Variable-Length Codes with Bursty Feedback

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Feedback codes

- Feedback from the receiver to the transmitter does not increase the capacity of a memoryless channel (Shannon '56)
- Feedback aids significantly in the construction of codes
  - Variable-length feedback (VLF) codes
  - Naghshvar *et al.'s* small enough difference (SED) code
  - Schalkwijk and Kailath's code for continuous channels
- Improves the higher-order terms of achievable rate, as well as the error exponent (Burnashev '76)
  - When the rate is less than the channel capacity, the error probability decays exponentially with blocklength following the error exponent

VLF codes

Non-asymptotic feedback codes

- Polyanskiy *et al.* demonstrate that at short blocklengths, variable-length feedback (VLF) codes can achieve much higher rates compared to their fixed-length counterparts
  - VLF codes have no restriction on the content of the feedback signal
  - In variable-length stop feedback (VLSF) codes, there is a single bit of noiseless feedback that informs the transmitter whether a decoding decision has been made
  - Feedback occurs after every channel use

Sparse feedback codes

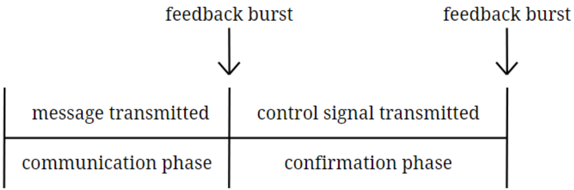
- In practice, frequent feedback brings significant operational challenges:
  - Increased power consumption
  - Issues with half-duplex devices
- In automatic repeat request (ARQ) codes, there is only a single bit of feedback following the entire transmission.
- In order to make feedback codes more practical, it is important to reduce the frequency of feedback instances.

Sparse feedback codes

- Kim *et al.* consider VLSF codes with L periodic feedback instances.
- Vakilinia *et al.* optimize a schedule of feedback instances for a binary-input additive white Gaussian noise (AWGN) channel using a Gaussian approximation.
- Yavas *et al.* derive a non-asymptotic achievability bound for VLSF codes with a limited number of feedback instances L. They optimize over the schedule of feedback instances, and demonstrate performance close to the unrestricted $L = \infty$ case with just $L = 4$.

Two-Phase Feedback Codes

- Yamamoto and Itoh present a two-phase feedback code that alternates between communication and confirmation phases



- If decoding does not occur after the confirmation phase, the transmission is repeated
- Later on, it was shown that it achieves Burnashev's optimal error exponent
  - Their analysis shows that as $n \to \infty$, the probability of entering the second communication phase goes to zero.
  - Communication phase dominates transmission length as $n \to \infty$.
- Lalitha and Javidi consider "almost-fixed-length" codes with a communication phase, confirmation phase, and then an extra long communication phase after which decoding is required

- Our work focuses on variable-length **bursty feedback** codes, which are a subset of VLF codes with a **limited number** of instances L of **unlimited-rate** feedback.

	Max number of feedback instances L		
Max symbols per feedback instance	$L = 1$	$1 < L < \infty$	$L = \infty$
1	ARQ codes	Yavas *et al.* (VLSF) Kim *et al.* (VLSF)	SED code Schalkwijk-Kailtah code Polyanskiy *et al.* (VLF, VLSF)
> 1		Vakilinia *et al.* (AWGN) VLBF codes	Yamamoto-Itoh Lalitha and Javidi

Theorem 1

Let $L \geq 3$ *be an odd number. Fix a DMC* $P_{Y|X} : \mathcal{X} \to \mathcal{Y}$*, a distribution* P_X *on* $\mathcal{X}$*, control symbols* $x_{\mathsf{A}}, x_{\mathsf{R}} \in \mathcal{X}$*, positive integers* M *and* $n_1 < \cdots < n_L$*, and type-I error probabilities* $\epsilon^{(i)} \in (0,1)$ *for* $i \in [\frac{L-1}{2}]$*. There exists an* (N, L, M, ϵ)*-VLBF code with*

$$N \leq n_2 + \sum_{j=1}^{\frac{L-1}{2}} \left[(n_{2j+2} - n_{2j}) \left(\mathrm{rcu}\left(\sum_{i=1}^{j} n_{2i-1} - n_{2i-2}, M \right) \left(1 - \beta^{(j)}\right) + \epsilon^{(j)} \right) \prod_{i=0}^{j-1} p^{(i)} \right] \tag{1}$$

$$\epsilon \leq \sum_{j=1}^{\frac{L-1}{2}} \mathrm{rcu}\left(\sum_{i=1}^{j} n_{2i-1} - n_{2i-2}, M \right) \beta^{(j)} \prod_{i=0}^{j-1} p^{(i)} + \mathrm{rcu}\left(\sum_{i=1}^{\frac{L+1}{2}} n_{2i-1} - n_{2i-2}, M \right) \prod_{i=0}^{\frac{L-1}{2}} p^{(i)}, \tag{2}$$

where $n_0 = 0$, $n_{L+1} = n_L$*, and*

$$\beta^{(i)} \triangleq \beta_{\epsilon^{(i)}}\left(P_{Y|X=x_{\mathsf{A}}}^{n_{2i}-n_{2i-1}} \| P_{Y|X=x_{\mathsf{R}}}^{n_{2i}-n_{2i-1}} \right), \ i \in \left[\frac{L-1}{2} \right] \tag{3}$$

$$p^{(i)} \triangleq \begin{cases} \max\{\epsilon^{(i)}, 1 - \beta^{(i)}\} & \textit{if } i \in [\frac{L-1}{2}] \\ 1 & \textit{if } i = 0. \end{cases} \tag{4}$$

References

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Thank you!