Funktionen

Was: ordnet jedem Element der Definitionsmenge eindeutig ein Element der Wertemenge zu.

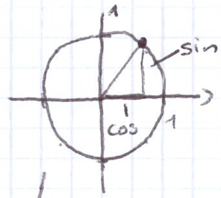
$\mathbb{D}$ = alle x-Werte $\mathbb{W}$ = y-Werte NST: Schnittpunkte mit x-Achse

Umkehrfunktion: Spiegelung an Identität $x = y$

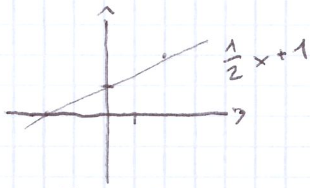
Transformationen: Verschiebung, Spiegelung

Grenzwert: siehe Stetigkeit

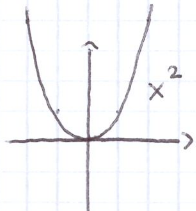
Trigonometrie:



Typen:

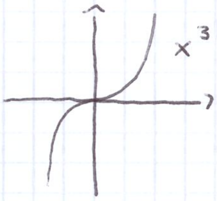


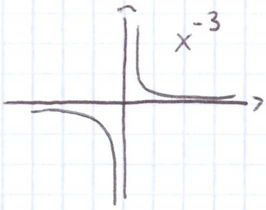
linear

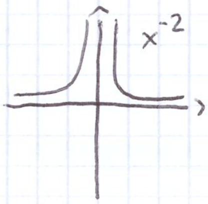


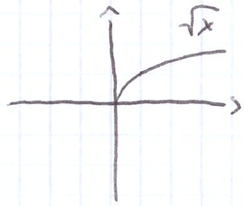
$a(x-u)^2 + v \leftarrow S(u|v)$

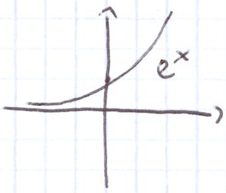
quadratisch

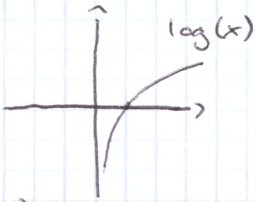


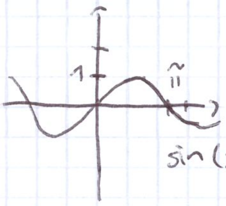




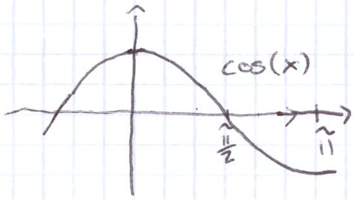


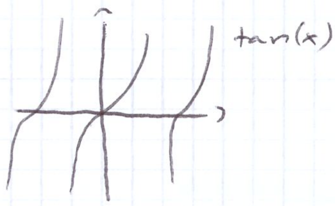




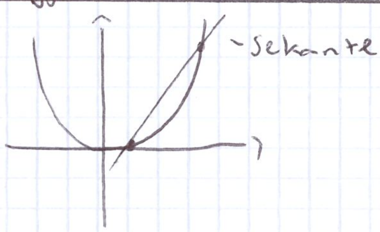


sin(x)





Differentialrechnung

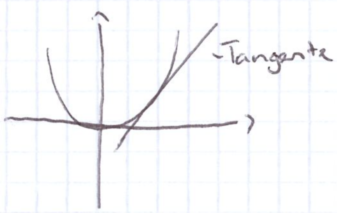


Differenzenquotient (mittlere Änderungsrate)

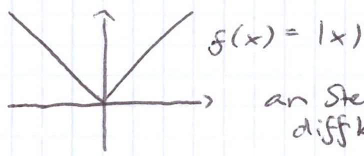
Differentialquotient $\leftarrow$ Näherungsverfahren (lokale Ä.) (immer kleineres Intervall)

