Nested Sparse Feedback Codes for Point-to-Point, Multiple Access, and Random Access Channels

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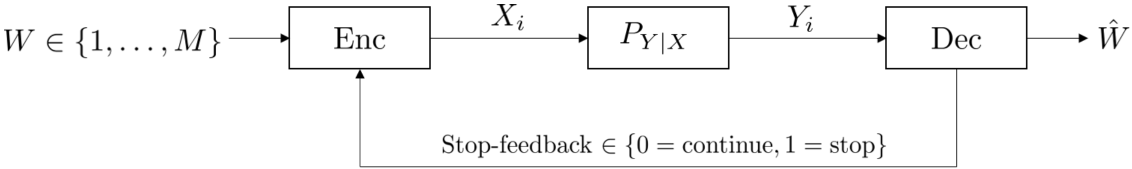
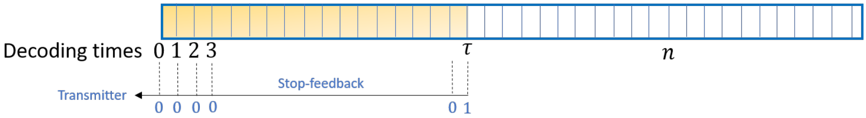
Variable-length stop-feedback (VLSF) codes



- Variable-length: decoding at arbitrary times $\tau \in \{0, 1, 2, \ldots\}$
- Advantage: Higher reliability compared to fixed-length codes
- Disadvantage: Feedback at each time instant is impractical!

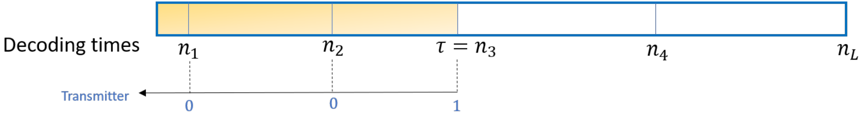
VLSF codes with $L = \infty$ vs. $L = O(1)$

- VLSF code ($L = \infty$): Impractical!



  - Transmitter constantly listens to the feedback signal

- VLSF code with $L = O(1)$ decoding times



  - Sporadic feedback
  - Practical codes: Incremental redundancy hybrid automatic repeat request codes

Channel models and VLSF codes

- Discrete memoryless point-to-point channel (DM-P2P)
- Discrete memoryless multiple access channel (DM-MAC)
- Discrete memoryless random access channel (DM-RAC) [Yavas et al. 2021]:
  - An unknown number of active transmitters out of K transmitters
  - Defined by a family of DM-MACs $\{P_{Y_k|X_1,\ldots,X_k}\}_{k=1}^{K}$.

Definition

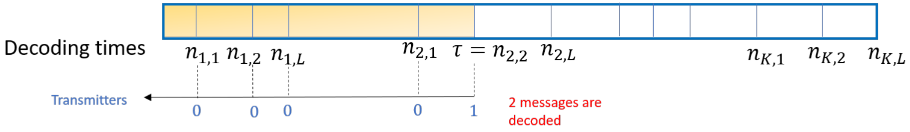
An $(N, \{n_i\}_{i=1}^{L}, M, \epsilon)$ VLSF code satisfies

$$\text{Average decoding time:} \quad \mathbb{E}[\tau] \leq N$$

$$\text{Average error probability:} \quad \mathbb{P}[\mathrm{g}_\tau(Y^\tau) \neq W] \leq \epsilon$$

where the message W is uniformly distributed on the set $[M]$ for the DM-P2P and on the set $[M_1] \times [M_2]$ for the DM-MAC.

VLSF code for the DM-RAC



Definition

An $(\{N_k\}_{k=1}^K, \{n_{k,\ell}\}_{k\in[K],\ell\in[L]}, M, \epsilon)$ VLSF code satisfies

$$\text{Average decoding time:} \quad \mathbb{E}[\tau_k] \leq N_k \qquad \text{for } k \in [K]$$

$$\text{Average error probability:} \quad \mathbb{P}\left[\mathrm{g}_{\tau_k}(Y^{\tau_k}) \neq W_{[k]}\right] \leq \epsilon \qquad \text{for } k \in [K]$$

where the messages $W_{[k]}$ are independent with each uniformly distributed on the set $[M]$.

