left(t-2\right)\right] \\ y^{''}(t) &= 6\left[\delta\left(t+\frac{1}{2}\right) * \delta(t) - \delta\left(t+\frac{1}{2}\right) * \delta(t-2) - \delta\left(t-\frac{1}{2}\right) * \delta(t) \right. \\ &\quad \left. + \delta\left(t-\frac{1}{2}\right) * \delta(t-2)\right] \end{aligned} \tag{17}$$

$$y''(t) = 6\left[\delta\left(t+\frac{1}{2}\right) - \delta\left(t-\frac{3}{2}\right) - \delta\left(t-\frac{1}{2}\right) + \delta\left(t-\frac{5}{2}\right)\right]$$

$$y'(t) = 6\left[\mathtt{u}\left(t+\frac{1}{2}\right) - \mathtt{u}\left(t-\frac{3}{2}\right) - \mathtt{u}\left(t-\frac{1}{2}\right) + \mathtt{u}\left(t-\frac{5}{2}\right)\right]$$

$$\boxed{y(t) = 6\left[\mathtt{ramp}\left(t+\frac{1}{2}\right) - \mathtt{ramp}\left(t-\frac{3}{2}\right) - \mathtt{ramp}\left(t-\frac{1}{2}\right) + \mathtt{ramp}\left(t-\frac{5}{2}\right)\right]} \tag{18}$$

(iii.) Table of values

t	$y(t)$
$t=\frac{-1}{2}$	$y\left(\frac{-1}{2}\right) = 6\left(0-0-0+0\right) = 0$
$t=0$	$y\left(0\right) = 6\left(\frac{1}{2}-0-0+0\right) = 3$
$t=\frac{1}{2}$	$y\left(\frac{1}{2}\right) = 6\left(1-0-0+0\right) = 6$
$t=1$	$y\left(1\right) = 6\left(\frac{3}{2}-0-\frac{1}{2}+0\right) = 6$
$t=\frac{3}{2}$	$y\left(\frac{3}{2}\right) = 6\left(2-0-1+0\right) = 6$
$t=2$	$y\left(2\right) = 6\left(\frac{5}{2}-\frac{1}{2}-\frac{3}{2}+0\right) = 3$
$t=\frac{5}{2}$	$y\left(\frac{5}{2}\right) = 6\left(3-1-2+0\right) = 0$

(iv.) Plot of the convolution of $x_1(t)$ and $x_2(t)$:

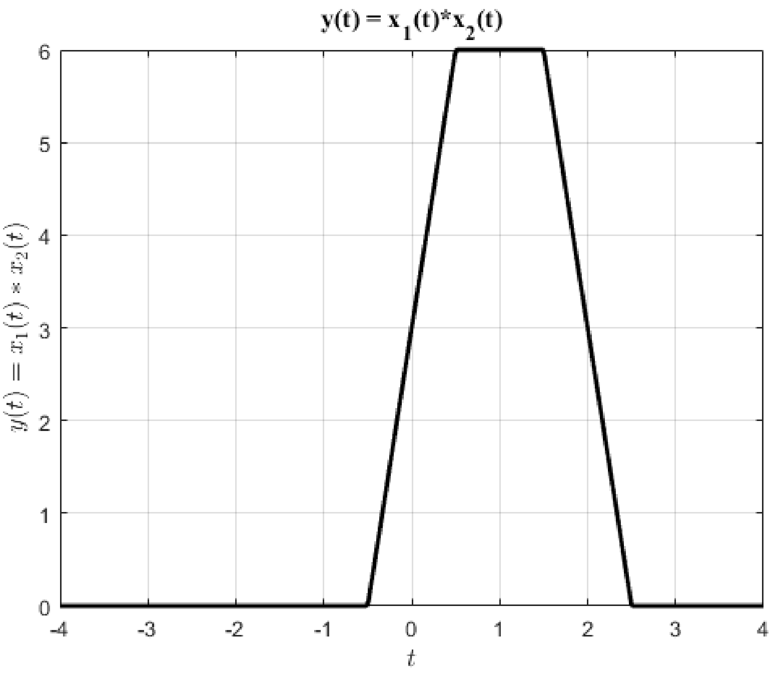


Figure 3: Plot of the convolution of $x_1(t) = 3\mathtt{rect}(t)$ and $x_2(t) = 2\mathtt{rect}(\frac{t-1}{2})$