Grenzwert

$$m = \frac{f(x) - f(x_0)}{x - x_0}$$



Differenzierbarkeit:



an Stelle 0 nicht diffbar

Ableitungsfunktion: $f'(x_0) = \lim\limits_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

x-Methode: $f(x) = x^2$ $\qquad x_0 = \frac{1}{2}$

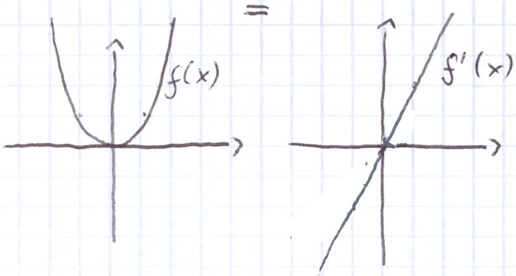
$$f'\left(\tfrac{1}{2}\right) = \lim_{x \to \frac{1}{2}} \frac{x^2 - \frac{1}{4}}{x - \frac{1}{2}} = \lim_{x \to \frac{1}{2}} \frac{\cancel{\left(x - \frac{1}{2}\right)}\left(x + \frac{1}{2}\right)}{\cancel{x - \frac{1}{2}}} \overset{x \to \frac{1}{2}}{=} \frac{1}{2} + \frac{1}{2} = \underline{\underline{1}}$$

h-Methode: $f(x) = x^2 - 4x$ $\qquad x_0 = 3$

$$f'(3) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \to 0} \frac{f(3+h) + 3}{h} = \lim_{h \to 0} \frac{9 + 6h + h^2 - 12 - 4h + 3}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{\cancel{h}} \overset{h \to 0}{=} 2 + 0 = \underline{\underline{2}}$$

graphisch:



Stetigkeit: existiert ein Grenzwert für $f(x_0)$ $\qquad \lim_{x \to x_0} f(x) = f(x_0)$

Bedingungen:
- Funktionswert muss existieren
- Grenzwert muss existieren
- Funktionswert & Grenzwert müssen übereinstimmen

Zusammenhang Stetigkeit & Differenzierbarkeit: differenzierbar → stetig

Beweise: **Summenregel:**

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{u(x) + v(x) - u(x_0) - v(x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{u(x) - u(x_0)}{x - x_0} + \lim_{x \to x_0} \frac{v(x) - v(x_0)}{x - x_0} = u'(x_0) + v'(x_0)$$

Faktorregel: $c \in \mathbb{R}$

$$f'(x_0) = \lim_{x \to x_0} c \cdot \frac{u(x) - u(x_0)}{x - x_0} = c \cdot \lim_{x \to x_0} \frac{u(x) - u(x_0)}{x - x_0} = c \cdot u'(x_0)$$

Produktregel: $f(x) = u(x) \cdot v(x)$ $\leftarrow$ u & v diffbar

$$f'(x_0) = \lim_{x \to x_0} \frac{u(x) \cdot v(x) - u(x_0) \cdot v(x_0)}{x - x_0}$$

Erweiterungsterm: $-u(x_0) \cdot v(x) + u(x_0) \cdot v(x)$

$$= \lim_{x \to x_0} \frac{v(x)\left(u(x) - u(x_0)\right)}{x - x_0} + \lim_{x \to x_0} \frac{u(x_0)\left(v(x) - v(x_0)\right)}{x - x_0}$$

$$= \lim_{x \to x_0} v(x) \cdot \lim_{x \to x_0} \frac{u(x) - u(x_0)}{x - x_0} + \lim_{x \to x_0} u(x_0) \cdot \lim_{x \to x_0} \frac{v(x) - v(x_0)}{x - x_0}$$

$$\overset{x \to x_0}{=} v(x_0) \cdot u'(x_0) + u(x_0) \cdot v'(x_0)$$

Kettenregel: $f(x) = u(v(x))$

$$f'(x_0) = \lim_{x \to x_0} \frac{u(v(x)) - u(v(x_0))}{x - x_0}$$

Erweiterung: $\frac{v(x) - v(x_0)}{v(x) - v(x_0)}$

$$= \lim_{x \to x_0} \frac{u(v(x)) - u(v(x_0))}{v(x) - v(x_0)} \cdot \lim_{x \to x_0} \frac{v(x) - v(x_0)}{x - x_0} \overset{x \to x_0}{=} u'(v(x_0)) \cdot v'(x_0)$$