Main Result (DM-P2P)

Theorem (Achievability)

Fix $L = O(1) \geq 2$, $\epsilon \in (0,1)$, and a distribution P_X. There exists a VLSF code with L decoding times for the DM-P2P provided that

$$\ln M \leq \frac{N I_1}{1-\epsilon} - \sqrt{N \ln_{(L-1)}(N) \frac{V_1}{1-\epsilon}} + o(\sqrt{N})$$

The decoding times satisfy $n_1 = 0$ and the equations

$$\ln M = n_\ell I_1 - \sqrt{n_\ell \ln_{(L-\ell+1)}(n_\ell) V(P)} - \ln n_\ell + O(1) \qquad \forall \ell \in \{2, \dots, L\}$$

$I_1 =$ mutual information, $V_1 =$ dispersion, $\ln_{(L)}(\cdot) \triangleq \overbrace{\ln(\ln(\dots(\ln(\cdot))))}^{L \text{ times}}$

- We optimize the choices of the decoding times $n_1, \dots, n_L$ to minimize average decoding time N for a given ϵ and M.
- The proof uses the non-asymptotic achievability bound in [Yavas et al. (ISIT 2021)], a moderate deviations theorem, and Karush-Kuhn-Tucker conditions.

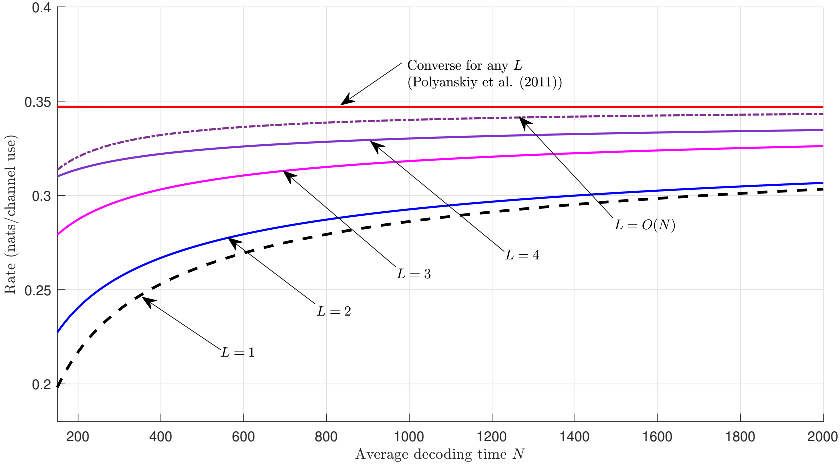


Figure: BSC(0.11), $\epsilon = 10^{-3}$

- Diminishing performance improvement as L increases!

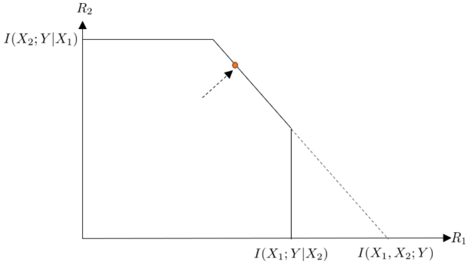
Main Result (DM-MAC)

Theorem (Achievability $L = O(1)$)

Fix $L = O(1) \geq 2$, $\epsilon \in (0,1)$, and distributions P_{X_1}, P_{X_2}. Let $\left(\frac{\ln M_1}{N}, \frac{\ln M_2}{N}\right)$ lie on the interior of the sum-rate boundary. There exists a VLSF code with L decoding times for the DM-MAC with

$$\ln M_1 + \ln M_2 \leq \frac{N I_2}{1-\epsilon} - \sqrt{N \ln_{(L-1)}(N) \frac{V_2}{1-\epsilon}} + o(\sqrt{N})$$

$I_2 = I(X_1, X_2; Y)$, $V_2 = \mathrm{Var}\left[\imath(X_1, X_2; Y)\right] =$ dispersion



Main Result (DM-MAC and DM-RAC)

Theorem (Achievability $L = \Omega(N)$)

Let $\left(\frac{\ln M_1}{N}, \frac{\ln M_2}{N}\right)$ lie on the interior of the sum-rate boundary. There exists a VLSF code with $L = \Omega(N)$ decoding times for the DM-MAC with

$$\ln M_1 + \ln M_2 \leq \frac{N\, I_2}{1-\epsilon} - \ln N + O(1)$$

- For $L = \Omega(N)$, we improve the second-order term $-O(\sqrt{N})$ in [Truong and Tan (2018)] to $-\ln N$ by employing a single threshold rule $\imath(X_1^n, X_2^n; Y^n) \geq \gamma$.

Theorem (Achievability)

Fix $L \geq 2$, $\epsilon \in (0,1)$, and a distribution P_X. Let the RAC satisfy the symmetry conditions in [Yavas et al. 2020]. There exists a VLSF code for the DM-RAC with L decoding times for each $k \in [K]$ provided that

$$k \ln M \leq \frac{N_k\, I_k}{1-\epsilon} - \sqrt{N_k \ln_{(L-1)}(N_k) \frac{V_k}{1-\epsilon}} + o(\sqrt{N_k}) \qquad k \in [K]$$

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