(c) Find the complex CTFS harmonic function of $x(t) = 10\texttt{rect}(t/2) * \delta_4(t)$ using

$$c_x[k] = \frac{1}{T}\int_T x(t)e^{\frac{-j2\pi kt}{T}}dt \tag{19}$$

Solution

(i.) From $x(t) = 10\texttt{rect}(t/2) * \delta_{\mathbf{T}}(t) = 10\texttt{rect}(t/2) * \delta_4(t)$, the period can be written as $T = 4$. Also, the width of the rectangle function $\texttt{rect}(t/2)$ is $w = 2$.

(ii.) Thus, the harmonic function can be calculated as:

$$\begin{aligned}
c_x[k] &= \frac{1}{T}\int_T x(t)e^{\frac{-j2\pi kt}{T}}dt \\
&= \frac{1}{T}\int_{-T/2}^{T/2} 10\texttt{rect}(t/2) * \delta_4(t)e^{\frac{-j2\pi kt}{T}}dt \\
&= \frac{10}{4}\int_{-4/2}^{4/2} \texttt{rect}(t/2)e^{\frac{-j2\pi kt}{4}}dt \\
&= \frac{5}{2}\int_{-1}^{1} e^{\frac{-j2\pi kt}{4}}dt \\
&= \frac{5}{2}\left(\frac{e^{\frac{-j2\pi kt}{4}}}{-j2\pi k/4}\right|_{-1}^{1} \\
&= \frac{5}{2}\left(\frac{e^{-j2\pi k/4} - e^{j2\pi k/4}}{-j2\pi k/4}\right) \\
&= \frac{5}{2}\left(\frac{\sin(2\pi k/4)}{\pi k/4}\right) = \frac{5}{2}\left(\frac{\sin(\pi k/2)}{\pi k/4}\right)\cdot\frac{2}{2} \\
&= 5\left(\frac{\sin(\pi k/2)}{\pi k/2}\right) \\
&\boxed{c_x[k] = 5\texttt{sinc}(\texttt{k}/2)\cdot\delta_1[k]}
\end{aligned} \tag{20}$$

(d) Find the CTFT of $x_t = 24\cos(100\pi t)\sin(10{,}000\pi t)$.

Table 6.4 More Fourier transform pairs

$\delta(t) \xleftrightarrow{\mathcal{F}} 1$	$1 \xleftrightarrow{\mathcal{F}} \delta(f)$
$\mathrm{sgn}(t) \xleftrightarrow{\mathcal{F}} 1/j\pi f$	$\mathrm{u}(t) \xleftrightarrow{\mathcal{F}} (1/2)\delta(f) + 1/j2\pi f$
$\mathrm{rect}(t) \xleftrightarrow{\mathcal{F}} \mathrm{sinc}(f)$	$\mathrm{sinc}(t) \xleftrightarrow{\mathcal{F}} \mathrm{rect}(f)$
$\mathrm{tri}(t) \xleftrightarrow{\mathcal{F}} \mathrm{sinc}^2(f)$	$\mathrm{sinc}^2(t) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(f)$
$\delta_{T_0}(t) \xleftrightarrow{\mathcal{F}} f_0\delta_{f_0}(f),\ f_0 = 1/T_0$	$T_0\delta_{T_0}(t) \xleftrightarrow{\mathcal{F}} \delta_{f_0}(f),\ T_0 = 1/f_0$
$\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} (1/2)[\delta(f-f_0)+\delta(f+f_0)]$	$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} (j/2)[\delta(f+f_0)-\delta(f-f_0)]$

Solution

(i.) $\cos(2\pi f_0 t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$. Therefore,

$$\begin{aligned} 24\cos(100\pi t) \overset{\mathcal{F}}{\leftrightarrow} & \frac{24}{2}[\delta(f-50)+\delta(f+50)] \\ &= 12[\delta(f-50)+\delta(f+50)] \end{aligned} \tag{21}$$

