Random Access Channel Coding in the Finite Blocklength Regime

Michelle Effros*, Victoria Kostina*, Recep Can Yavas*

California Institute of Technology

Background

Wireless communication networks are often accessed through WiFi hotspots and cellular base stations. The number of users employing such access point may be unknown and time varying. We call such a communication environment a random access network.

A memoryless random access channel (RAC) models the random access network using a family of stationary, memoryless multiple access channels (MACs) representing different numbers of active transmitters.

Motivation

State-of-the-art methods for multiple-transmitter channels use orthogonalization (e.g., TDMA, FDMA, CDMA) or collision avoidance (e.g., CSMA, slotted ALOHA). Even when the number of active transmitters is known, both methods can at best attain a sum-rate for all users equal to the single-transmitter capacity. This poor performance can cause a huge waste of valuable bandwidth resources.

Our solution: We employ a rateless code with sporadic feedback to achieve rate that is identical in the long run to the optimal rate for the MAC in operation. We achieve this performance without either the transmitters or the receiver knowing the number of active transmitters.

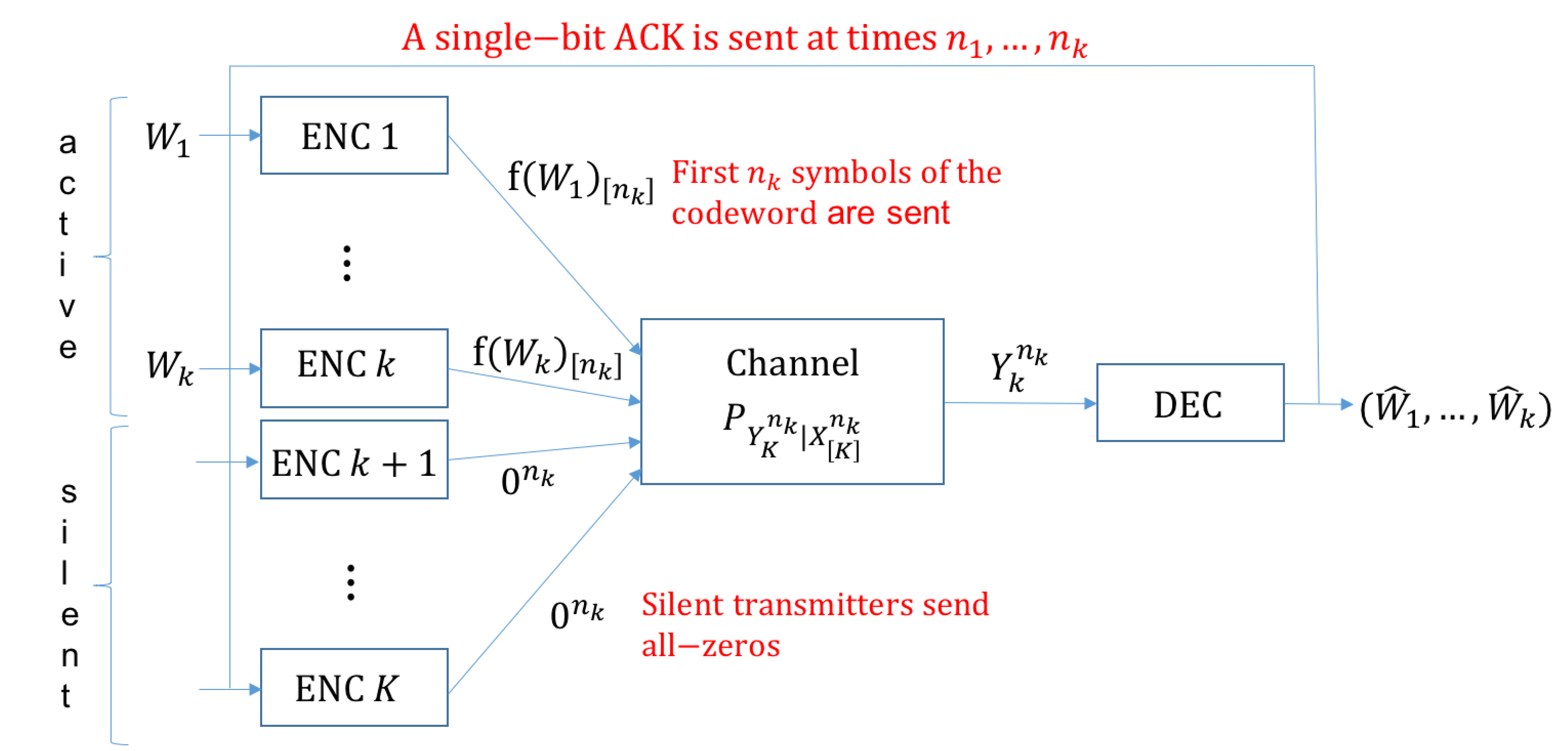
Problem Setup

- Neither the transmitters nor the receiver know which or how many transmitters are active.
- The decoder is tasked with determining the number of active transmitters (k) and their messages but need not determine which transmitter sent which message.
- Sporadic feedback is employed to synchronize the transmitters during epochs.
- An $(M, \{(n_k, \epsilon_k)\}_{k=0}^K)$ is a code for which the maximal number of transmitters is K, the message set size for each transmitter is M, n_k channel uses are used to decode the messages of k active transmitters, and the probability of error when k transmitters are active is at most ϵ_k.

$*$Alphabeticalorder

Rateless Coding Scheme

- Communication occurs in epochs.
- At the beginning of each epoch, each transmitter decides to be active or silent in that epoch.
- Each active transmitter uses the epoch to describe its message $W \in \{1, \ldots, M\}$.
- The receiver makes a decision at each time $n_0, n_1, \ldots$, choosing to end the epoch at time n_t if it believes that the number of active transmitters is t.
- The decoder uses a single threshold rule to decode messages and broadcasts a single-bit ACK, Z_i at time n_i, with $Z_i = 0$ if the epoch should continue, and $Z_i = 1$ if it should end.
- If the channel is "permutation-invariant," then all transmitters employ identical encoders. This reduces the complexity of the code design.