Extremas: - notwendige Bedingung $f'(x_0) = 0$

- hinreichende Bedingung $f''(x_0) \neq 0$

Wendepunkte: $f''(x_0) = 0$ $\quad f'''(x_0) \neq 0$

Symmetrien: Funktion ist Achsensymmetrisch, wenn nur gerade Exponenten

Funktion ist Punktsymmetrisch, wenn nur ungerade Exponenten

Ableitungen:

$f(x) = \sin(x) \qquad f'(x) = \cos(x)$

$f(x) = \cos(x) \qquad f'(x) = -\sin(x)$

$f(x) = \tan(x) \qquad f'(x) = \frac{1}{\cos^2(x)}$ (← Quotientenregel)

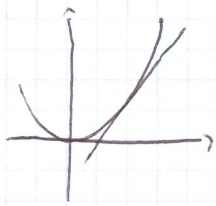
$f(x) = \frac{u(x)}{v(x)} \qquad f'(x) = \frac{u'(x)\cdot v(x) - u(x)\cdot v'(x)}{v^2(x)}$

$f(x) = x^n \qquad f'(x) = n\cdot x^{n-1}$

$f(x) = e^x \qquad f'(x) = e^x$

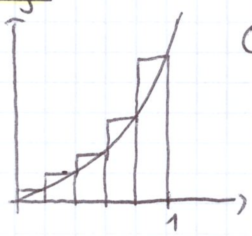
$f(x) = \ln(x) \qquad f'(x) = \frac{1}{x}$

Schnittwinkel:



Integralrechnung

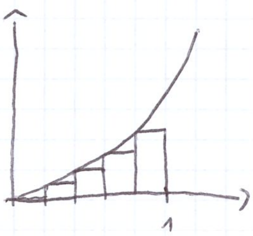
Obersumme:



$$O_5 = \sum_{i=1}^{5} \frac{1}{5} \cdot f\left(\frac{i}{5}\right)$$

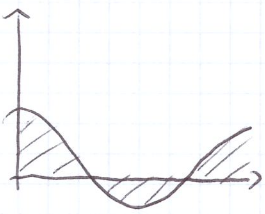
$$\lim_{n\to\infty} O_n = \lim_{n\to\infty} U_n = \int f(x)\,dx$$

Untersumme:



$$U_5 = \sum_{i=0}^{4} \frac{1}{5} \cdot f\left(\frac{i}{5}\right)$$

Flächenbilanz:



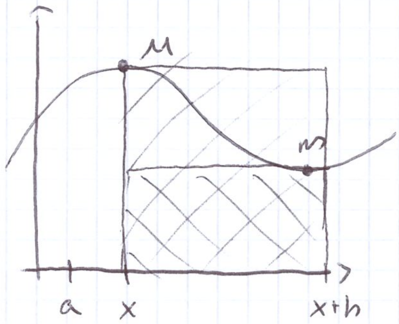
positiv & negativ

Integralfunktion: $I_a(x) = \int_a^x f(t)\,dt$

^ untere Grenze a

unbestimmtes Integral: $\int f(x)\,dx = F(x) + c$

HDI:



$$m \cdot h \leq I_a(x+h) - I_a(x) \leq M \cdot h \qquad | : h$$

$$m \leq \frac{I_a(x+h) - I_a(x)}{h} \leq M \qquad | h \to 0$$

$$f(x) \leq \lim_{h \to 0} \text{"} \leq f(x)$$

$$I_a'(x) = f(x)$$

Summenregel: $\int_a^b f(x) + g(x)\, dx = [F(x) + G(x)]_a^b$

$$= F(b) + G(b) - F(a) - G(a)$$

$$= F(b) - F(a) + G(b) - G(a)$$

$$= [F(x)]_a^b + [G(x)]_a^b = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

Faktorregel: $\int_a^b c \cdot f(x)\, dx = [c \cdot F(x)]_a^b = c \cdot F(b) - c \cdot F(a) = c\,(F(b) - F(a))$

$$= c \cdot \int_a^b f(x)\, dx$$

Fläche zwischen x-Achse: über NST

Fläche zwischen zwei Kurven: obere minus untere

Rotationskörper: $V = \pi \cdot \int_a^b f^2(x)\, dx$

Uneigentliche Integrale: $\lim_{u \to \infty} \int_a^u f(x)\, dx$ z. B. $\lim_{u \to \infty} \left(-\frac{2}{u} + 2\right) \overset{u \to \infty}{=} 2$