(ii.) $\sin(2\pi f_0 t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2}[\delta(f+f_0)-\delta(f-f_0)]$. Therefore,

$$\sin(10{,}000\pi t) \overset{\mathcal{F}}{\leftrightarrow} \frac{j}{2}[\delta(f+5{,}000)-\delta(f-5{,}000)] \tag{22}$$

(iii.) $g(t)h(t) \overset{\mathcal{F}}{\leftrightarrow} G(f)*H(f)$. Therefore,

$$\begin{aligned} 24\cos(100\pi t)\sin(10{,}000\pi t) \overset{\mathcal{F}}{\leftrightarrow} & 12[\delta(f-50)+\delta(f+50)] * \frac{j}{2}[\delta(f+5{,}000) \\ & - \delta(f-5{,}000)] \\ &= j6[\delta(f-50)*\delta(f+5{,}000) \\ & \quad - \delta(f-50)*\delta(f-5{,}000) \\ & \quad + \delta(f+50)*\delta(f+5{,}000) \\ & \quad - \delta(f+50)*\delta(f-5{,}000)] \\ 24\cos(100\pi t)\sin(10{,}000\pi t) \overset{\mathcal{F}}{\leftrightarrow} & \quad j6[\delta(f+4950)-\delta(f-5050) \\ & \quad + \delta(f+5050)-\delta(f-4950)] \end{aligned} \tag{23}$$

(e) Find and graph the inverse DTFT of

$$X(F) = \left[\texttt{rect}\left(50\left(F-\frac{1}{4}\right)\right) + \texttt{rect}\left(50\left(F+\frac{1}{4}\right)\right)\right] * \delta_1(F) \tag{24}$$

Table 7.5 More DTFT pairs

$\delta[n] \xleftrightarrow{\mathcal{F}} 1$	
$\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1-e^{-j2\pi F}} + (1/2)\delta_1(F),$	$\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1-e^{-j\Omega}} + \pi\delta_1(\Omega)$
$\mathrm{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{rect}(wF) * \delta_1(F),$	$\mathrm{sinc}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{rect}(w\Omega/2\pi) * \delta_{2\pi}(\Omega)$
$\mathrm{tri}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{drcl}^2(F,w),$	$\mathrm{tri}(n/w) \xleftrightarrow{\mathcal{F}} w\,\mathrm{drcl}^2(\Omega/2\pi,w)$
$1 \xleftrightarrow{\mathcal{F}} \delta_1(F),$	$1 \xleftrightarrow{\mathcal{F}} 2\pi\delta_{2\pi}(\Omega)$
$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (1/N_0)\delta_{1/N_0}(F),$	$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (2\pi/N_0)\delta_{2\pi/N_0}(\Omega)$
$\cos(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (1/2)[\delta_1(F-F_0)+\delta_1(F+F_0)],$	$\cos(\Omega_0 n) \xleftrightarrow{\mathcal{F}} \pi[\delta_{2\pi}(\Omega-\Omega_0)+\delta_{2\pi}(\Omega+\Omega_0)]$
$\sin(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (j/2)[\delta_1(F+F_0)-\delta_1(F-F_0)],$	$\sin(\Omega_0 n) \xleftrightarrow{\mathcal{F}} j\pi[\delta_{2\pi}(\Omega+\Omega_0)-\delta_{2\pi}(\Omega-\Omega_0)]$
$\mathrm{u}[n-n_0]-\mathrm{u}[n-n_1] \xleftrightarrow{\mathcal{Z}} \frac{e^{j2\pi F}}{e^{j2\pi F}-1}(e^{-j2\pi n_0 F}-e^{-j2\pi n_1 F}) = \frac{e^{-j\pi F(n_0+n_1)}}{e^{-j\pi F}}(n_1-n_0)\mathrm{drcl}(F,n_1-n_0)$	
$\mathrm{u}[n-n_0]-\mathrm{u}[n-n_1] \xleftrightarrow{\mathcal{Z}} \frac{e^{j\Omega}}{e^{j\Omega}-1}(e^{-jn_0\Omega}-e^{-jn_1\Omega}) = \frac{e^{-j\Omega(n_0+n_1)/2}}{e^{-j\Omega/2}}(n_1-n_0)\mathrm{drcl}(\Omega/2\pi,n_1-n_0)$	