Main Theorem

For any permutation-invariant and reducible RAC, any $K < \infty$, and any input distribution P_X satisfying the symmetry assumptions in [?], there exists an $(M, \{(n_k, \epsilon_k)\}_{k=0}^K)$ code provided that

$$\log M \le \frac{1}{k}\left(n_k I_k - \sqrt{n_k V_k} Q^{-1}(\epsilon_k) - \frac{1}{2}\log n_k + O(1)\right), \tag{1}$$

for all $1 \le k \le K$, and

$$n_0 \ge C(\log n_1) + o(\log n_1). \tag{2}$$

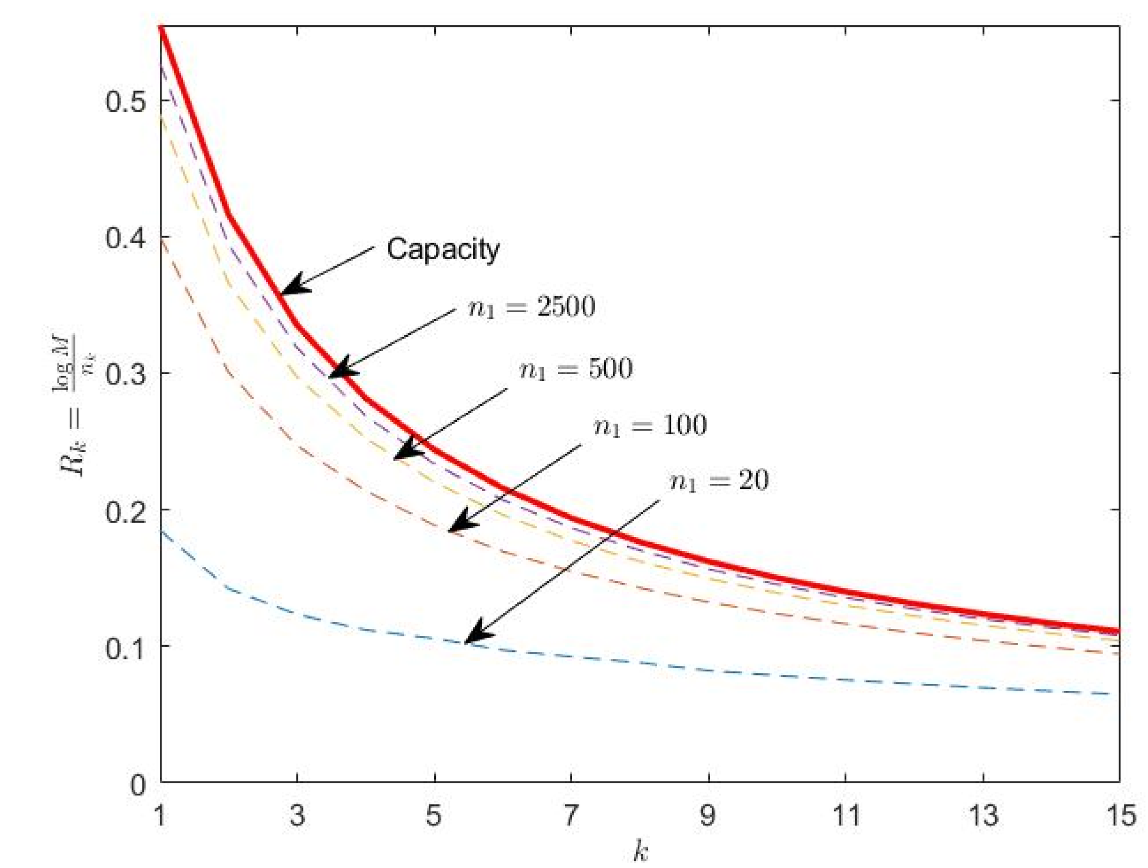
Here $C > 0$ is a positive constant, I_k and V_k denote the mutual information and the dispersion, respectively, evaluated according to P_X when k transmitters are active, and Q^{-1} denotes the functional inverse of the Q-function.

Example Channel

Consider the adder-erasure RAC

$$Y_k = \begin{cases} \sum_{i=1}^k X_i, & \text{w.p. } 1-\delta \\ \mathsf{e} & \text{w.p. } \delta, \end{cases}$$

where $X_i \in \{0,1\}$ and $Y_k \in \{0, \ldots, k\} \cup \{\mathsf{e}\}$.



We here plot capacity and approximate achievable rates per user for this adder-erasure RAC with erasure probability $\delta = 0.2$. Achievable rates are given for target error probability $\epsilon_k = 10^{-6}$ for all k. For each curve, the message set size M is chosen so that the rate $\frac{\log M}{n_1}$ is achievable with error probability $\epsilon_1 = 10^{-6}$ when n_1 equals 20, 100, 500, and 2500, respectively.

Summary

- Current methods do not achieve the theoretical limits of random access communication, where the transmitter activity is unknown.
- We propose a new RAC code and bound its performance.
- Our proposed RAC code design with limited feedback performs as well in its first and second order terms as the best known code for the k-transmitter MAC for every k.

Acknowledgments

This work is supported in part by the National Science Foundation under Grant CCF-1817241.

References

[1] M. Effros, V. Kostina, and R. C. Yavas, "Random access channel coding in the finite blocklength regime," submitted to IEEE Transactions on Information Theory on July 23, 2019. [Online]. Available: http://arxiv.org/abs/1801.09018.