Solution

(i.) $\texttt{sinc}\left(\frac{n}{w}\right) \overset{\mathcal{F}}{\leftrightarrow} w\texttt{rect}\,(wF) * \delta_1(F)$. Therefore, if $w = 50$, then

$$\frac{1}{50}\texttt{sinc}\left(\frac{n}{50}\right) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{50}\cdot 50\texttt{rect}\,(50F) * \delta_1(F) = \texttt{rect}(50F) * \delta_1(F) \tag{25}$$

(ii.) The frequency shifting property is given by $e^{j2\pi F_0 n}x[n] \overset{\mathcal{F}}{\leftrightarrow} X(F-F_0)$. Therefore,
$F_0 = \frac{1}{4}$:

$$e^{j2\pi n/4}\cdot\frac{1}{50}\texttt{sinc}\left(\frac{n}{50}\right) \overset{\mathcal{F}}{\leftrightarrow} \texttt{rect}\left(50\left(F-\frac{1}{4}\right)\right) * \delta_1(F) \tag{26}$$

$F_0 = -\frac{1}{4}$:

$$e^{-j2\pi n/4}\cdot\frac{1}{50}\texttt{sinc}\left(\frac{n}{50}\right) \overset{\mathcal{F}}{\leftrightarrow} \texttt{rect}\left(50\left(F+\frac{1}{4}\right)\right) * \delta_1(F) \tag{27}$$

(iii.) Adding eqs. (26) and (27) gives

$$\left(e^{j2\pi n/4} + e^{-j2\pi n/4}\right)\frac{1}{50}\texttt{sinc}\left(\frac{n}{50}\right) \overset{\mathcal{F}}{\leftrightarrow} \left[\texttt{rect}\left(50\left(F-\frac{1}{4}\right)\right) + \texttt{rect}\left(50\left(F+\frac{1}{4}\right)\right)\right] * \delta_1(F) \tag{28}$$

Therefore, the inverse DTFT of

$$X(F) = \left[\texttt{rect}\left(50\left(F-\frac{1}{4}\right)\right) + \texttt{rect}\left(50\left(F+\frac{1}{4}\right)\right)\right] * \delta_1(F) \tag{29}$$

is:

$$\begin{aligned} x[n] &= \left(e^{j2\pi n/4} + e^{-j2\pi n/4}\right) \frac{1}{50}\texttt{sinc}\left(\frac{n}{50}\right) \\ &= 2\cos\left(\frac{2\pi n}{4}\right) \cdot \frac{1}{50}\texttt{sinc}\left(\frac{n}{50}\right) \\ &\boxed{x[n] = \frac{1}{25}\cos\left(\frac{\pi n}{2}\right)\texttt{sinc}\left(\frac{n}{50}\right)} \end{aligned} \tag{30}$$

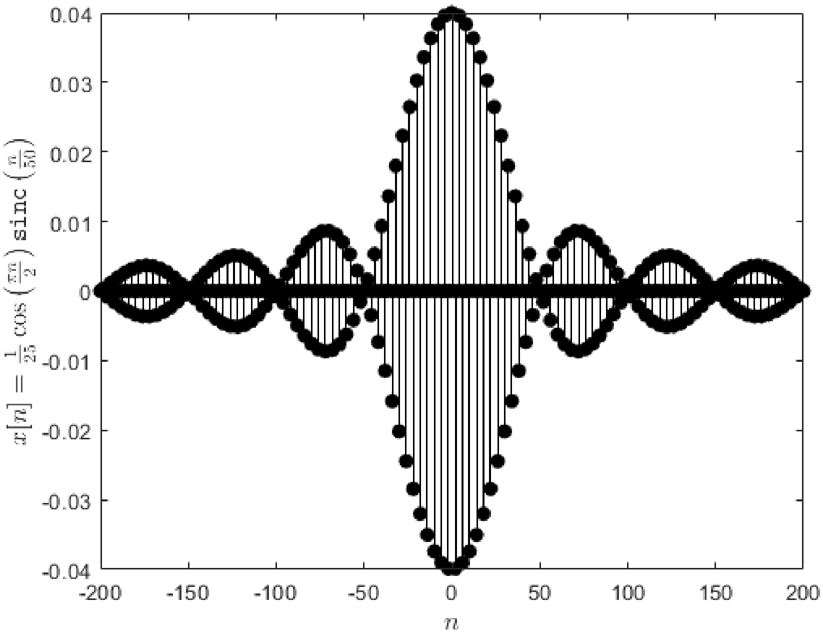


Figure 4: The inverse DTFT of $X(F) = \left[\texttt{rect}\left(50\left(F - \frac{1}{4}\right)\right) + \texttt{rect}\left(50\left(F + \frac{1}{4}\right)\right)\right] * \delta_1(F)$

3. MATLAB coding

(a) Using the DFT, find the approximate CTFT of

$$x(t) = \begin{Bmatrix} t(1-t), & 0 < t < 1 \\ 0, & \texttt{otherwise} \end{Bmatrix} = t(1-t)\texttt{rect}\left(t - \frac{1}{2}\right)$$

Source Code

```
% BRI509 (Introduction to Brain Signal Processing)
% Assignment # 2
% Author: Raymart Jay E. Canoy
% Student ID #: 2020021376

%% Problem 3a
N = 64;         % Sample 128 times
Ts = 2/N;       % Sampling interval

fs = 1/Ts;      % Sampling rate
df = fs/N;      % Frequency-domain resolution

n = [0: N-1]; % Indices

t = n*Ts;       % Time vector
x = t.*(1-t).*prob3a_rect(t - 0.5); % x(t)

num = 2*N;
X = fftshift(Ts*fft(x, num, 2)); % X(f) with zero-padding

k = [-N/2:N/(num):N/2-N/(num)]';

figure;
subplot(3,1,1);
p = plot([t, linspace(3, num-1, num-1)], [x, zeros(1, num
    -1)], 'k');
set(p, 'LineWidth', 2);
grid on
xlim([0 128])
xlabel('Time, $t \;\; (s)$', 'Interpreter', 'Latex', '
    FontName', 'Times', 'FontSize', 12);
ylabel('$x(t)$', 'Interpreter', 'Latex', 'FontName', '
    Times', 'FontSize', 12)
subplot(3,1,2)
p = plot(k*df, abs(X), 'k')
set(p, 'LineWidth', 2);
grid on
xlim([-16 16])
xlabel('Frequency, $f \;\; (Hz)$', 'Interpreter', 'Latex',
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    'FontName', 'Times', 'FontSize', 12);
ylabel('$|X(f)|$', 'Interpreter', 'Latex', 'FontName', '
    Times', 'FontSize', 12)
subplot(3,1,3)
p = plot(k*df, angle(X), 'k')
set(p, 'LineWidth', 2);
grid on
xlim([-16 16])
xlabel('Frequency, $f \;\; (Hz)$', 'Interpreter', 'Latex',
    'FontName', 'Times', 'FontSize', 12);
ylabel('Phase of $|X(f)|$', 'Interpreter', 'Latex', '
    FontName', 'Times', 'FontSize', 12)
```

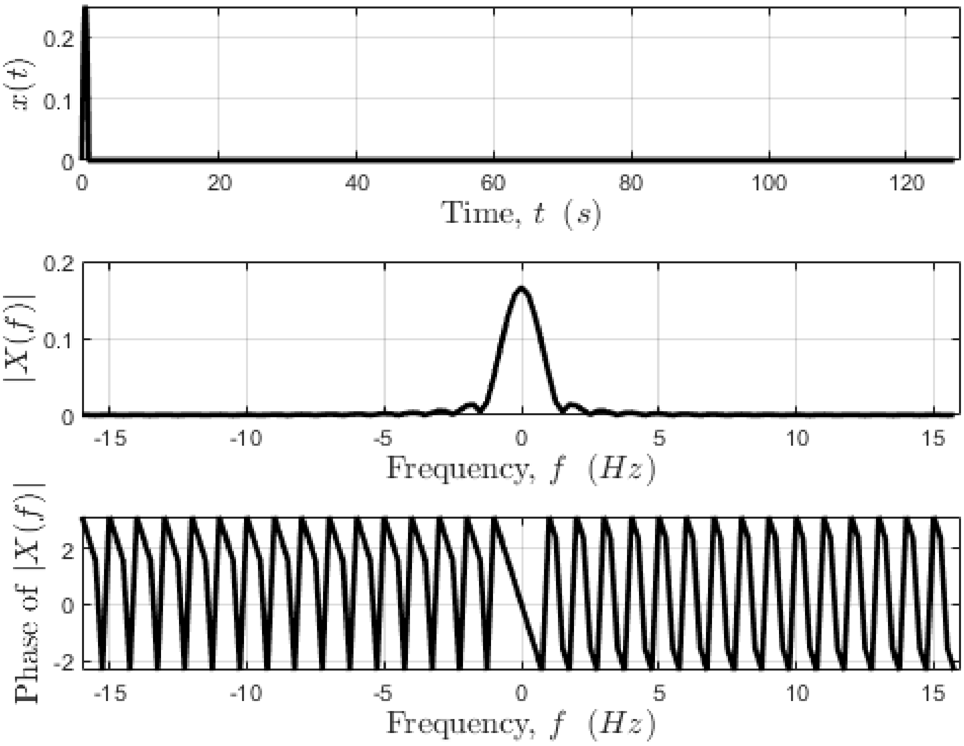

(b.1) Graph the CTFT of of C4(523.25/2Hz), D4 (587.33/2Hz), E4 (659.26/2Hz), F4 (698.46/2Hz), G4 (784/2Hz), A4 (440Hz), B4 (493.8Hz), C5 (523.25Hz).

Source Code

```matlab
% BRI509 (Introduction to Brain Signal Processing)
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% Student ID #: 2020021376

% Notes:
% "The frequencies 440 Hz and 880 Hz both correspond to the musical note A,
% but one octave apart. In the western musical scale, there are 12 notes in
% every octave. These notes are evenly distributed (geometrically), so the
% next note above A, which is B flat, has frequency (440 x beta), where
% beta is the twelfth root of two."
% Source: https://ptolemy.berkeley.edu/eecs20/week8/scale.html

%% Part I: CTFT of the musical scales C4,D4,E4,F4,G4,A4,A5,B5,C5

%% Having the container for the musical scales
clear; clc;

fs = 1200;                          % Sampling Frequency
T = 1/fs;                            % Sampling Period
dur  = 1;                            % duration, s
L = dur*fs;                          % Length of signal
t = (0:L-1)*T;          % One-second duration

% Sinusoid Signal
% y(t) = sin(2*pi*f*t), where f = 440*beta^(index), beta = 2^(1/12), and
% n is the index, i.e., n = [1, 13]
beta = 2^(1/12);
beta_exponents = [-9, -7, -5, -4, -2, 0, 2, 3];
note = @(index) sin(2*pi*(440*(beta^(index)))*t); % a sinusoid function

% musical_scale_keys: Names of the notes
musical_scale_keys = {'C4', 'D4', 'E4', 'F4', 'G4', 'A4', 'B4', 'C5'};
% musical_scale_values: An empty 13x1 cell, and will hold the notes' values
musical_scale_values = cell(length(musical_scale_keys), 1);

% Assigns the values of each note to musical_scale_values
for ind = 1 : length(musical_scale_keys)
    musical_scale_values{ind} = note(beta_exponents(ind));
end
```

```
% Dictionary-type container -- keys: notes's names; values
    : notes' values
music_octave = containers.Map(musical_scale_keys,
    musical_scale_values);

%% Fourier Transform of the musical scales
musical_scale_CTFT_values = cell(length(musical_scale_keys
    ),1);

n = 2*(2^nextpow2(L)); % So that I will get an amplitude
    of 0.5

for ind = 1 :length(musical_scale_keys)
    x = music_octave(musical_scale_keys{ind});
    X = fftshift(fft(x, n, 2));
    P2 = abs(X/L);
    musical_scale_CTFT_values{ind} = P2;
end

% Dictionary-type container -- keys: notes's names; values
    : CTFT values
music_scale_CTFT = containers.Map(musical_scale_keys,
    musical_scale_CTFT_values);


for set_num = 1 : 2
    figure;
    disp(['set = ' num2str(set_num)])
    for ind = (floor(length(musical_scale_keys)/2))*(
    set_num - 1) + 1 : set_num*floor(length(
    musical_scale_keys)/2)
        disp(['ind = ' num2str(ind)])
        subplot(floor(length(musical_scale_keys)/4), 2,
    ind - (set_num - 1)*floor(length(musical_scale_keys)
    /2))
        p = plot([-fs/2:fs/n:(fs/2)-(fs/n)],
    music_scale_CTFT(musical_scale_keys{ind}), 'k');
        set(p, 'LineWidth', 1.5);
        ylim([0 0.5])
        yticks([0 0.1 0.2 0.3 0.4 0.5]);
        xticks([-600 -500 -400 -300 -200 -100 0 100 200
    300 400 500 600])
        xlabel('Frequency, $f \;\; (Hz)$', 'Interpreter',
    'Latex', 'FontName', 'Times', 'FontSize', 12);
        ylabel('$|X(f)|$', 'Interpreter', 'Latex', '
    FontName', 'Times', 'FontSize', 12)
        title([musical_scale_keys{ind}, '(f = ', num2str
    (440*(beta^(beta_exponents(ind)))), ' Hz)'], 'FontName
    ', 'Times', 'FontSize', 12);
    end
end
```

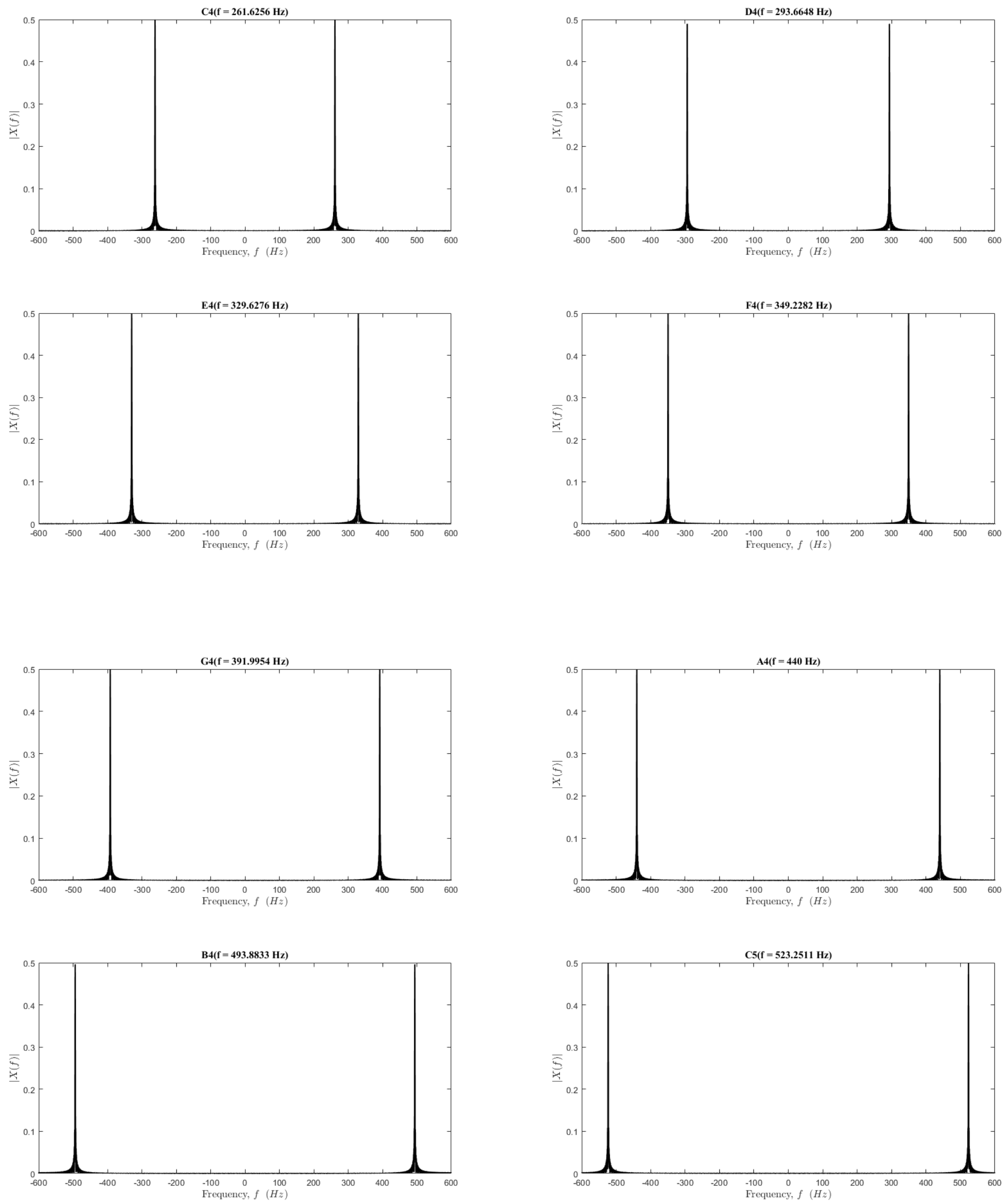

Figure 5: CTFT of the solfeges of C Major.

(b.2) Make an MP3 file containing C4, D4, E4, F4, G4, A4, B4, C5.

Source Code

```matlab
% BRI509 (Introduction to Brain Signal Processing)
% Assignment # 2
% Author: Raymart Jay E. Canoy
% Student ID #: 2020021376

% Notes:
% "The frequencies 440 Hz and 880 Hz both correspond to the musical note A,
% but one octave apart. In the western musical scale, there are 12 notes in
% every octave. These notes are evenly distributed (geometrically), so the
% next note above A, which is B flat, has frequency (440 x beta), where
% beta is the twelfth root of two."
% Source: https://ptolemy.berkeley.edu/eecs20/week8/scale.html

%% Part II: MP4 file containing the musical scales C4,D4,E4,F4,G4,A4,A5,B5,C5 one second each
clear; clc;

fs = 48000;                    % Sampling Frequency
T = 1/fs;                      % Sampling Period
dur  = 1;                      % duration, s
L = dur*fs;                    % Length of signal
t = (0:L-1)*T;         % One-second duration

% Sinusoid Signal
% y(t) = sin(2*pi*f*t), where f = 440*beta^(n - 1), beta = 2^(1/12), and
% n is the index, i.e., n = [1, 13]
beta = 2^(1/12);
beta_exponents = [-9, -7, -5, -4, -2, 0, 2, 3];
note = @(index) sin(2*pi*(440*(beta^(index)))*t); % a sinusoid function

% musical_scale_keys: Names of the notes
musical_scale_keys = {'C4', 'D4', 'E4', 'F4', 'G4', 'A4', 'B4', 'C5'};
% musical_scale_values: An empty 13x1 cell, and will hold the notes' values
musical_scale_values = cell(length(musical_scale_keys), 1);

% Assigns the values of each note to musical_scale_values
for ind = 1 : length(musical_scale_keys)
    musical_scale_values{ind} = note(beta_exponents(ind));
end

% Dictionary-type container -- keys: notes's names; values
```

```
    : notes' values
music_octave = containers.Map(musical_scale_keys,
    musical_scale_values);

mp3_file = [];
filename = 'prob3b.mp4';
for ind = 1 : length(musical_scale_keys)
    mp3_file = [mp3_file; [music_octave(musical_scale_keys
    {ind}) zeros(1, 100)]'];
end

soundsc(mp3_file, fs)
audiowrite(filename, mp3_file, fs)
```